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+ S. Vitenti & A. Valentini

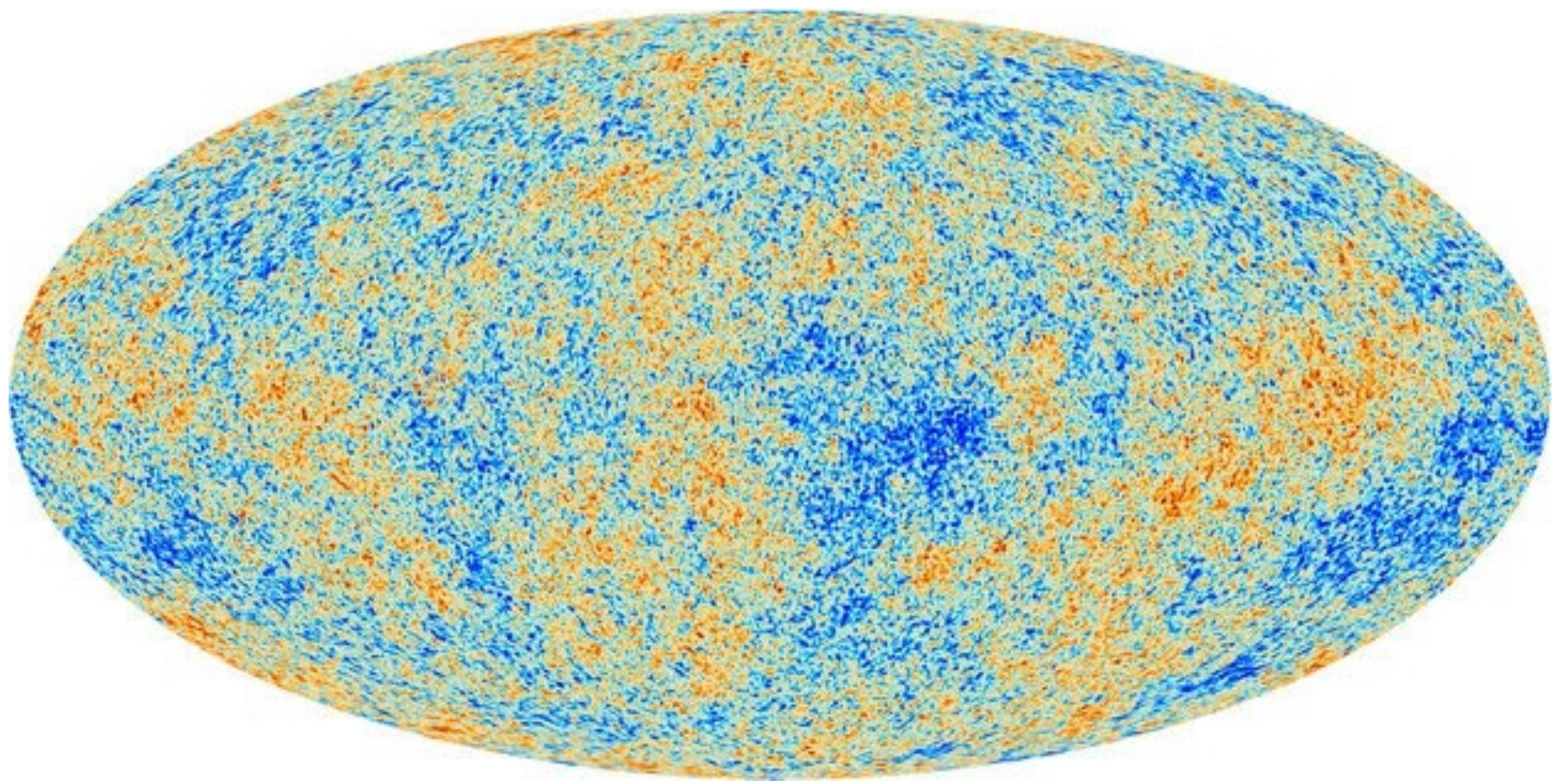
Out-of-equilibrium quantum cosmology

569. Wilhelm and Else Heraeus Seminar

Bad Honnef - Jul. 28, 2014

$$\mathrm{d}s^2 = a^2(\eta)\left\{(1+2\Phi)\,\mathrm{d}\eta^2 - \left[(1-2\Phi)\,\gamma_{ij} + h_{ij}\right]\mathrm{d}x^i\mathrm{d}x^j\right\}$$

$$i\hbar\frac{\mathrm{d}}{\mathrm{d}t}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle$$



Inflationary perturbations: quantum fluctuations / expanding background

Classical temperature fluctuations

$$\frac{\Delta T}{T} \propto v \ \sim \Phi \sim \delta g_{00}$$

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Classical temperature fluctuations promoted to quantum operators

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second order perturbed Einstein action $\quad {}^{(2)}\delta S = \frac{1}{2}\int \mathrm{d}^4 x \left[\left(v'\right)^2 - \delta^{ij}\partial_i v \partial_j v + \frac{\left(a\sqrt{\epsilon_1}\right)''}{a\sqrt{\epsilon_1}} v^2 \right]$

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$$\epsilon_1 = 1 - \mathcal{H}'/\mathcal{H}^2$$

slow-roll parameter

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variable-mass scalar fields in Minkowski spacetime

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variable-mass scalar fields in Minkowski spacetime

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slow-roll parameter

+ Fourier transform $\quad v\left(\eta, \boldsymbol{x}\right) = \frac{1}{\left(2\pi\right)^{3/2}} \int_{\mathbb{R}^3} \mathrm{d}^3 \boldsymbol{k}\, v_{\boldsymbol{k}}\left(\eta\right) \mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{x}}$

$$\Longrightarrow \quad {}^{(2)}\delta S = \int \mathrm{d}\eta \int \mathrm{d}^3 \boldsymbol{k} \left\{ v'_{\boldsymbol{k}} v_{\boldsymbol{k}}^{*\prime} + v_{\boldsymbol{k}} v_{\boldsymbol{k}}^{*} \left[\frac{\left(a\sqrt{\epsilon_1}\right)''}{a\sqrt{\epsilon_1}} - k^2 \right] \right\}$$

Lagrangian formulation...

Hamiltonian

$$H = \int \mathrm{d}^3\boldsymbol{k} \left\{ p_{\boldsymbol{k}} p^*_{\boldsymbol{k}} + v_{\boldsymbol{k}} v^*_{\boldsymbol{k}} \left[\overbrace{k^2 - \frac{\left(a\sqrt{\epsilon_1}\right)''}{a\sqrt{\epsilon_1}}}^{\omega^2(\eta,\boldsymbol{k})} \right] \right\}$$

collection of parametric oscillators with time dependent frequency

factorization of the full wave function

real and imaginary parts

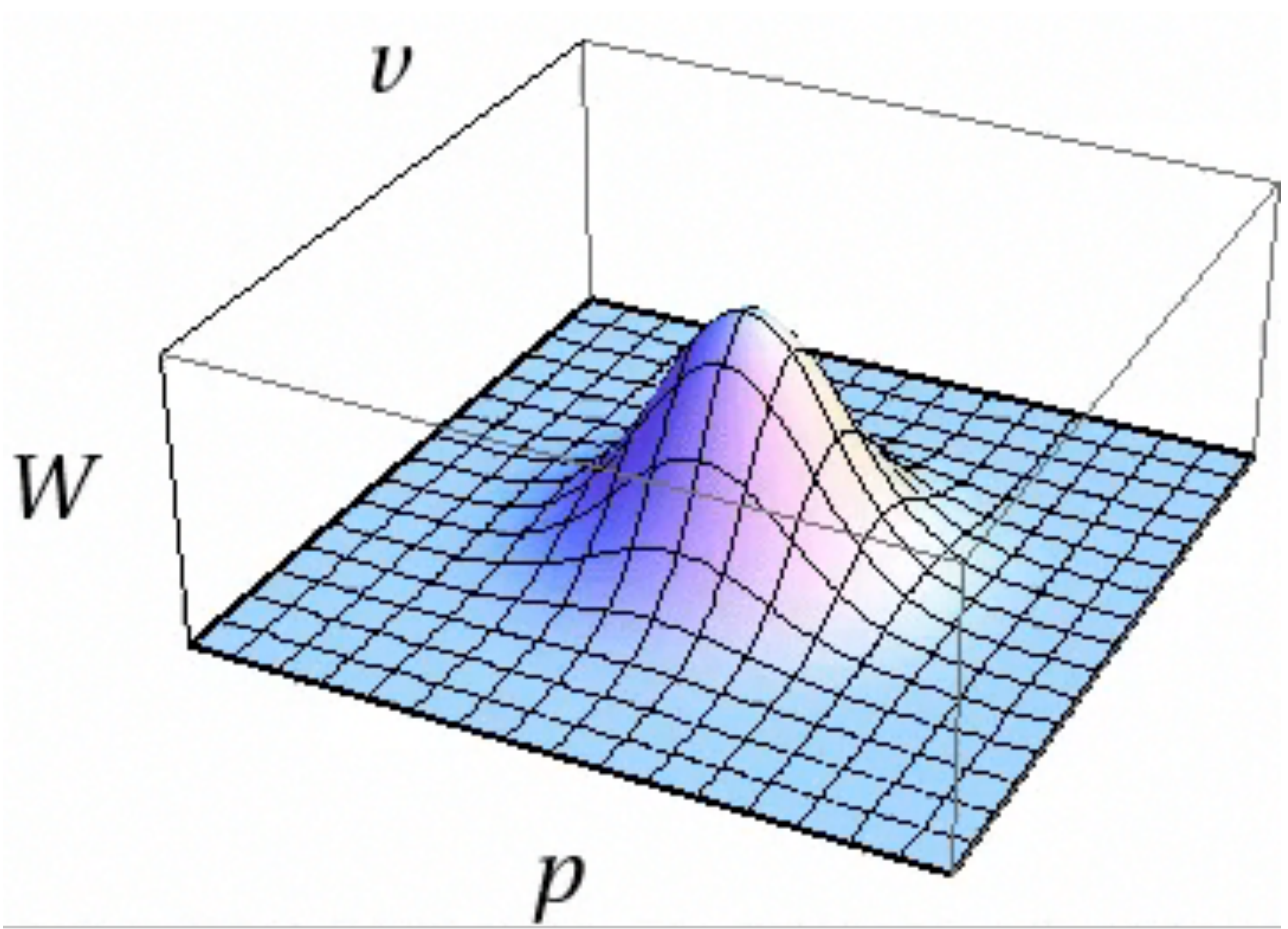
$$\Psi\left[v(\eta,\boldsymbol{x})\right] = \prod_{\boldsymbol{k}} \Psi_{\boldsymbol{k}}\left(v^{\mathrm{R}}_{\boldsymbol{k}}, v^{\mathrm{I}}_{\boldsymbol{k}}\right) = \prod_{\boldsymbol{k}} \Psi^{\mathrm{R}}_{\boldsymbol{k}}\left(v^{\mathrm{R}}_{\boldsymbol{k}}\right) \Psi^{\mathrm{I}}_{\boldsymbol{k}}\left(v^{\mathrm{I}}_{\boldsymbol{k}}\right)$$

$$\left.\begin{aligned} i\frac{\Psi^{\mathrm{R,I}}_{\boldsymbol{k}}}{\partial\eta} &= \hat{\mathcal{H}}^{\mathrm{R,I}}_{\boldsymbol{k}} \Psi^{\mathrm{R,I}}_{\boldsymbol{k}} \\ \hat{\mathcal{H}}^{\mathrm{R,I}}_{\boldsymbol{k}} &= -\frac{1}{2}\frac{\partial^2}{\partial\left(v^{\mathrm{R,I}}_{\boldsymbol{k}}\right)^2} + \frac{1}{2}\omega^2(\eta,\boldsymbol{k})\left(\hat{v}^{\mathrm{R,I}}_{\boldsymbol{k}}\right)^2 \end{aligned}\right\}$$

Gaussian state solution $\Psi(\eta, v_{\boldsymbol{k}}) = \left[\frac{2\Re\mathrm{e}\,\Omega_{\boldsymbol{k}}(\eta)}{\pi}\right]^{1/4} \mathrm{e}^{-\Omega_{\boldsymbol{k}}(\eta) v_{\boldsymbol{k}}^2}$

Wigner function $W(v_{\boldsymbol{k}}, p_{\boldsymbol{k}}) = \int \frac{\mathrm{d}x}{2\pi^2} \Psi^*\left(v_{\boldsymbol{k}} - \frac{x}{2}\right) \mathrm{e}^{-ip_{\boldsymbol{k}}x} \Psi\left(v_{\boldsymbol{k}} + \frac{x}{2}\right)$

large squeezing limit ➡ $W \propto \delta\left(p_{\boldsymbol{k}} + k \tan\phi_{\boldsymbol{k}} v_{\boldsymbol{k}}\right)$



Stochastic distribution of classical processes

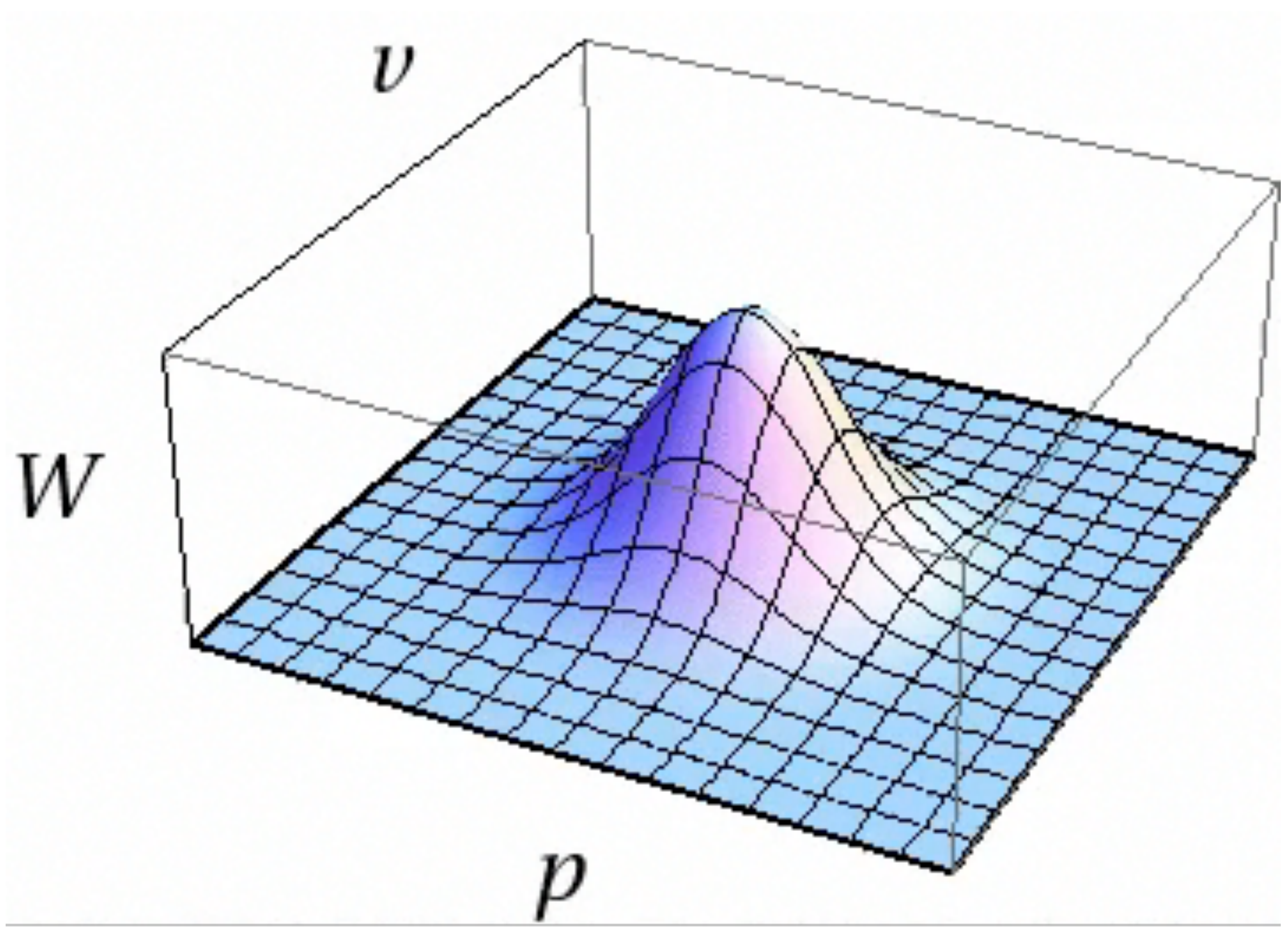
Ergodicity $\left\langle \frac{\Delta T(\xi, \boldsymbol{e})}{T} \right\rangle_{\xi} \simeq \left\langle \frac{\Delta T(\xi, \boldsymbol{e})}{T} \right\rangle_{\boldsymbol{e}}$

realization spatial direction

Gaussian state solution $\Psi\left(\eta, v_{\boldsymbol{k}}\right)=\left[\frac{2 \Re \mathrm{e}\, \Omega_{\boldsymbol{k}}(\eta)}{\pi}\right]^{1/4} \mathrm{e}^{-\Omega_{\boldsymbol{k}}(\eta) v_{\boldsymbol{k}}^2}$

Wigner function $W\left(v_{\boldsymbol{k}}, p_{\boldsymbol{k}}\right)=\int \frac{\mathrm{d} x}{2 \pi^2} \Psi^*\left(v_{\boldsymbol{k}}-\frac{x}{2}\right) \mathrm{e}^{-i p_{\boldsymbol{k}} x} \Psi\left(v_{\boldsymbol{k}}+\frac{x}{2}\right)$

large squeezing limit $\Rightarrow$ $W \propto \delta\left(p_{\boldsymbol{k}}+k \tan \phi_{\boldsymbol{k}} v_{\boldsymbol{k}}\right)$



Stochastic distribution of classical processes

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Animation provided by V. Vennin

Primordial Power Spectrum

Standard case

Quantization in the Schrödinger picture
(functional representation)

$$i\frac{\mathrm{d}|\Psi_{\boldsymbol{k}}\rangle}{\mathrm{d}\eta} = \hat{\mathcal{H}}_{\boldsymbol{k}}\,|\Psi_{\boldsymbol{k}}\rangle$$

with $$\hat{\mathcal{H}}_{\boldsymbol{k}} = \frac{\hat{p}_{\boldsymbol{k}}^2}{2} + \omega^2(\boldsymbol{k},\eta)\hat{v}_{\boldsymbol{k}}^2$$

$$\hat{v}_{\boldsymbol{k}} = v_{\boldsymbol{k}}$$

$$\hat{p}_{\boldsymbol{k}} = i\frac{\partial}{\partial v_{\boldsymbol{k}}}$$

and $$\omega^2(\boldsymbol{k},\eta) = k^2 - \frac{(a\sqrt{\epsilon_1})''}{a\sqrt{\epsilon_1}}$$

$$= k^2 - \frac{\beta(\beta+1)}{\eta^2}$$

Power-law inflation example

$$a(\eta) = \ell_0(-\eta)^{1+\beta}$$

$$\beta \lesssim -2$$

(de Sitter: $\beta = -2$)

Parametric Oscillator System

Primordial Power Spectrum

Standard case

Quantization in the Schrödinger picture (functional representation)

$$\Psi_{\boldsymbol{k}}(\eta, v_{\boldsymbol{k}}) = \left[\frac{2\,\Re\mathrm{e}\Omega_{\boldsymbol{k}}(\eta)}{\pi}\right]^{1/4} \mathrm{e}^{-\Omega_{\boldsymbol{k}}(\eta)\, v_{\boldsymbol{k}}^2}$$

$$i\frac{\mathrm{d}|\Psi_{\boldsymbol{k}}\rangle}{\mathrm{d}\eta} = \hat{\mathcal{H}}_{\boldsymbol{k}}\,|\Psi_{\boldsymbol{k}}\rangle$$

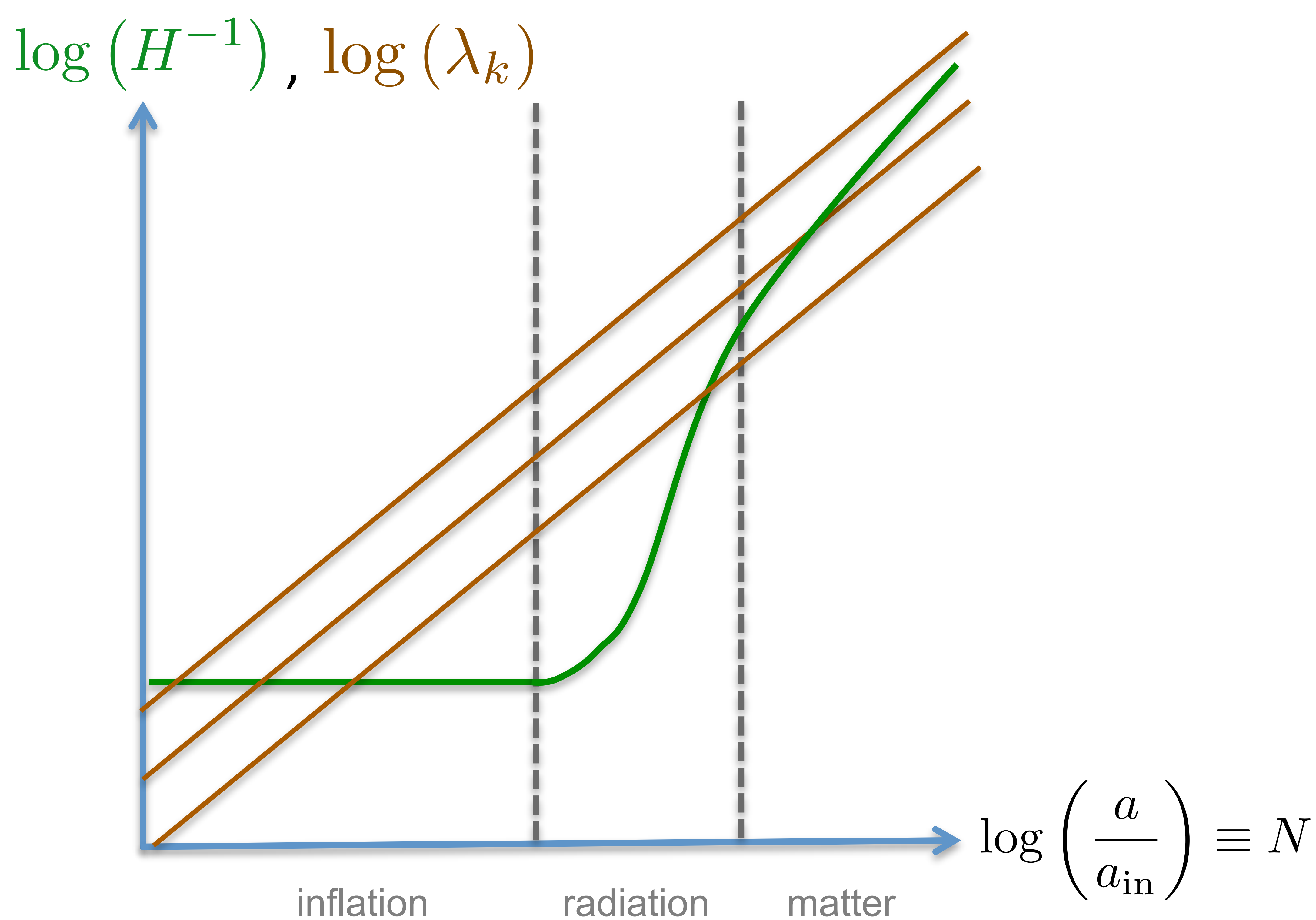
with

$$\hat{\mathcal{H}}_{\boldsymbol{k}} = \frac{\hat{p}_{\boldsymbol{k}}^2}{2} + \omega^2(\boldsymbol{k},\eta)\hat{v}_{\boldsymbol{k}}^2$$

$$\Omega_{\boldsymbol{k}}' = -2i\Omega_{\boldsymbol{k}}^2 + \frac{i}{2}\omega^2(\eta,\boldsymbol{k})$$

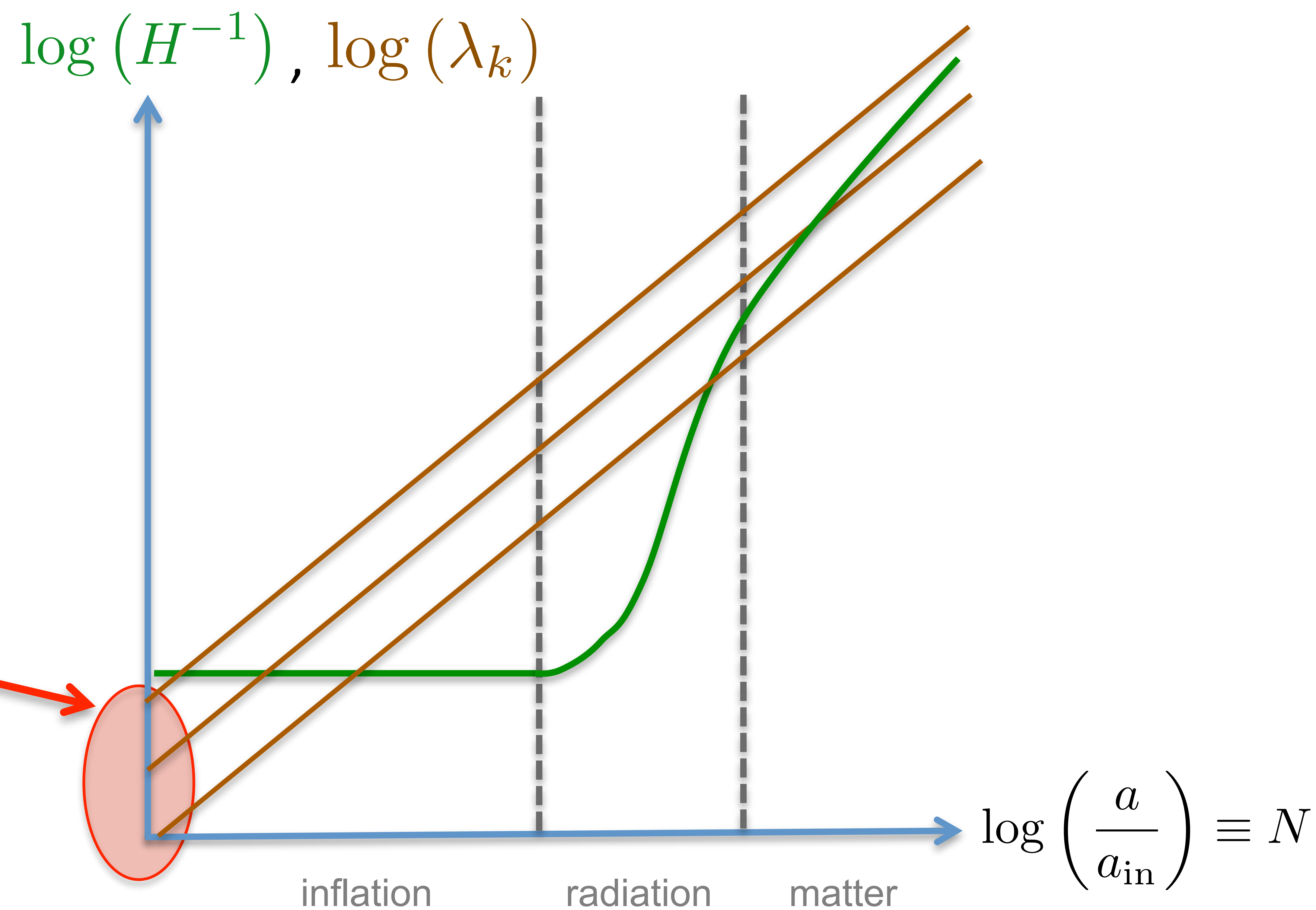
$$\Omega_{\boldsymbol{k}} = -\frac{i}{2}\frac{f_{\boldsymbol{k}}'}{f_{\boldsymbol{k}}}$$

$$\boxed{f_{\boldsymbol{k}}'' + \omega^2(\boldsymbol{k},\eta)\,f_{\boldsymbol{k}} = 0}$$


$\log\left(H^{-1}\right), \log\left(\lambda_k\right)$
$\log\left(\frac{a}{a_{\rm in}}\right) \equiv N$
inflation
radiation
matter

Harmonic oscillator fundamental state

$$\Psi_{\boldsymbol{k}} = \left(\frac{k}{\pi}\right)^{\frac{1}{4}} \mathrm{e}^{-\frac{k}{2} v_{\boldsymbol{k}}^2}$$



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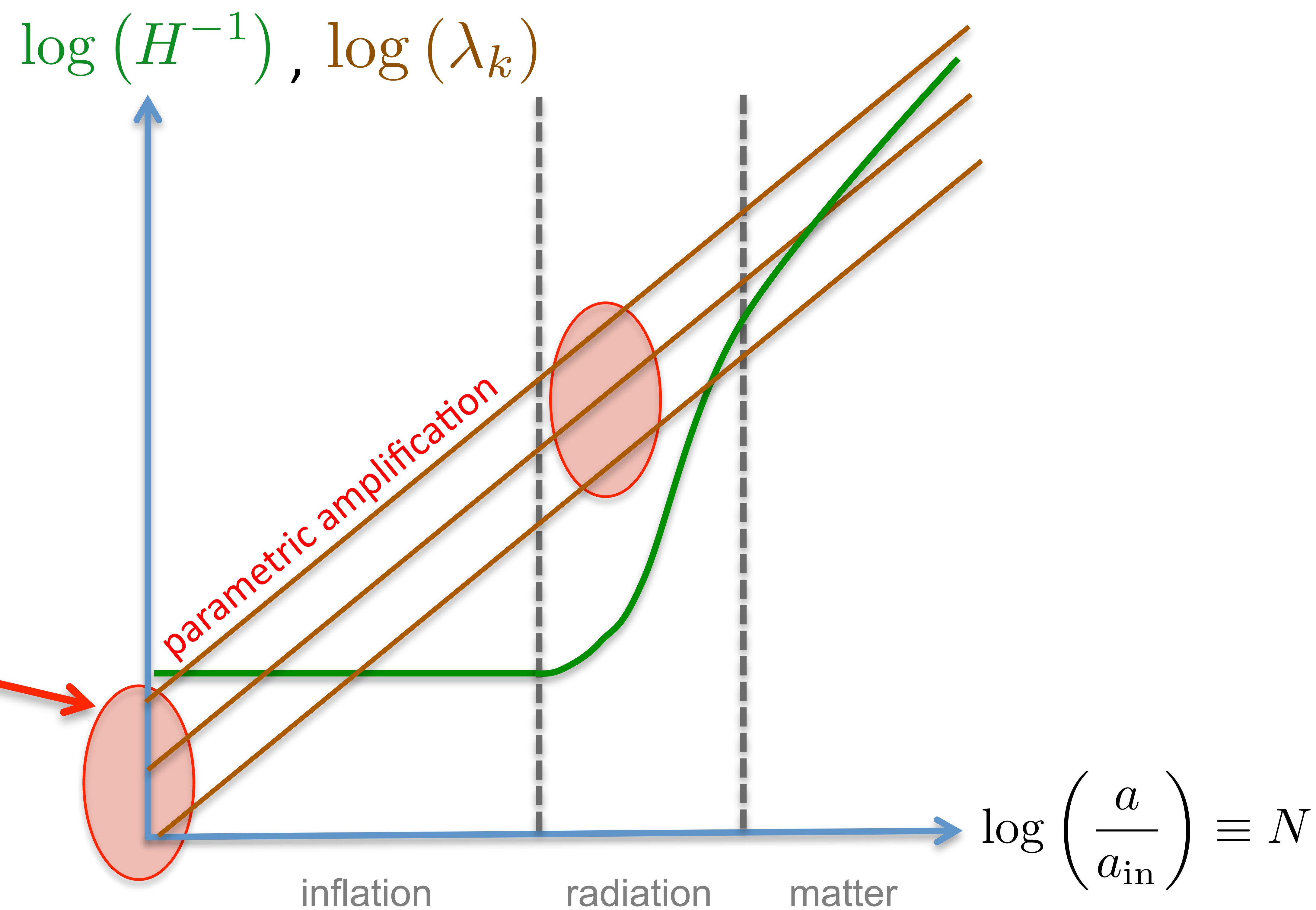

