CFT driven cosmology:
initial conditions, Higgs inflation and temperature of the CMB temperature

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Plan

Cosmological initial conditions – density matrix of the Universe:

- microcanonical ensemble in cosmology
- initial conditions for the Universe via statistical sum – EQG path integral

CFT driven cosmology:

- constraining landscape of Λ
- initial conditions for inflation – selection of inflaton potential maxima and Higgs inflation
- thermally corrected CMB spectrum – temperature of the CMB temperature
- a-theorem and CFT cosmology – role of higher-spin conformal fields and renormalization group flow

Two known prescriptions for a *pure* initial state:

no-boundary wavefunction (Hartle-Hawking);

$$\Psi[\,g_{ij},\varphi\,] = \int D[\,g_{\mu\nu},\phi\,]\; e^{-S_E[\,g_{\mu\nu},\phi\,]}$$

“tunneling” wavefunction (Linde, Vilenkin, Rubakov, Zeldovich-Starobinsky, …)

semiclassical solution of the minisuperspace
Wheeler-DeWitt equation,
outgoing wave prescription,
etc.

Both no-boundary (EQG path integral) and tunneling (WKB approximation) do not have a clear operator interpretation

The *path integral* formulation *for the microcanonical ensemble* in quantum cosmology

A.B. & A.Kamenshchik,
JCAP, 09, 014 (2006)
Phys. Rev. D74, 121502 (2006);
A.B., Phys. Rev. Lett.
99, 071301 (2007)

We apply it to the Universe dominated by massless matter conformally coupled to gravity →

thermal version of the no-boundary state;

bounded range of the **primordial** Λ with band structure;

$\Lambda \rightarrow$ ***V(φ),*** initial conditions for inflation at **maxima** of ***V(φ),*** Higgs inflation;

thermal corrections to the CMB power spectrum **– temperature of the CMB temperature**

application of ***a*-theorem** – RG flow and enhancement of thermal effect

A.B, J JCAP 1310 (2013) 059,
arXiv:1308.4451

Microcanonical ensemble in cosmology and EQG path integral

$H_\mu = 0$ constraints on initial value data – corner stone of any diffeomorphism invariant theory.

Physical states: $\hat{H}_\mu|\Psi\rangle = 0 \qquad \hat{H}_\mu \equiv \underbrace{\hat{H}_\perp(\mathbf{x}),\ \hat{H}_i(\mathbf{x})}$

operators of the Wheeler-DeWitt equations

$\mu = (\perp\mathbf{x},\ i\mathbf{x}),\ i = 1,2,3$
$\mathbf{x}$ – spatial coordinates

Microcanonical density matrix – projector onto subspace of quantum gravitational constraints

$$|\Psi\rangle \to \hat{\rho}, \qquad \hat{H}_\mu\,\hat{\rho} = 0$$

$$\hat{\rho} = e^{\Gamma}\ \prod_\mu \delta(\hat{H}_\mu)$$

A.B., Phys. Rev. Lett. 99, 071301 (2007)

Statistical sum

$$e^{-\Gamma} = \mathrm{Tr}\ \prod_\mu \delta(\hat{H}_\mu)$$

Motivation: aesthetical (minimum of assumptions – Occam razor)

A simple analogy — an unconstrained system with a conserved Hamiltonian $\widehat{H}$ in the microcanonical state with a fixed energy E

$$\hat{\rho} \sim \delta(\widehat{H} - E)$$

Spatially closed cosmology does not have *freely specifiable* constants of motion. The only conserved quantities are the Hamiltonian and momentum constraints H_μ, all having a particular value --- zero

$$\hat{\rho} \sim \prod_{\mu} \delta(\widehat{H}_\mu)$$

is a natural candidate for the quantum state of the closed Universe – ultimate equipartition in the physical phase space of the theory --- *Sum over Everything*.

A.B., Phys.Rev.Lett. 99, 071301 (2007)

Path integral representation of the statistical sum

$$-i\infty < N < i\infty, \quad g^{44} = +N^2$$

↑

Euclidean metric

↓

$$e^{-\Gamma} \equiv \mathrm{Tr} \prod_{\mu} \delta(\hat{H}_\mu) = \int_{\text{periodic}} D[\,g_{\mu\nu}, \phi\,]\, e^{-S_E[\,g_{\mu\nu}, \phi\,]}$$

EQG density matrix D.Page (1986)

