CFT driven cosmology: initial conditions, Higgs inflation and temperature of the CMB temperature

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Plan

Cosmological initial conditions – density matrix of the Universe:

- microcanonical ensemble in cosmology
- initial conditions for the Universe via statistical sum – EQG path integral

CFT driven cosmology:

- constraining landscape of \( \Lambda \)
- initial conditions for inflation – selection of inflaton potential maxima and Higgs inflation
- thermally corrected CMB spectrum – temperature of the CMB temperature
- a-theorem and CFT cosmology – role of higher-spin conformal fields and renormalization group flow
Two known prescriptions for a *pure* initial state:

no-boundary wavefunction (Hartle-Hawking);

\[ \Psi[g_{ij}, \varphi] = \int D[g_{\mu\nu}, \phi] e^{-S_E[g_{\mu\nu}, \phi]} \]

“tunneling” wavefunction (Linde, Vilenkin, Rubakov, Zeldovich-Starobinsky, …)

semiclassical solution of the minisuperspace Wheeler-DeWitt equation, outgoing wave prescription, etc.

Both no-boundary (EQG path integral) and tunneling (WKB approximation) do not have a clear operator interpretation.
The path integral formulation for the microcanonical ensemble in quantum cosmology

We apply it to the Universe dominated by massless matter conformally coupled to gravity

thermal version of the no-boundary state;

bounded range of the primordial $\Lambda$ with band structure;

$\Lambda \rightarrow V(\phi)$, initial conditions for inflation at maxima of $V(\phi)$, Higgs inflation;

thermal corrections to the CMB power spectrum – temperature of the CMB temperature

application of a-theorem – RG flow and enhancement of thermal effect

Microcanonical ensemble in cosmology and EQG path integral

\[ H_\mu = 0 \] constraints on initial value data – corner stone of any diffeomorphism invariant theory.

Physical states:

\[ \hat{H}_\mu |\Psi\rangle = 0 \quad \hat{H}_\mu \equiv \hat{H}_\perp (x), \hat{H}_i (x) \]

operators of the Wheeler-DeWitt equations

\[ \mu = (\perp x, ix), \quad i = 1, 2, 3 \]
\[ x \text{ – spatial coordinates} \]

Microcanonical density matrix – projector onto subspace of quantum gravitational constraints

\[ |\Psi\rangle \rightarrow \hat{\rho}, \quad \hat{H}_\mu \hat{\rho} = 0 \]

\[ \hat{\rho} = e^{\Gamma} \prod_{\mu} \delta (\hat{H}_\mu) \]

\[ e^{-\Gamma} = \text{Tr} \prod_{\mu} \delta (\hat{H}_\mu) \]

99, 071301 (2007)
Motivation: aesthetical (minimum of assumptions – Occam razor)

A simple analogy — an unconstrained system with a conserved Hamiltonian \( \hat{H} \) in the microcanonical state with a fixed energy \( E \)

\[ \hat{\rho} \sim \delta(\hat{H} - E) \]

Spatially closed cosmology does not have \textit{freely specifiable} constants of motion. The only conserved quantities are the Hamiltonian and momentum constraints \( H_\mu \), all having a particular value --- zero

\[ \hat{\rho} \sim \prod_\mu \delta(H_\mu) \]

is a natural candidate for the quantum state of the closed Universe – ultimate equipartition in the \textit{physical} phase space of the theory --- \textit{Sum over Everything}.

Path integral representation of the statistical sum

\[ e^{-\Gamma} \equiv \text{Tr} \sum_{\mu} \delta(\hat{H}_\mu) = \int D[g_{\mu\nu}, \phi] e^{-S_E[g_{\mu\nu}, \phi]} \]

\[ -i\infty < N < i\infty, \quad g^{44} = +N^2 \]

Euclidean metric

EQG density matrix

D. Page (1986)

BFV/BRST method

Spacetime topology in the statistical sum:

\[ e^{-\Gamma} = \int_{\text{periodic}} D[g_{\mu\nu}, \phi] e^{-S_E[g_{\mu\nu}, \phi]} \]

\( S^3 \) topology of spatially closed cosmology

on \( S^3 \times S^1 \) (thermal)

including as a limiting (vacuum) case \( S^4 \)
Hartle-Hawking state as a vacuum member of the microcanonical ensemble:

\[ \hat{\rho}_{\text{mixed}} = \rho_{\text{mixed}}(q, q') \]

pinching a tubular spacetime

\[ |\Psi_{HH}\rangle \langle \Psi_{HH}| = \rho_{HH}(q, q') \]

density matrix representation of a pure Hartle-Hawking state – vacuum state of zero temperature \( T \sim 1/\eta = 0 \)

\[ \eta = \int_{a=0}^{a=\infty} d\tau \frac{N}{a} \rightarrow \infty \]
(see below)
**Path integral calculation**

