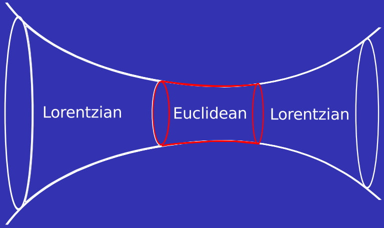
Euclidean Big Bounce



Jakub Mielczarek

Jagiellonian University, Cracow
National Centre for Nuclear Research, Warsaw

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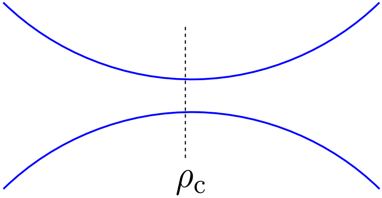
In loop quantum cosmology (LQC) the holonomy corrections deform the classical Friedmann equation to

$$H^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right),$$

where the critical energy density $\rho_c = \frac{3}{8\pi G\lambda^2\gamma^2} \sim \rho_{Pl}$. Because $\rho \leq \rho_c$, the classical Big Bang singularity is replaced by the non-singular Big Bounce[1].



Contracting and expanding branches are causally connected.

[1] Bojowald, Ashtekar, Pawłowski, Singh, ...

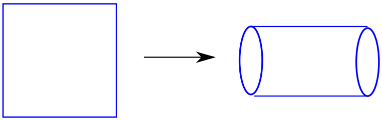
The holonomy corrections mimic the loop quantization.

For the FRW model, the holonomy corrections can be introduced by the following replacement in the classical Hamiltonian:

$$\bar{k} \to \mathbb{K}[n] := \frac{\sin(n\bar{\mu}\gamma\bar{k})}{n\bar{\mu}\gamma},$$

where $\bar{k}$ is canonically conjugated with $\bar{p} = a^2$. Furthermore, $\bar{\mu} \propto \bar{p}^{\delta}$ where $-1/2 \leq \delta \leq 0$ and $n \in \mathbb{N}$.

Holonomy corrections can be seen as bending (periodification) of the phase space. For the FRW model, the classical phase space $\Gamma_G = \mathbb{R} \times \mathbb{R}$ is deformed onto $\Gamma_G^Q = \mathbb{R} \times U(1)$.



Inhomogeneities

Problems:

- The procedure of introducing quantum corrections suffers from ambiguities.
- In general, the algebra of modified constraints is not closed:

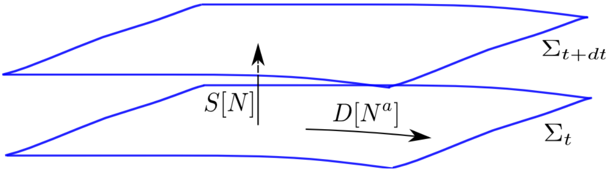
$$\{\mathcal{C}_I^Q, \mathcal{C}_J^Q\} = g^K{}_{IJ}(A_b^j, E_i^a)\mathcal{C}_K^Q + \mathcal{A}_{IJ}.$$

Can we introduce quantum corrections in the anomaly-free manner (i.e. such that $\mathcal{A}_{IJ} = 0$)? We found that, at least for linear inhomogeneities on the flat FRW background, the answer is affirmative:

- scalar perturbations - T. Cailleteau, J. Mielczarek, A. Barrau, J. Grain, Class. Quantum Grav. **29** (2012) 095010,
- vector perturbations - J. Mielczarek, T. Cailleteau, A. Barrau and J. Grain, Class. Quant. Grav. **29** (2012) 085009,
- tensor perturbations - no problem with anomalies.

Some details...

The constraints generate gauge transformations:



The algebra of constraints (hypersurface deformation algebra):

$$
\begin{aligned}
\{D[N_1^a], D[N_2^a]\} &= D[N_1^b \partial_b N_2^a - N_2^b \partial_b N_1^a], \\
\{S[N], D[N^a]\} &= -S[N^a \partial_a N], \\
\{S[N_1], S[N_2]\} &= sD\left[g^{ab}(N_1 \partial_b N_2 - N_2 \partial_b N_1)\right],
\end{aligned}
$$

where $s = 1$ corresponds to the Lorentzian signature and $s = -1$ to the Euclidean one. Due to the factor g^{ab} the algebra of constraints is not a Lie algebra.