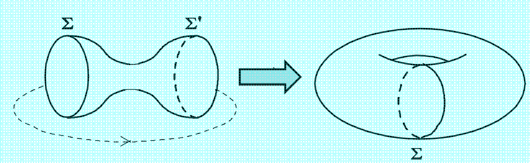
↑

BFV/BRST method
A.B. JHEP 1310 (2013) 051,
arXiv:1308.3270

Spacetime topology in the statistical sum:

S^3 topology of spatially closed cosmology

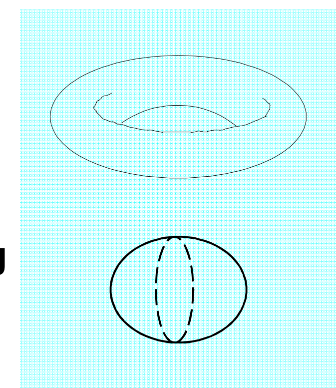
$$\mathrm{tr}\,\hat{\rho} = 1$$



$$e^{-\Gamma} = \int_{\text{periodic}} D[\,g_{\mu\nu},\phi\,]\; e^{-S_E[\,g_{\mu\nu},\phi\,]}$$

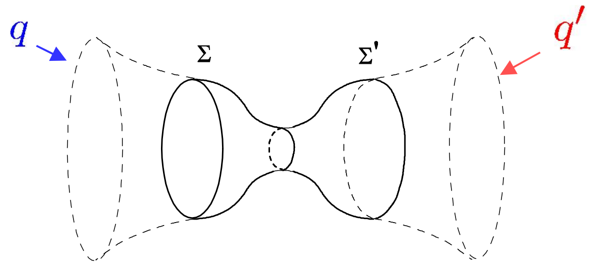
on $S^3 \times S^1$ (thermal)

including as a limiting (vacuum) case S^4



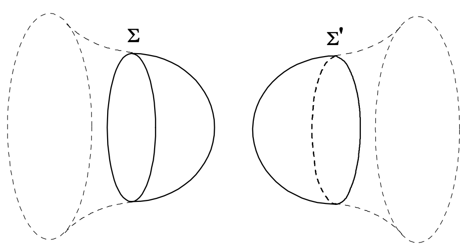
Hartle-Hawking state as a vacuum member of the microcanonical ensemble:

$$\hat{\rho}_{\mathrm{mixed}} = \rho_{\mathrm{mixed}}(q, q')$$

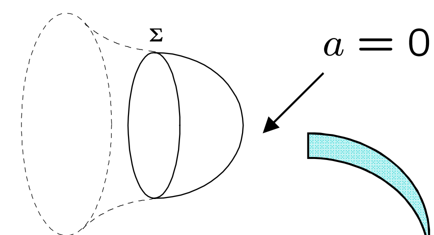


pinching a tubular spacetime

$$|\Psi_{HH}\rangle\langle\Psi_{HH}| = \rho_{HH}(q, q')$$



$$|\Psi_{HH}\rangle$$



$$\eta = \int_{a=0} d\tau \, \frac{N}{a} \rightarrow \infty$$

(see below)

density matrix representation of a pure Hartle-Hawking state – vacuum state of zero temperature $T \sim 1/\eta = 0$

Path integral calculation

Disentangling the minisuperspace sector:

Euclidean FRW metric

3-sphere of a unit size

$$ds^2 = N^2\, d\tau^2 + a^2\, d^2\Omega^{(3)}$$

lapse — scale factor

$$[\, g_{\mu\nu}, \phi\,] = [\, a(\tau), N(\tau); \Phi(x)\,]$$

minisuperspace background

quantum "matter" – cosmological perturbations:

$$\Phi(x) = (\varphi(x), \psi(x), A_\mu(x), h_{\mu\nu}(x), \ldots)$$

Decomposition of the statistical sum path integral:

$$e^{-\Gamma} = \int_{\text{periodic}} D[\,a, N\,]\; e^{-S_{\text{eff}}[\,a,N\,]}$$

$$e^{-S_{\text{eff}}[\,a,N\,]} = \int_{\text{periodic}} D\Phi(x)\; e^{-S_E[\,a,N;\Phi(x)\,]}$$

quantum effective action of Φ on minisuperspace background

Application to the CFT driven cosmology

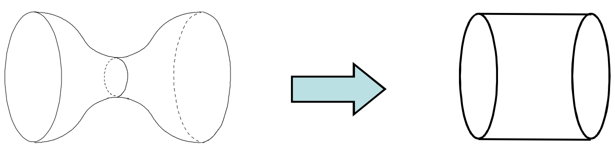
$$S_E[g_{\mu\nu},\phi] = -\frac{1}{16\pi G}\int d^4x\, g^{1/2}\left(R - 2\Lambda\right) + S_{CFT}[g_{\mu\nu},\phi]$$