Disentangling the minisuperspace sector:

Euclidean FRW metric

\[ ds^2 = N^2 \, d\tau^2 + a^2 \, d^2\Omega^{(3)} \]

- lapse
- scale factor

3-sphere of a unit size

\[ [ g_{\mu\nu}, \phi ] = [ a(\tau), N(\tau); \Phi(x) ] \]

minisuperspace background

quantum “matter” – cosmological perturbations:

\[ \Phi(x) = (\varphi(x), \psi(x), A_\mu(x), h_{\mu\nu}(x), \ldots) \]
Decomposition of the statistical sum path integral:

\[ e^{-\Gamma} = \int D[a, N] e^{-S_{\text{eff}}[a, N]} \]

(periodic)

\[ e^{-S_{\text{eff}}[a, N]} = \int D\Phi(x) e^{-S_E[a, N; \Phi(x)]} \]

(periodic)

quantum effective action of \( \Phi \) on minisuperspace background
Application to the CFT driven cosmology

\[ S_E[g_{\mu\nu}, \phi] = -\frac{1}{16\pi G} \int d^4x \, g^{1/2} \left( R - 2\Lambda \right) + S_{\text{CFT}}[g_{\mu\nu}, \phi] \]

\( \Lambda = 3H^2 \) -- primordial cosmological constant

Conformal invariance \( \rightarrow \) exact calculation of \( S_{\text{eff}} \)

Assumption of \( N_{\text{cdf}} \) conformally invariant, \( N_{\text{cdf}} \gg 1 \), quantum fields and recovery of the action from the conformal anomaly and the action on a static Einstein Universe

\[ ds^2 = a^2(\eta)(d\eta^2 + d^2\Omega^{(3)}) \quad \rightarrow \quad ds^2 = d\eta^2 + d^2\Omega^{(3)} \]

\( \text{conformal time} \)

Starobinsky (1980);
Fischetty, Hartle, Hu;
Riegert; Tseytlin;
Antoniadis, Mazur & Mottola;
......
\[ S_{\text{eff}} = \text{classical part} + \Gamma_A + \Gamma_{EU} \]

\[
g_{\mu\nu} \frac{\delta \Gamma_A}{\delta g_{\mu\nu}} = \frac{1}{4(4\pi)^2 g^{1/2}} \left( \alpha \Box R + \beta E + \gamma C_{\mu\nu\alpha\beta}^2 \right)
\]

- \(\alpha, \beta, \gamma\) -- spin-dependent coefficients

\[
\beta = \frac{1}{360} (2N_0 + 11N_{1/2} + 124N_1)
\]

\(N_s\) # of fields of spin \(s\)
Full quantum effective action on FRW background

\[ S_{\text{eff}}[a, N] = \int d\tau N \mathcal{L}(a, a') + F(\eta) \]

\[ \mathcal{L}(a, a') = -\alpha a r^2 - a + \frac{\Lambda}{3} a^3 + B \left( \frac{a'^2}{a} - \frac{a'^4}{6a} + \frac{1}{2a} \right) \]

- classical part
- conformal anomaly part
- vacuum (Casimir) energy – from static EU

\[ B = \frac{3\beta}{4m_P^2} \quad \text{-- coefficient of the Gauss-Bonnet term in the conformal anomaly} \]

nonlocal (thermal) part

\[ F(\eta) = \pm \sum_{\omega} \ln \left( 1 \mp e^{-\omega \eta} \right) \]

energies of field oscillators on \( S^3 \)

Inverse (comoving) temperature

\[ \eta = \int d\tau \frac{N}{a} \quad \text{-- time reparameterization invariance (1D diffeomorphism)} \]

\[ a' \equiv \frac{1}{N} \frac{da}{d\tau} \]
Effective Friedmann equation for saddle points of the path integral:

\[ \frac{\delta S_{\text{eff}}[a, N]}{\delta N(\tau)} = 0 \]

amount of radiation constant

\[ \frac{1}{a^2} - \frac{a'^2}{a^2} - \frac{B}{2} \left( \frac{1}{a^2} - \frac{a'^2}{a^2} \right)^2 = \frac{\Lambda}{3} + \frac{C}{a^4}, \quad C = \sum \frac{\omega}{e^{\omega \eta} \pm 1} \]

“bootstrap” equation:

\[ \eta = \eta[a(\tau)] \]

Is the lapse \( N \) (or time \( \tau \)) imaginary or real?
Calculating the path integral over *imaginary* $N$ by semiclassical saddle point approximation:

$$\frac{\delta S_{\text{eff}}[N]}{\delta N} = 0$$

No periodic solutions of effective equations with *imaginary* Euclidean lapse $N$ (Lorentzian spacetime geometry). Saddle points exist for *real* $N$ (Euclidean geometry):