Gravitational part of the holonomy-modified scalar constraint:

$$S_G^Q[N] = \frac{1}{2\kappa}\int_\Sigma d^3x \left[\bar{N}(\mathcal{H}_G^{(0)} + \mathcal{H}_G^{(2)}) + \delta N \mathcal{H}_G^{(1)}\right], \text{ where}$$

$$\begin{aligned}
\mathcal{H}_G^{(0)} &= -6\sqrt{\bar{p}}(\mathbb{K}[1])^2, \\
\mathcal{H}_G^{(1)} &= -4\sqrt{\bar{p}}\left(\mathbb{K}[s_1] + \alpha_1\right)\delta_j^c \delta K_c^j - \frac{1}{\sqrt{\bar{p}}}\left(\mathbb{K}[1]^2 + \alpha_2\right)\delta_c^j \delta E_j^c \\
&+ \frac{2}{\sqrt{\bar{p}}}(1+\alpha_3)\partial_c \partial^j \delta E_j^c, \\
\mathcal{H}_G^{(2)} &= \sqrt{\bar{p}}(1+\alpha_4)\delta K_c^j \delta K_d^k \delta_k^c \delta_j^d - \sqrt{\bar{p}}(1+\alpha_5)(\delta K_c^j \delta_j^c)^2 \\
&- \frac{2}{\sqrt{\bar{p}}}\left(\mathbb{K}[s_2] + \alpha_6\right)\delta E_j^c \delta K_c^j - \frac{1}{2\bar{p}^{3/2}}\left(\mathbb{K}[1]^2 + \alpha_7\right)\delta E_j^c \delta E_k^d \delta_c^k \delta_d^j \\
&+ \frac{1}{4\bar{p}^{3/2}}\left(\mathbb{K}[1]^2 + \alpha_8\right)(\delta E_j^c \delta_c^j)^2 \\
&- \frac{1}{2\bar{p}^{3/2}}(1+\alpha_9)\delta^{jk}(\partial_c \delta E_j^c)(\partial_d \delta E_k^d).
\end{aligned}$$

Here, $\alpha_i(\bar{p},\bar{k})$ are the counter-terms ($\alpha_i(\bar{p},\bar{k}) \to 0$ for $\bar{\mu} \to 0$).

Exemplary bracket

$$
\begin{aligned}
\left\{S_G^Q[N_1], S_G^Q[N_2]\right\} = {} & (1+\alpha_3)(1+\alpha_5) D_G \left[\frac{\bar{N}}{\bar{p}} \partial^a (\delta N_2 - \delta N_1)\right] \\
& + \frac{\bar{N}}{\kappa} \int_\Sigma d^3x \partial^a (\delta N_2 - \delta N_1)(\partial_i \delta K_a^i)(1+\alpha_3)\mathcal{A}_5 \\
& + \frac{\bar{N}}{\kappa \bar{p}} \int_\Sigma d^3x (\delta N_2 - \delta N_1)(\partial^i \partial_a \delta E_i^a)\mathcal{A}_6 \\
& + \frac{\bar{N}}{\kappa} \int_\Sigma d^3x (\delta N_2 - \delta N_1)(\delta_i^a \delta K_a^i)\mathcal{A}_7 \\
& + \frac{\bar{N}}{\kappa \bar{p}} \int_\Sigma d^3x (\delta N_2 - \delta N_1)(\delta_a^i \delta E_i^a)\mathcal{A}_8
\end{aligned}
$$

The $\mathcal{A}_5, \ldots, \mathcal{A}_8$ are anomaly functions. The diffeomorphism constraint is multiplied by the factor $(1+\alpha_3)(1+\alpha_5)$.

Anomaly freedom

The requirement of anomaly freedom is equivalent to the conditions $\mathcal{A}_i = 0$ for $i = 1, \dots, 13$. Furthermore $(1+\alpha_3)(1+\alpha_5) = (1+\alpha_{10})$. These conditions uniquely determine form of the counter terms α_i for $i = 1, \dots, 10$.

Moreover, we have

$$\begin{aligned} \mathcal{A}_7 &= 2(1+2\delta)(\Omega\mathbb{K}[1]^2 - \mathbb{K}[2]^2), \\ \mathcal{A}_8 &= \bar{k}(1+2\delta)(\mathbb{K}[2]^2 - \Omega\mathbb{K}[1]^2). \end{aligned}$$

The anomaly freedom conditions for those terms ($\mathcal{A}_7 = 0$ and $\mathcal{A}_8 = 0$) are fulfilled if and only if $\delta = -1/2$. The choice $\delta = -1/2$ is called the $\bar{\mu}-$scheme ('new quantization scheme').

Our results show that the $\bar{\mu}-$scheme is embedded in the structure of the theory and this gives a new motivation for this particular choice of quantization scheme.

Cosmological perturbations

Tedious calculations lead us to deformed EOM for the Munhanov variable:

$$\frac{d^2}{d\tau^2}v - c_s^2\nabla^2 v - \frac{z''}{z}v = 0,$$

where the holonomy corrections are introduced through

$$c_s^2 = \Omega = \cos(2\gamma\bar{\mu}\bar{k}) = 1 - 2\frac{\rho}{\rho_c}.$$

Similarly for the tensor modes and scalar matter[2]. The EOMs are of the mixed type:

- hyperolic - for $\rho < \rho_c/2$ $(\Omega > 0)$,
- parabolic - for $\rho = \rho_c/2$ $(\Omega = 0)$,
- elliptic - for $\rho > \rho_c/2$ $(\Omega < 0)$.