$\Lambda = 3H^2$ -- primordial cosmological constant

$N_s \gg 1$ conformal fields of spin s=0,1,1/2

Conformal invariance → exact calculation of S_{eff}

Assumption of N_{cdf} conformally invariant, $N_{cdf} \gg 1$, quantum fields and recovery of the action from the conformal anomaly and the action on a static Einstein Universe



Starobinsky (1980);
Fischetty,Hartle,Hu;
Riegert; Tseytlin;
Antoniadis, Mazur & Mottola;
……

$$ds^2 = a^2(\eta)(d\eta^2 + d^2\Omega^{(3)}) \quad \Longrightarrow \quad d\bar{s}^2 = d\eta^2 + d^2\Omega^{(3)}$$

conformal time

$$S_{\text{eff}} = \text{classical part} + \Gamma_A + \Gamma_{EU}$$

anomaly contribution **Einstein universe contribution**

$$g_{\mu\nu}\frac{\delta\Gamma_A}{\delta g_{\mu\nu}} = \frac{1}{4(4\pi)^2}g^{1/2}\left(\alpha\Box R + \beta E + \gamma C^2_{\mu\nu\alpha\beta}\right)$$

Gauss-Bonnet term **Weyl term**

α,β,γ **-- spin-dependent coefficients**

$$\beta = \tfrac{1}{360}(2N_0 + 11N_{1/2} + 124N_1)$$

N_s* # of fields of spin *s

Full quantum effective action on FRW background

$$S_{\rm eff}[\,a,N\,]=\int d\tau\,N\mathcal{L}(a,a')+F(\eta)$$

$$\mathcal{L}(a,a')=-aa'^2-a+\frac{\Lambda}{3}a^3+B\left(\frac{a'^2}{a}-\frac{a'^4}{6a}+\frac{1}{2a}\right)$$

classical part

conformal anomaly part

vacuum (Casimir) energy – from static EU

$B=\dfrac{3\beta}{4m_P^2}$ **-- coefficient of the Gauss-Bonnet term in the conformal anomaly**

nonlocal (thermal) part $F(\eta)=\pm\sum_\omega \ln\left(1\mp e^{-\omega\eta}\right),$

energies of field oscillators on S^3

Inverse (comoving) temperature

$$\eta=\oint d\tau\,\frac{N}{a}$$

$$a'\equiv\frac{1}{N}\frac{da}{d\tau}$$

-- time reparameterization invariance (1D diffeomorphism)

Effective Friedmann equation for saddle points of the path integral:

$$\frac{\delta S_{\text{eff}}[\,a, N\,]}{\delta N(\tau)} = 0$$

amount of radiation constant

$$\frac{1}{a^2} - \frac{a'^2}{a^2} - \frac{B}{2}\left(\frac{1}{a^2} - \frac{a'^2}{a^2}\right)^2 = \frac{\Lambda}{3} + \frac{\mathcal{C}}{a^4}, \quad \mathcal{C} = \sum_\omega \frac{\omega}{e^{\omega\eta} \pm 1}$$

"bootstrap" equation:

$$\eta = \eta[\,a(\tau)\,]$$

Is the lapse *N* (or time τ) imaginary or real?

Calculating the path integral over *imaginary* N by semiclassical saddle point approximation:

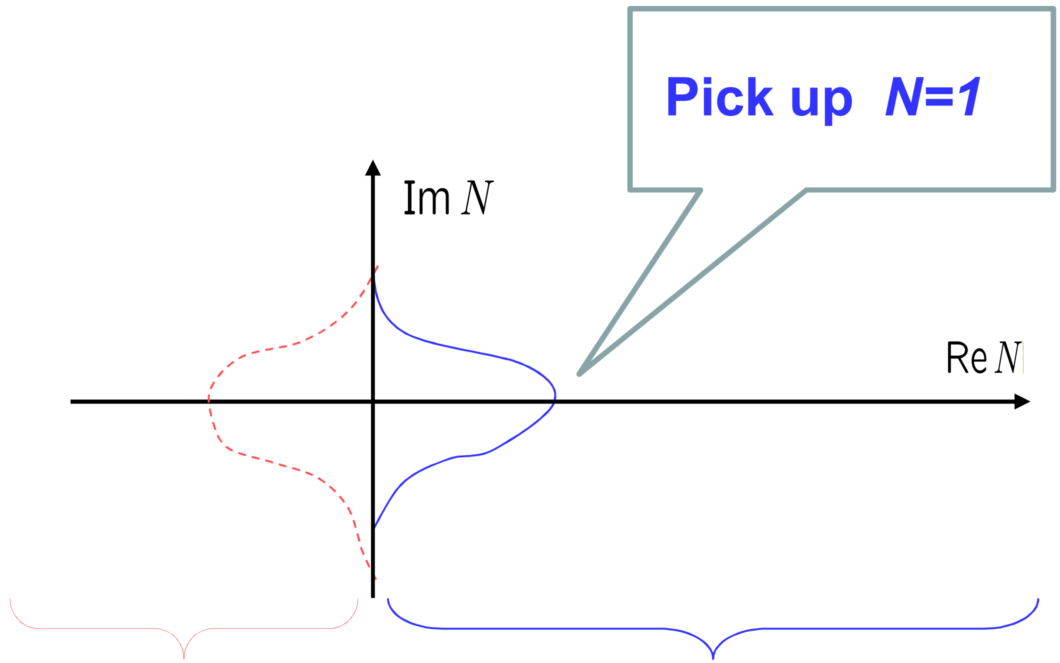
$$\frac{\delta S_{\text{eff}}[\,N\,]}{\delta N} = 0$$