Deformation of the original contour of integration

$$-i\infty < N < i\infty$$

into the complex plane to pass through the saddle point with real $N>0$ or $N<0$
Saddle point solutions --- set of periodic (thermal) garland-type instantons with oscillating scale factor \((S^1 \times S^3)\) and vacuum Hartle-Hawking instantons \((S^4)\)

- 1-fold, \(k=1\)
- \(k\)-folded garland, \(k=1,2,3,\ldots\)
- \(S^4\)

does not contribute: weighted by \(e^{-\infty}\)
Band structure of the bounded cosmological constant range

\[ 4C H^2 < 1 \]
\[ C \geq B - B^2 H^2 \]

New QG scale at \( k \to \infty \): instantons with temperatures

\[ T(k) = \frac{1}{\sqrt{B} \ln k^2} \to 0 \]

\[ \Lambda_{\text{min}} < \Lambda < \Lambda_{\text{max}} = \frac{2m_P^2}{\beta} \]

\( \Delta_k \) — cosmological constant band for \( k \)-folded garland

Constraining string landscape?
Inflationary evolution and “hot” CMB

Lorentzian Universe with initial conditions set by the saddle-point instanton. Analytic continuation of the instanton solutions:

$$\tau = it, \quad a(t) = a_E(it)$$

Expansion and quick dilution of primordial radiation

Generalization to $\Lambda$ as a composite operator – inflaton potential and a “slow roll”

$$\Lambda \rightarrow \frac{V(\varphi)}{M_P^2}$$

decay of a composite $\Lambda$, exit from inflation and particle creation of conformally non-invariant matter:

$$\frac{\Lambda}{3} + \frac{C}{a^4} \Rightarrow \frac{8\pi G}{3} \varepsilon$$

energy density of non-conformal matter
Selection of inflaton potential maxima as initial conditions for inflation

\[ \frac{d}{d\tau} a^3 \dot{a} = a^3 \frac{\partial V}{\partial \varphi} \Rightarrow \int d\tau a^3 \frac{\partial V}{\partial \varphi} = 0 \Rightarrow \frac{\partial V}{\partial \varphi} \geq 0 \]

grad V and \( \dot{a} \) change sign – this is not a slow roll -- two coupled nonlinear and oscillators.

Do we have such potentials in realistic cosmology? Yes!
Higgs inflation with non-minimally coupled inflaton

The effective inflaton potential for $M_{\text{Higgs}} = 134$ GeV. Inflationary domain for a $N = 60$ CMB perturbation is marked by dashed lines.

2-loop RG contribution leads to $M_{\text{Higgs}} \rightarrow M_{\text{LHC}} = 126$ GeV (Bezrukov & Shaposhnikov 2009)
Primordial CMB spectrum with thermal corrections:

\( T_{\text{CMB}} \) is subject to vacuum fluctuations

Now \( T_{\text{CMB}} \) is subject to thermal distribution with the temperature \( T = 1/\eta \)

\[-\text{temperature of the CMB temperature}\]

\[
\delta_{\phi}^2(k) = \langle \tilde{\phi}_k(t)\tilde{\phi}_k(t) \rangle_{\text{thermal}}
\]

\[
\sim \langle \tilde{a}_k^{\dagger} \tilde{a}_k + \tilde{a}_k \tilde{a}_k^{\dagger} \rangle_{\text{thermal}} |u_k(t)|^2 = |u_k(t)|^2 \left( 1 + 2N_k(\eta) \right)
\]

\[
N_k(\eta) = \frac{1}{e^{k\eta} - 1}
\]

\[
kl \leftrightarrow l \Rightarrow C_l^2 \rightarrow C_l^2 \left( 1 + \frac{2}{e^{k\eta} - 1} \right)
\]

additional reddening of the CMB spectrum
Vacuum part of $\Delta T_{CMB}$ is under investigation:

\[ g_{\mu \nu} = e^\sigma \bar{g}_{\mu \nu}, \]

\[
\Gamma_R[g] - \Gamma_R[\bar{g}] = \frac{\gamma}{4(4\pi)^2} \int d^4x \bar{g}^{1/2} \sigma \bar{C}_{\mu \nu \alpha \beta}^2 \\
+ \frac{\beta}{2(4\pi)^2} \int d^4x \bar{g}^{1/2} \left\{ \frac{1}{2} \sigma \bar{E} - \left( \bar{R}^{\mu \nu} - \frac{1}{2} \bar{g}^{\mu \nu} \bar{R} \right) \partial_\mu \sigma \partial_\nu \sigma \\
- \frac{1}{2} \bar{\Box} \sigma \left( \bar{\nabla}^\mu \sigma \bar{\nabla}_\mu \sigma \right) - \frac{1}{8} \left( \bar{\nabla}^\mu \sigma \bar{\nabla}_\mu \sigma \right)^2 \right\}
\]

\[ u_k(t) ? \]