Nontrivial boundary/initial conditions à la Tricomi problem.

[2] T. Cailleteau, A. Barrau, J. Grain and F. Vidotto, Phys. Rev. D **86** (2012) 087301, J. Mielczarek, PhD thesis (2012).

Algebra of constraints:

$$
\begin{aligned}
\{D[N_1^a], D[N_2^a]\} &= D[N_1^b \partial_b N_2^a - N_2^b \partial_b N_1^a], \\
\left\{S^Q[N], D[N^a]\right\} &= -S^Q[N^a \partial_a N], \\
\left\{S^Q[N_1], S^Q[N_2]\right\} &= \Omega D\left[g^{ab}(N_1 \partial_b N_2 - N_2 \partial_b N_1)\right],
\end{aligned}
$$

The algebra is closed but deformed with respect to the classical case due to presence of the factor

$$\Omega = \cos(2\bar{\mu}\gamma\bar{k}) = 1 - 2\frac{\rho}{\rho_c} \in [-1, 1] \quad \text{where} \quad \rho_c = \frac{3}{8\pi G \Delta \gamma^2} \sim \rho_{\mathrm{Pl}}.$$

What is the interpretation? Classically, we have

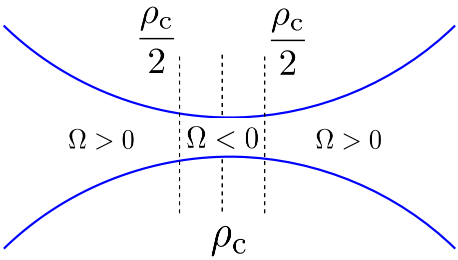
$$\{S[N_1], S[N_2]\} = sD\left[\frac{\bar{N}}{\bar{p}} \partial^a (\delta N_2 - \delta N_1)\right],$$

where $s = 1$ corresponds to the Lorentzian signature and $s = -1$ to the Euclidean one.

Signature change

The sign of Ω reflects a signature of space: $\Omega > 0$ ($\rho < \rho_c/2$) - Lorentzian signature, $\Omega < 0$ ($\rho > \rho_c/2$) - Euclidean signature.

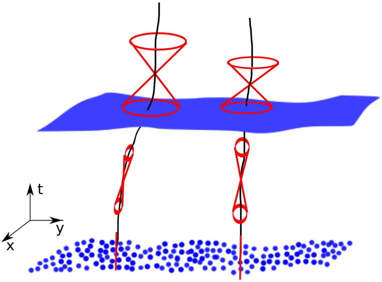
Based on this, a new picture of the Big Bounce emerges: In the Planck epoch, space-time becomes a four dimensional Euclidean space. This behavior resembles the famous Harte-Hawking no-boundary proposal (1983).



Causal connection between contracting and expanding branches is limited.

Silence in LQC

The light cones are collapsing onto the time lines for. $\rho \to \rho_c/2$.
The effective speed of light $c_{eff} = \sqrt{\Omega} = \sqrt{1 - 2\frac{\rho}{\rho_c}} \to 0$.
Communication between different space points is forbidden[3].



The similar behavior is expected at the classical level by virtue of the famous BKL conjecture (Belinsky-Khalatnikov-Lifshitz).

[3] J. Mielczarek, "Asymptotic silence in loop quantum cosmology," AIP Conf. Proc. **1514** (2012) 81

Carrollian limit

"A slow sort of country ..., ... now, here, you see, it takes all the running you can do to stay in the same place ..." Lewis Carroll

General relativity → Hypersurface deformation algebra

$$
\begin{aligned}
\{D[N_1^a], D[N_2^a]\} &= D[N_1^b \partial_b N_2^a - N_2^b \partial_b N_1^a], \\
\{S[N], D[N^a]\} &= -S[N^a \partial_a N], \\
\{S[N_1], S[N_2]\} &= D\left[g^{ab}(N_1 \partial_b N_2 - N_2 \partial_b N_1)\right].
\end{aligned}
$$

Special relativity → Poincaré algebra

$$
\begin{aligned}
\{P_\mu, P_\nu\} &= 0, \\
\{M_{\mu\nu}, P_\nu\} &= \eta_{\mu\rho} P_\nu - \eta_{\nu\rho} P_\mu, \\
\{M_{\mu\nu}, M_{\rho\sigma}\} &= \eta_{\mu\rho} M_{\nu\sigma} - \eta_{\mu\sigma} M_{\nu\rho} - \eta_{\nu\rho} M_{\mu\sigma} + \eta_{\nu\sigma} M_{\mu\rho}.
\end{aligned}
$$

From DHDA to deformed Poincaré algebra

In the limit of vanishing space-time curvaure, the loop-deformed algebra of constraints, reduces to the loop-deformed Poincaré algebra. This corresponds to linear deformations of the hypersurface[4]:

$$N(x) = \Delta t + v_a x^a, \quad N^a(x) = \Delta x^a + R^a{}_b x^b, \text{ and } g_{ab} = \delta_{ab}.$$