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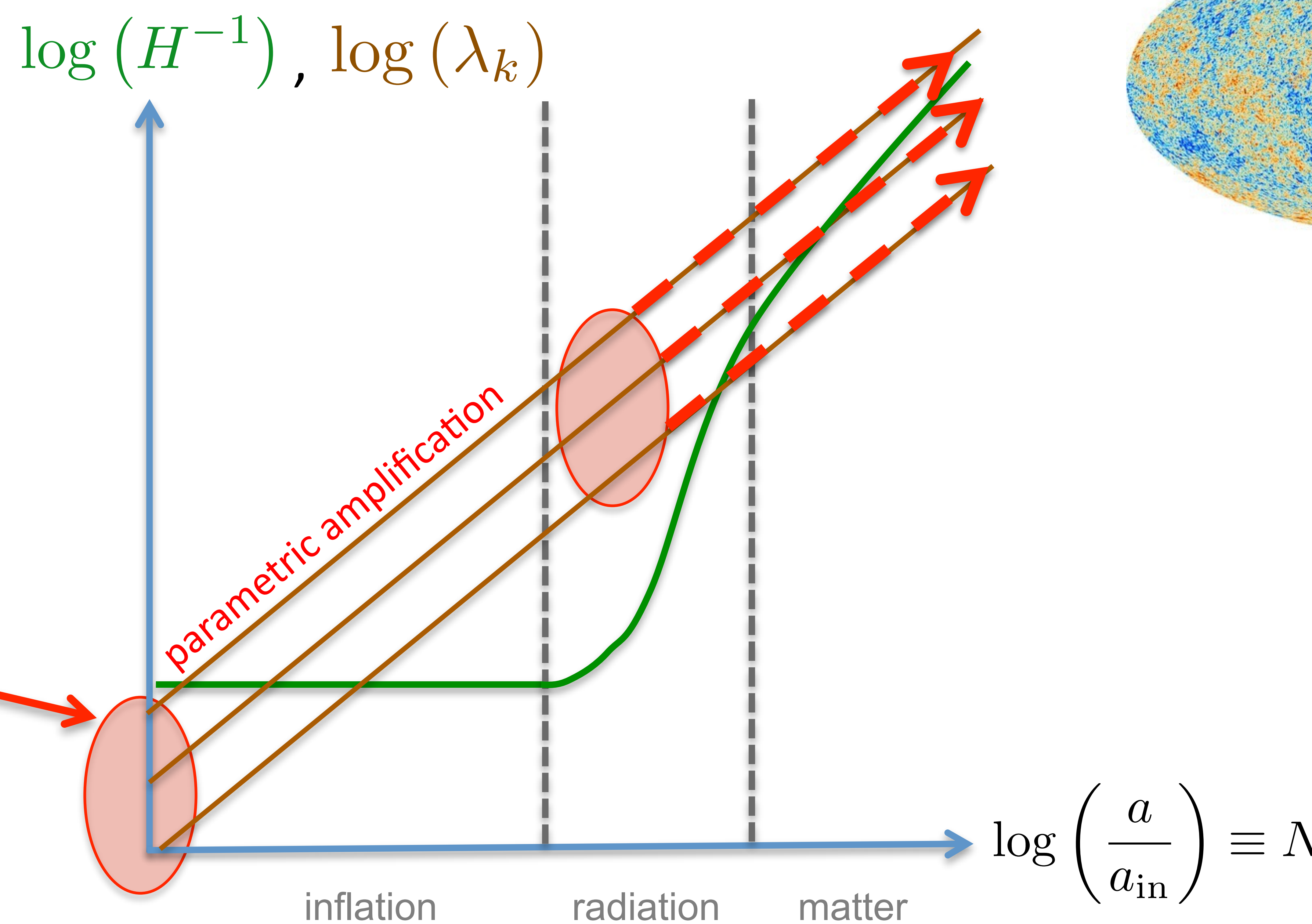
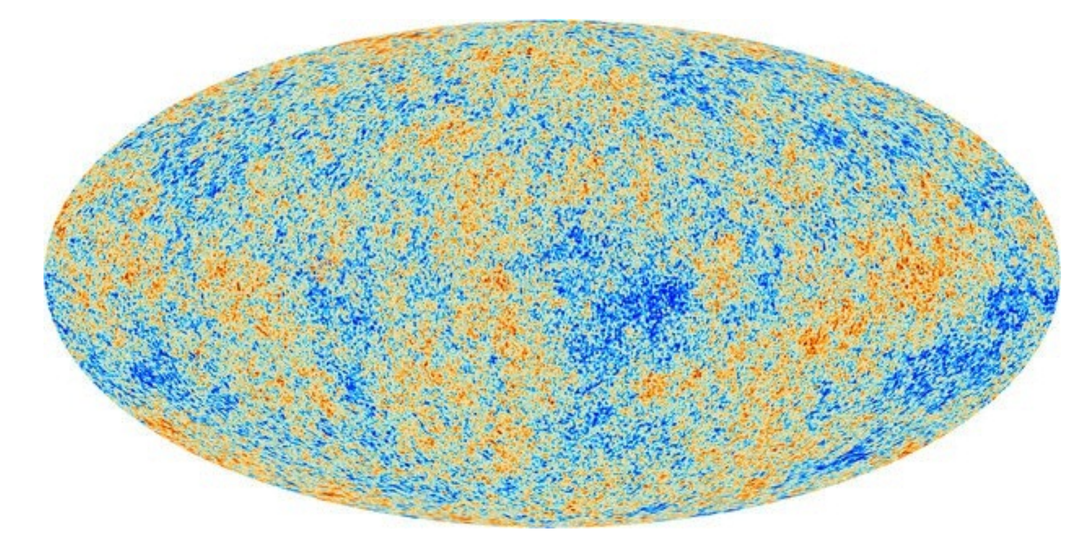




Primordial Power Spectrum

Standard case

Two physical scales

Hubble radius $H^{-1} = \frac{a^2}{a'} \underset{\beta\sim-2}{\simeq} \ell_0$

wavelength $\lambda = \frac{a}{k} \underset{\beta\sim-2}{\simeq} \frac{\ell_0}{-k\eta}$

Sub-Hubble regime

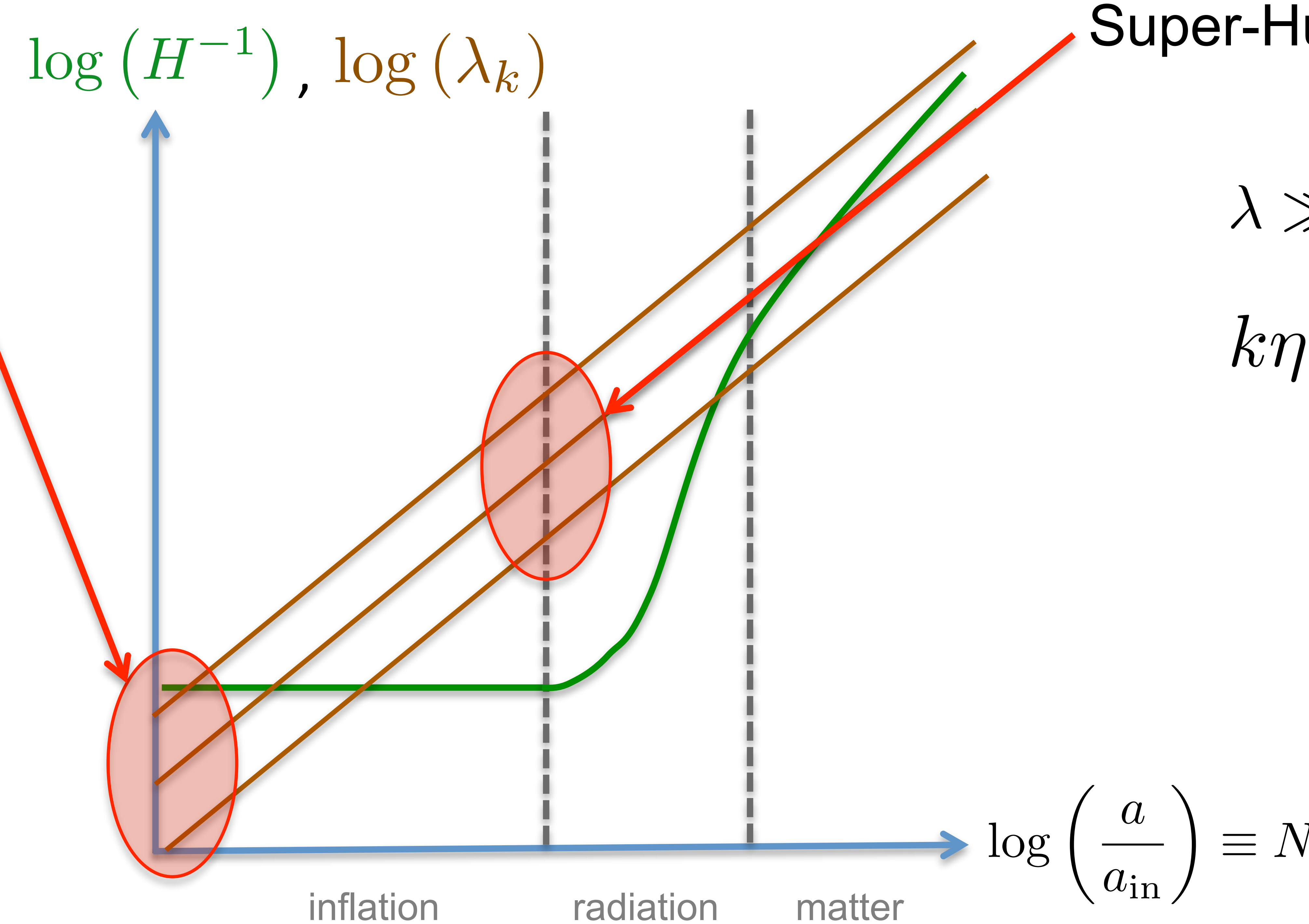
$\lambda \ll H^{-1}$

$k\eta \to -\infty$

$\omega \simeq k$
harmonic oscillator

$$\Psi_{\boldsymbol{k}} = \left(\frac{k}{\pi}\right)^{\frac{1}{4}} \mathrm{e}^{-\frac{k}{2}v_{\boldsymbol{k}}^2}$$

Bunch Davis vacuum



Super-Hubble regime

$\lambda \gg H^{-1}$

$k\eta \to 0^-$

sets initial conditions $f_{\boldsymbol{k}}(k\eta \to -\infty) = \mathrm{e}^{ik\eta}/\sqrt{2k}$

$$v_{\boldsymbol{k}}'' + \left[\boldsymbol{k}^2 - U(\eta)\right] v_{\boldsymbol{k}} = 0$$

Vacuum state

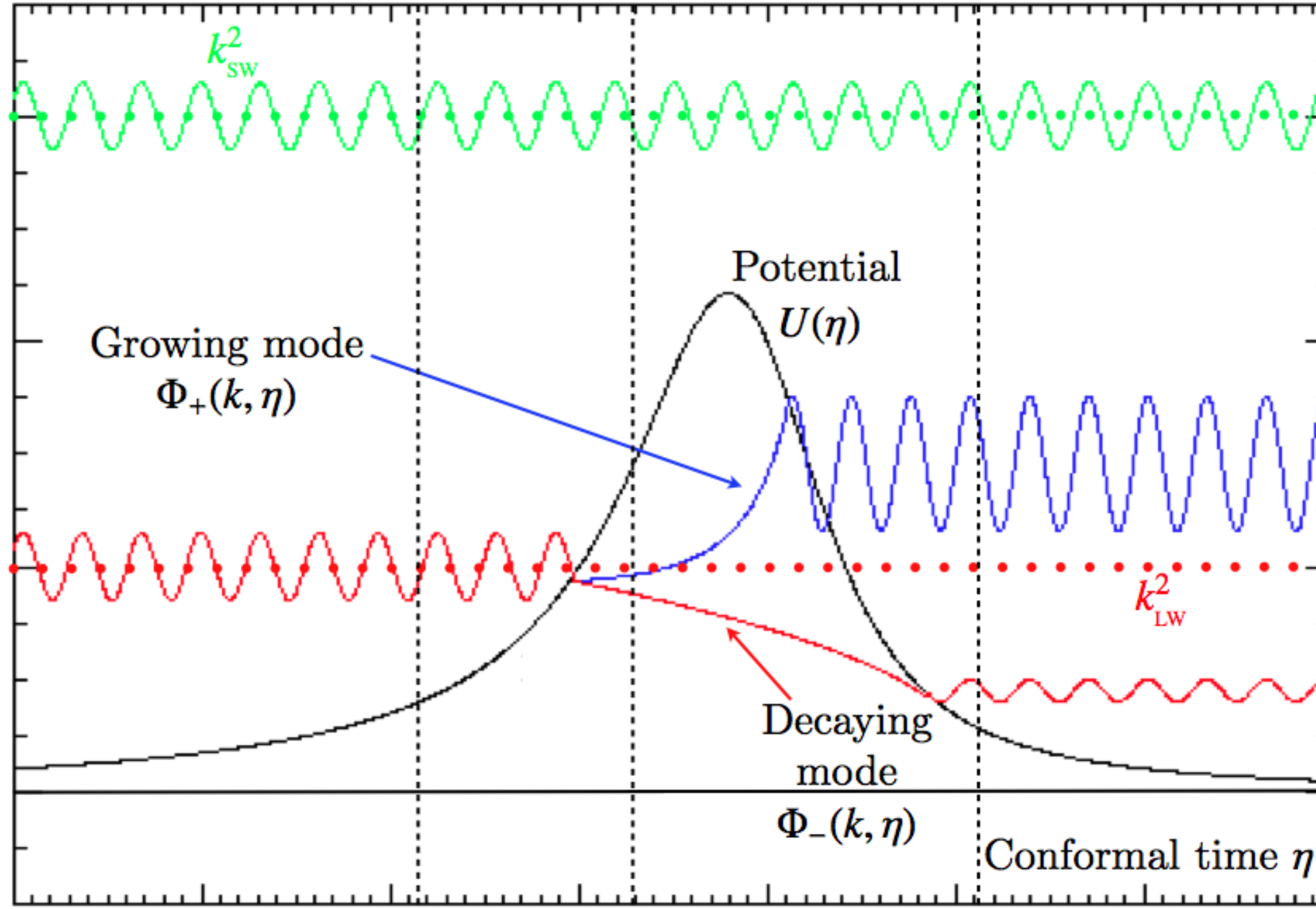
$$v_{\boldsymbol{k}} \xrightarrow[|k\eta|\to\infty]{} \frac{e^{-ik\eta}}{\sqrt{2k}}$$

Initial conditions fixed!

compare

$$\frac{d^2\Psi}{dx^2} + \frac{2m}{\hbar^2}\left[E - U(x)\right]\Psi = 0$$

(time independent Schrödinger equation)



Transmission & Reflexion coefficients!

$$v_{\boldsymbol{k}}'' + \left[\boldsymbol{k}^2 - U(\eta)\right] v_{\boldsymbol{k}} = 0$$

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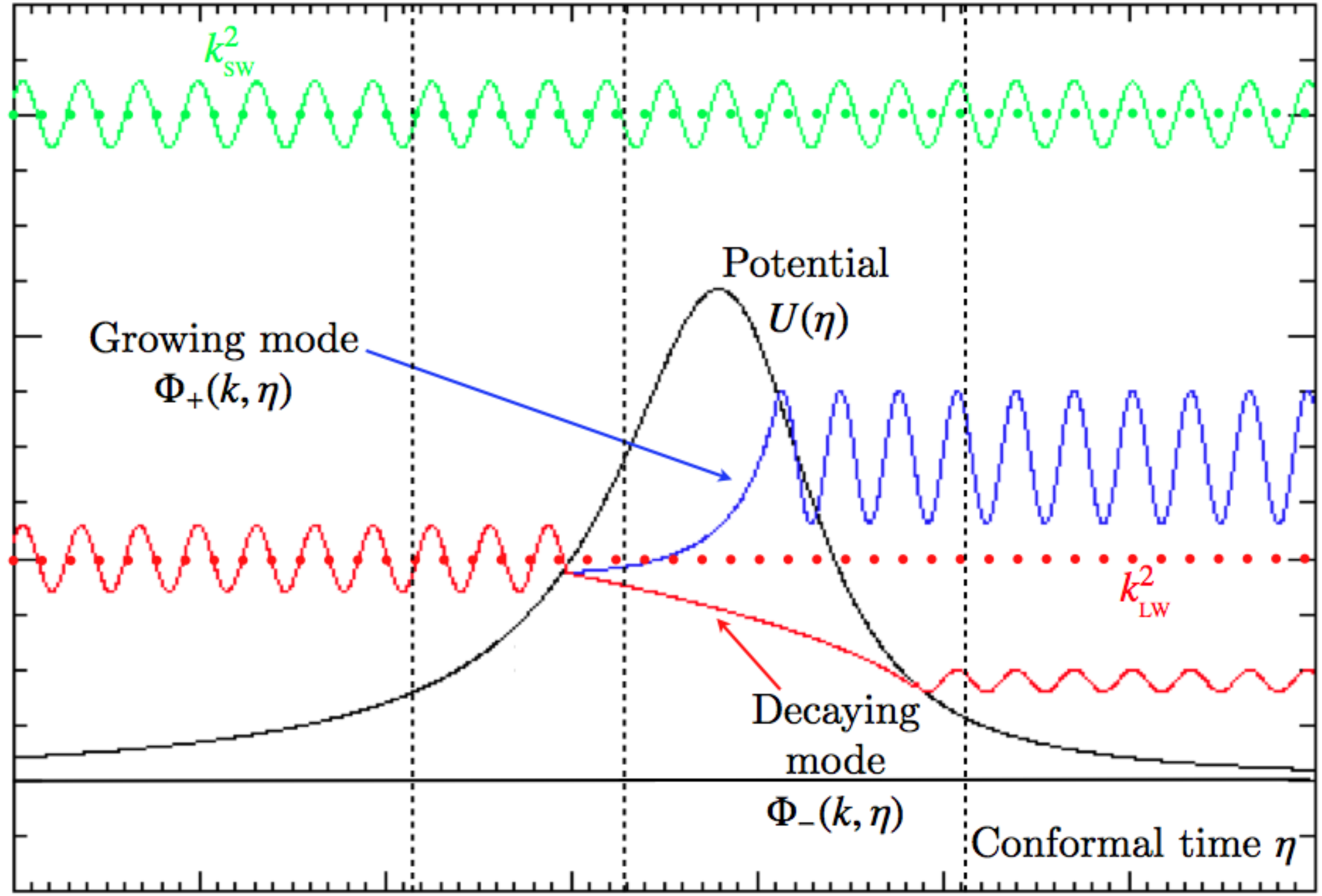


Transmission & Reflexion coefficients!

Primordial Power Spectrum

Standard case

$\boxed{f_{\boldsymbol{k}}'' + \omega^2(\boldsymbol{k},\eta) f_{\boldsymbol{k}} = 0}$ with $\omega^2(\boldsymbol{k},\eta) = k^2 - \dfrac{\beta(\beta+1)}{\eta^2}$ and $f_{\boldsymbol{k}}(k\eta \to -\infty) = \mathrm{e}^{ik\eta}/\sqrt{2k}$

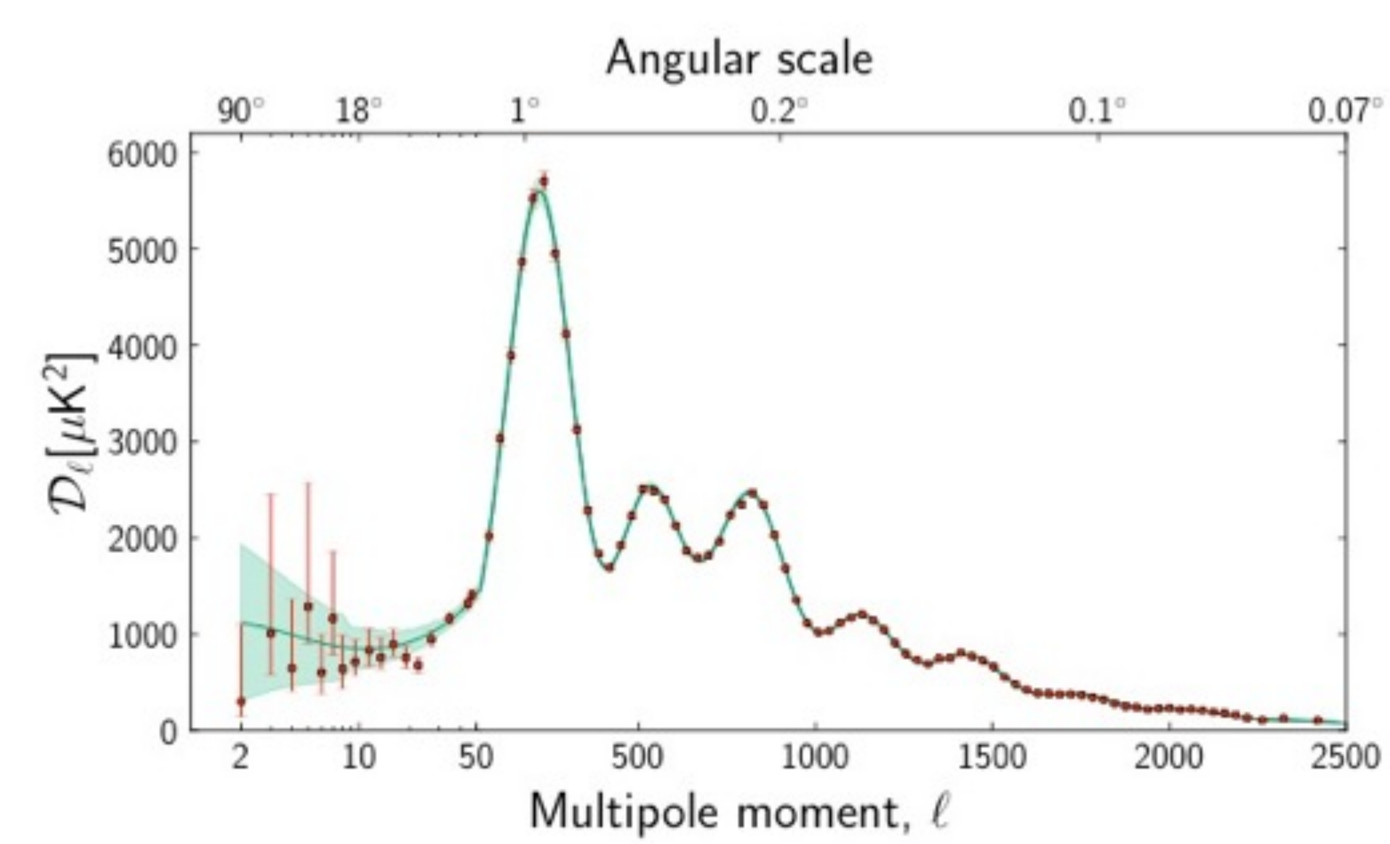
Uniquely determines $f_{\boldsymbol{k}}$ $\xrightarrow{\Omega_{\boldsymbol{k}} = -\frac{i}{2}\frac{f_{\boldsymbol{k}}'}{f_{\boldsymbol{k}}}}$ $\Re\mathrm{e}\,\Omega_{\boldsymbol{k}} = \langle \hat{v}_{\boldsymbol{k}}^2 \rangle - \langle \hat{v}_{\boldsymbol{k}} \rangle^2$

Evaluated at the end of inflation($k\eta \to 0^-$), this gives $P_v(k) = \dfrac{k^3}{2\pi^3}\left(\langle \hat{v}_{\boldsymbol{k}}^2 \rangle - \langle \hat{v}_{\boldsymbol{k}} \rangle^2\right)$

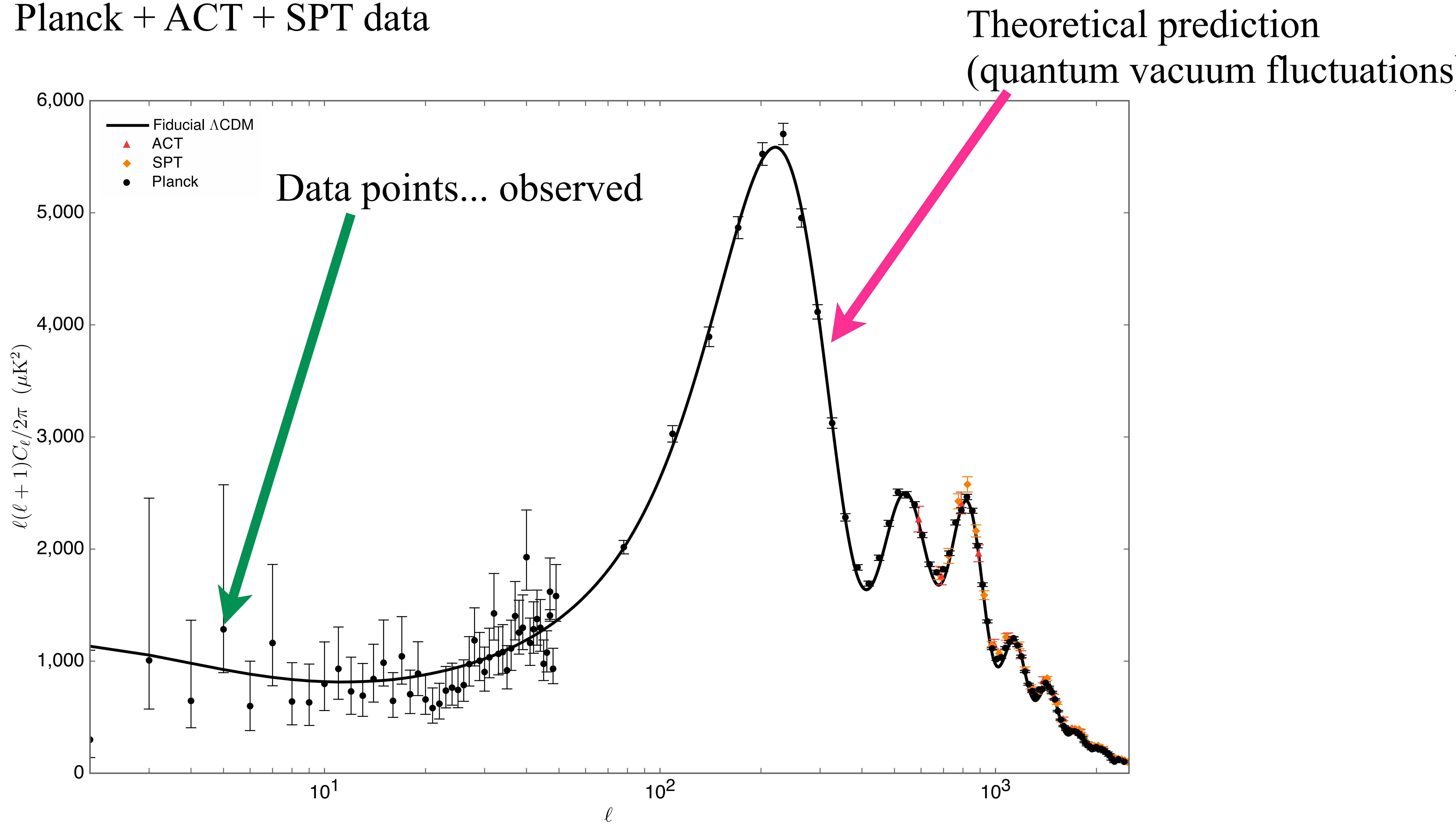
and eventually $P_\zeta(k) = \dfrac{1}{2a^2 M_{\mathrm{Pl}}^2 \epsilon_1} P_v(k) = A_S k^{n_{\mathrm{S}}-1}$

with $n_{\mathrm{S}} = 2\beta + 5 \underset{\beta\sim -2}{\simeq} 1$

Planck: $1 - n_{\mathrm{s}} = 0.0389 \pm 0.0054$



Planck + ACT + SPT data



Quantum mechanics

Physical system = Hilbert space of configurations

State vectors

Observables = self-adjoint operators

Measurement = eigenvalue $A|a_n\rangle = a_n|a_n\rangle$

Evolution = Schrödinger equation (time translation invariance) $i\hbar\frac{\mathrm{d}}{\mathrm{d}t}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle$

Hamiltonian

Born rule $\mathrm{Prob}[a_n;t] = |\langle a_n|\psi(t)\rangle|^2$

Collapse of the wavefunction: $|\psi(t)\rangle$ before measurement, $|a_n\rangle$ after

Schrödinger equation = linear (superposition principle) / unitary evolution

Wavepacket reduction = non linear / stochastic

Mutually incompatible

+ ***External observer***

Predictions for a quantum theory

Calculated by quantum average $\langle \Psi | \hat{O} | \Psi \rangle$

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Usually in a lab:
repeat the experiment