No periodic solutions of effective equations with imaginary Euclidean lapse N (Lorentzian spacetime geometry). Saddle points exist for real N (Euclidean geometry):

Deformation of the original contour of integration

$$-i\infty < N < i\infty$$

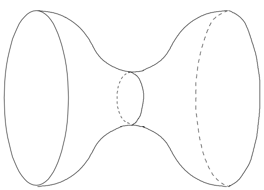
into the complex plane to pass through the saddle point with real $N>0$ or $N<0$



gauge equivalent
N<0 (tunneling proposal)

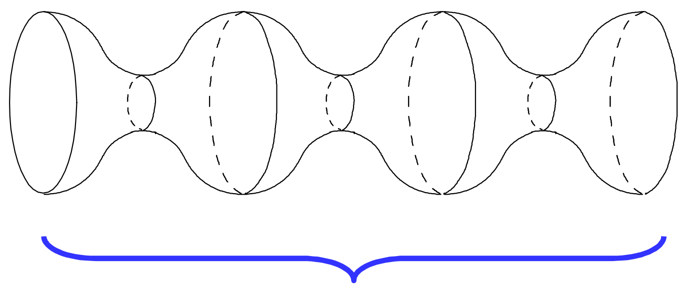
gauge equivalent
N>0 (no-boundary proposal)

Saddle point solutions --- set of periodic (thermal) garland-type instantons with oscillating scale factor ($S^1 \times S^3$) and vacuum Hartle-Hawking instantons (S^4)

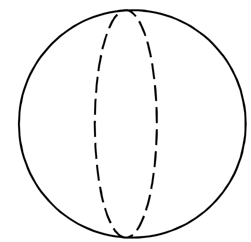


1- fold, *k*=1

, ...