Based on this structure of deformed Poincaré algebra can be inferred. Schematically, we have $S \sim P_0 + K_i$ and $D \sim P_i + J_i$. The fact that only the $\{S^Q, S^Q\} = \Omega D$ bracket is deformed imposes constraints on the possible deformations of the Poincaré algebra:

$$\cancel{\{P_0, P_0\}} + \underline{\{K_i, P_0\}} + \underline{\underline{\{K_i, K_j\}}} \sim \underline{\Omega P_i} + \underline{\underline{\Omega J_i}}$$

[4]M. Bojowald and G. M. Paily, Phys. Rev. D **87** (2013) 044044

Loop-deformed Poincaré algebra

$$\begin{aligned} [J_i, J_j] &= i\epsilon_{ijk}J_k, \quad [J_i, K_j] = i\epsilon_{ijk}K_k, \quad [K_i, K_j] = -i\Omega(P_0, P_i^2)\epsilon_{ijk}J_k, \\ [J_i, P_j] &= i\epsilon_{ijk}P_k, \quad [K_i, P_j] = i\delta_{ij}P_0, \quad [J_i, P_0] = 0, \\ [K_i, P_0] &= i\Omega(P_0, P_i^2)P_i, \quad [P_i, P_j] = 0, \quad [P_i, P_0] = 0. \end{aligned}$$

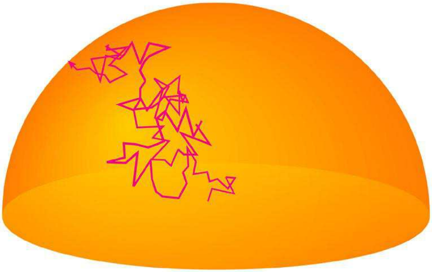
Generalized version of the $\Omega = \Omega(P_0)$ case studied in Ref.[5]. Assuming that $\Omega(P_0, P_i^2) = A(P_0)B(P_i^2)$, the Jacobi identities are satisfied if

$$\Omega = \frac{P_0^2 - \alpha}{P_i^2 - \alpha}.$$

The constant of integration α plays a role of the deformation parameter.

[5] J. Mielczarek, arXiv:1304.2208 [gr-qc]. Essay honorably mentioned in the 2013 essay competition of the Gravity Research Foundation.

We consider random walk



which on average is described by the diffusion process described by the heat kernel equation

$$\frac{\partial}{\partial \sigma} K(x, y; \sigma) = \Delta_x K(x, y; \sigma),$$

where σ is a diffusion time.

By introducing the average return probability

$$P(\sigma) = \mathrm{tr}K = \int dxK(x,x;\sigma) = \int d\mu(p)e^{\sigma\Delta_p}$$

the spectral dimension can be defined:

$$d_S \equiv -2\frac{\partial \log P(\sigma)}{\partial \log \sigma}.$$

The Laplace operator in the momentum space Δ_p can be expressed in terms of the Wick rotated ($P_0 \to iP_0$) first Casimir operator

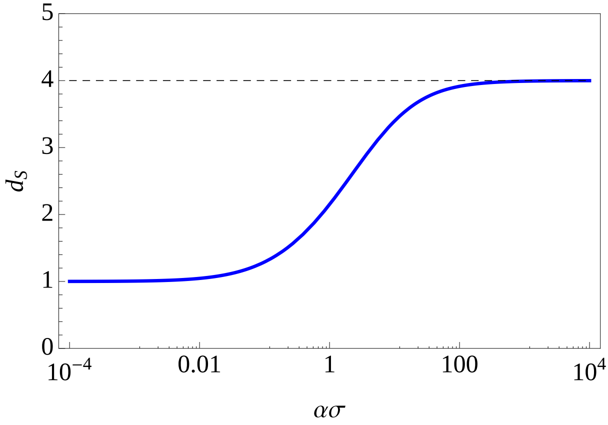
$$\Delta_p = -\mathcal{C}_1 + \sum_{n=2} c_n \alpha \left(\frac{\mathcal{C}_1}{\alpha}\right)^n.$$

In what follows we set $\forall_{n\geq 2} c_n = 0$, $d\mu = \frac{d^4p}{(2\pi)^4}$ and use

$$\mathcal{C}_1 = \alpha(\Omega - 1) = \frac{-P_0^2 + P_i^2}{1 - \frac{P_i^2}{\alpha}}.$$

Spectral dimension:

$$d_S = -2\frac{\partial \log P(\sigma)}{\partial \log \sigma}.$$



Dimensional reduction from $d_S = 4$ at the large scales to $d_S = 1$ at the short scales.

The dimensional reduction reflects the ultralocality (Carollian limit, $c \to 0$) recovered in the high energy limit ($\Omega \to 0$). The four dimensional spacetime becomes a congruence of the time lines.

This is consistent with what was observed in the cosmological case for $\Omega \to 0$ which corresponds to the state of silence.