Ensemble average over experiments $\Longleftrightarrow$ Quantum average

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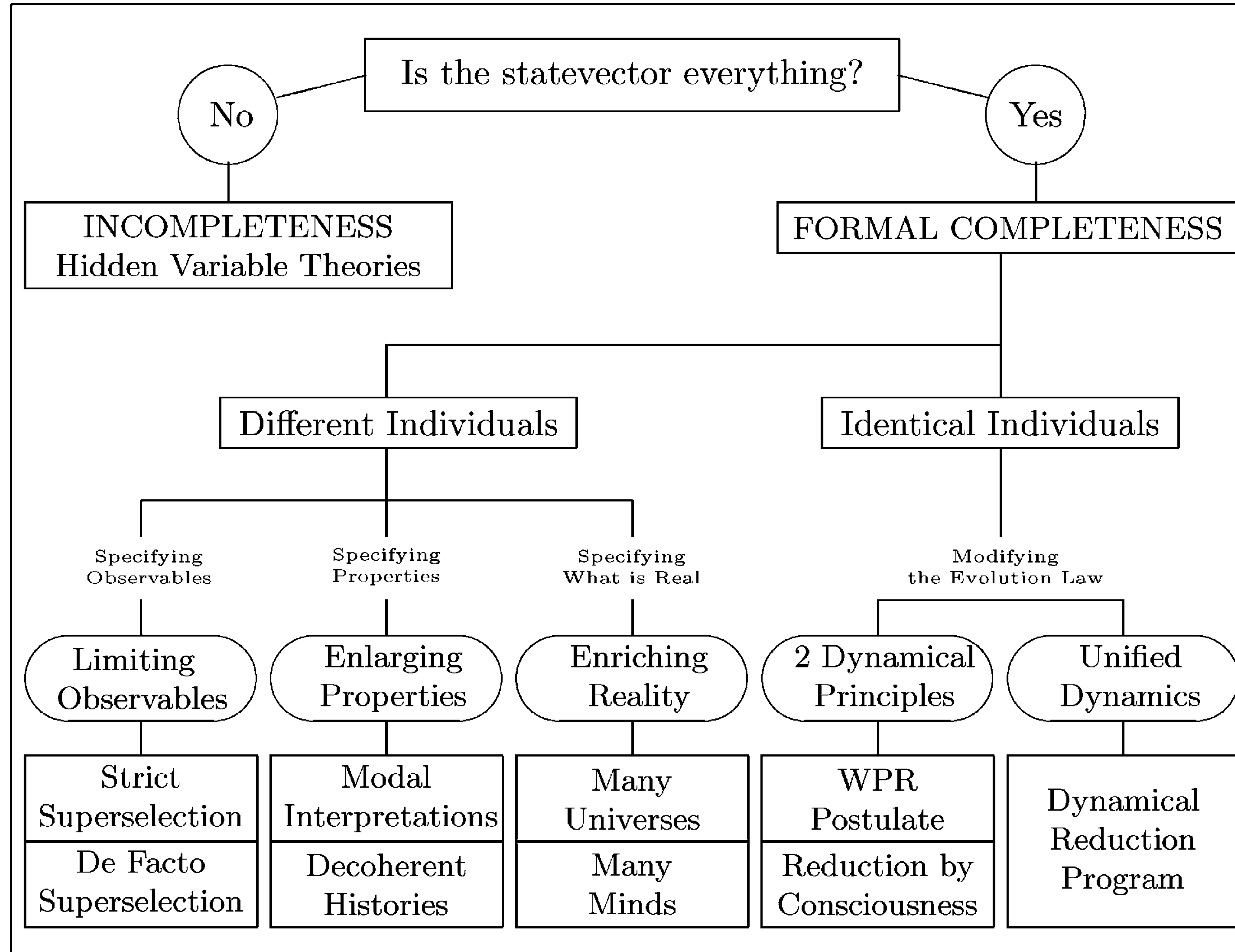
Here one has a single experiment (a single universe)

Ergodicity

Spatial average over directions in the sky $\Longleftrightarrow$ Quantum average

- Possible extensions and a criterion: the Born rule



A. Bassi & G.C. Ghirardi, *Phys. Rep.* **379**, 257 (2003)

▲ ***Superselection rules***

▲ ***Modal interpretation***

▲ ***Consistent histories***

▲ ***Many worlds / many minds***

▲ ***Hidden variables***

▲ ***Modified Schrödinger dynamics***

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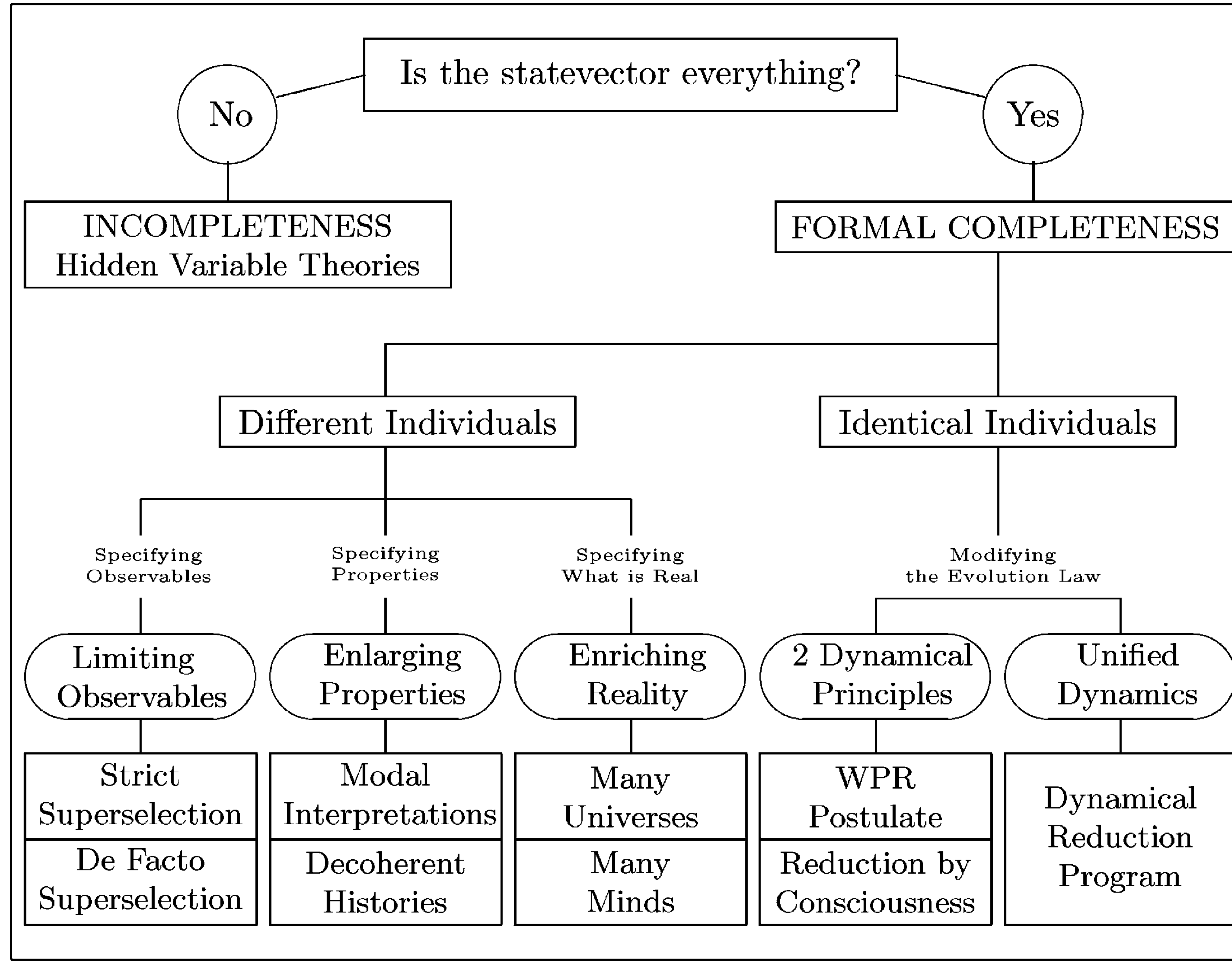

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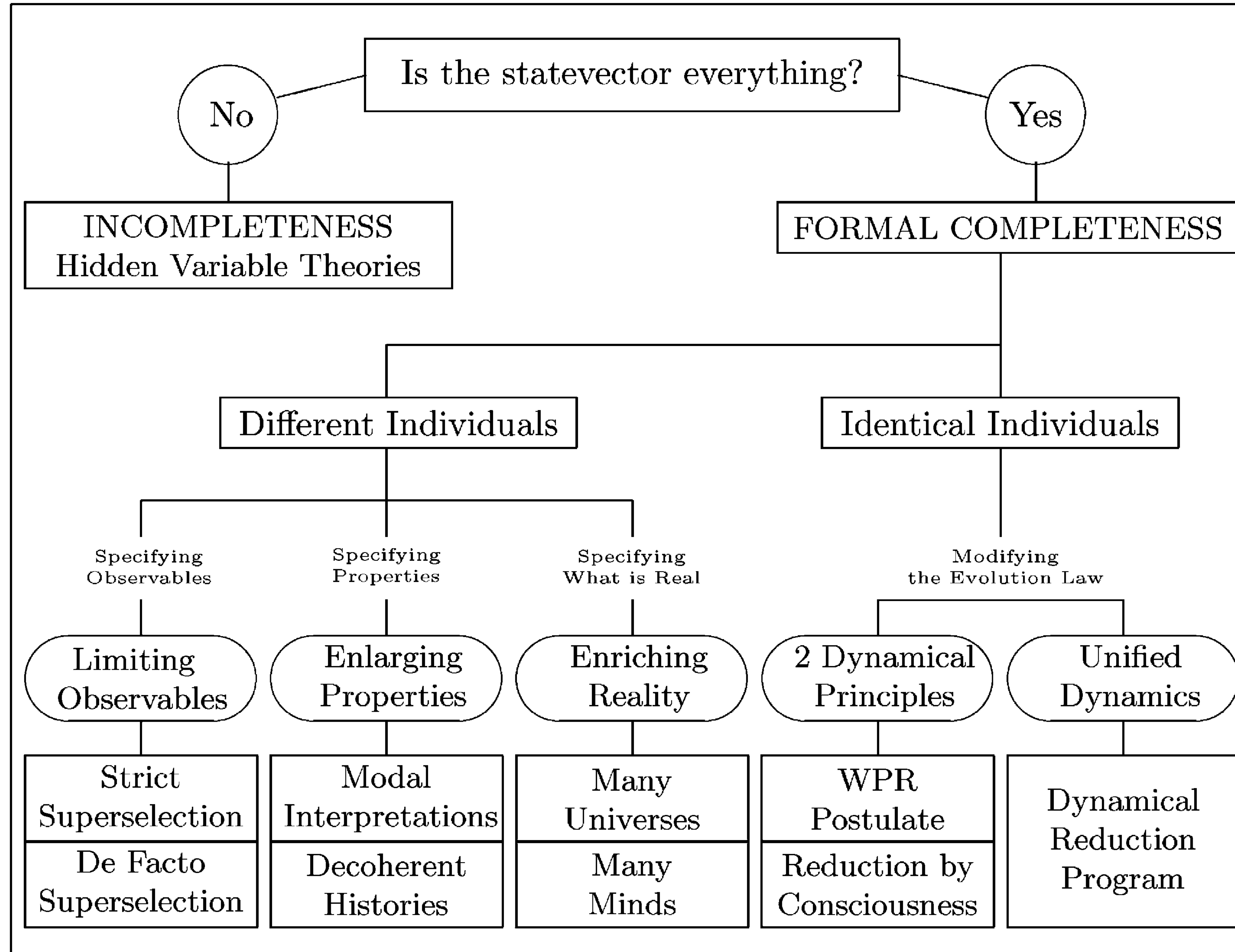

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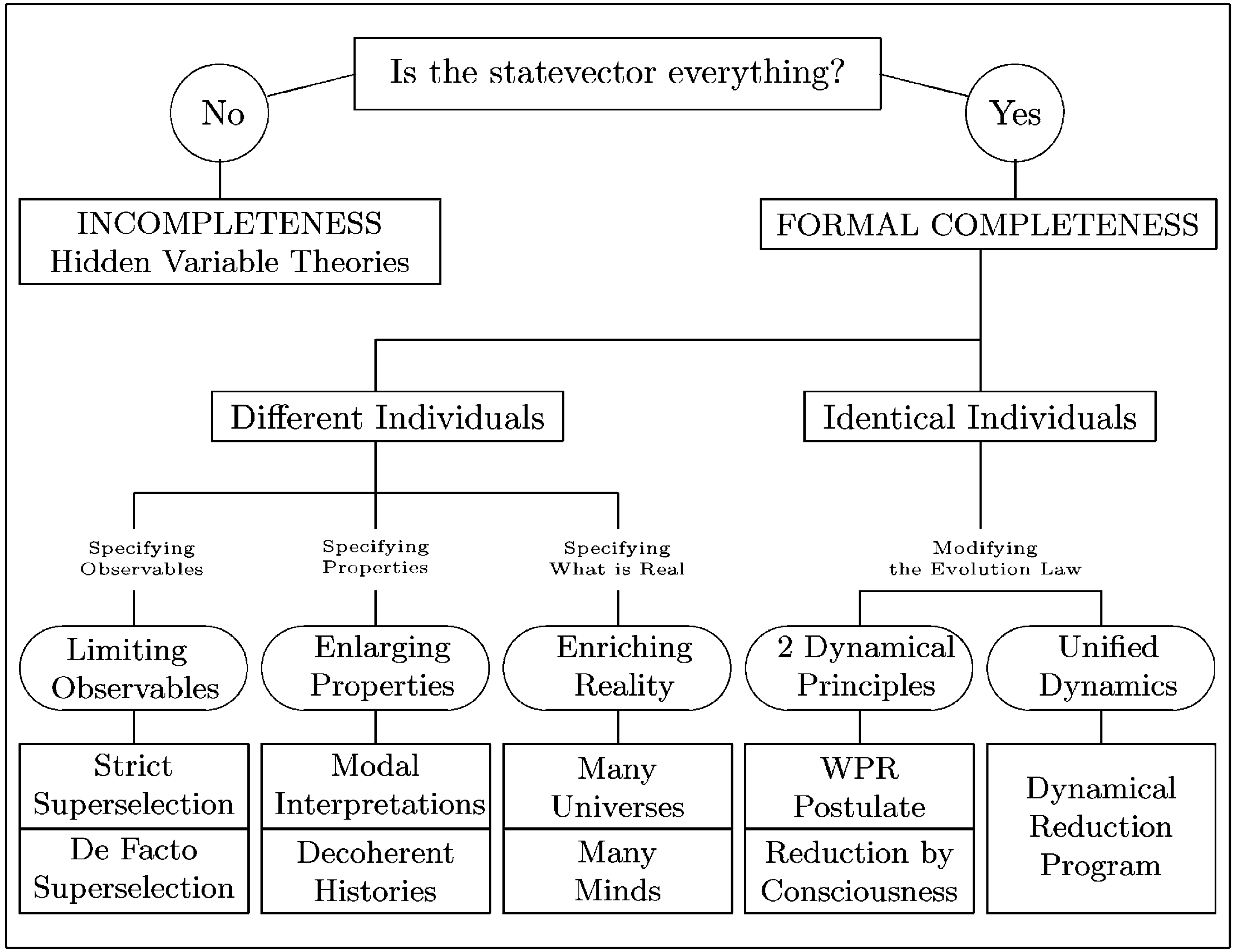

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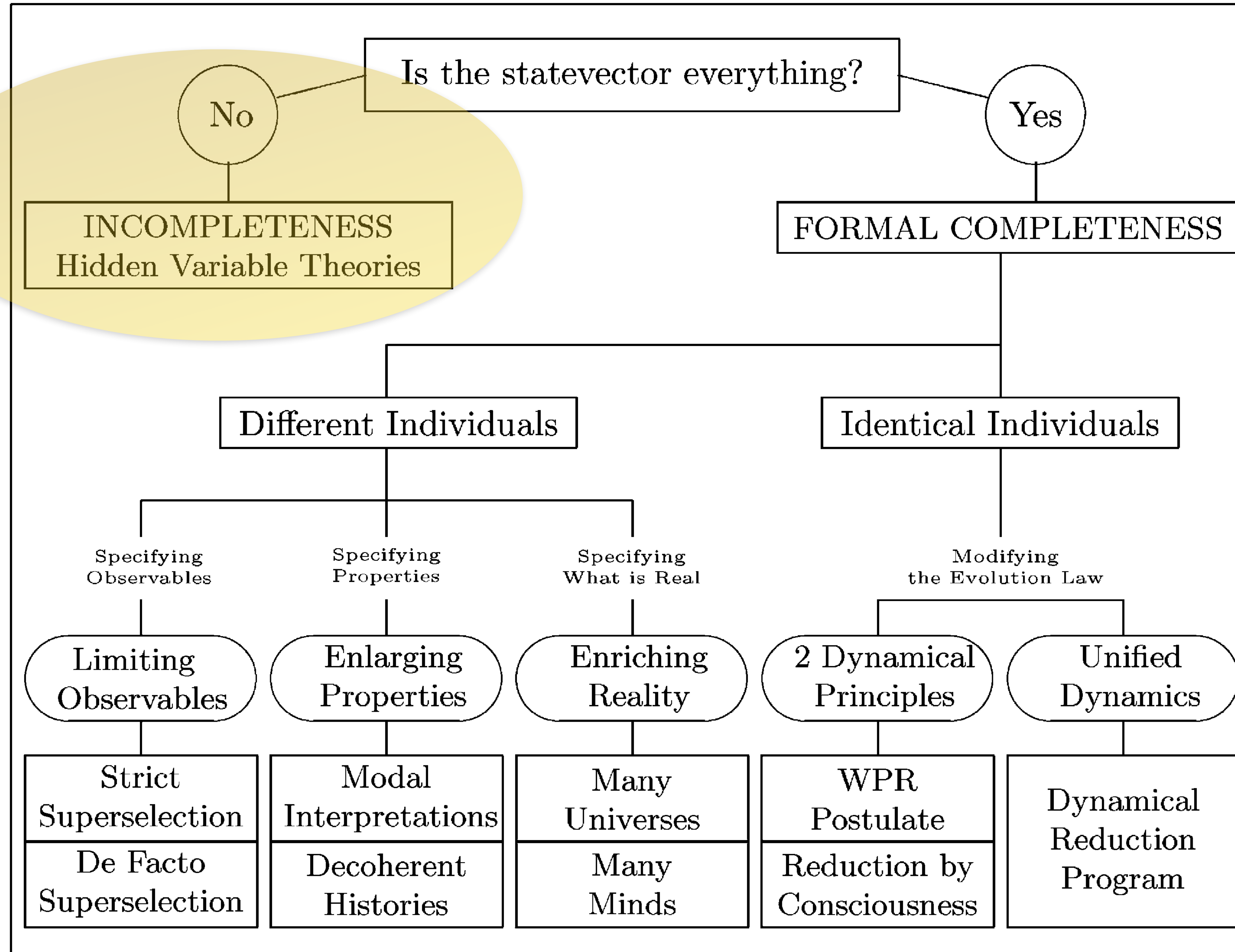

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Hidden Variable Theories

Schrödinger $$i\frac{\partial\Psi}{\partial t} = \left[-\frac{\nabla^2}{2m} + V(\boldsymbol{r})\right]\Psi$$

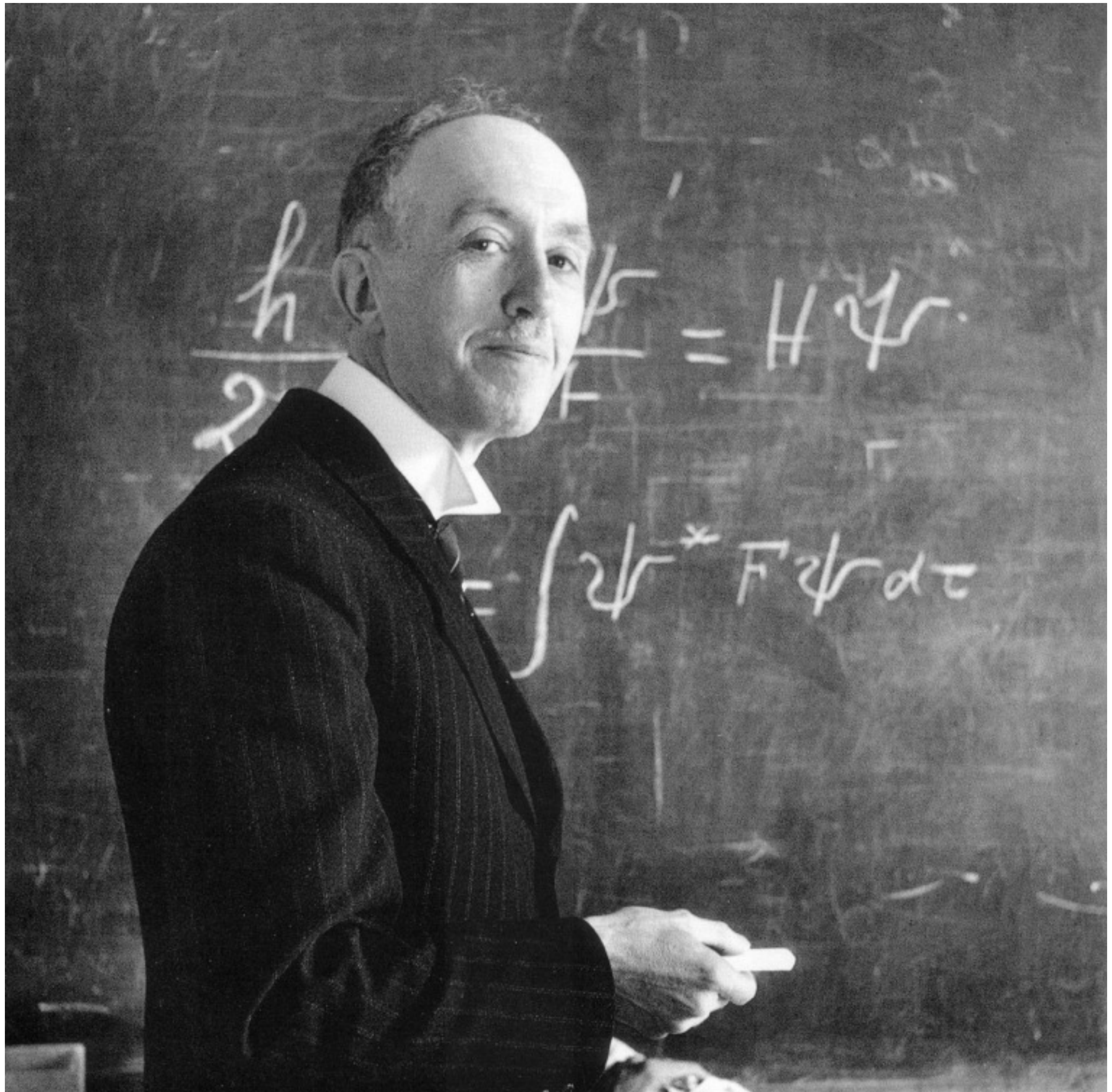
Polar form of the wave function $$\Psi = A(\boldsymbol{r},t)\,\mathrm{e}^{iS(\boldsymbol{r},t)}$$

Hamilton-Jacobi $$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V(\boldsymbol{r}) + Q(\boldsymbol{r},t) = 0$$

quantum potential

$$\equiv -\frac{1}{2m}\frac{\nabla^2 A}{A}$$

Ontological interpretation (dBB)



Louis de Broglie

David Bohm

1927 Solvay meeting and von Neuman mistake ... '*In 1952, I saw the impossible done*' (J. Bell)

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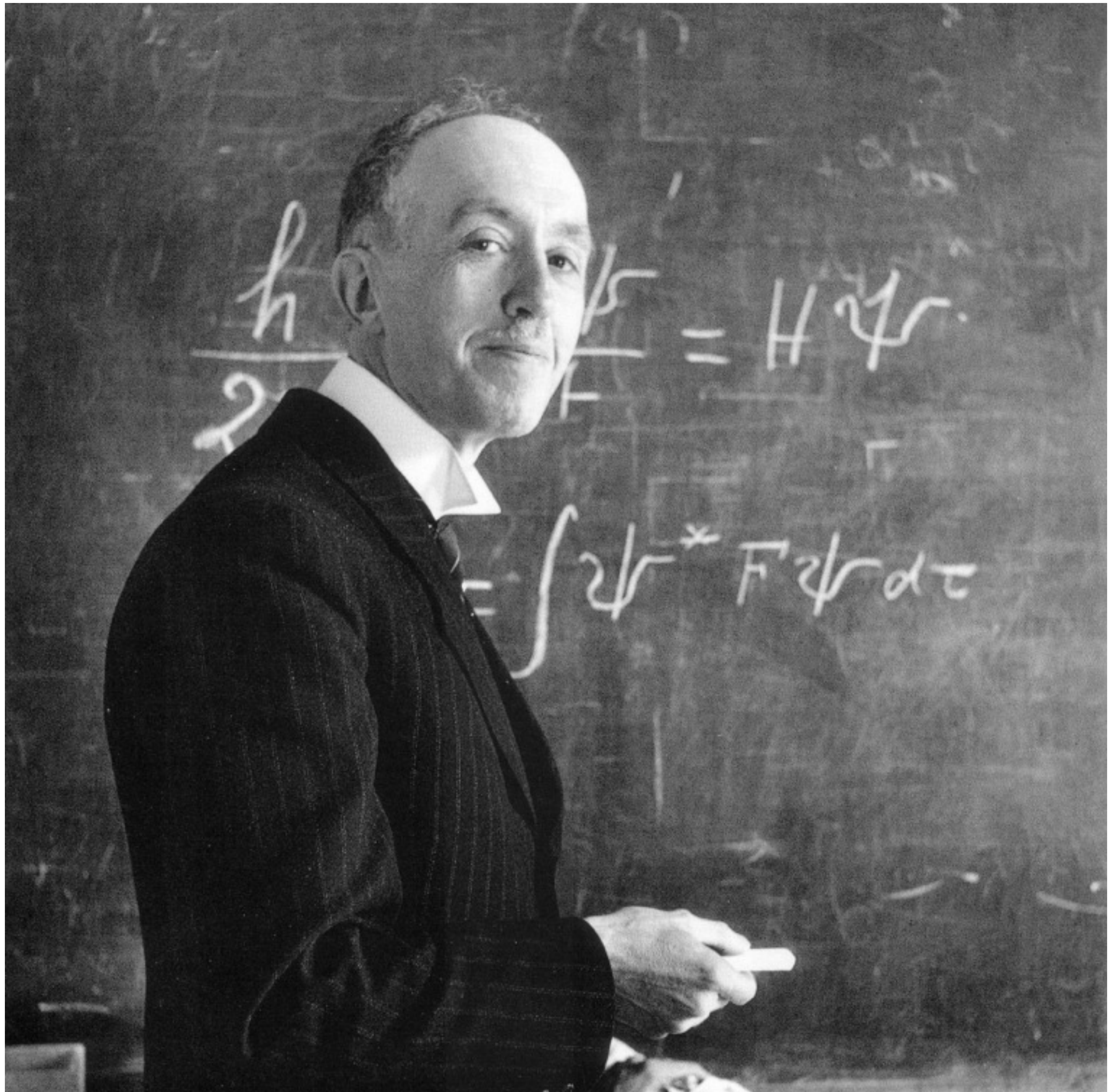

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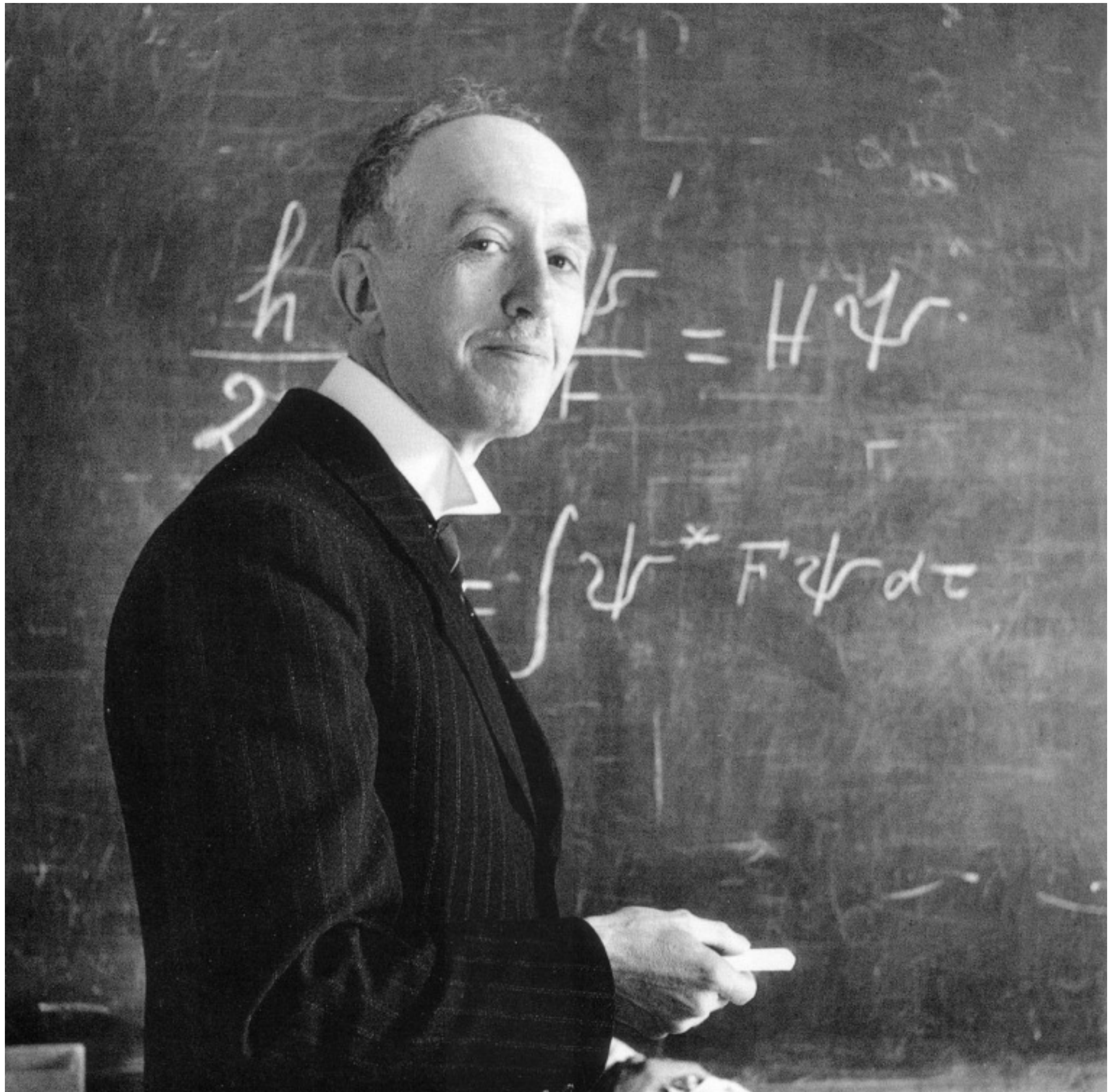