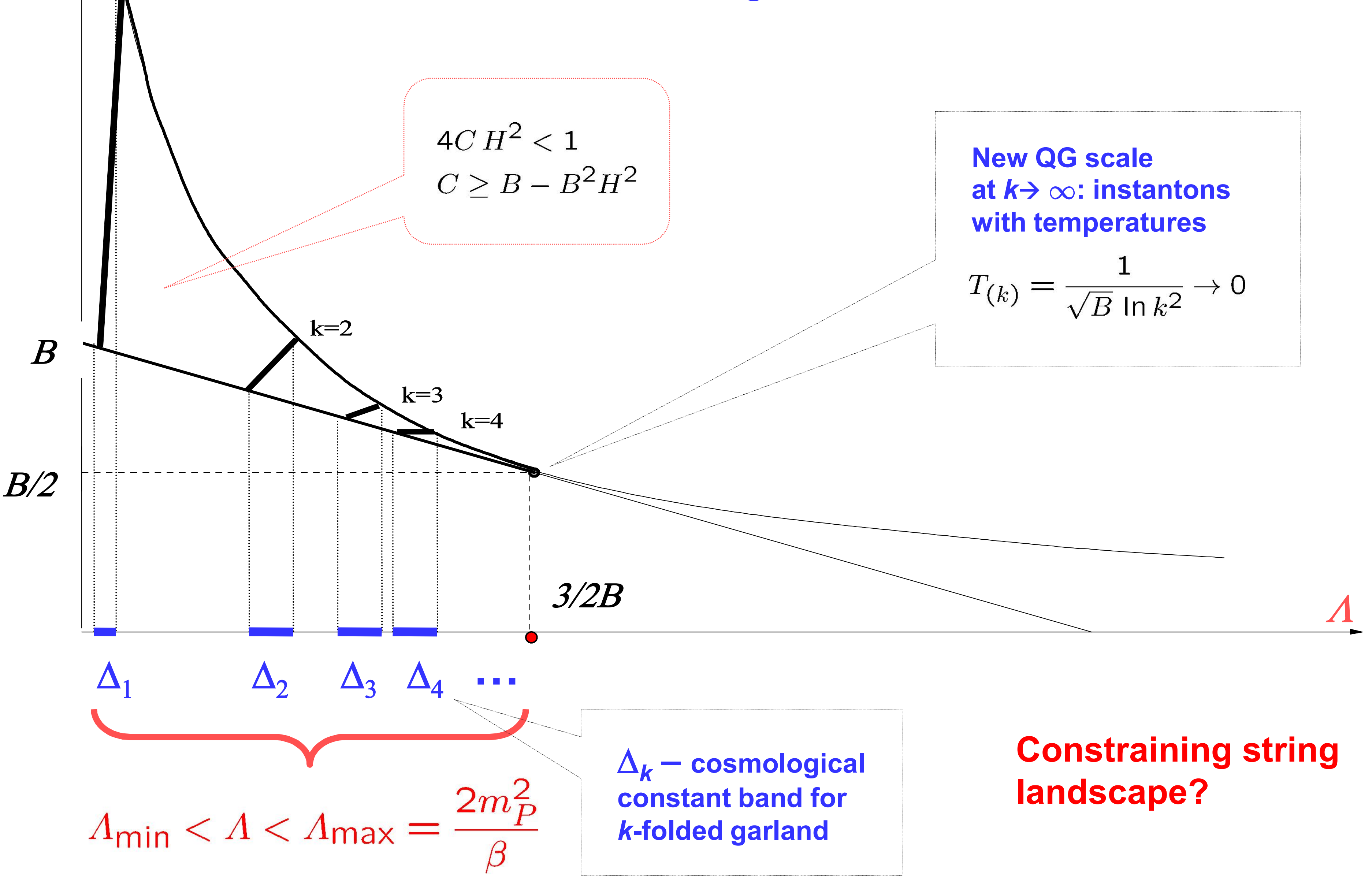
k- folded garland, *k*=1,2,3,…



S^4

does not contribute:
weighted by $e^{-\infty}$

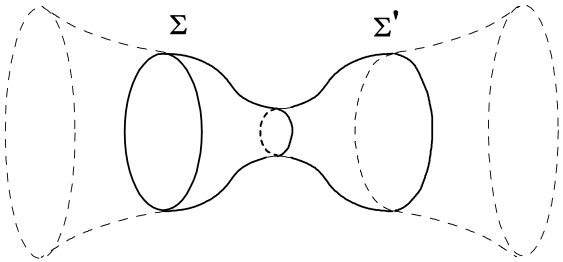
Band structure of the bounded cosmological constant range



Inflationary evolution and "*hot*" CMB

Lorentzian Universe with initial conditions set by the saddle-point instanton. Analytic continuation of the instanton solutions:

$$\tau = it, \ a(t) = a_E(it)$$



Expansion and quick dilution of primordial radiation

Generalization to Λ as a composite operator – inflaton potential and a "slow roll"

$$\Lambda \to \frac{V(\varphi)}{M_P^2}$$

⇩

decay of a composite Λ, exit from inflation and particle creation of conformally non-invariant matter:

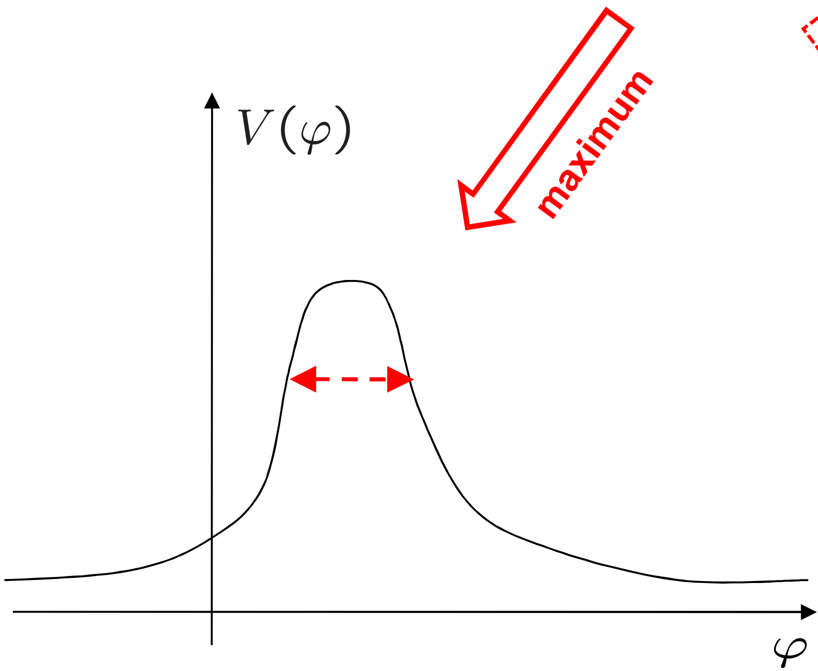
$$\frac{\Lambda}{3} + \frac{\mathcal{C}}{a^4} \Rightarrow \frac{8\pi G}{3}\varepsilon$$