Silence is everywhere

- Classical GR $\rightarrow$ Belinsky-Khalatnikov-Lifshitz conjecture $\rightarrow$ Close to the initial singularity spatial derivatives are suppressed $\rightarrow$ Asymptotic silence.
- Spatial derivatives are suppressed also in the strong coupling limit $G_N \rightarrow \infty$, which leads to the ultralocal gravity $\{S[N_1], S[N_2]\} = 0$ (Isham).
- Causal Dynamical Triangulations $\rightarrow$ In the so-called phase A of gravity, universe factorizes into independent components $\rightarrow$ in this phase the strong coupling limit is realized.
- Hořava-Lifshitz gravity with anisotropic scaling $\mathbf{x} \rightarrow b\mathbf{x}$ and $t \rightarrow b^z t$. Ultralocal gravity is recovered for $z \rightarrow 0$. However, then $d_S \rightarrow \infty$. On the other hand $d_S \rightarrow 1$ for $z \rightarrow \infty$. The relation $d_S = 1 + \frac{3}{z}$ has been used.

Silent beginning?

The Lorentzian phase of expansion begins with silence...

We need observational verification → e.g. analysis of primordial perturbations and their relevance for the CMB aniostropy and polarization.

Quantum generation of perturbations

Let us focus on the tensor modes (gravitational waves) $h_{+,\times} = \sqrt{16\pi G}\phi$ for which we have the following EOM:

$$\frac{d^2}{d\eta^2}\phi + 2\left(\mathcal{H} - \frac{1}{2\Omega}\frac{d\Omega}{d\eta}\right)\frac{d}{d\eta}\phi - \Omega\nabla^2\phi = 0.$$

By introducing $u = z\phi$ with $z = a/\sqrt{\Omega}$ we get

$$\frac{d^2}{d\eta^2}u - \Omega\nabla^2 u - \frac{z''}{z}u = 0,$$

which after Fourier transform $u = \int \frac{d^3k}{(2\pi)^{3/2}} u_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}}$ takes the form

$$\frac{d^2}{d\eta^2}u_{\mathbf{k}} + \left(\Omega k^2 - \frac{z''}{z}\right)u_{\mathbf{k}} = 0.$$

The equations of motion can be derived by considering wave equations $\Box\phi = 0$ on the effective FRW line element

$$ds^2_{eff} = g^{eff}_{\mu\nu} dx^\mu dx^\nu = -\sqrt{\Omega} a^2 d\eta^2 + \frac{a^2}{\sqrt{\Omega}} \delta_{ab} da^a dx^b.$$

Based on this, action of a given mode is

$$\begin{aligned} S &= -\frac{1}{2} \int d^4x \sqrt{-g} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \\ &= \frac{1}{2} \int d\eta d^3x a^3 \left[\frac{1}{\Omega} \left(\phi'\right)^2 - \left(\partial_i \phi\right)^2 \right] \\ &= \frac{1}{2} \int d\eta d^3x \left[v'^2 - \Omega \left(\partial_i v\right)^2 + v^2 \frac{z''}{z} \right] = \int d\eta L, \end{aligned}$$

which, using $\pi = \frac{\delta L}{\delta v'} = v'$, leads to

$$H = \int d^3x \pi v' - L = \int d^3x \left[\pi^2 + \Omega \left(\partial_i v\right)^2 - v^2 \frac{z''}{z} \right].$$

Characteristic scale for the modes is

$$\lambda_H = \frac{a}{k_H},$$

defined such that

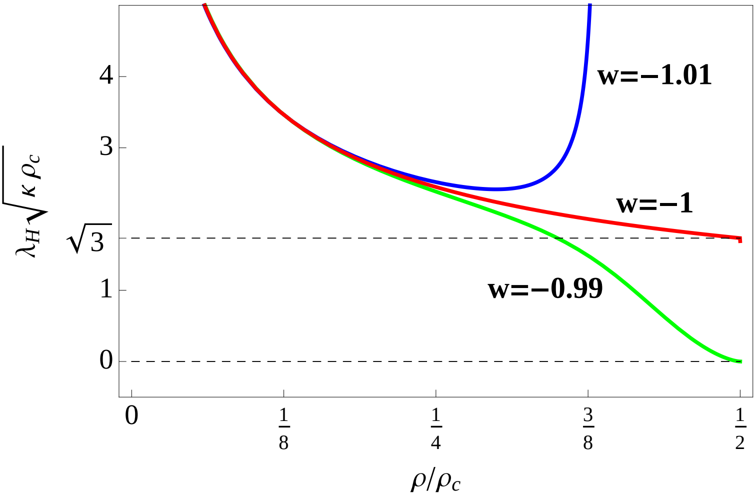
$$\left|\Omega k_H^2\right| = \left|\frac{z''}{z}\right|.$$

The modes are called super-horizontal if $\lambda \gg \lambda_H$ and sub-horizontal if $\lambda \ll \lambda_H$. The modes are "frozen" at the super-horizontal scales and decaying at the sub-horizontal scales.