Galileon type mode -- no higher derivative ghosts in the scalar sector

Basis function of the cosmological perturbations (Mukhanov-Sasaki mode)
Thermal contribution to the spectral index:

\[ n_s(k) = 1 + \frac{d}{d \ln k} \ln \delta^2_\phi(k) = n_s^{\text{vac}}(k) + \Delta n_s^{\text{thermal}}(k), \]

\[ \Delta n_s^{\text{thermal}}(k) = \frac{d}{d \ln k} \ln \left( 1 + 2N_k(\eta) \right) \]

\[ N_{k_l} \approx \exp \left[ -\frac{2l}{(\Omega_0 - 1)^{1/2}} \left( \frac{1}{180\bar{\beta}} \right)^{1/6} \right] \approx \exp \left[ -\frac{10l}{(3\bar{\beta})^{1/6}} \right] \]

\[ \Delta n_s^{\text{thermal}}(k_l) \approx -\frac{20l}{(3\bar{\beta})^{1/6}} e^{-10l/(3\bar{\beta})^{1/6}} \ll 1 \]

\[ \bar{\beta} \sim \frac{\sum s \beta_s N_s}{\sum s N_s} \quad \text{specific } \beta \text{ per one degree of freedom} \]

\[ \frac{1}{60} \leq 3\bar{\beta}_{\text{low-spin}} \leq \frac{31}{30} \approx 1 \]
\[ \beta_s = \begin{cases} \frac{1}{180} & s = 0 \\ \frac{11}{360} & s = \frac{1}{2} \\ \frac{31}{90} & s = 1 \\ -\frac{274}{90} & s = \frac{3}{2} \text{ conformal gravitino} \\ \frac{87}{10} & s = 2 \text{ conformal graviton} \\ \ldots & s > 1 \end{cases} \]

For each \( s \), the thermal part is negligible:

\[ 3\tilde{\beta} \lesssim 1 \]

Contribution of higher conformal spins might have large thermal imprint on CMB:

\[ \Delta n_s < e^{-\frac{l}{\sqrt{\Omega_0 - 1}}} \sim e^{-10l} \]

\[ \beta_s = \frac{1}{360} N_s^2(3 + 14N_s), \quad N_s = s(s + 1), \quad s = 1, 2, 3, \ldots, \]

\[ \beta_s = \frac{1}{720} N_s(12 + 45N_s + 14N_s^2), \quad N_s = -2\left(s + \frac{1}{2}\right)^2, \quad s = 1, \frac{3}{2}, \frac{5}{2}, \ldots \]

A.A. Tseytlin, arXiv:1309.0785
CFT cosmology and the a-theorem

Free CFT \rightarrow \text{interacting CFT}

\[ e^{iS_{\text{eff}}[g_{\mu\nu}]} = \int D\phi e^{iS_{\text{CFT}}[g_{\mu\nu},\phi]} \]

\[ \langle T^\mu_\mu \rangle \equiv \frac{2}{g^{1/2}} g^{\mu\nu} \frac{\delta S_{\text{eff}}}{\delta g_{\mu\nu}} = aE - c C^{2}_{\mu\nu\alpha\beta} + b \Box R. \]

Renormalization group flow: \quad a, b, c \Rightarrow a(\mu^2), b(\mu^2), c(\mu^2)

\begin{align*}
\text{UV} & \quad \downarrow & \quad \text{IR} \\
\downarrow & & \downarrow \\
\text{UV} & & \text{IR} \\
\sigma(s) = s \, \text{Im} A(s, t)_{t=0} > 0
\end{align*}

Total cross section of the forward \(2 \rightarrow 2\) dilaton scattering

Z. Komargodski and A. Schwimmer, arXiv:1107.3987
Z. Komargodski, arXiv:1112.4538
Cosmological expansion: $\text{UV} \rightarrow \text{IR}$

$$a(\mu^2) = \frac{\beta(\mu^2)}{32\pi^2}, \quad \beta_{\text{early}} \gg 1 \rightarrow \beta_{\text{now}} \sim O(1)$$

Observable thermal imprint on CMB of invisible now higher spin fields?
Conclusions

Microcanonical density matrix of the Universe

Application to the CFT driven cosmology with a large # of quantum species – thermal version of the no-boundary state

Initial conditions for inflation with a limited range of $\mathcal{A}$ -- cosmological landscape -- and generation of the thermal CMB spectrum, but rather cold

SOME LIKE IT HOT

SOME LIKE IT COOL