energy density of non-conformal matter

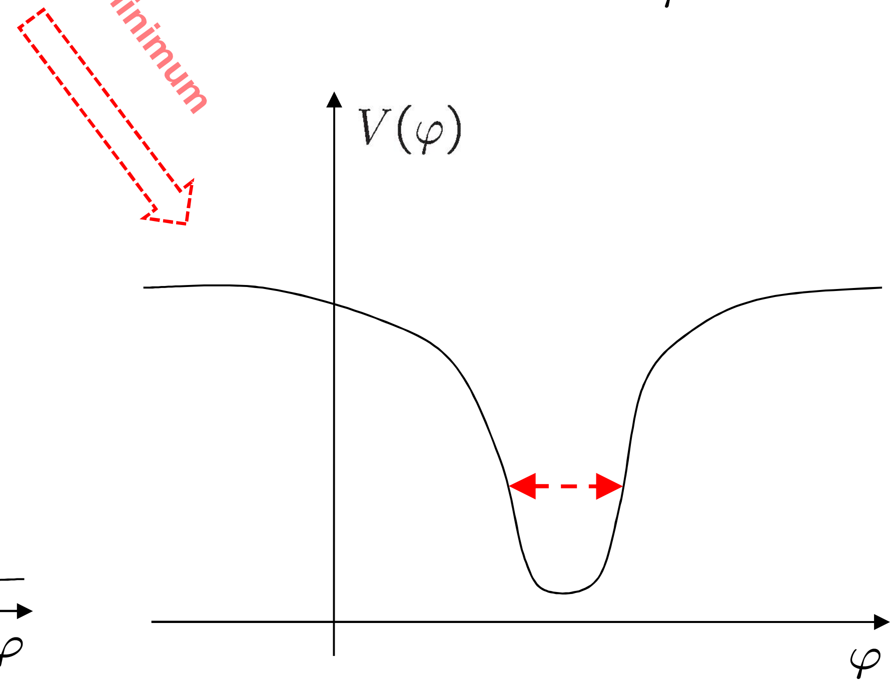
Selection of inflaton potential maxima as initial conditions for inflation

$$\frac{d}{d\tau} a^3 \dot{a} = a^3 \frac{\partial V}{\partial \varphi} \Rightarrow \oint d\tau\, a^3 \frac{\partial V}{\partial \varphi} = 0 \Rightarrow \frac{\partial V}{\partial \varphi} \gtrless 0$$

grad V and $\dot{a}$ change sign – this is not a slow roll -- two coupled nonlinear a and φ oscillators.