Let us investigate properties of the modes in the case of barotropic fluid $p = w\rho$ for which

$$\begin{aligned} \frac{z''}{z} &= \rho_c \kappa a^2 \frac{x}{\Omega^2}\left[\frac{1}{2}\left(\frac{1}{3} - w\right) + \left(\frac{9}{2} + 13w + \frac{9}{2}\right)x \right. \\ &- \left.(3 + 30w + 15w^2)x^2 + \left(\frac{11}{3} + 21w + 12w^2\right)x^3\right], \end{aligned}$$

where $x = \rho/\rho_c$. Danger of a divergence at $\Omega \to 0$.



For $w > -1$ all wavelengths are super-horizontal at $\Omega = 0$.
For $w < -1$ (phantom case) all modes can be made sub-horizontal close to $\Omega = 0$.

Quantization of modes

Let us quantize the Fourier modes $u_{\mathbf{k}}(\eta)$. This follows the standard canonical procedure. Promoting this quantity to be an operator, one performs the decomposition

$$\hat{u}_{\mathbf{k}}(\eta) = f_k(\eta)\hat{a}_{\mathbf{k}} + f_k^*(\eta)\hat{a}^{\dagger}_{-\mathbf{k}},$$

where $f_k(\eta)$ is the so-called mode function which satisfies the same equation as $u_{\mathbf{k}}(\eta)$. The creation ($\hat{a}^{\dagger}_{\mathbf{k}}$) and annihilation ($\hat{a}_{\mathbf{k}}$) operators fulfill the commutation relation $[\hat{a}_{\mathbf{k}}, \hat{a}^{\dagger}_{\mathbf{q}}] = \delta^{(3)}(\mathbf{k} - \mathbf{q})$. Two-point correlation function for the ϕ filed is given by

$$\langle 0|\hat{\phi}(\mathbf{x},\eta)\hat{\phi}(\mathbf{y},\eta)|0\rangle = \int_0^{\infty} \frac{dk}{k}\mathcal{P}_{\phi}(k,\eta)\frac{\sin kr}{kr},$$

where the power spectrum

$$\mathcal{P}_{\phi}(k,\eta) = \frac{k^3}{2\pi^2}\left|\frac{f_k}{z}\right|^2,$$

and $r = |\mathbf{x} - \mathbf{y}|$.

Wronskian condition

Based on the Klein-Gordon inner product:

$$
\begin{aligned}
\langle\phi|\phi\rangle &= i\int_\Sigma d^3x\sqrt{q}\left(\phi^* n^\mu\partial_\mu\phi - \phi n^\mu\partial_\mu\phi^*\right) \\
&= i\int_\Sigma d^3x \frac{a^3}{\Omega^{3/4}}\frac{1}{a\Omega^{1/4}}\frac{\Omega}{a^2}\left(u^*\frac{du}{d\tau} - u\frac{du^*}{d\tau}\right) \\
&= i\int_\Sigma d^3x\left(u^*\frac{du}{d\tau} - u\frac{du^*}{d\tau}\right) \\
&= -i\int_\Sigma d^3x W(u,u') = \int_\Sigma d^3x = V_0 = 1,
\end{aligned}
$$

where

$$
W(u,u') = u\frac{du^*}{d\eta} - u^*\frac{du}{d\eta} = i.
$$

The Wronskian condition preserves its classical form.

Vacuum

Vacuum expectation value of the Hamiltonian

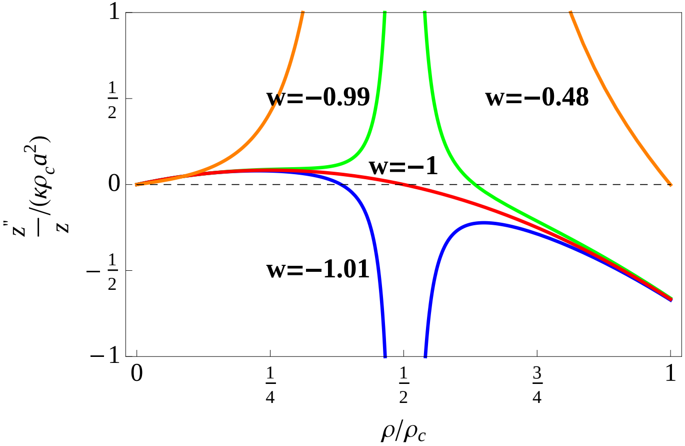
$$\hat{H} = \frac{1}{2}\int d^3k \left[\hat{a}_{\mathbf{k}}\hat{a}_{-\mathbf{k}}F_k + \hat{a}^{\dagger}_{\mathbf{k}}\hat{a}^{\dagger}_{-\mathbf{k}}F^*_k + \left(2\hat{a}^{\dagger}_{\mathbf{k}}\hat{a}_{\mathbf{k}} + \delta^{(3)}(0)\right)E_k\right],$$