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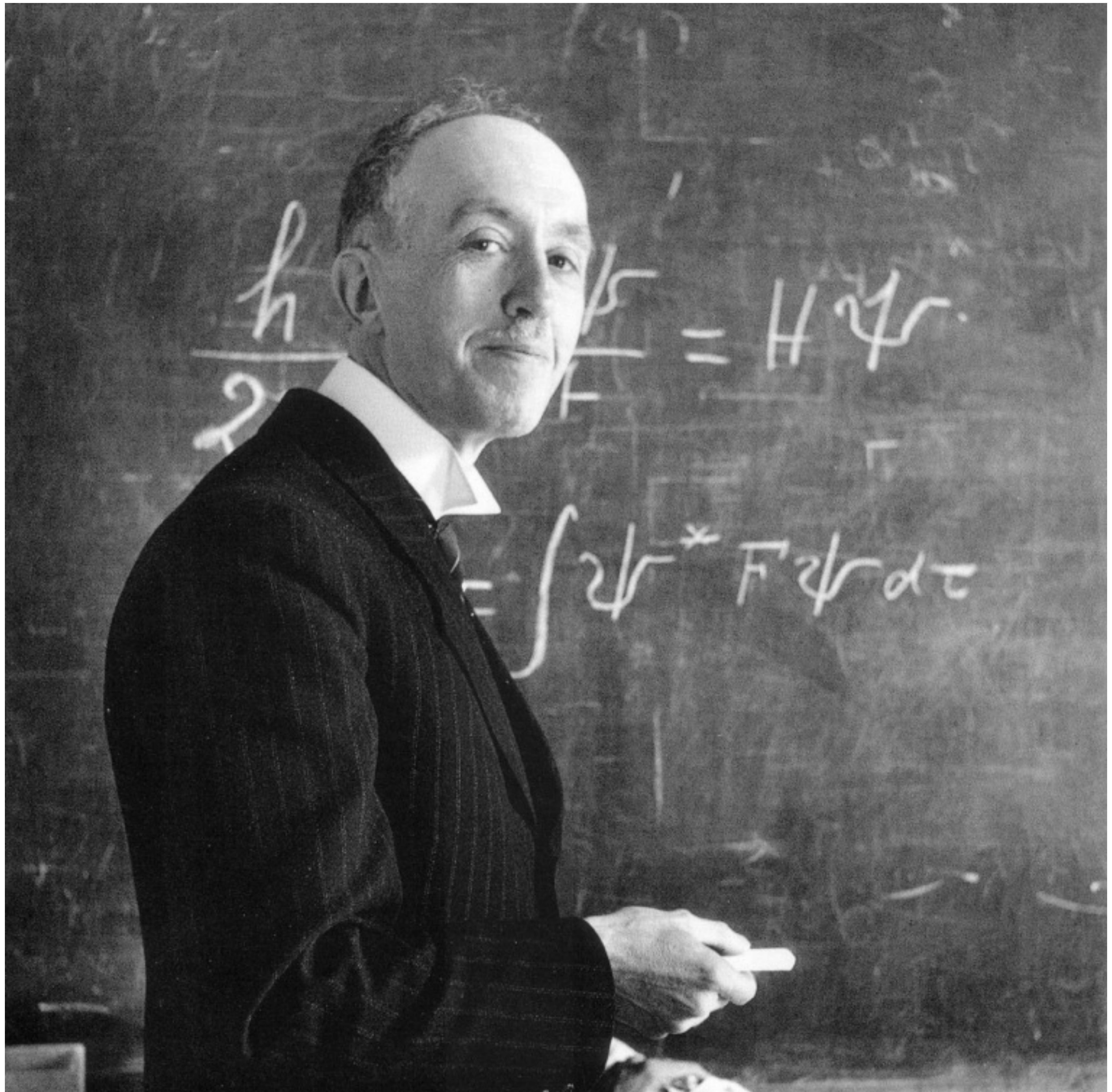
Ontological *formulation* (dBB)



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$$\Psi = A\left(\boldsymbol{r}, t\right) \mathrm{e}^{iS(\boldsymbol{r}, t)}$$

Ontological *formulation* (dBB) $\exists\, \boldsymbol{x}(t)$

Trajectories satisfy (de Broglie) $$m \frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = \Im\mathrm{m}\, \frac{\Psi^{*} \boldsymbol{\nabla} \Psi}{|\Psi(\boldsymbol{x}, t)|^{2}} = -\boldsymbol{\nabla} S$$

$$\Psi = A\left(\boldsymbol{r},t\right)\mathrm{e}^{iS(\boldsymbol{r},t)}$$

Ontological *formulation* (BdB) $\exists\, \boldsymbol{x}(t)$

Trajectories satisfy (Bohm)

$$m\frac{\mathrm{d}^2\boldsymbol{x}}{\mathrm{d}t^2} = -\boldsymbol{\nabla}(V+Q)$$

$$Q \equiv -\frac{1}{2m}\frac{\boldsymbol{\nabla}^2|\Psi|}{|\Psi|}$$

$$\Psi = A\left(\boldsymbol{r},t\right)\mathrm{e}^{iS(\boldsymbol{r},t)}$$

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<u>Properties:</u>

- strictly equivalent to Copenhagen QM
  - probability distribution (attractor)

    $\exists t_0; \rho\left(\boldsymbol{x},t_0\right) = \left|\Psi\left(\boldsymbol{x},t_0\right)\right|^2$
- classical limit well defined $Q \longrightarrow 0$
- state dependent
- $\exists$ intrinsic reality
  - non local …
- no need for external classical domain/observer!

$$\Psi = A\left(\boldsymbol{r},t\right)\mathrm{e}^{iS(\boldsymbol{r},t)}$$

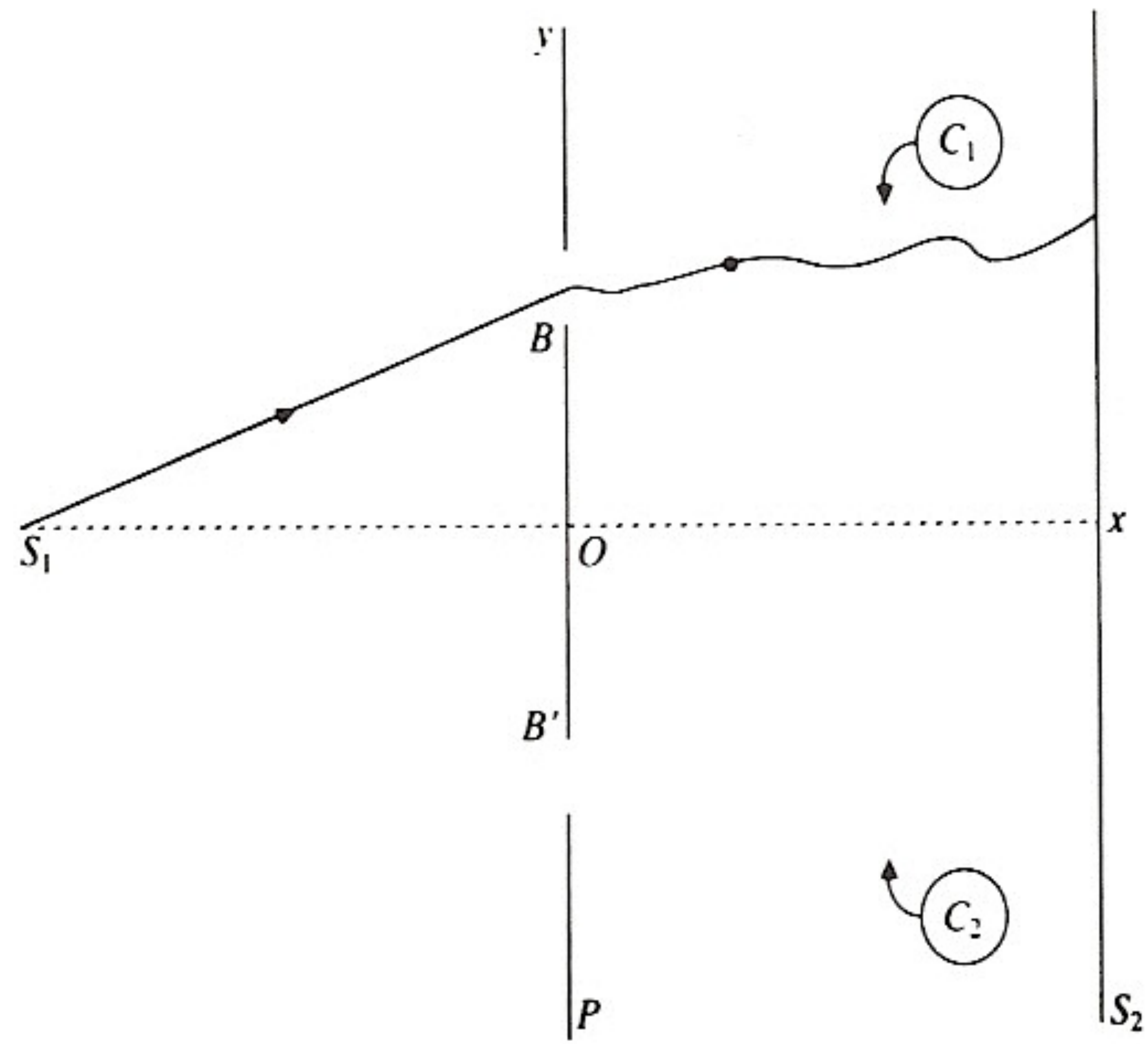
Ontological *formulation* (dBB) $\exists\, \boldsymbol{x}(t)$

Trajectories satisfy (de Broglie) $$m\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = \Im\mathrm{m}\,\frac{\Psi^*\boldsymbol{\nabla}\Psi}{|\Psi(\boldsymbol{x},t)|^2} = -\boldsymbol{\nabla}S$$

<u>Properties:</u>

- ☺ strictly equivalent to Copenhagen QM
  - probability distribution (attractor)
    $\exists t_0 ; \rho\left(\boldsymbol{x},t_0\right) = \left|\Psi\left(\boldsymbol{x},t_0\right)\right|^2$
- ☺ classical limit well defined $Q \longrightarrow 0$
- ☺ state dependent
- ☺ $\exists$ intrinsic reality
  - non local …
- ☺ no need for external classical domain/observer!

The two-slit experiment:



... a phenomenon which is impossible, *absolutely* impossible, to explain in any classical way, and which has in it the heart of quantum mechanics.

R. P. Feynman (1961)

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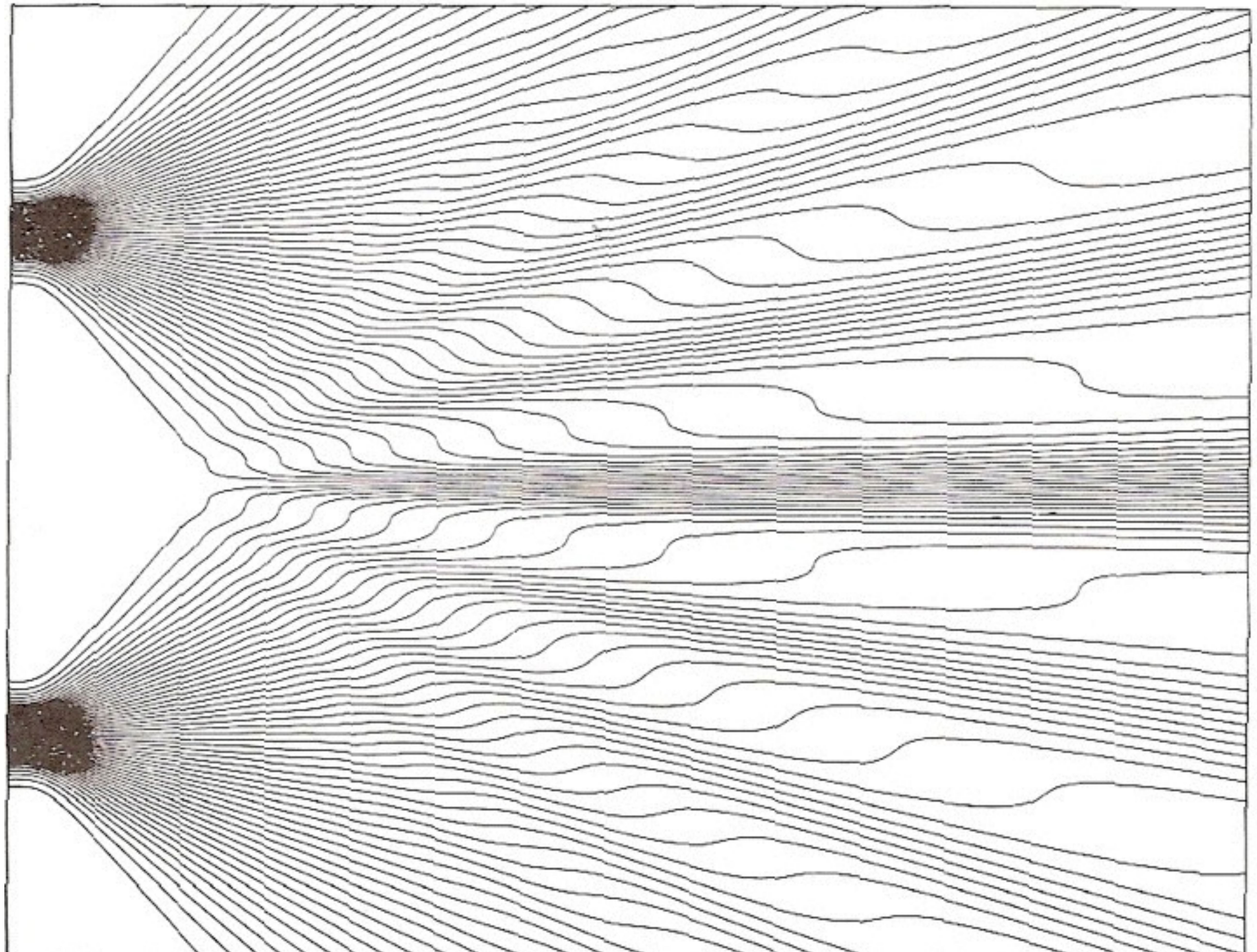

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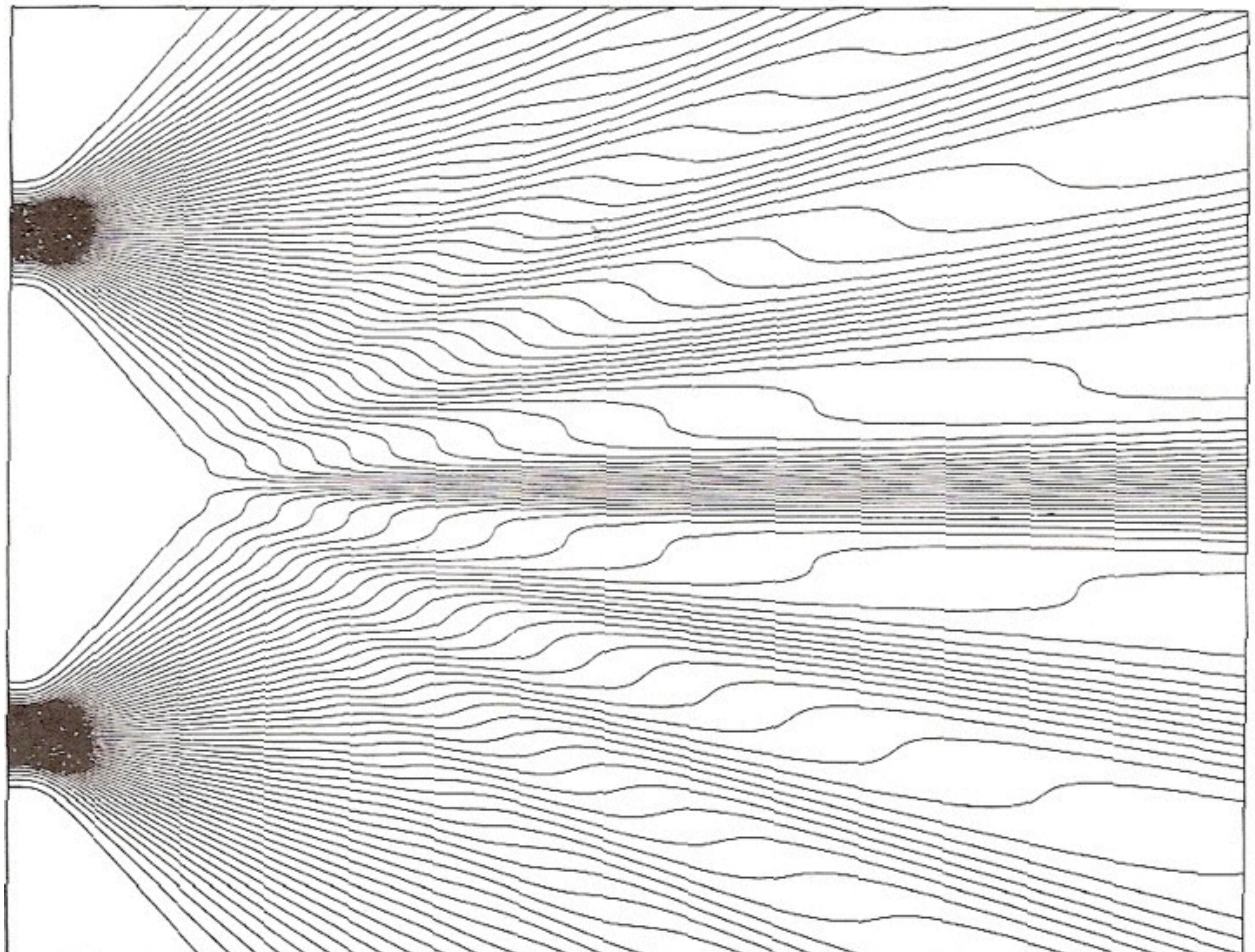
The two-slit experiment:

Surrealistic trajectories?



Non straight in vacuum...

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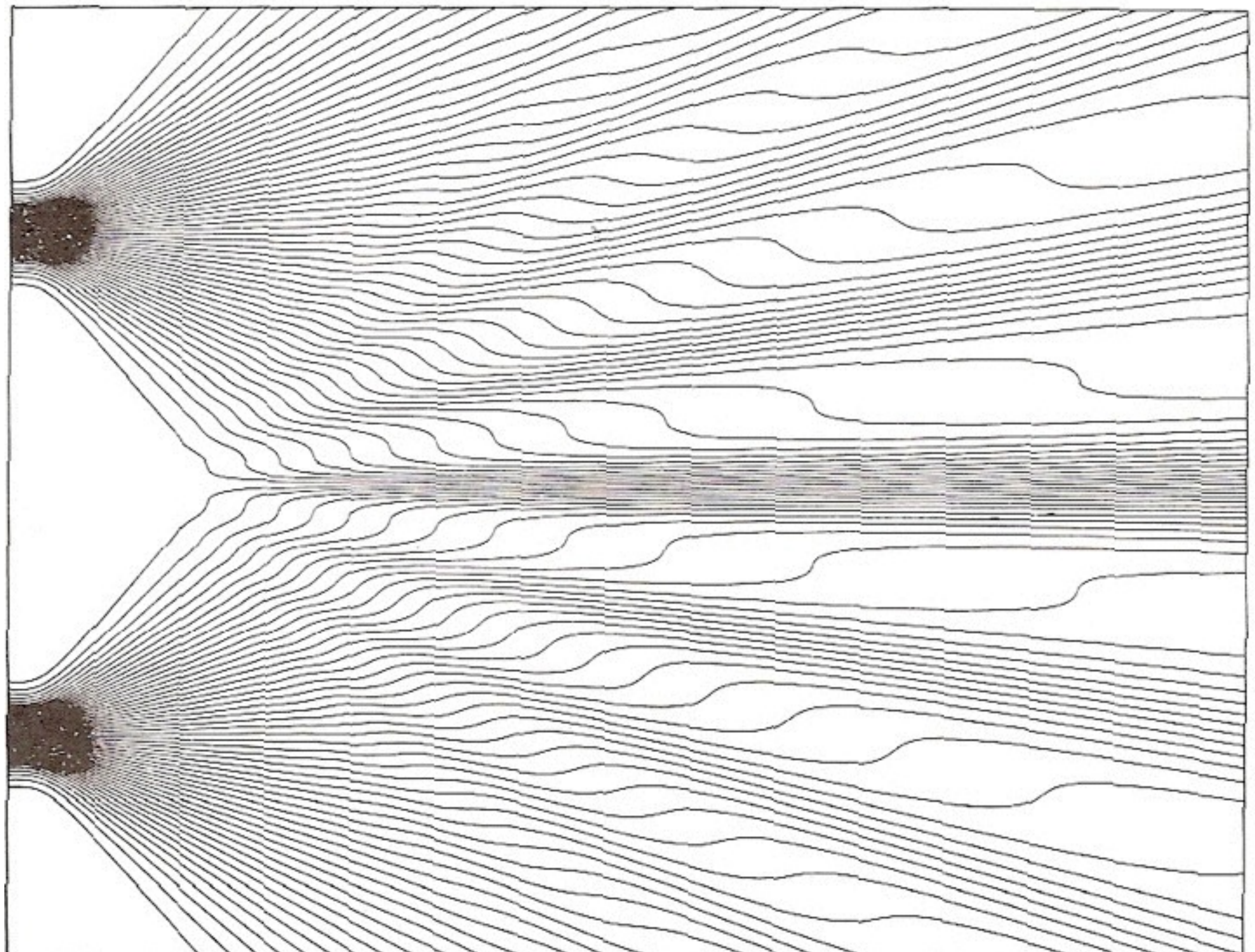

Surrealistic trajectories?

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$$m\frac{\mathrm{d}^2 x(t)}{\mathrm{d}t^2} = -\nabla\left(V+Q\right)$$

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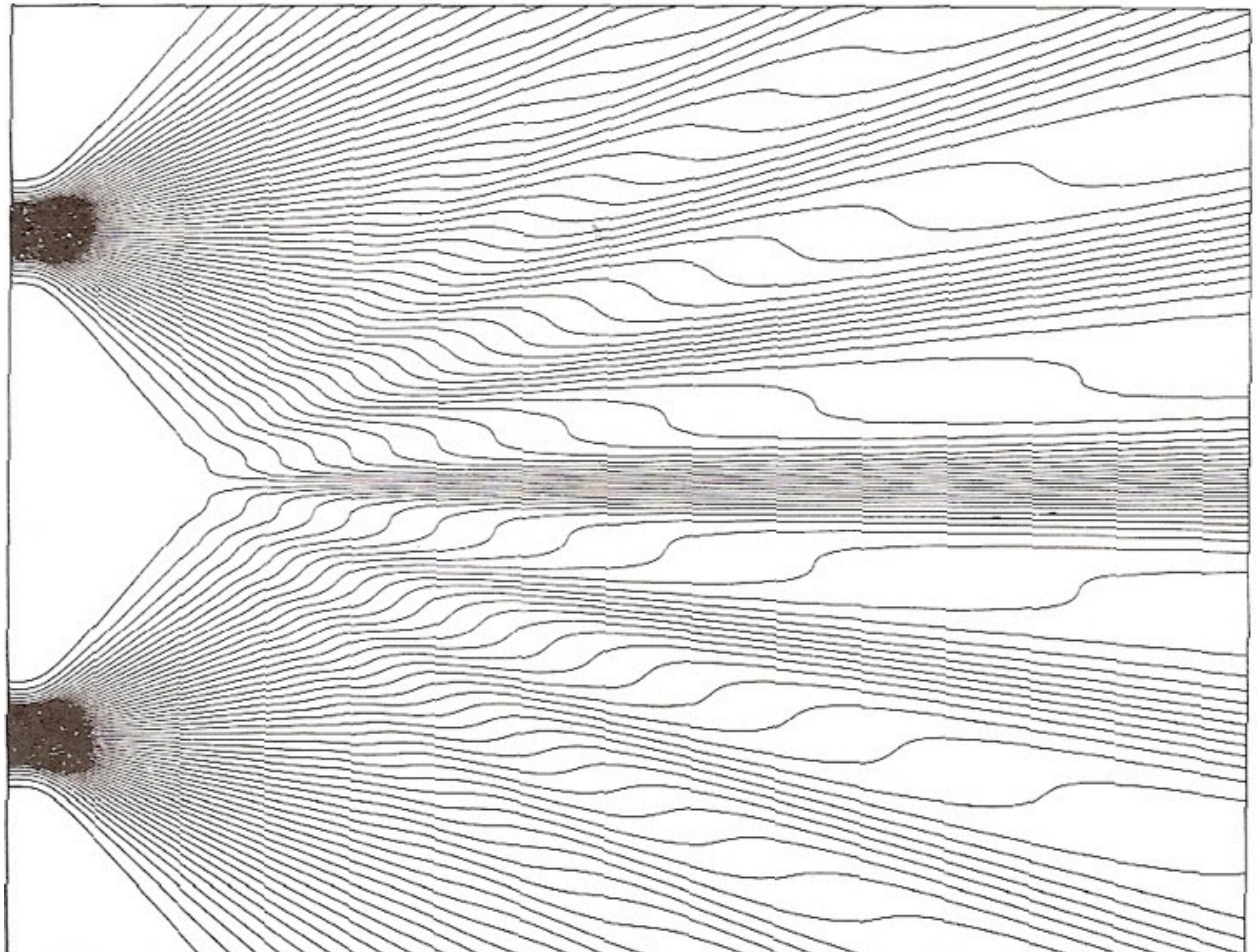

Surrealistic trajectories?

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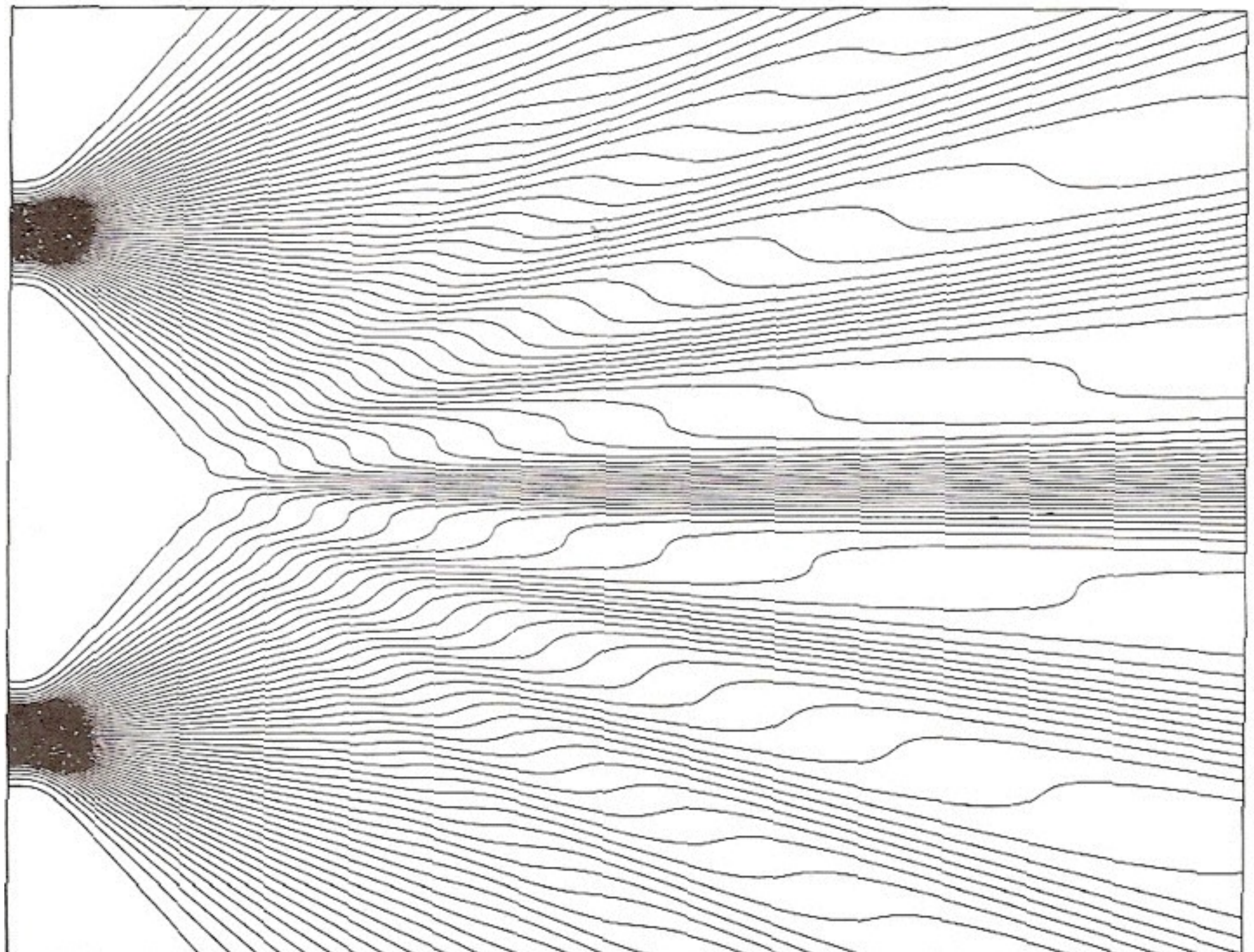
The two-slit experiment:

Surrealistic trajectories?



Non straight in vacuum...