classically forbidden (underbarrier) oscillation

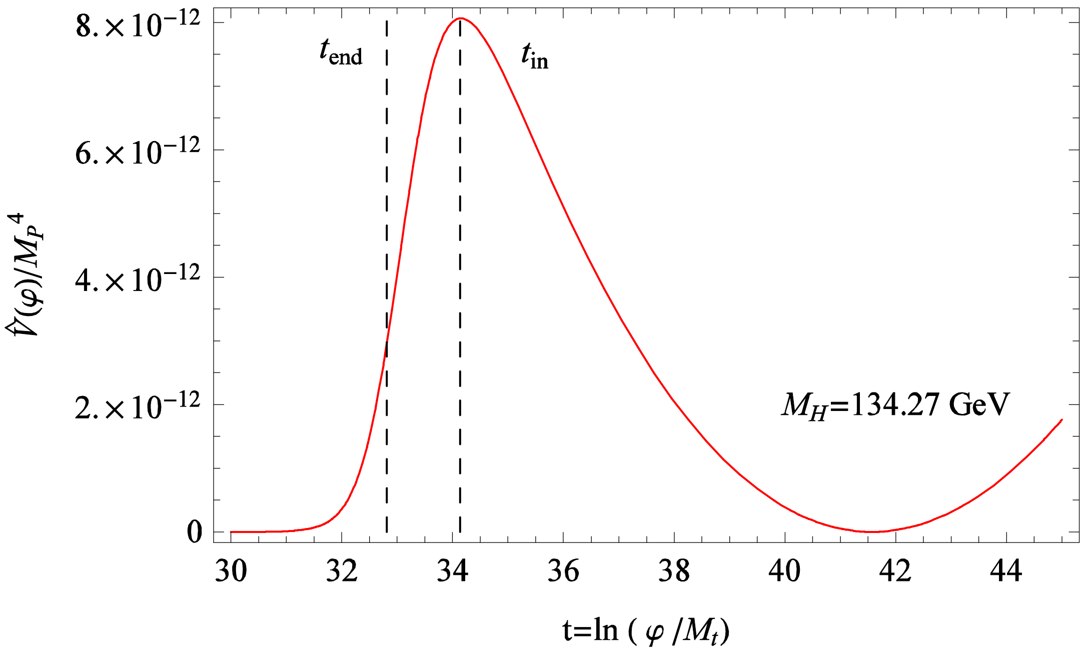


classically allowed (overbarrier) oscillation

Do we have such potentials in realistic cosmology? Yes!

Higgs inflation with non-minimally coupled inflaton

B.Spokoiny 1986,
A.Kamenshchik & A.B 1991,
Bezrukov,Shaposhnikov 2008,
A.Kamenshchik, A.Starobinsky & A.B 2008



The effective inflaton potential for $M_{\rm Higgs} = 134$ GeV.
Inflationary domain for a $N = 60$ CMB perturbation
is marked by dashed lines.

A.Kamenshchik, C.Kiefer,
A.Starobinsky, C.Steinwachs,
& A.B., JCAP 12 (2009) 003,
arXiv:0904.1698

2-loop RG contribution leads to $M_{Higgs} \rightarrow M_{LHC}$= 126 GeV
(Bezrukov & Shaposhnikov 2009)

Primordial CMB spectrum with thermal corrections:

T_{CMB} is subject to vacuum fluctuations

Now T_{CMB} is subject to thermal distribution with the temperature $T = 1/\eta$
-- *temperature of the CMB temperature*

standard red CMB spectrum

thermal contribution

$$\delta^2_\phi(k) = \langle\, \hat{\phi}_k(t)\hat{\phi}_k(t)\,\rangle_{\text{thermal}}$$

$$\sim \langle\, \hat{a}^\dagger_k \hat{a}_k + \hat{a}_k \hat{a}^\dagger_k\,\rangle_{\text{thermal}} |u_k(t)|^2 = |u_k(t)|^2 \big(1 + 2N_k(\eta)\big)$$

$$N_k(\eta) = \frac{1}{e^{k\eta} - 1}$$

$$k_l \leftrightarrow l \;\Rightarrow\; C^2_l \to C^2_l \left(1 + \frac{2}{e^{k_l\eta} - 1}\right)$$

additional reddening of the CMB spectrum

Vacuum part of ΔT_{CMB} is under nvestigation:

dilaton = physical variable in cosmology

$$g_{\mu\nu} = e^{\sigma}\bar{g}_{\mu\nu},$$

dilaton action

$$\Gamma_R[g] - \Gamma_R[\bar{g}] = \frac{\gamma}{4(4\pi)^2}\int d^4x\bar{g}^{1/2}\,\sigma\,\bar{C}^2_{\mu\nu\alpha\beta} + \frac{\beta}{2(4\pi)^2}\int d^4x\bar{g}^{1/2}\Big\{\frac{1}{2}\sigma\bar{E} - \Big(\bar{R}^{\mu\nu} - \frac{1}{2}\bar{g}^{\mu\nu}\bar{R}\Big)\partial_\mu\sigma\,\partial_\nu\sigma - \frac{1}{2}\bar{\Box}\sigma\,(\bar{\nabla}^\mu\sigma\,\bar{\nabla}_\mu\sigma) - \frac{1}{8}(\bar{\nabla}^\mu\sigma\,\bar{\nabla}_\mu\sigma)^2\Big\}$$

Galileon type mode --no higher derivative ghosts in the scalar sector

$$u_k(t)\ ?$$

Basis function of the cosmological perturbations (Mukhanov-Sasaki mode)