where $F_k = (f'_k)^2 + \omega_k^2 f_k^2$, $E_k = |f'_k|^2 + \omega_k^2|f_k|^2$ and $\omega_k^2 = \Omega k^2 - \frac{z''}{z}$ is

$$\langle 0|\hat{H}|0\rangle = \delta^{(3)}(0)\frac{1}{2}\int d^3kE_k.$$

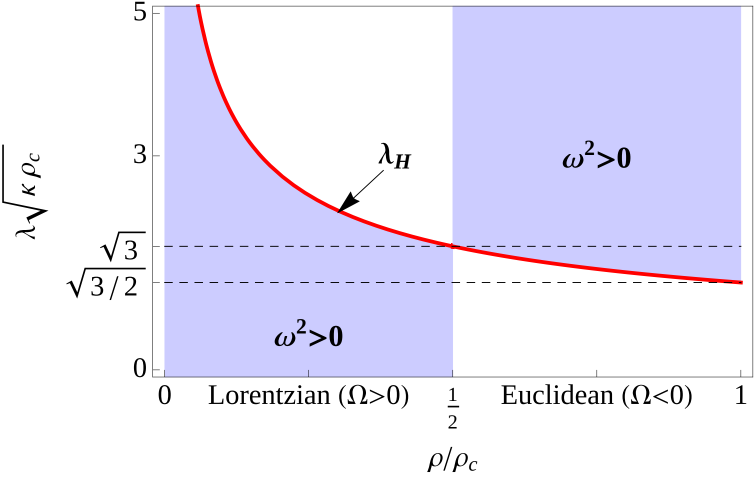
The ground state (vacuum) can be found by minimizing E_k with the Wronskian condition $f_k(f'_k)^* - f^*_k f'_k = i$ taken into account.

The energy can be minimized only if $\omega_k^2 > 0$ - interpretation in terms of particle-like excitations of the filed is possible.



Only for $w = -1$ (Cosmological constant) evolution of $\frac{z''}{z}$ across the moment of signature change is regular. No well defined vacuum state in any part of the Euclidean domain for $w > \frac{1}{6}(-7 + \sqrt{17}) \approx -0.48$.

Regions of $\omega_k^2 > 0$ for $w = -1$, where $\omega_k^2 = \Omega k^2 - \frac{z''}{z}$.



In the Euclidean region, the state of vacuum is well defined for the large scale modes. This is in opposite to the Lorentzian case.

- Vacuum at $\Omega = -1$ for $\lambda \gg \lambda_H$: $|f_k|^2 = \frac{\sqrt{3}}{2\sqrt{2\kappa\rho_c}a}$, which leads to the power spectrum

$$\mathcal{P}_\phi(k) \equiv \frac{k^3}{2\pi^2}\left|\frac{f_k}{z}\right|^2 = \frac{\sqrt{3}}{4\pi^2\sqrt{2\kappa\rho_c}}\left(\frac{k}{a}\right)^3 \propto k^3.$$

  The correlation function is vanishing $\langle 0|\hat{\phi}(\mathbf{x},\eta)\hat{\phi}(\mathbf{y},\eta)|0\rangle = 0$ (white noise).
- Vacuum at $\Omega = 0 \to$ Silent initial conditions? (in progress)
- Vacuum at $\Omega > 0$ for $\lambda \ll \lambda_H$: $|f_k|^2 = \frac{1}{2k\sqrt{\Omega}}$, which leads to the power spectrum

$$\mathcal{P}_\phi(k) \equiv \frac{k^3}{2\pi^2}\left|\frac{f_k}{z}\right|^2 = \frac{k^2}{4\pi^2}\frac{\Omega}{a} \propto k^2.$$

  This is $\Omega-$deformed analogue of the Bunch-Davies vacuum.

Inflationary power spectra for $\Omega > 0$

Using the $\Omega-$deformed Bunch-Davies vacuum the inflationary spectra can be derived[6]:

$$\mathcal{P}_{\mathrm{S}}(k) = A_{\mathrm{S}} \left(\frac{k}{aH}\right)^{n_{\mathrm{S}}-1} \quad \text{and} \quad \mathcal{P}_{\mathrm{T}}(k) = A_{\mathrm{T}} \left(\frac{k}{aH}\right)^{n_{\mathrm{T}}},$$

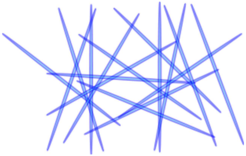
$$\begin{aligned} A_{\mathrm{S}} &= \frac{1}{\pi\epsilon}\left(\frac{H}{m_{Pl}}\right)^2 (1+2\delta_H) \quad \text{and} \quad n_{\mathrm{S}} = 1+2\eta-6\epsilon+\mathcal{O}(\delta_H^2), \\ A_{\mathrm{T}} &= \frac{16}{\pi}\left(\frac{H}{m_{Pl}}\right)^2 (1+\delta_H) \quad \text{and} \quad n_{\mathrm{T}} = -2\epsilon+\mathcal{O}(\delta_H^2). \end{aligned}$$

For the typical inflationary models $\delta_H := \frac{V}{\rho_c} \sim 10^{-12}$. The corrections are extremely hard to constrain observationally.