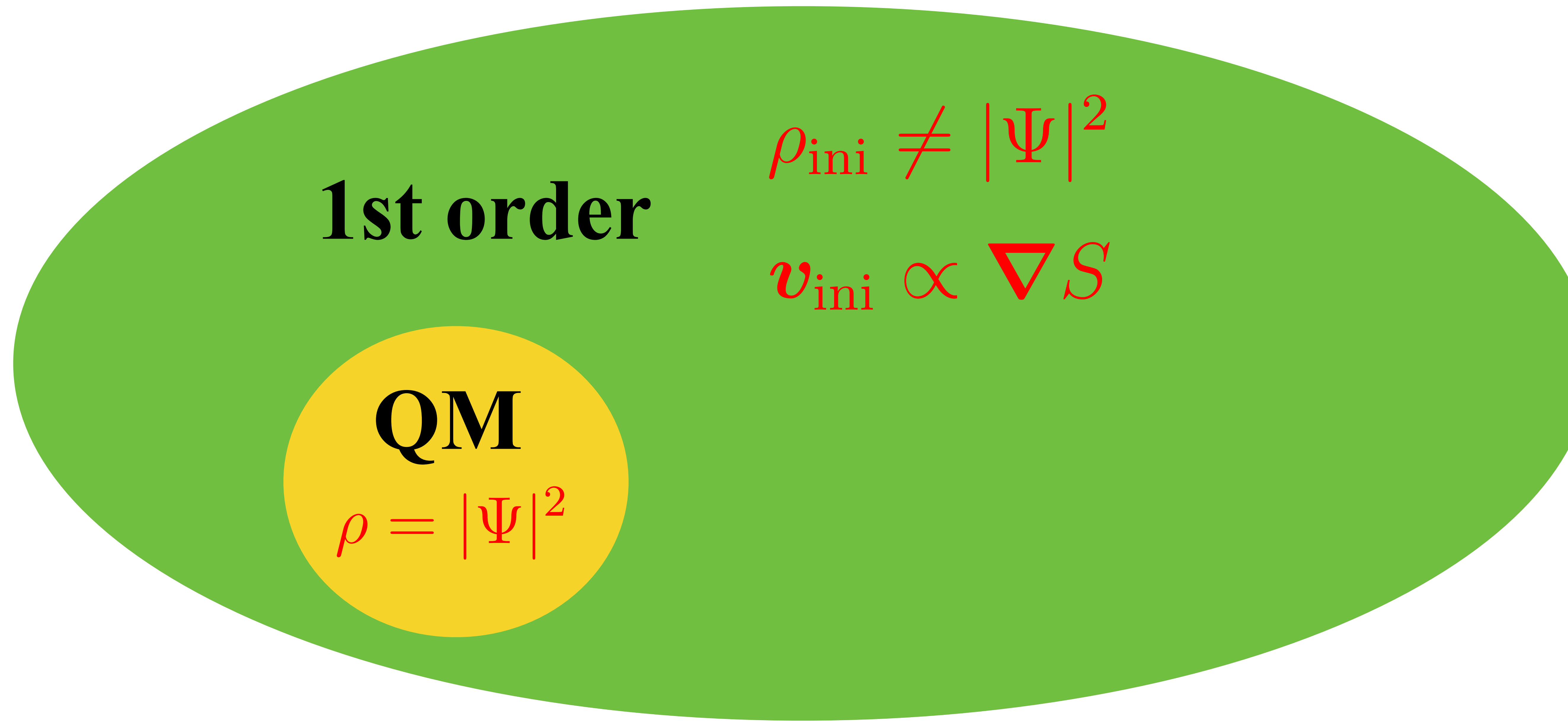
$$m\frac{\mathrm{d}^2x(t)}{\mathrm{d}t^2} = -\nabla\left(\cancel{X} + Q\right)$$

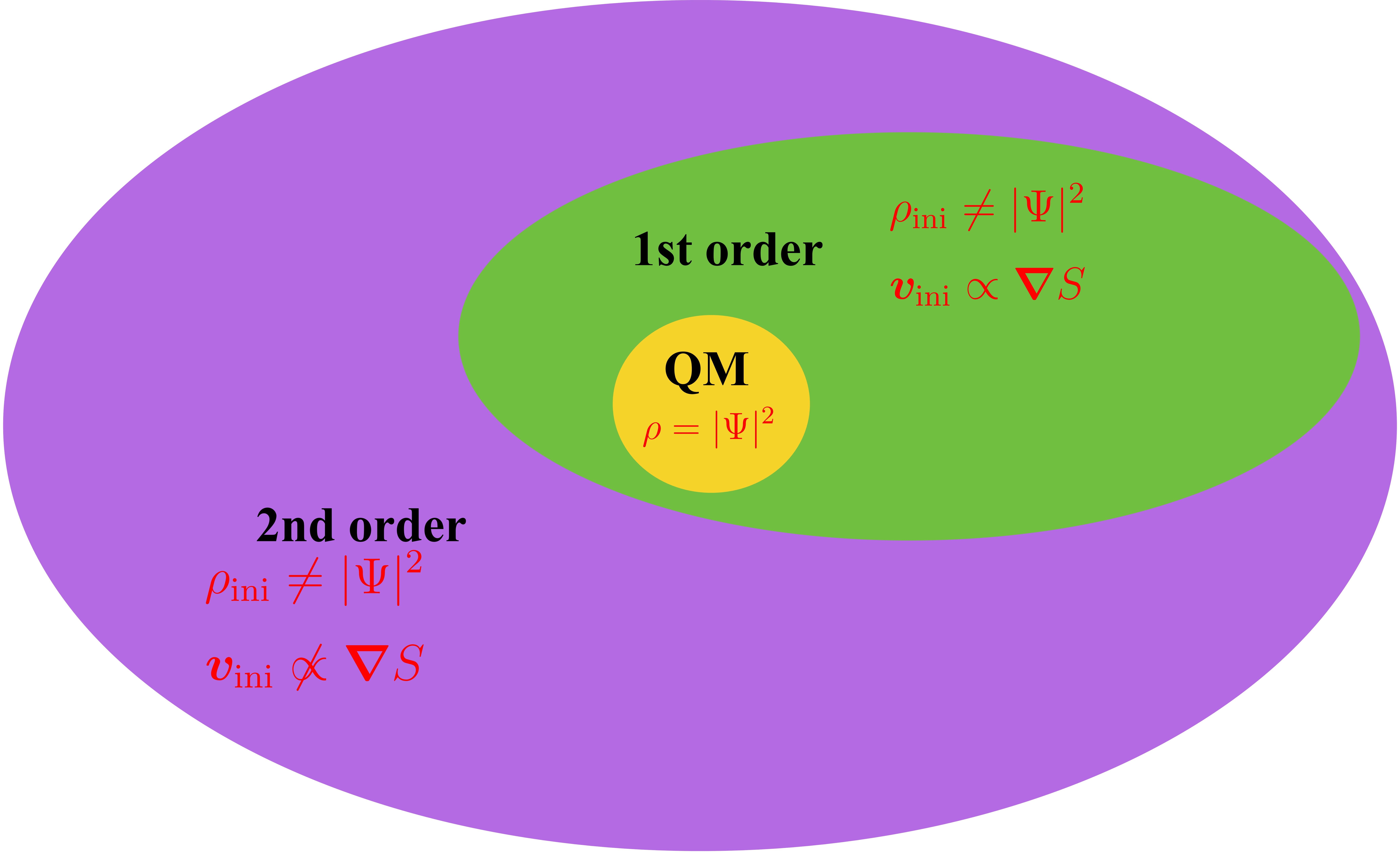
... a phenomenon which is impossible, *absolutely* impossible, to explain in any classical way, and which has in it the heart of quantum mechanics.

R. P. Feynman (1961)

QM

$$\rho = |\Psi|^2$$


1st order
$\rho_{\rm ini} \neq |\Psi|^2$
$\boldsymbol{v}_{\rm ini} \propto \boldsymbol{\nabla} S$
QM
$\rho = |\Psi|^2$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QM
$\rho = |\Psi|^2$
2nd order
$\rho_{\text{ini}} \neq |\Psi|^2$
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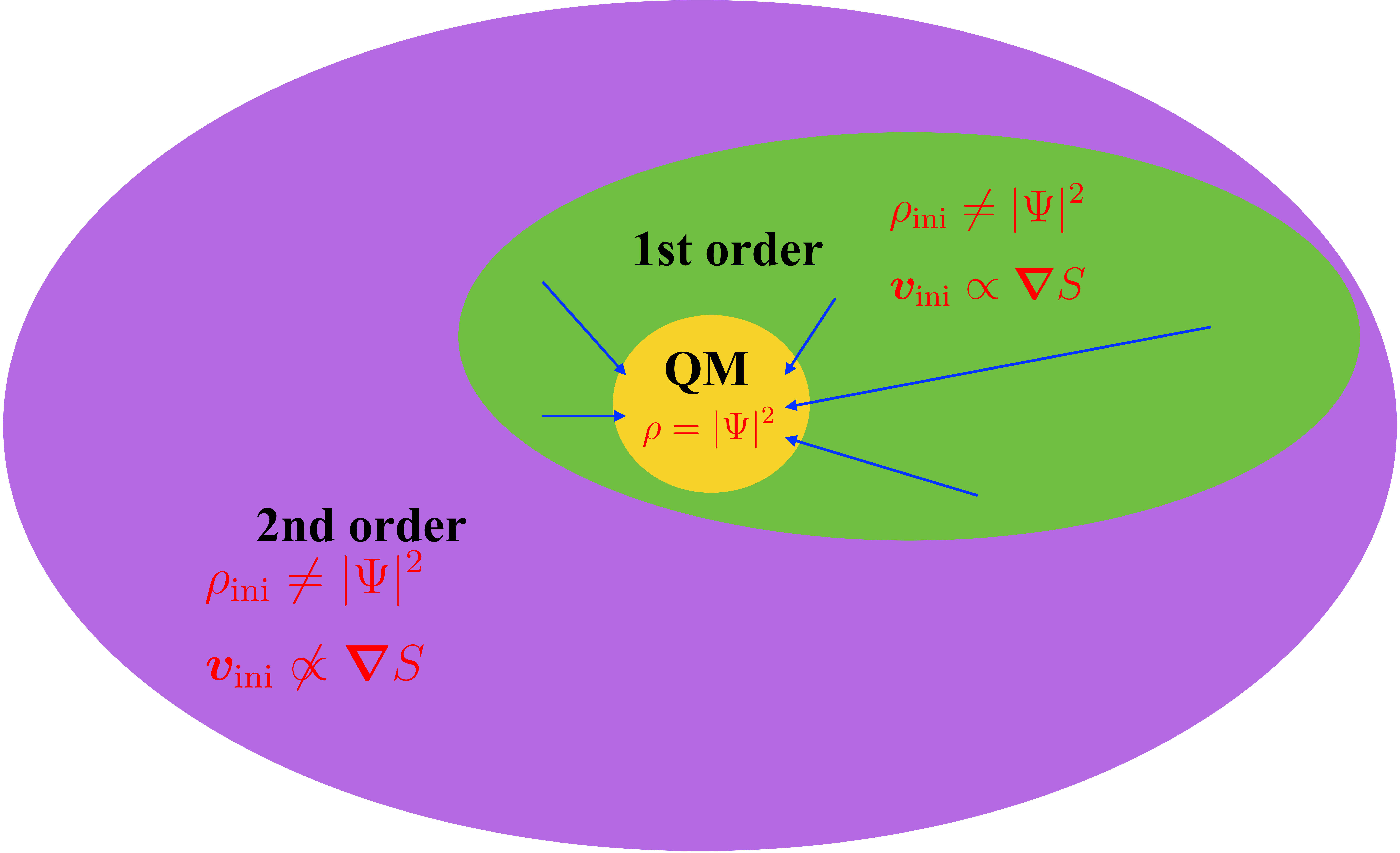
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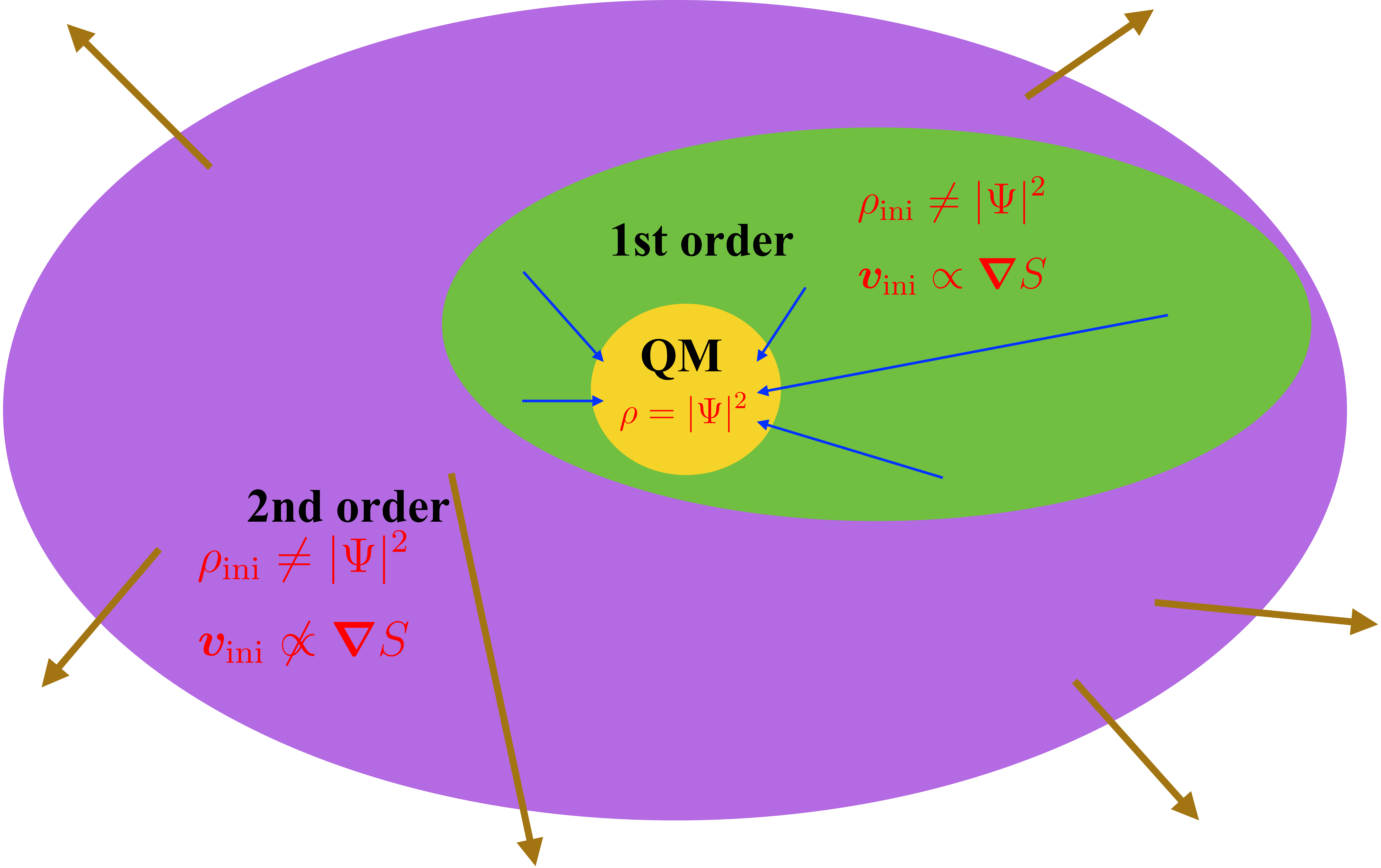
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1st order: can be tested

2nd order: has been tested…

and is ruled out!

1st order: can be tested *How????*

2nd order: has been tested…

and is ruled out!

1st order: can be tested

How????

Primordial
Perturbation
Theory

2nd order: has been tested…

and is ruled out!

Quantum equilibrium

(Valentini & Westman, 2005)

$$i \frac{\mathrm{d}}{\mathrm{d}t}|\Psi\rangle = \hat{H}|\Psi\rangle$$

Particle in a box - 2D

$$i \frac{\partial \psi}{\partial t} = -\frac{1}{2}\frac{\partial^2 \psi}{\partial x^2} - \frac{1}{2}\frac{\partial^2 \psi}{\partial y^2} + V\psi$$

infinite square well - size π

Density of actual configurations

$$\rho(x, y, t) \quad \Longrightarrow \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho \dot{x}) + \frac{\partial}{\partial y}(\rho \dot{y}) = 0$$

continuity equation

Energy eigenfunctions $\phi_{mn}(x, y) = \frac{2}{\pi} \sin(mx) \sin(ny)$

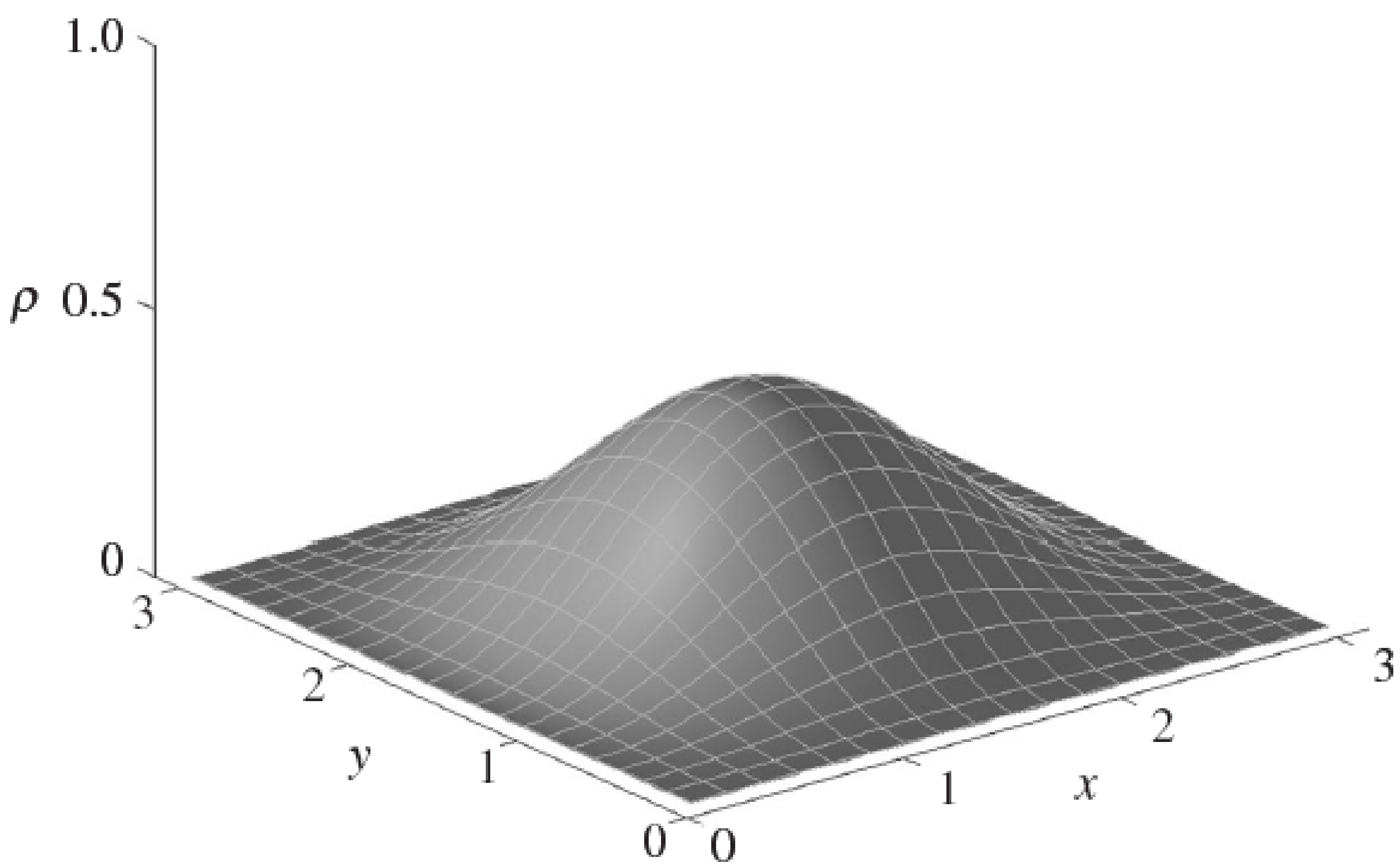
Energy levels $E_{mn} = \frac{1}{2}\left(m^2 + n^2\right)$

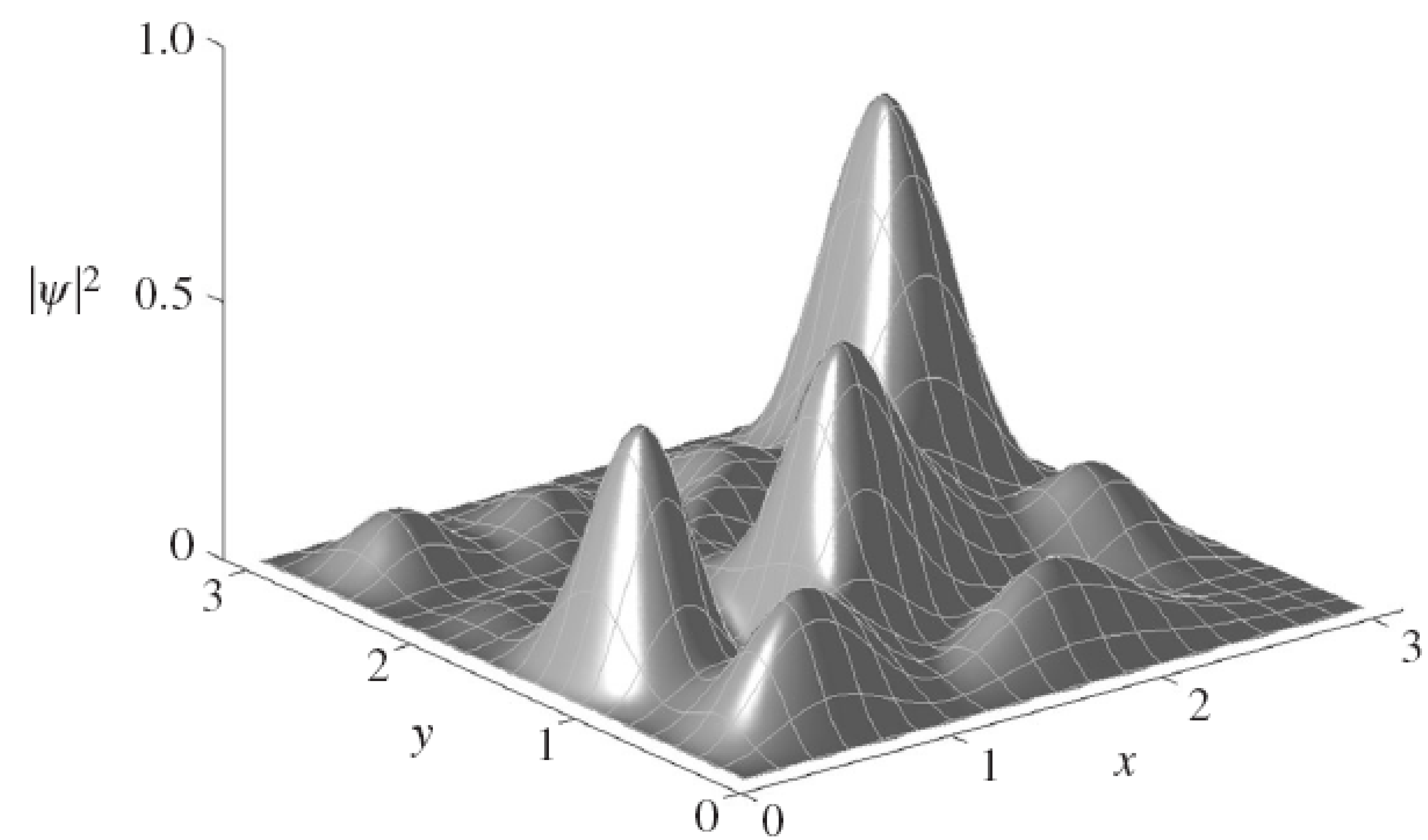
Initial configuration

$$\rho(x,y,0) = |\phi_{11}(x,y)|^2$$

$$\psi(x,y,0) = \sum_{m,n=1}^{4} \frac{1}{4}\phi_{mn}(x,y)\exp(i\theta_{mn})$$

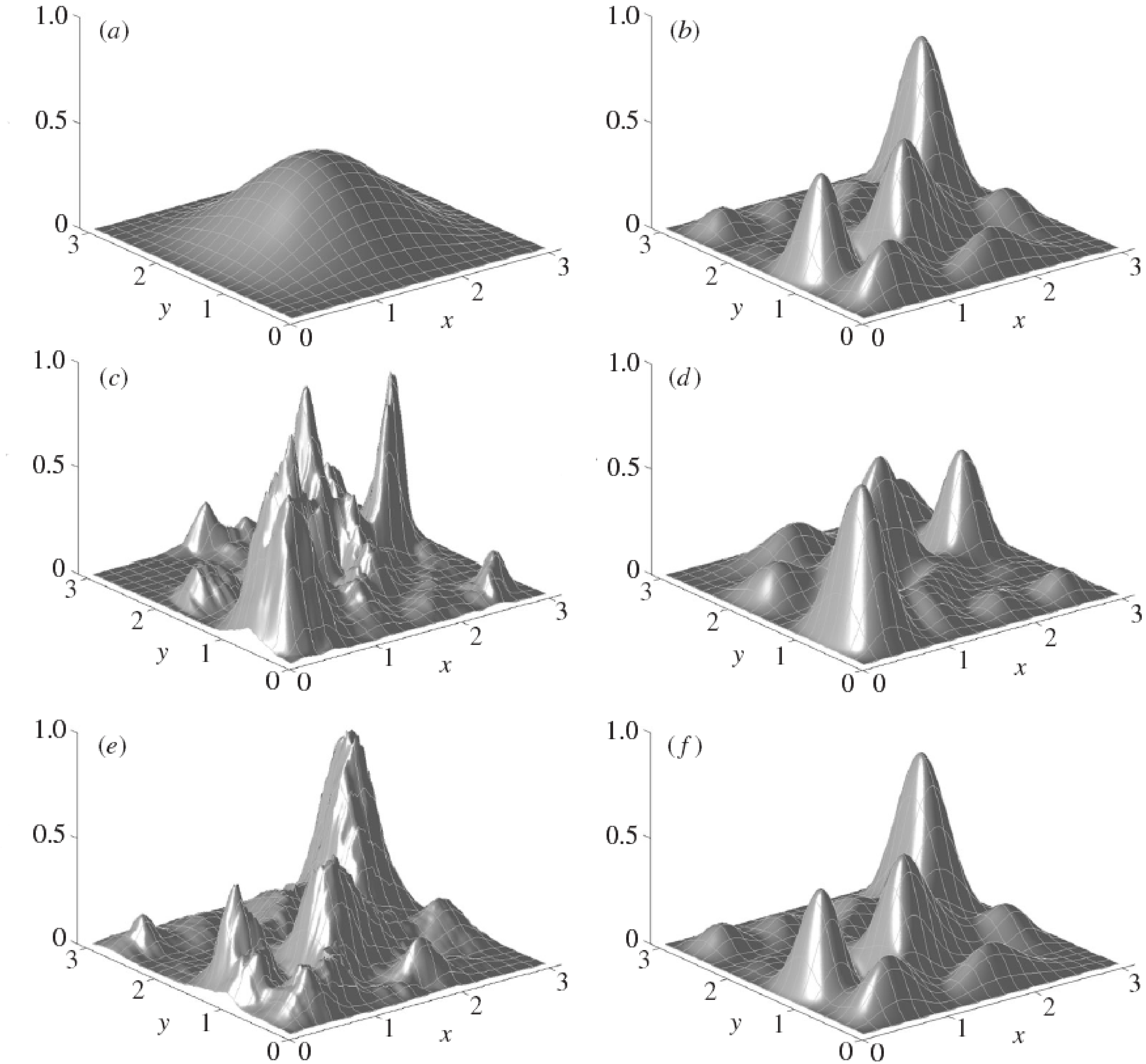
$$\psi(x,y,t) = \sum_{m,n=1}^{4} \frac{1}{4}\phi_{mn}(x,y)\exp i(\theta_{mn} - E_{mn}t)$$