Thermal contribution to the spectral index:

$$n_s(k) = 1 + \frac{d}{d\ln k}\ln \delta_\phi^2(k) = n_s^{\mathrm{vac}}(k) + \Delta n_s^{\mathrm{thermal}}(k),$$

$$\Delta n_s^{\mathrm{thermal}}(k) = \frac{d}{d\ln k}\ln\left(1 + 2N_k(\eta)\right)$$

$$N_{k_l} \simeq \exp\left[-\frac{2l}{(\Omega_0 - 1)^{1/2}}\left(\frac{1}{180\,\tilde{\beta}}\right)^{1/6}\right] \simeq \exp\left[-\frac{10\,l}{(3\,\tilde{\beta})^{1/6}}\right]$$

<<1

$$\Delta n_s^{\mathrm{thermal}}(k_l) \simeq -\frac{20\,l}{(3\,\tilde{\beta})^{1/6}} e^{-10\,l/(3\,\tilde{\beta})^{1/6}} \ll 1$$

$$\frac{1}{60} \leq 3\,\tilde{\beta}_{\mathrm{low-spin}} \leq \frac{31}{30} \simeq 1$$

$$\tilde{\beta} \sim \frac{\sum_s \beta_s N_s}{\sum_s N_s}$$

specific β per one degree of freedom

$$\beta_s = \begin{cases} \frac{1}{180} & s=0 \\ \frac{11}{360} & s=\frac{1}{2} \\ \frac{31}{90} & s=1 \\ -\frac{274}{90} & s=\frac{3}{2} \\ \frac{87}{10} & s=2 \\ \ldots & s>1 \end{cases}$$

For $s = 0, \frac{1}{2}, 1$ ⇒ $3\tilde{\beta} \lesssim 1$

thermal part is negligible: $\Delta n_s < e^{-\frac{l}{\sqrt{\Omega_0 - 1}}} \sim e^{-10\,l}$

$s = \frac{3}{2}$: **conformal gravitino**

$s = 2$: **conformal graviton** ⇒ **contribution of higher conformal spins might have large thermal imprint on CMB ?**

$s > 1$: $\ldots$ ↓ $s^6 \rightarrow \infty$ ⇒ $\tilde{\beta}_s = \frac{\beta_s}{N_s} \sim s^4 \rightarrow \infty$

$$\beta_s = \frac{1}{360} N_s^2 (3 + 14 N_s), \quad N_s = s(s+1), \quad s = 1, 2, 3, \ldots,$$

$$\beta_s = \frac{1}{720} N_s (12 + 45 N_s + 14 N_s^2), \quad N_s = -2\left(s + \frac{1}{2}\right)^2, \quad s = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$$

A.A.Tseytlin, arXiv:1309.0785

CFT cosmology and the a-theorem

Free CFT ⟶ interacting CFT

$$e^{iS_{\rm eff}[g_{\mu\nu}]} = \int D\phi \, e^{iS_{CFT}[g_{\mu\nu},\phi]}$$

$$\langle T^{\mu}_{\mu} \rangle \equiv \frac{2}{g^{1/2}} g^{\mu\nu} \frac{\delta S_{\rm eff}}{\delta g^{\mu\nu}} = aE - c\, C^2_{\mu\nu\alpha\beta} + b \Box R.$$

Renormlization group flow: $a,\, b,\, c \;\; \Rightarrow \;\; a(\mu^2),\, b(\mu^2),\, c(\mu^2)$

UV ↓ IR ↓

$$a(\infty) - a(0) = \frac{1}{4\pi} \int_{s>0} ds \, \frac{\sigma(s)}{s^2} > 0$$

$$\sigma(s) = s\, {\rm Im} \mathcal{A}(s,t)_{t=0} > 0$$

Total cross section of the forward
2 ⟶ 2 dilaton scattering

Z.Komargodski and A.Schwimmer, arXiv:1107.3987

Z.Komargodski, arXiv:1112.4538

Cosmological expansion: UV $\longrightarrow$ IR

$$a(\mu^2) = \frac{\beta(\mu^2)}{32\pi^2}, \qquad \beta_{\text{early}} \gg 1 \quad \rightarrow \quad \beta_{\text{now}} \sim O(1)$$

$$\Downarrow$$

Observable thermal imprint on CMB of invisible now higher spin fields ?

Conclusions

Microcanonical density matrix of the Universe

Application to the CFT driven cosmology with a large # of quantum species – thermal version of the no-boundary state

Initial conditions for inflation with a limited range of Λ -- cosmological landscape -- and generation of the thermal CMB spectrum, but rather cold

SOME LIKE IT HOT

SOME LIKE IT COOL