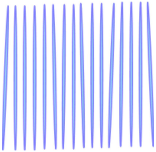
[6] J. Mielczarek, "Inflationary power spectra with quantum holonomy corrections," JCAP **1403** (2014) 048

Signature change due to SSB?

In metamaterials composed of ordered nanowires, electric permittivity in the direction of alignment can be negative[7]. Therefore, the Laplace equation becomes effectively hyperbolic - one of the spatial directions transforms int the effective time. Transition to the oriented phase from the random one can proceed as a result of the spontaneous symmetry breaking.



$T > T_C$



$T < T_C$

Rotational symmetry $SO(3)$ of the high temperature phase is broken onto $SO(2)$ symmetry below the critical temperature T_C.

Something similar in the case of gravity?

[7] I. I. Smolyaninov and E. E. Narimanov, *Phys. Rev. Lett.*, **105** (2010) 067402.

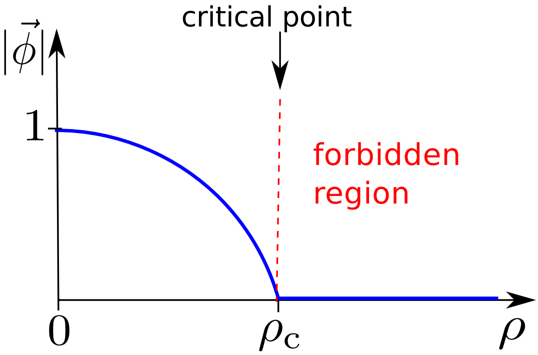
- Symmetry breaking from $SO(4)$ to $SO(3)$ at the level of an effective homogeneous vector field ϕ_μ. This translates to a symmetry change from $SO(4)$ to $SO(3,1)$, at the level of geometry described by the metric $g_{\mu\nu} = \delta_{\mu\nu} - 2\phi_\mu\phi_\nu$.
- Let us assume that the free energy for the model with a massless scalar field v is [8]

$$F = \int dV \underbrace{\left(\delta^{\mu\nu} + \frac{2\phi^\mu\phi^\nu}{1-2|\vec{\phi}|^2}\right)}_{g^{\mu\nu}} \partial_\mu v \partial_\nu v + \beta \underbrace{\left[\left(\frac{\rho}{\rho_c} - 1\right)|\vec{\phi}|^2 + \frac{1}{2}|\vec{\phi}|^4\right]}_{V(\phi^\mu,\rho)},$$

where $|\vec{\phi}| = \sqrt{\delta^{\mu\nu}\phi_\mu\phi_\nu}$ and β is a constant.

[8] J. Mielczarek, "Big Bang as a critical point," arXiv:1404.0228 [gr-qc].

Modulus of the filed ϕ^{μ} as a function of the energy density ρ. The region $\rho > \rho_c$ is forbidden within the model.



An interesting possibility is that the system has been maintained at the critical point before the energy density started to drop. This may not require a fine-tuning if the dynamics of the system exhibited Self Organized Criticality (SOC), which is observed in various complex systems.

- Without loss of generality, let us assume that the SSB takes place in direction ϕ_0, for which

$$g_{00} = 1 - 2\phi_0\phi_0 = 1 - 2\left(1 - \frac{\rho}{\rho_c}\right) = -1 + 2\frac{\rho}{\rho_c} = -\Omega, \quad g_{ii} = 1,$$

leading to the effective speed of light $c^2_{eff} = \Omega$.

- As a consequence, the equation of motion for the scalar field v takes the form

$$g^{\mu\nu}\partial_\mu\partial_\nu v = -\frac{1}{c^2_{eff}}\frac{\partial^2}{\partial t^2}v + \Delta v = 0,$$

manifesting $SO(4)$ symmetry at the critical point ($\Omega = -1$), and $SO(3,1)$ symmetry in the low temperature limit ($\Omega = 1$).

- The form of the above equation agrees with the one derived from the holonomy deformations of the HDA.

Summary and outlook

- Loop deformations of the space-time symmetries lead to a new scenario in the Planck epoch - the Euclidean Big Bounce.
- The smooth signature change and the Big Silence.
- The model of Euclidean Big Bounce unifies various ideas, such as signature change, the Hartle-Hawking proposal, ultralocality, variable speed of light, gravitational phase transitions ...
- Different approaches to quantum gravity meet in the silence.
- Attractive from the conceptual viewpoint.
- Phenomenology available thanks to loop-deformed Poinceré algebra. Complementary methods of testing the same quantum gravitational effects: cosmology and astrophysics.
- Signature change as a gravitational phase transition?