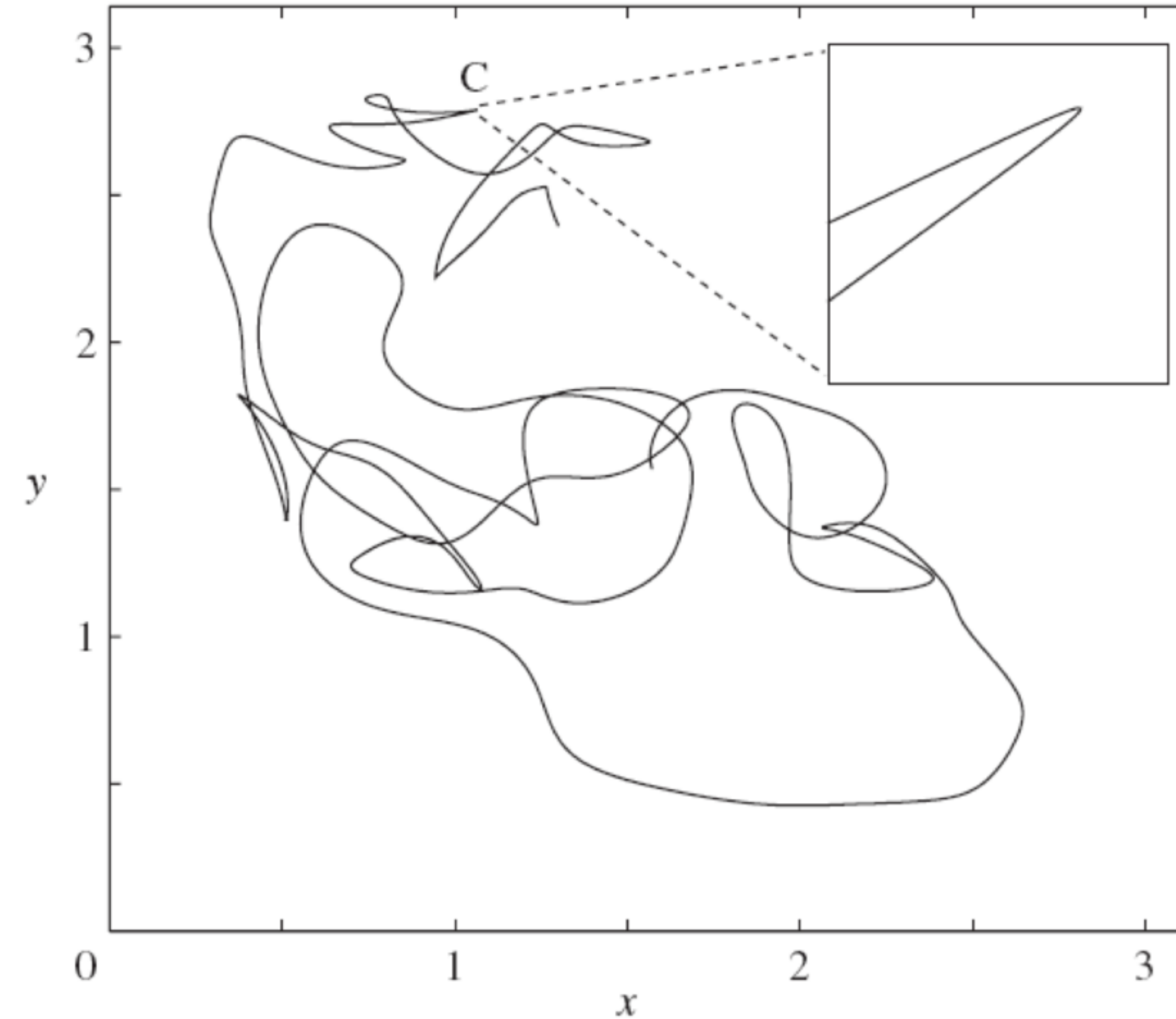
Dynamical evolutions

ρ

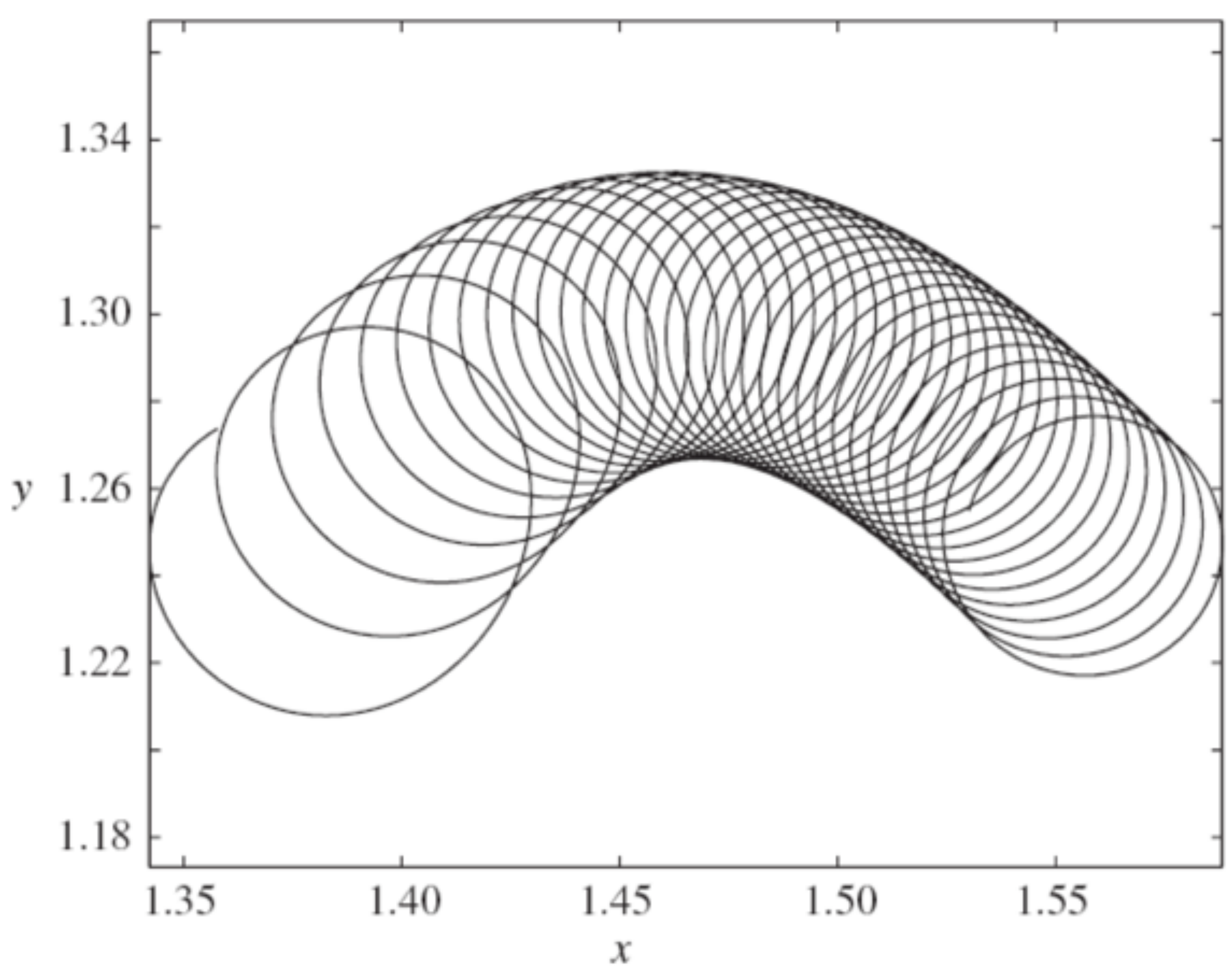


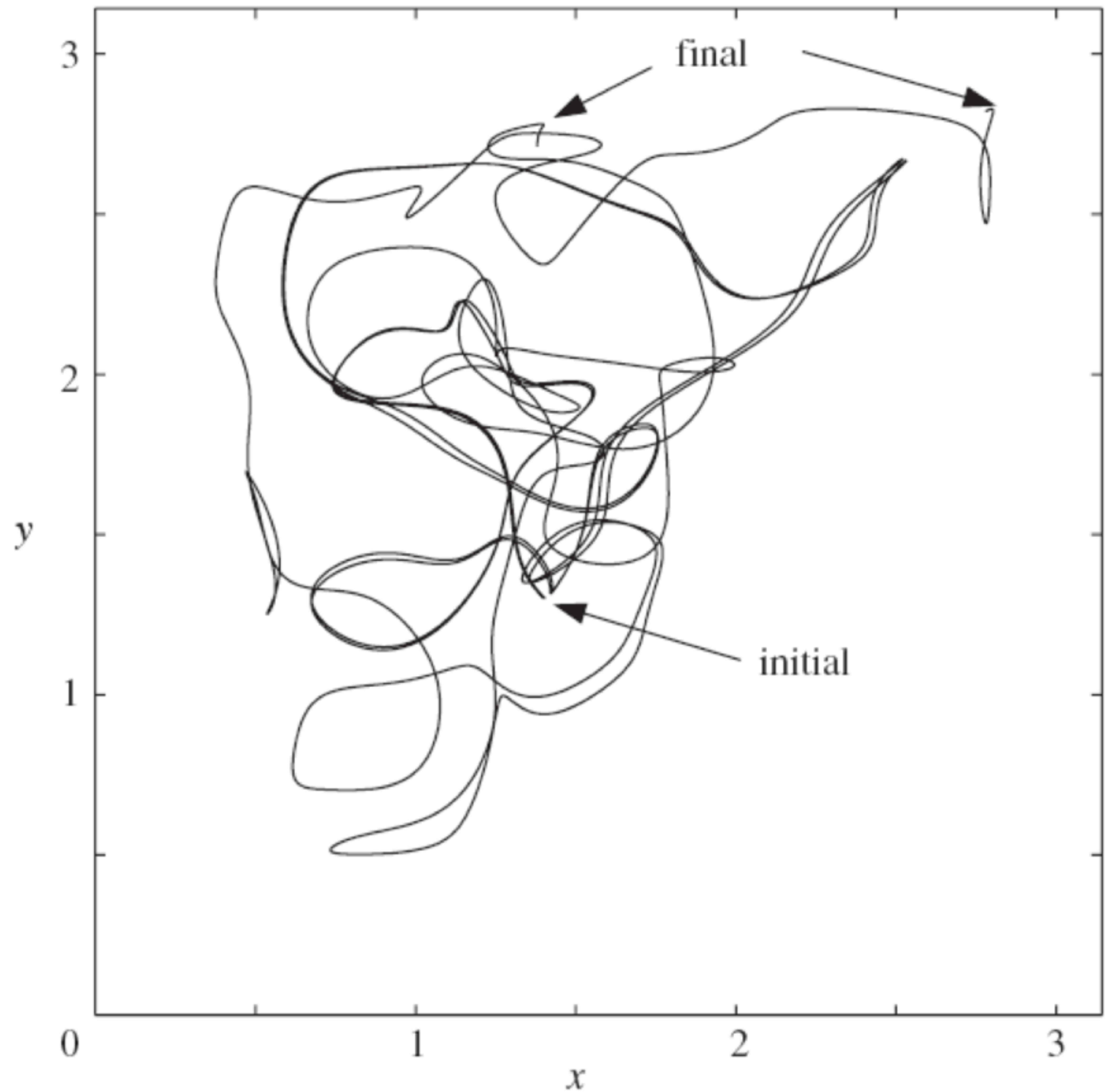
$|\Psi|^2$

Typical quantum trajectory…

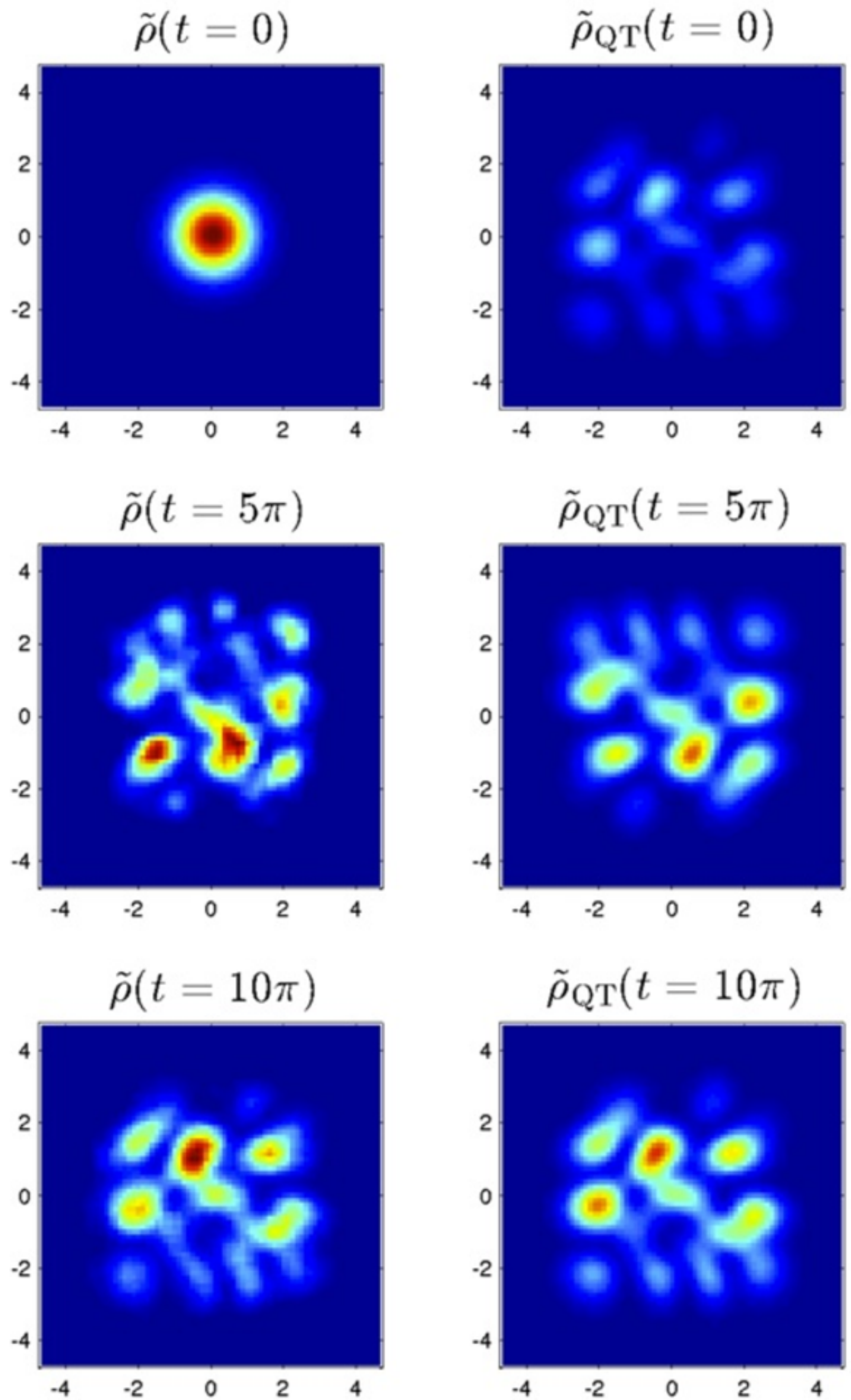


Close-up of a trajectory near a node





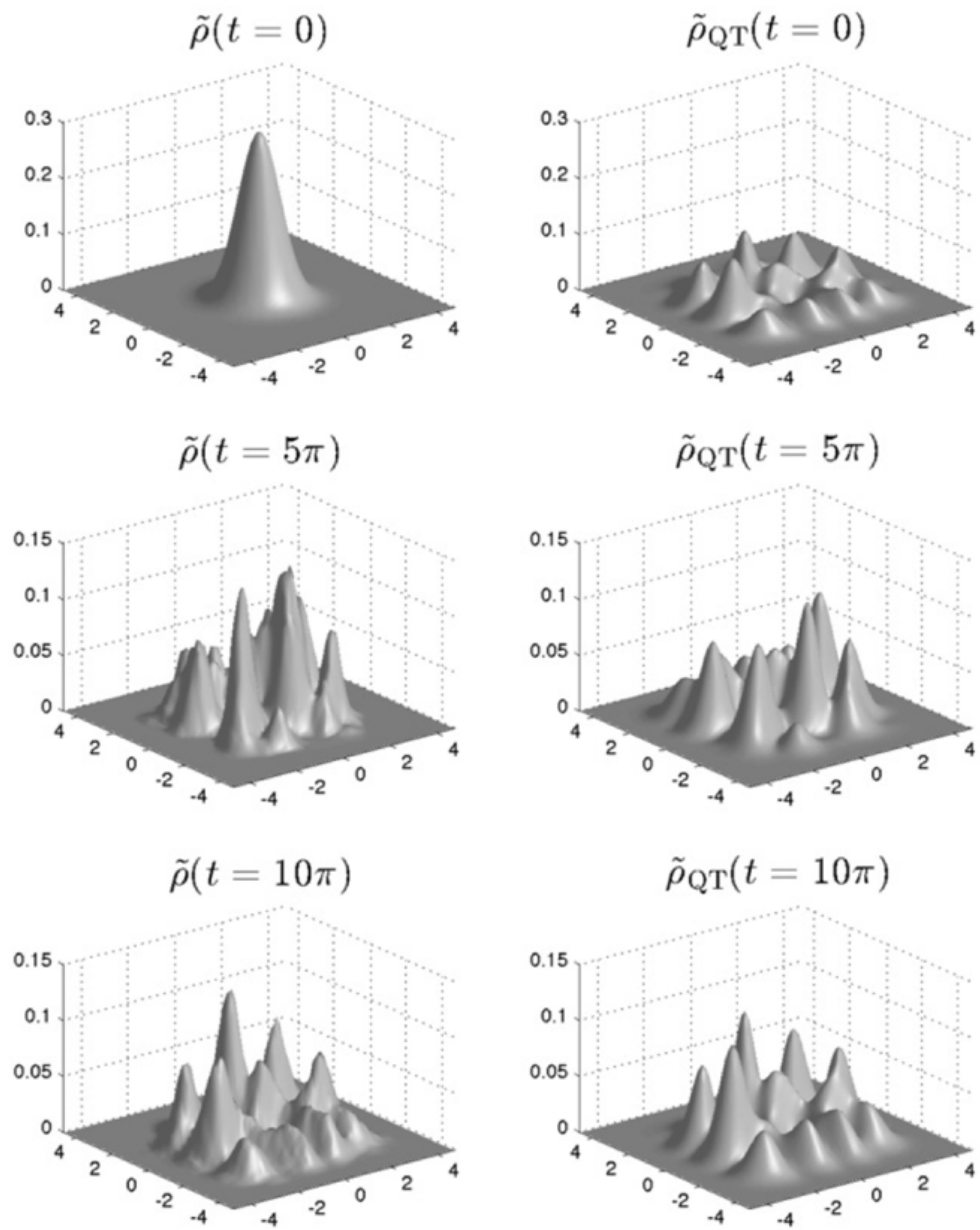
chaotic mixing…



chaotic mixing…

relaxation towards equilibrium

just like ordinary thermal equilibrium



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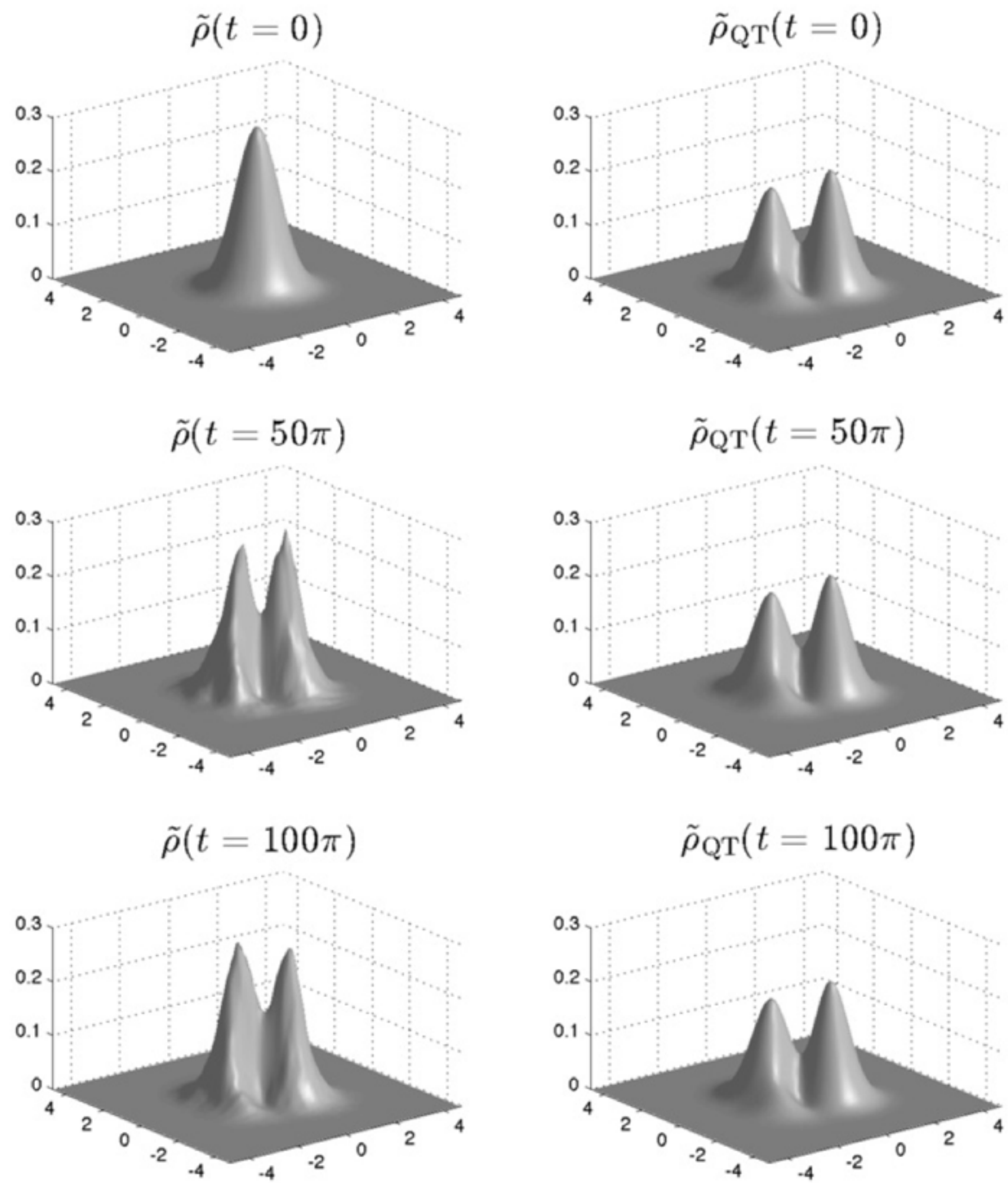

chaotic mixing…

relaxation towards equilibrium

just like ordinary thermal equilibrium

possibly slightly smaller width for low number of modes…

Recall: Hamiltonian

$$H = \int \mathrm{d}^3\boldsymbol{k} \left\{ p_{\boldsymbol{k}} p_{\boldsymbol{k}}^* + v_{\boldsymbol{k}} v_{\boldsymbol{k}}^* \left[\overbrace{k^2 - \frac{\left(a\sqrt{\epsilon_1}\right)''}{a\sqrt{\epsilon_1}}}^{\omega^2(\eta, \boldsymbol{k})} \right] \right\}$$

collection of parametric oscillators with time dependent frequency

Simpler model: spectator scalar field in an expanding and finite size Universe

$$\phi_{\boldsymbol{k}} = \frac{\sqrt{V}}{(2\pi)^{3/2}} \left(q_{\boldsymbol{k}1} + i q_{\boldsymbol{k}2}\right) \qquad H = \sum_{\boldsymbol{k},\ r=1,2} \frac{1}{2a^3} \pi_{\boldsymbol{k}r}^2 + \frac{1}{2} a k^2 q_{\boldsymbol{k}r}^2$$

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$$a^3 \to m$$

$$k/a \to \omega$$

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$$H = \int \mathrm{d}^3\boldsymbol{k} \left\{ p_{\boldsymbol{k}} p_{\boldsymbol{k}}^* + v_{\boldsymbol{k}} v_{\boldsymbol{k}}^* \Bigg[\overbrace{k^2 - \frac{\left(a\sqrt{\epsilon_1}\right)''}{a\sqrt{\epsilon_1}}}^{\omega^2(\eta, \boldsymbol{k})} \Bigg] \right\}$$

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$$a^3 \to m$$

$$k/a \to \omega$$

$$i\frac{\partial \psi}{\partial t} = \sum_{r=1}^{2} \left(-\frac{\partial_r^2}{2m} + \frac{1}{2} m \omega^2 q_r^2 \right)$$

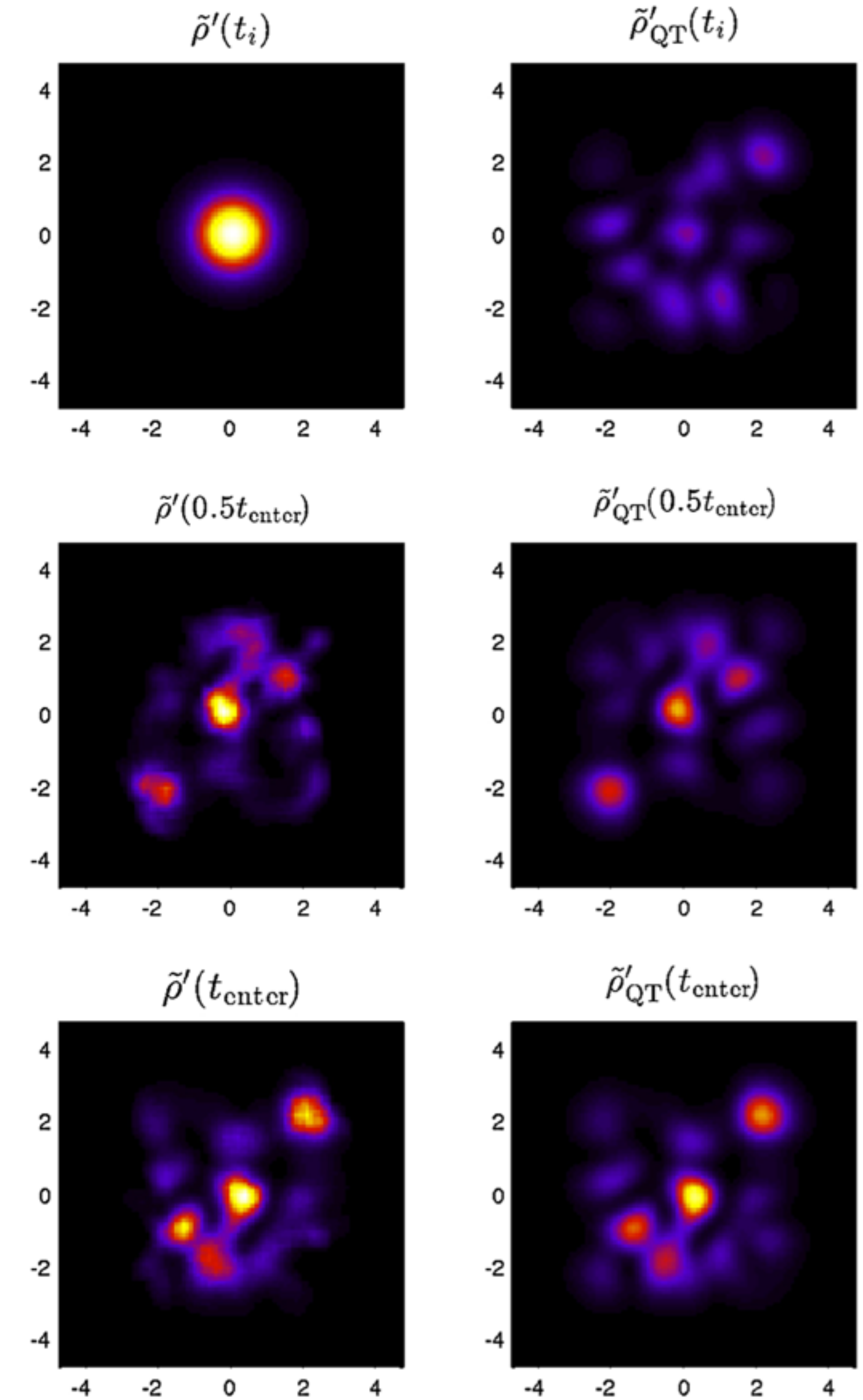
dBB trajectory of the field component $\dot{q}_r = m^{-1}\Im\mathrm{m}\,\frac{\partial_r \psi}{\psi}$

Statistical distribution $\frac{\partial \rho}{\partial t} + \sum_r \partial_r \left(\frac{\rho}{m} \Im\mathrm{m}\,\frac{\partial_r \psi}{\psi} \right)$

$$i\frac{\partial \psi}{\partial t} = \sum_{r=1}^{2} \left(-\frac{\partial_r^2}{2m} + \frac{1}{2} m\omega^2 q_r^2 \right)$$

Relaxation of a 2D harmonic oscillator

(time dependent mass & frequency)

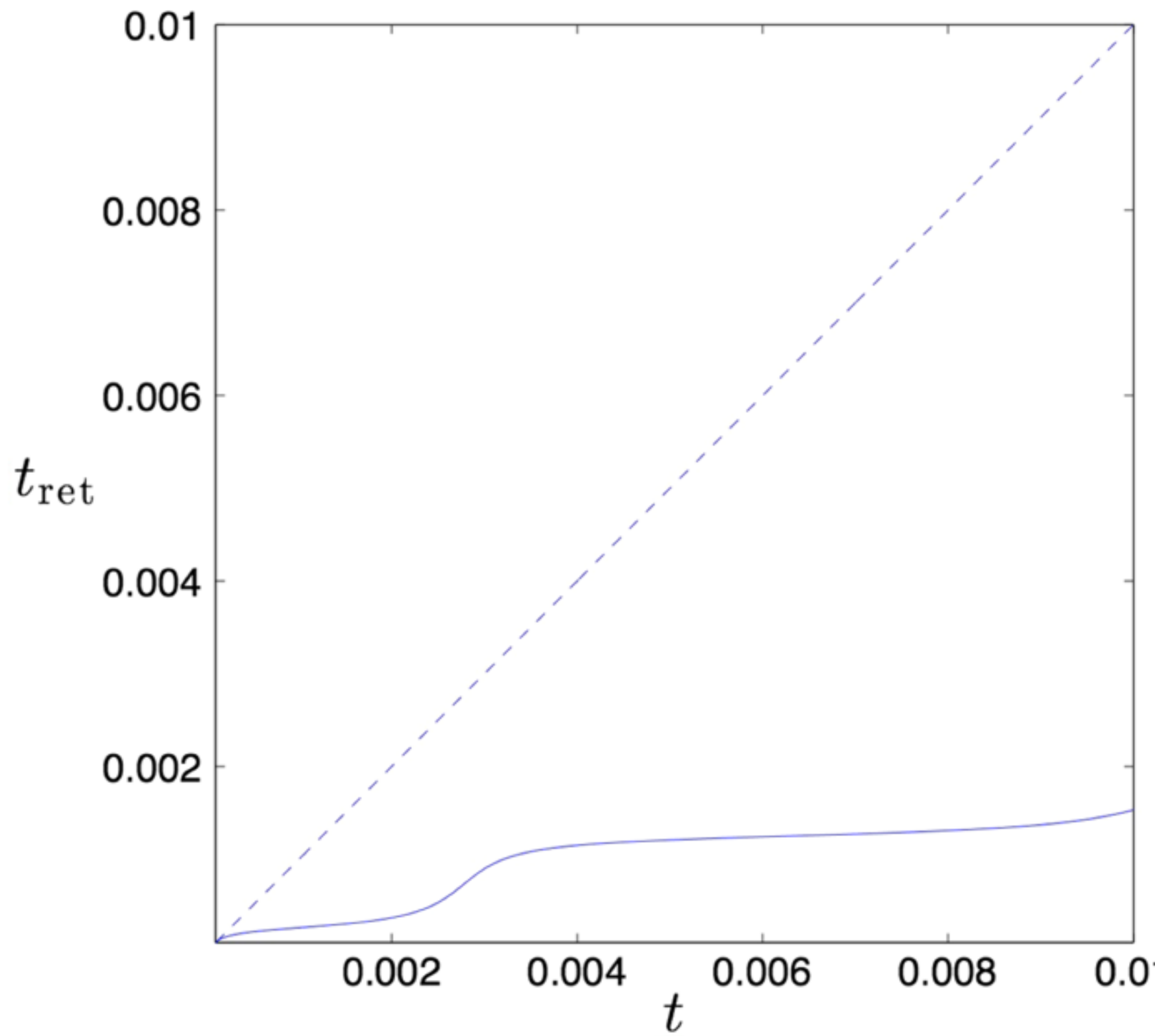


(constant mass & frequency)

Out-of-equilibrium time evolution

- Usual behaviour = evolves towards equilibrium (Minkowski or slowly expanding Universe)

- expansion: there is a retarded time…



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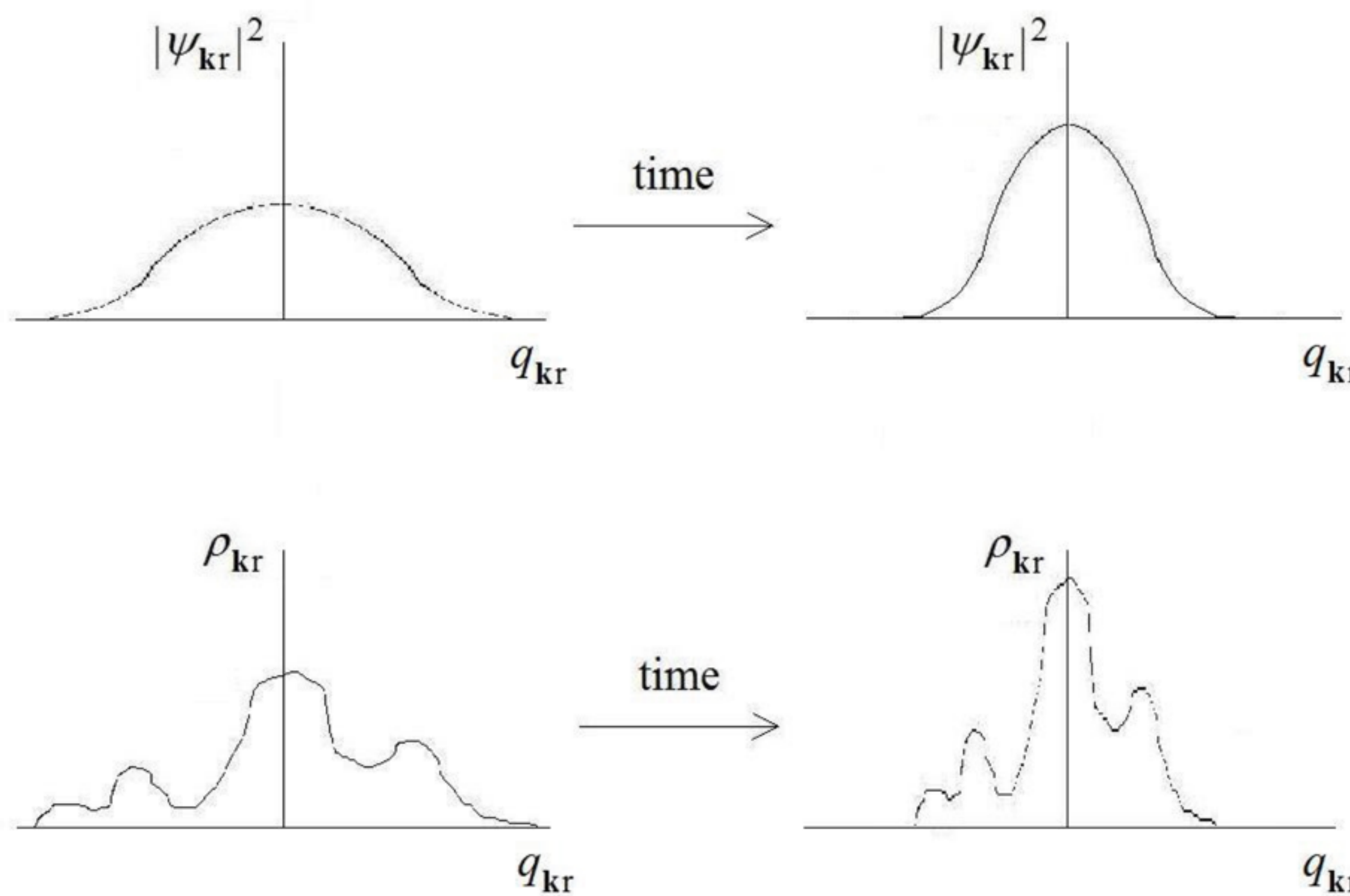

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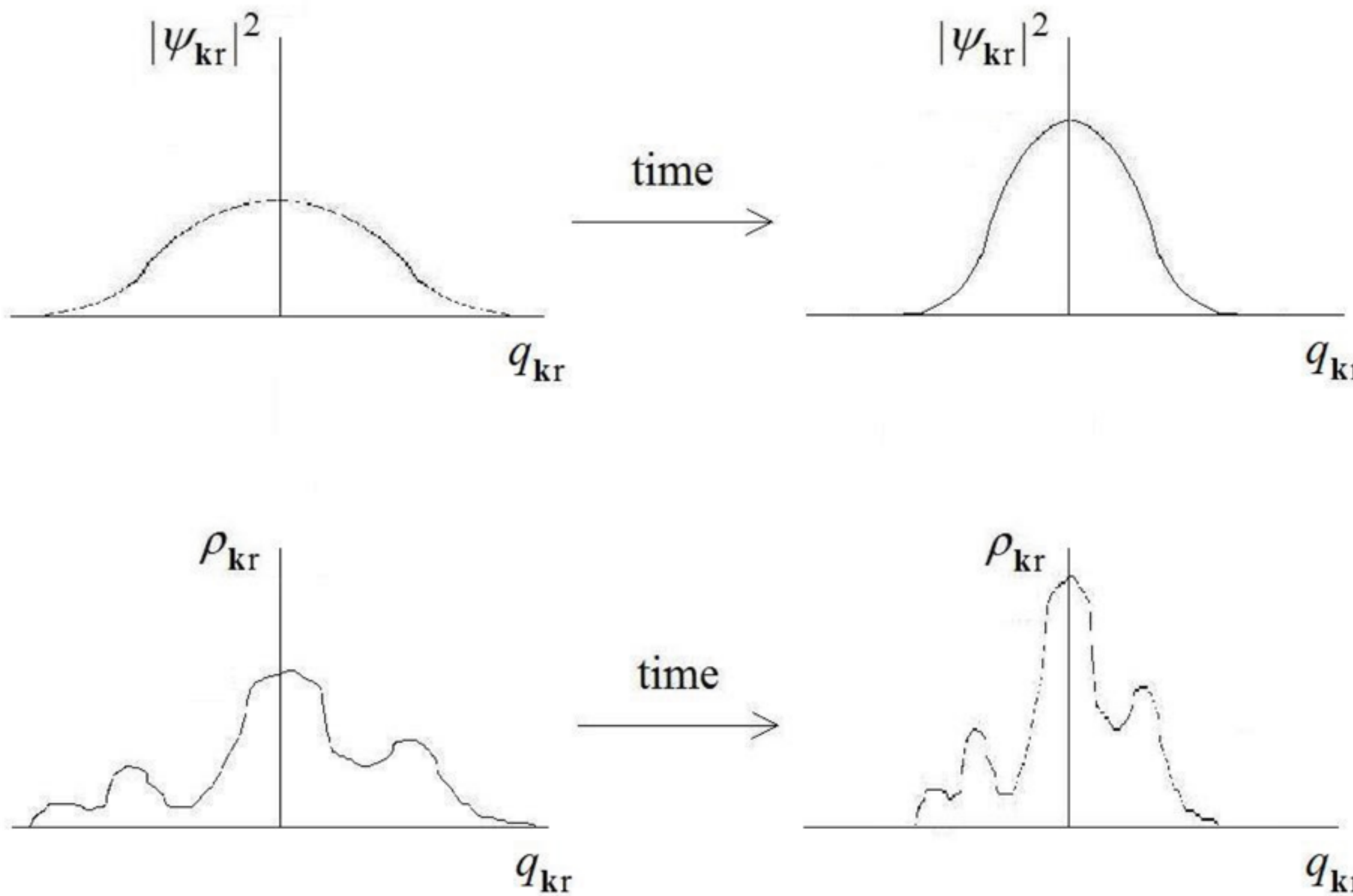
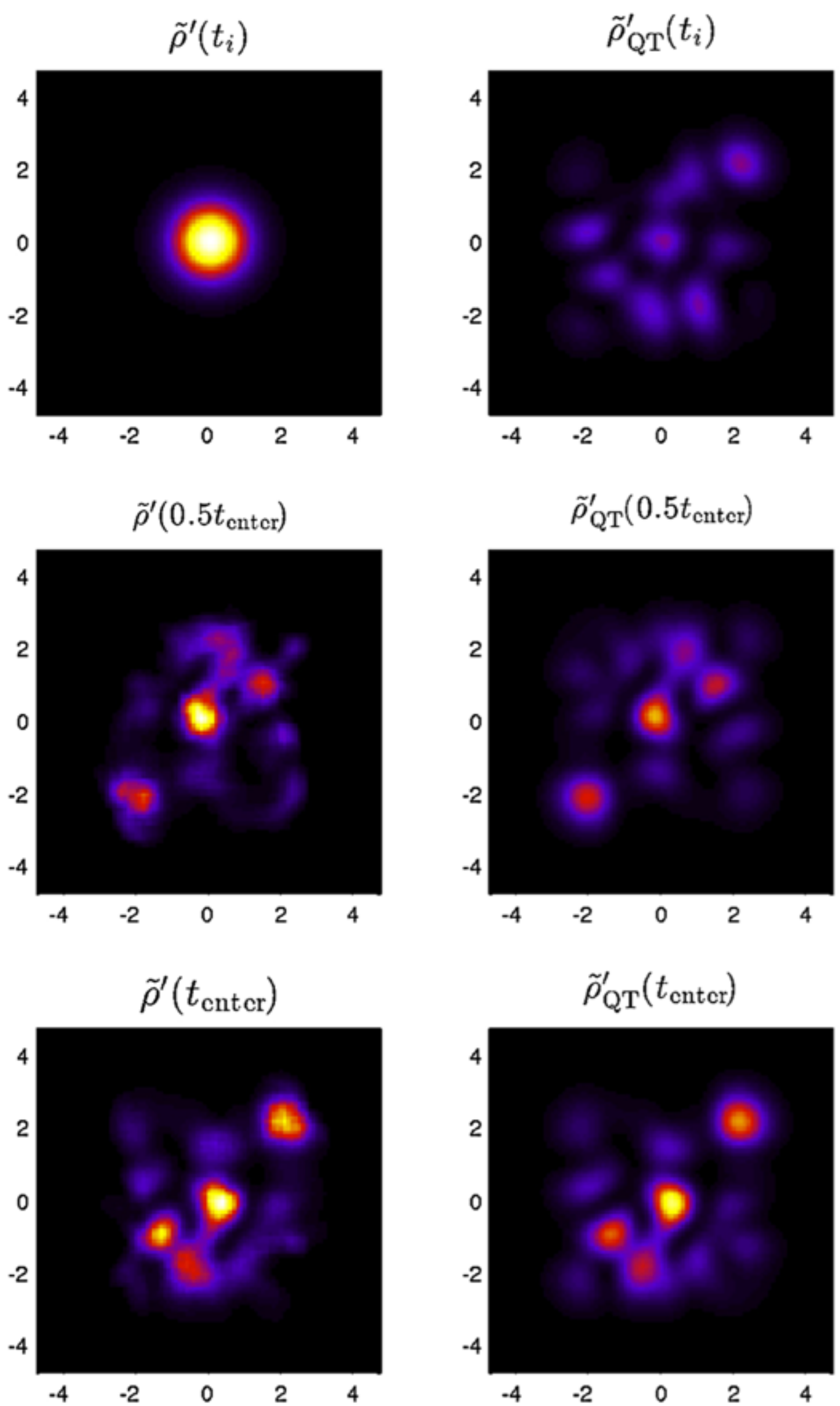
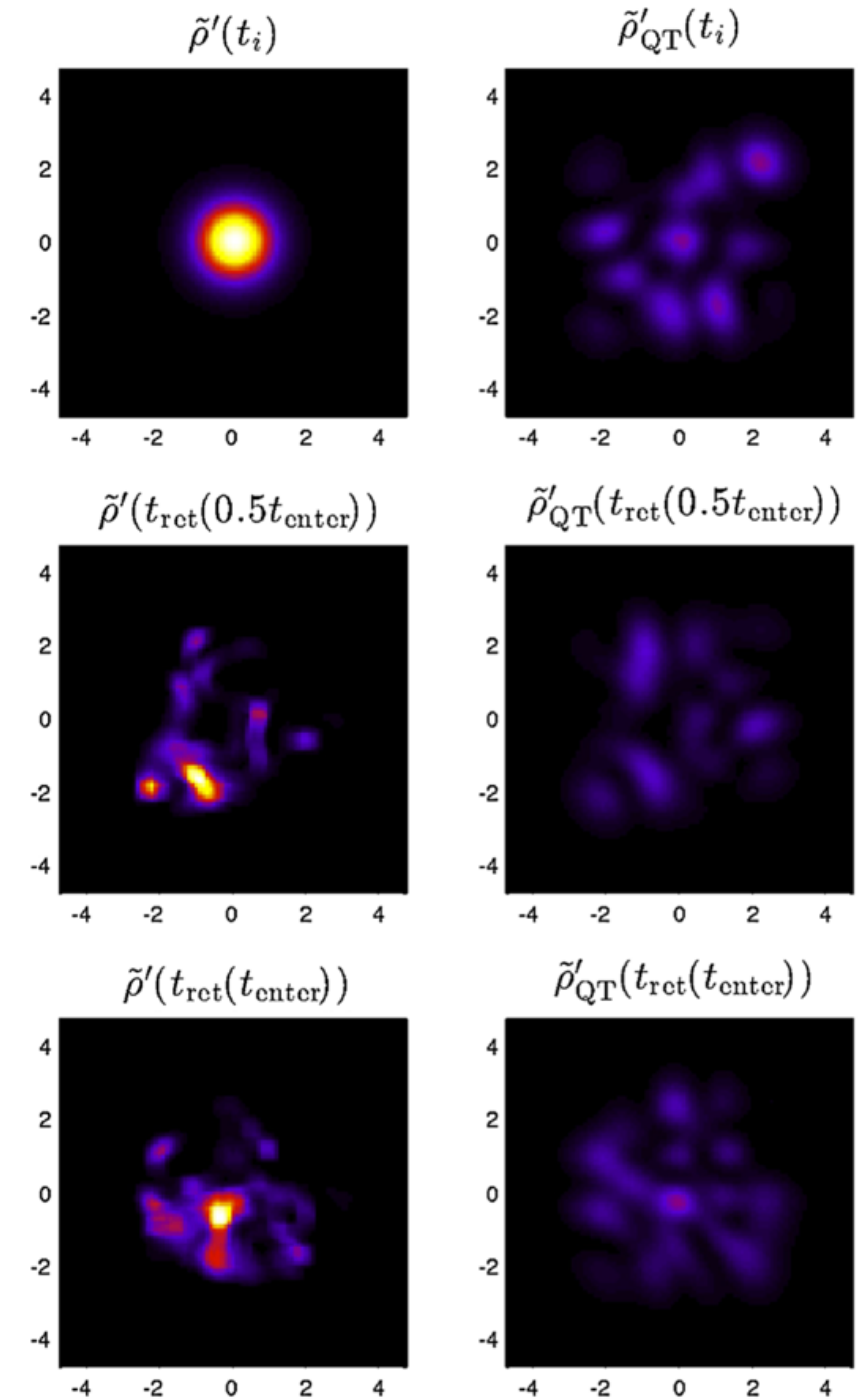


Freezing the pdf out of equilibrium



without expansion

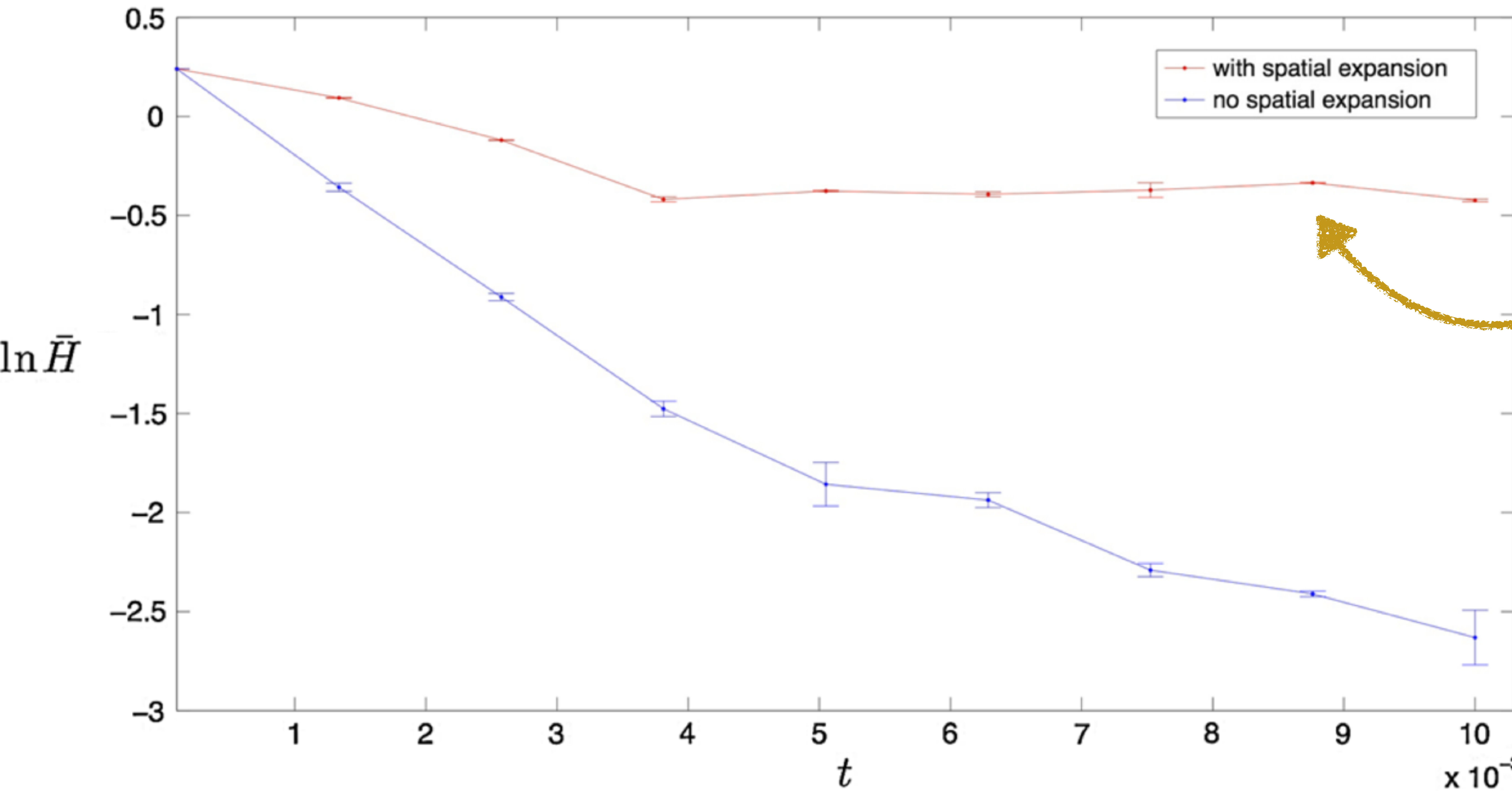


with expansion

S. Colin & A. Valentini, *Phys. Rev.* **D88** 103515 (2013)

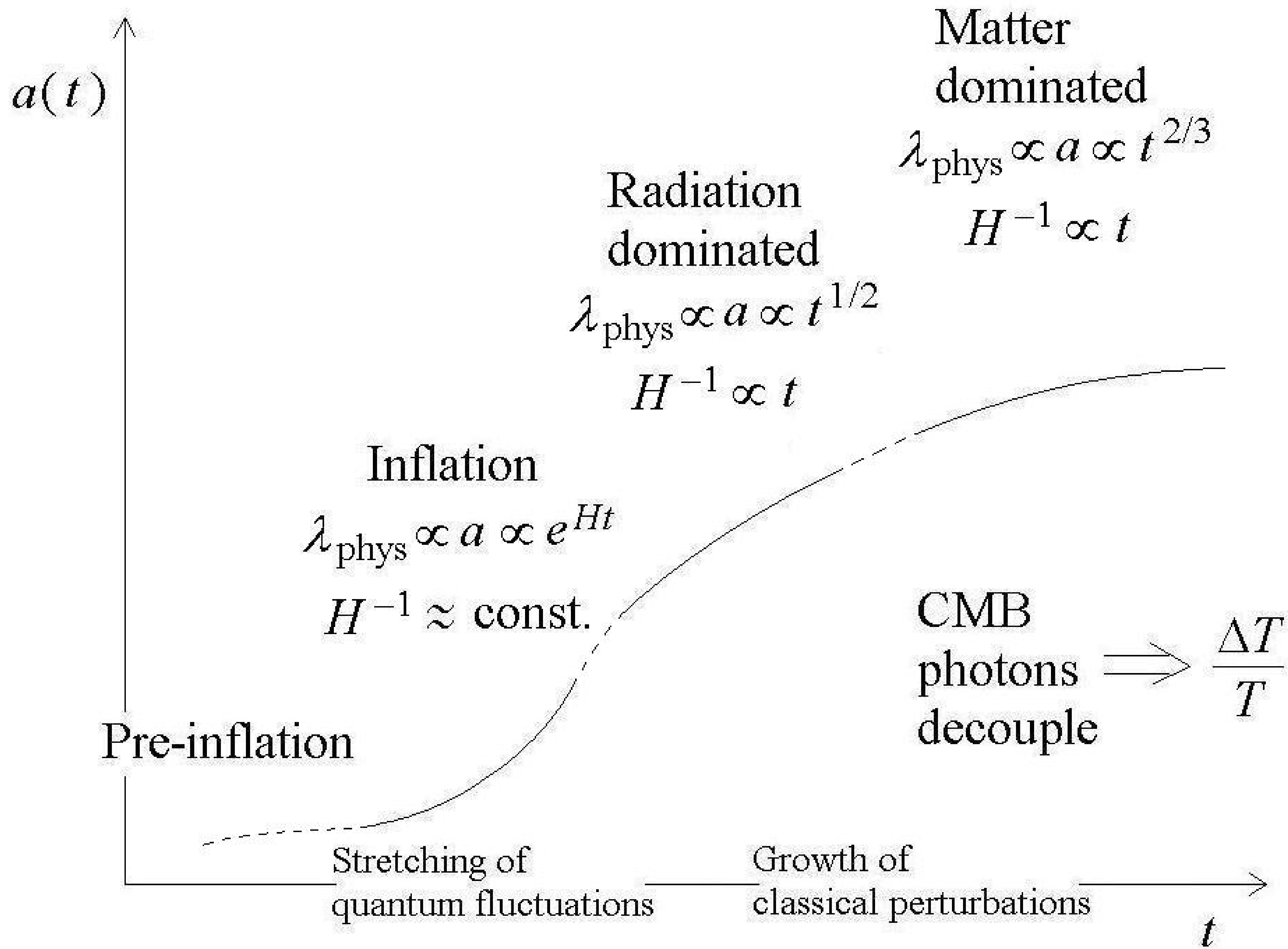
$$H \equiv \int \mathrm{d}q\, \rho \ln \left(\frac{\rho}{|\Psi|^2} \right)$$

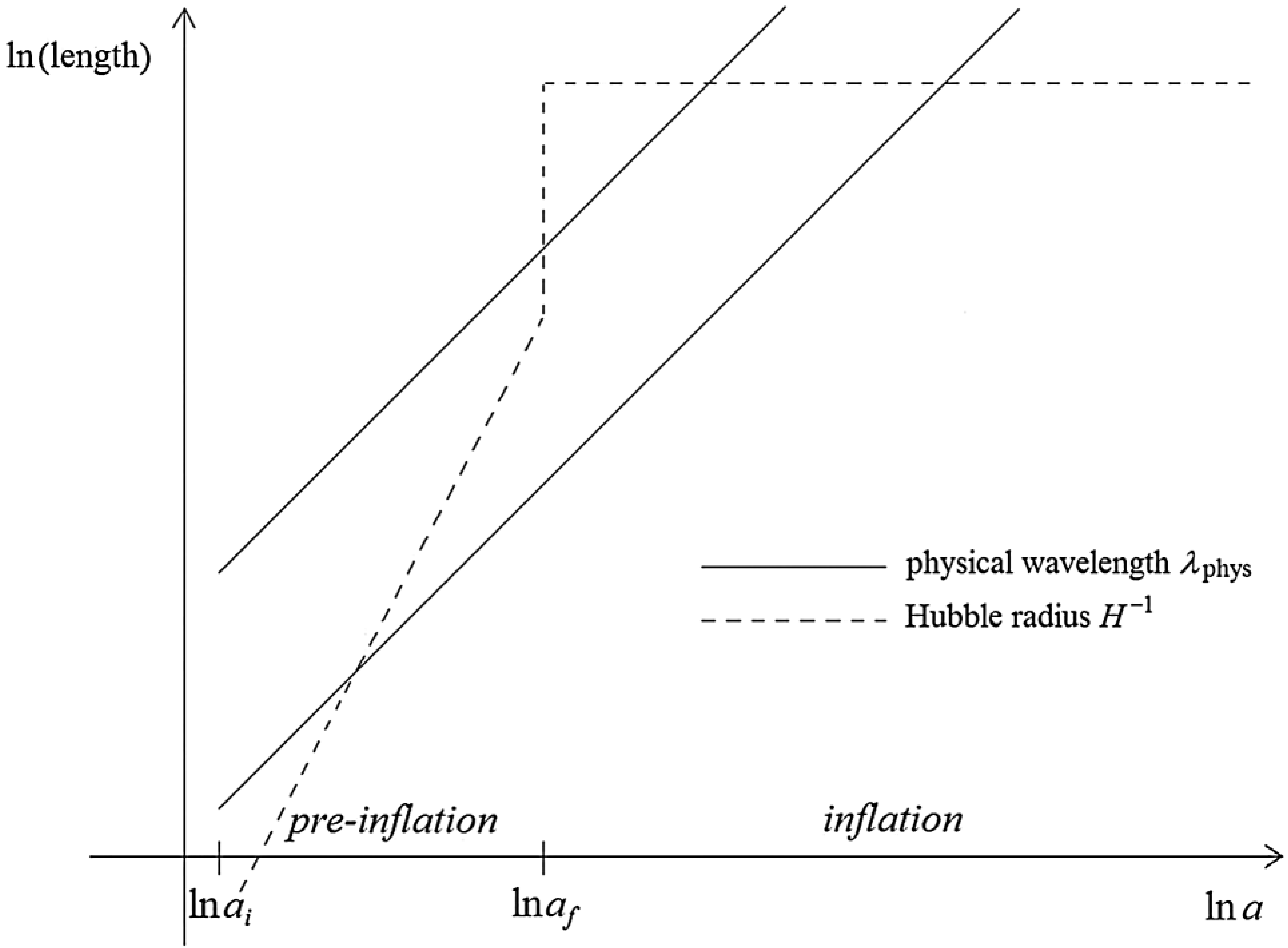
measures “out-of-equilibrium-ness”



freezing of out-of-equilibrium modes

A simplified model





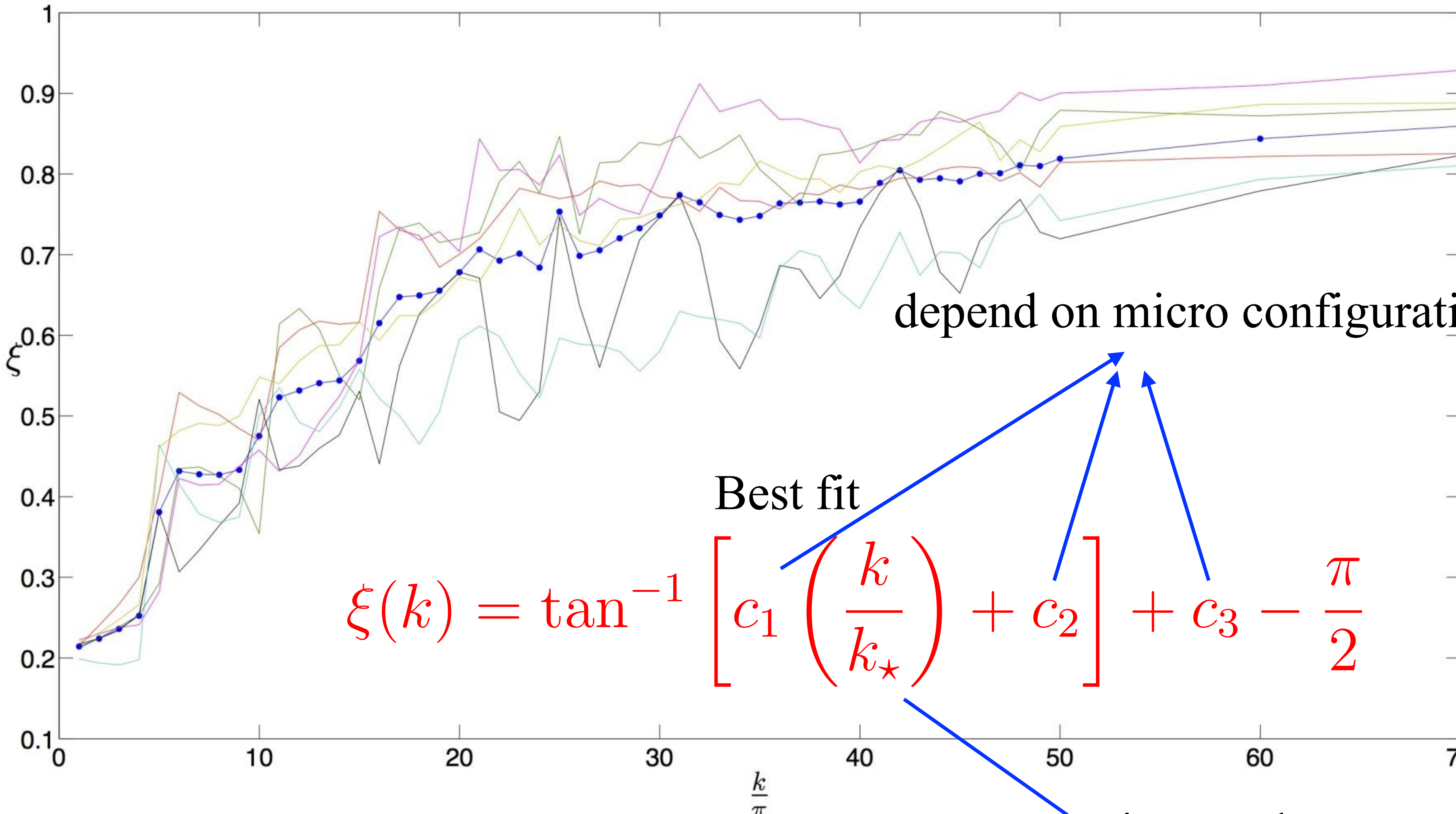
ln(length)
physical wavelength λ_{phys}
Hubble radius H^{-1}
pre-inflation
inflation
$\ln a_i$
$\ln a_f$
$\ln a$

Initial out-of-equilibrium conditions

S. Colin & A. Valentini, arXiv:1407.8262

$$\mathcal{P}(k) = \mathcal{P}(k)_{\mathrm{QE}}\,\xi(k)$$

width deficit

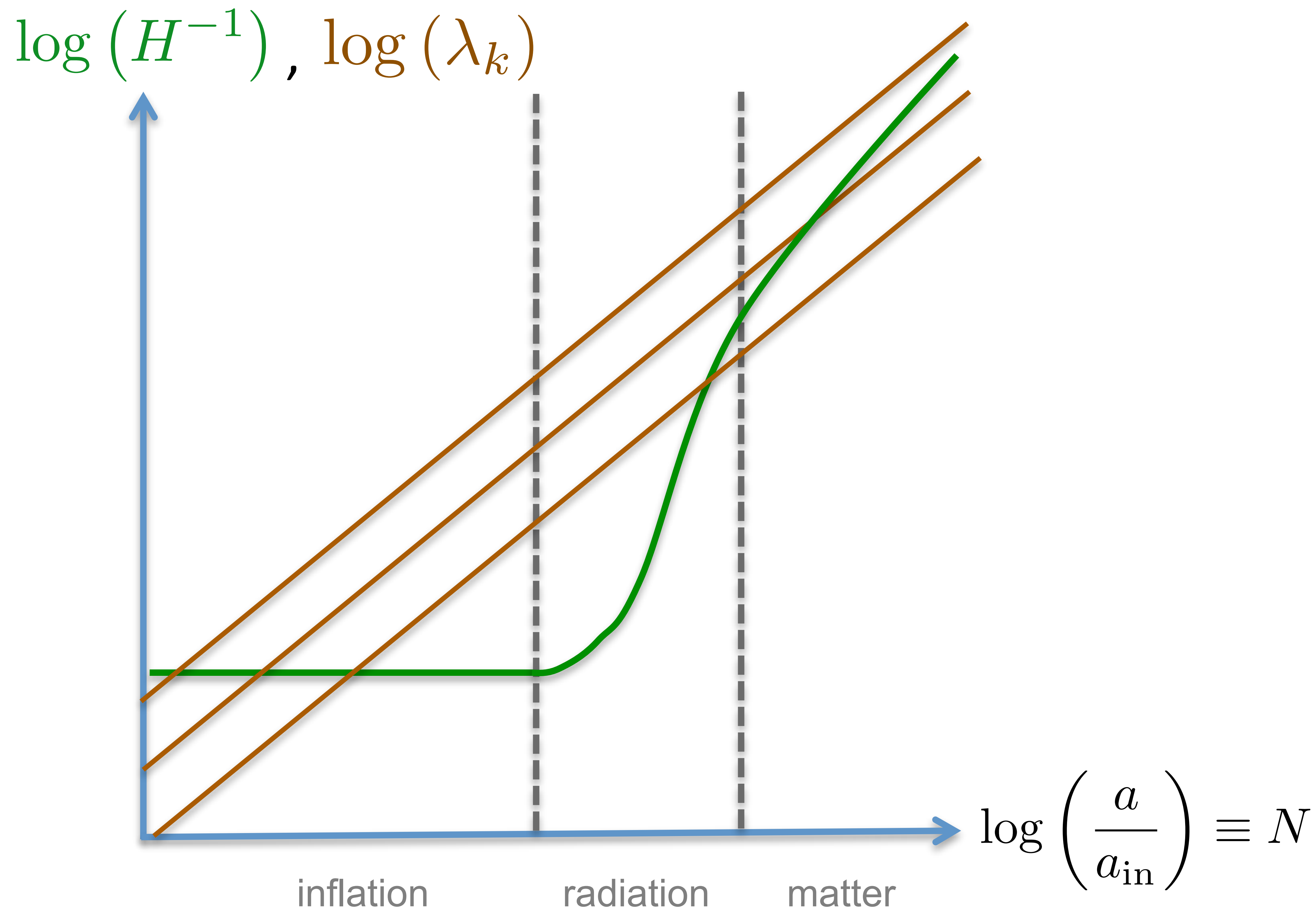


depend on micro configuration

Best fit

$$\xi(k) = \tan^{-1}\left[c_1\left(\frac{k}{k_\star}\right) + c_2\right] + c_3 - \frac{\pi}{2}$$

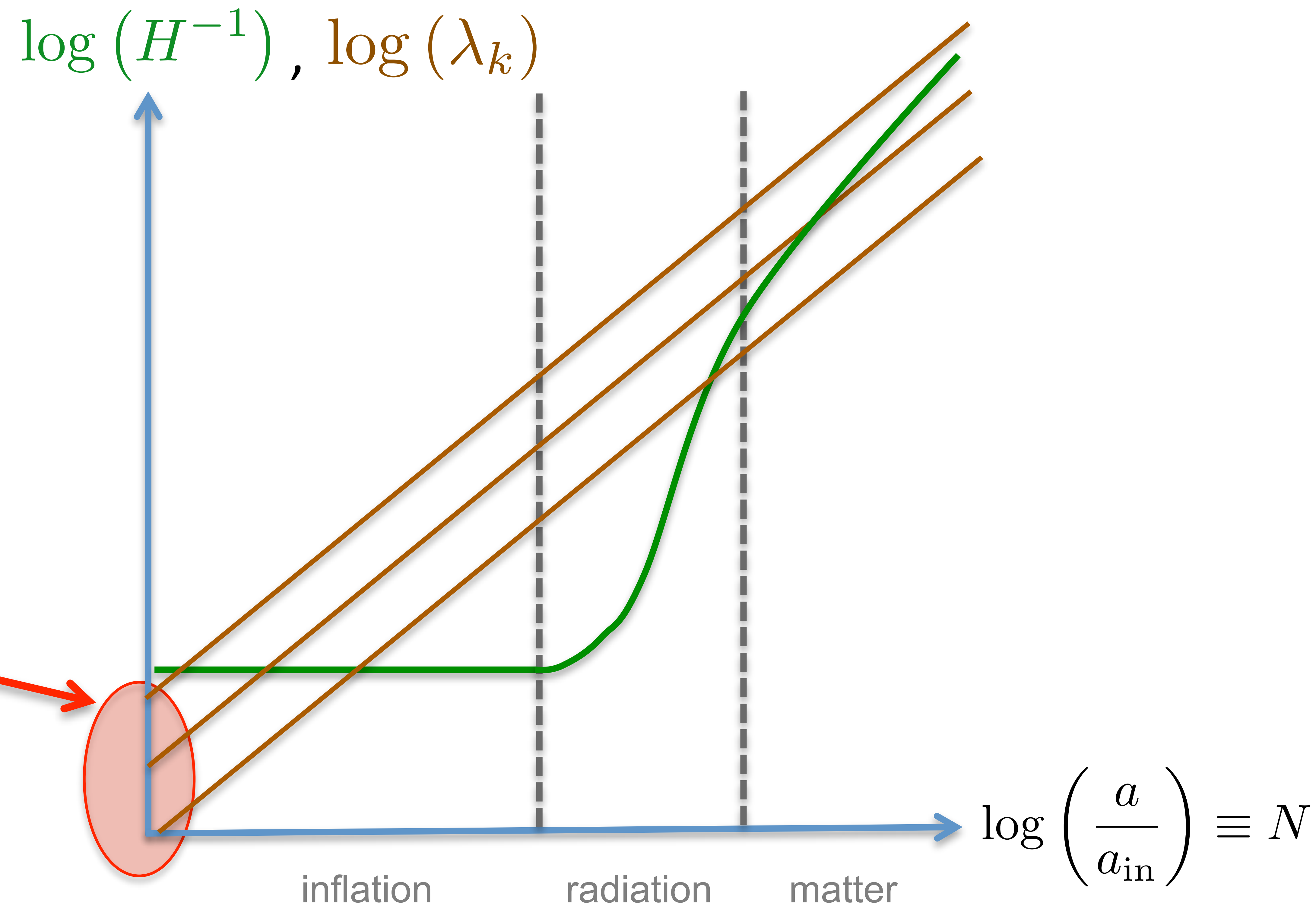
pivot scale



$\log\left(H^{-1}\right)$, $\log\left(\lambda_k\right)$
$\log\left(\frac{a}{a_{\rm in}}\right) \equiv N$
inflation
radiation
matter

Harmonic oscillator fundamental state

$$\Psi_{\boldsymbol{k}} = \left(\frac{k}{\pi}\right)^{\frac{1}{4}} \mathrm{e}^{-\frac{k}{2}v_{\boldsymbol{k}}^2}$$



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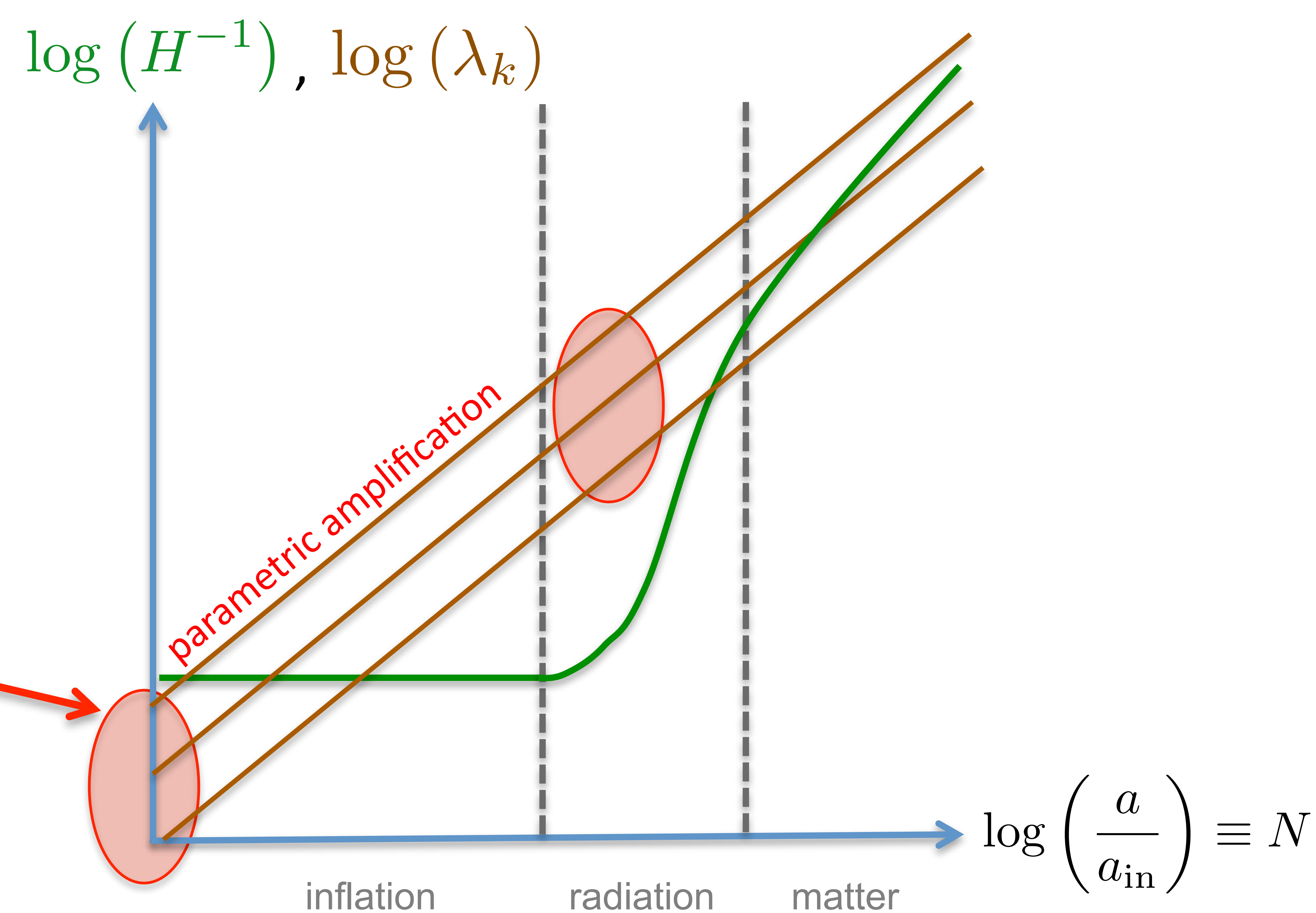

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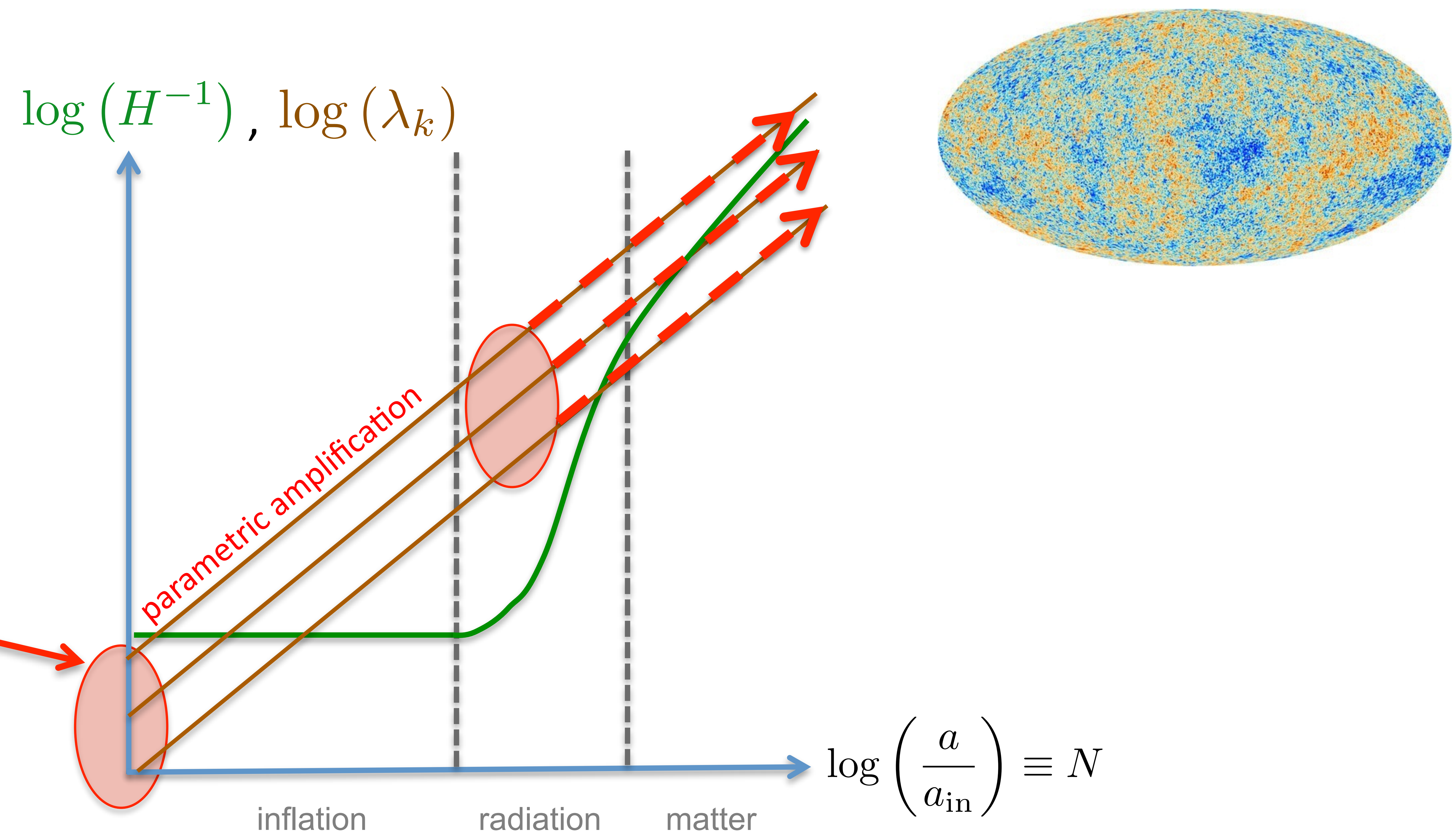

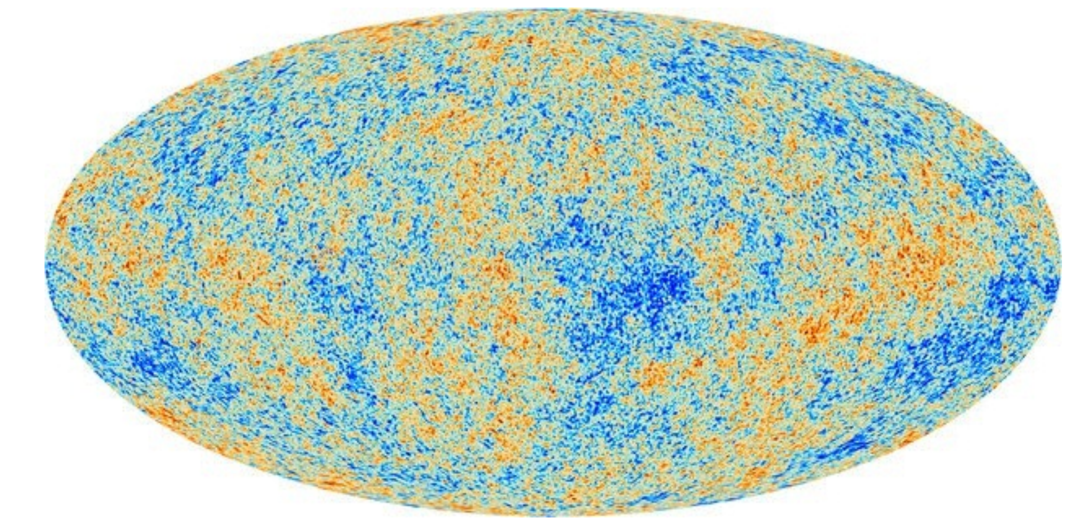

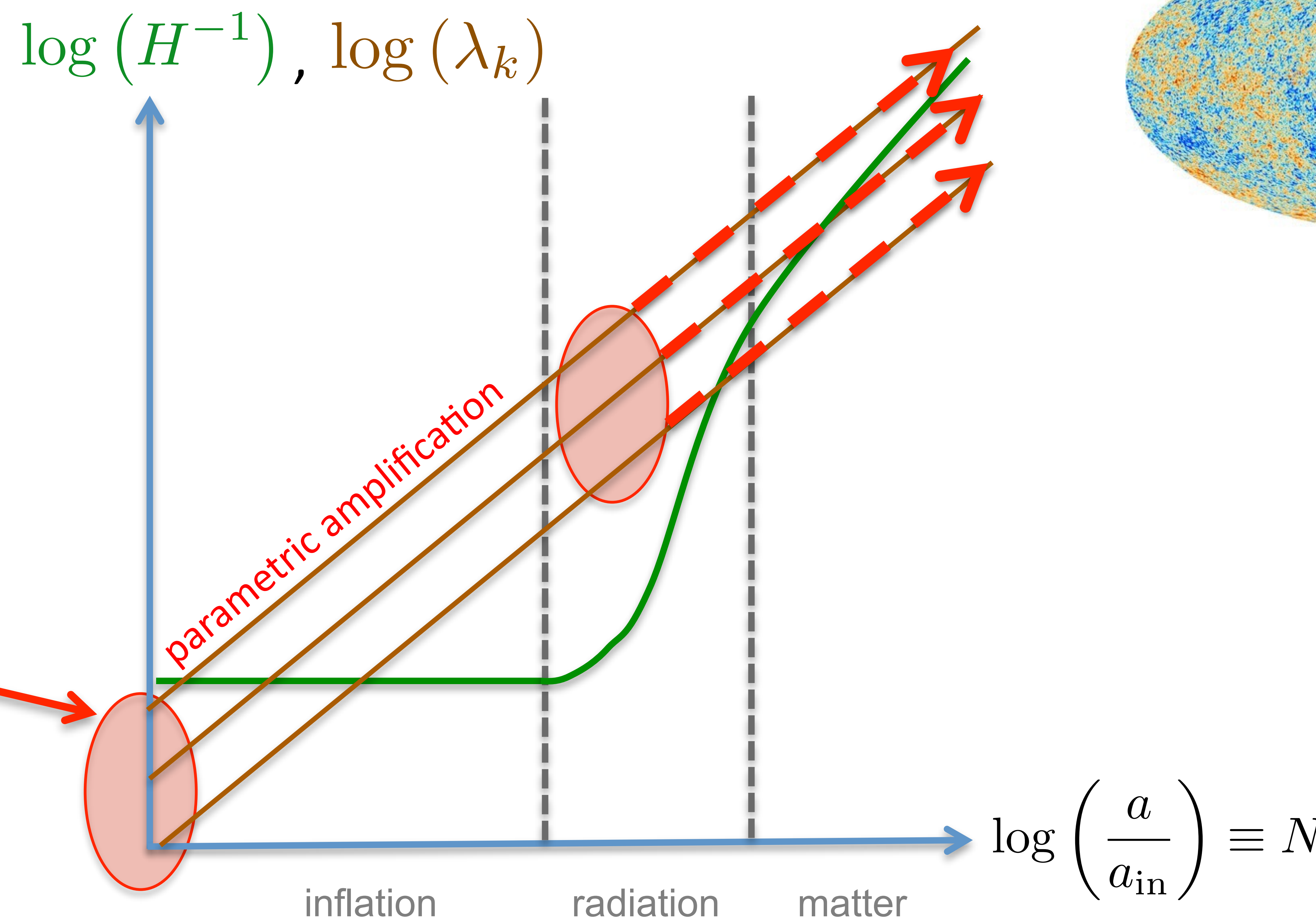

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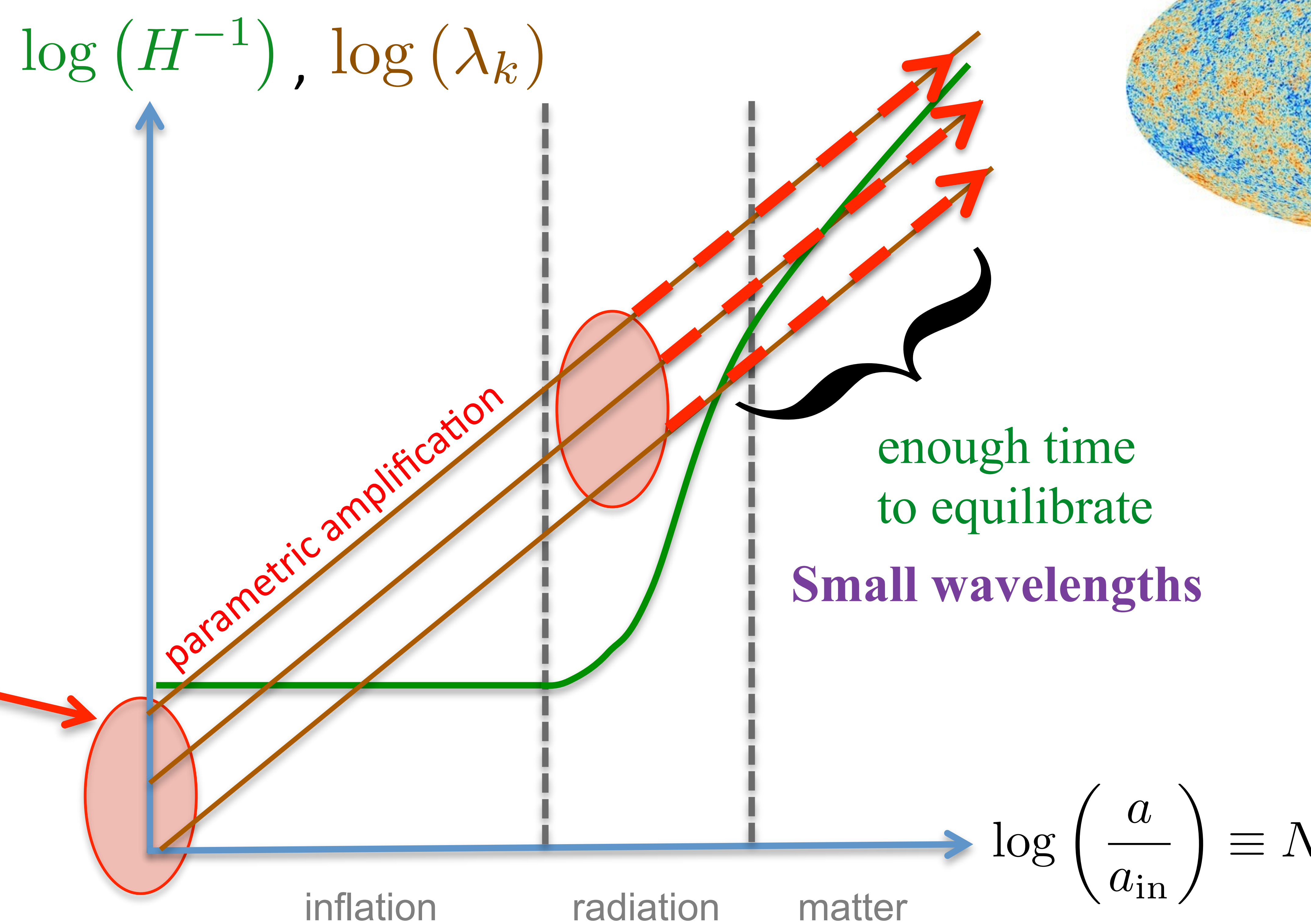
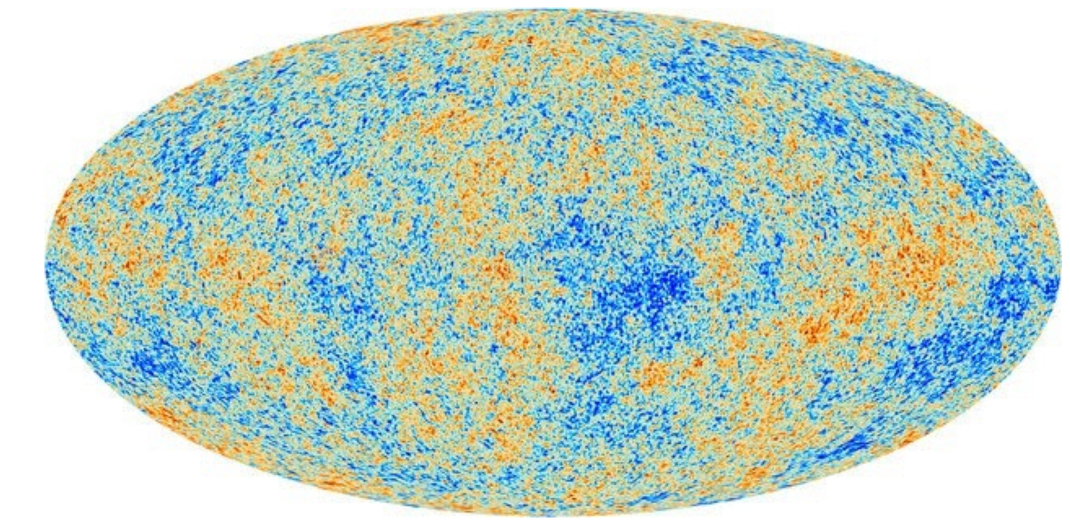
Out-of-Equilibrium
initial density:
less quantum noise





Harmonic oscillator
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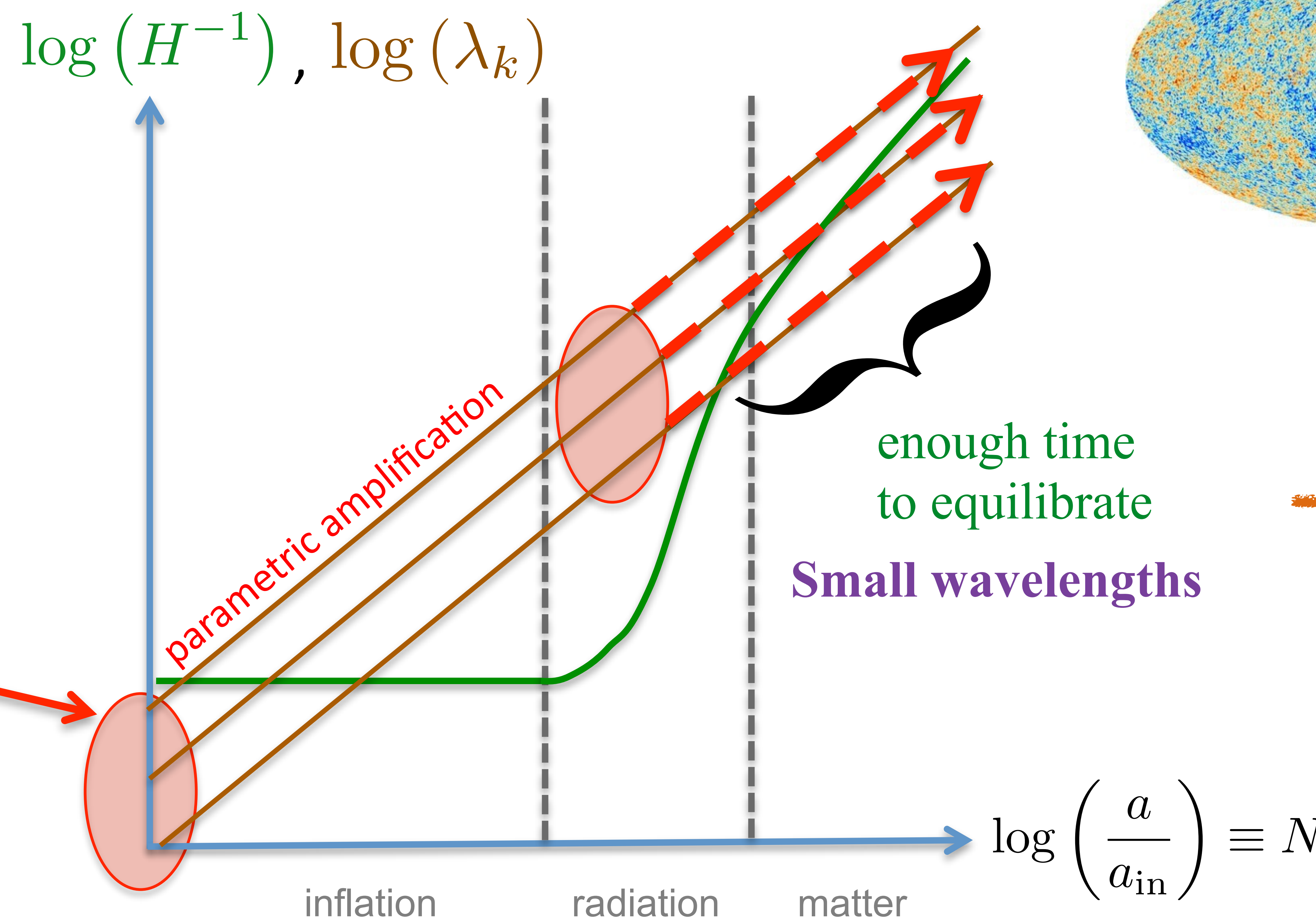

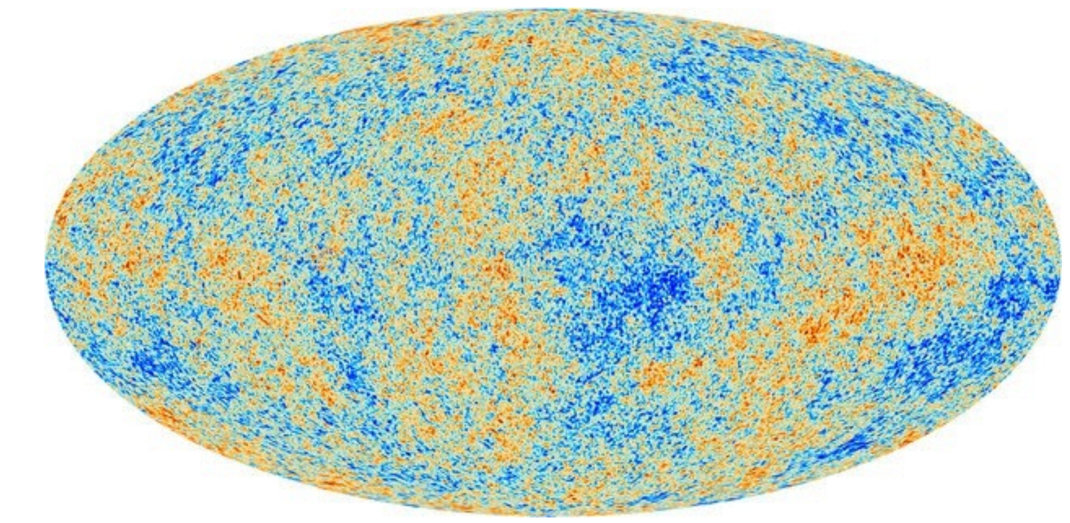

usual spectrum

Harmonic oscillator
fundamental state

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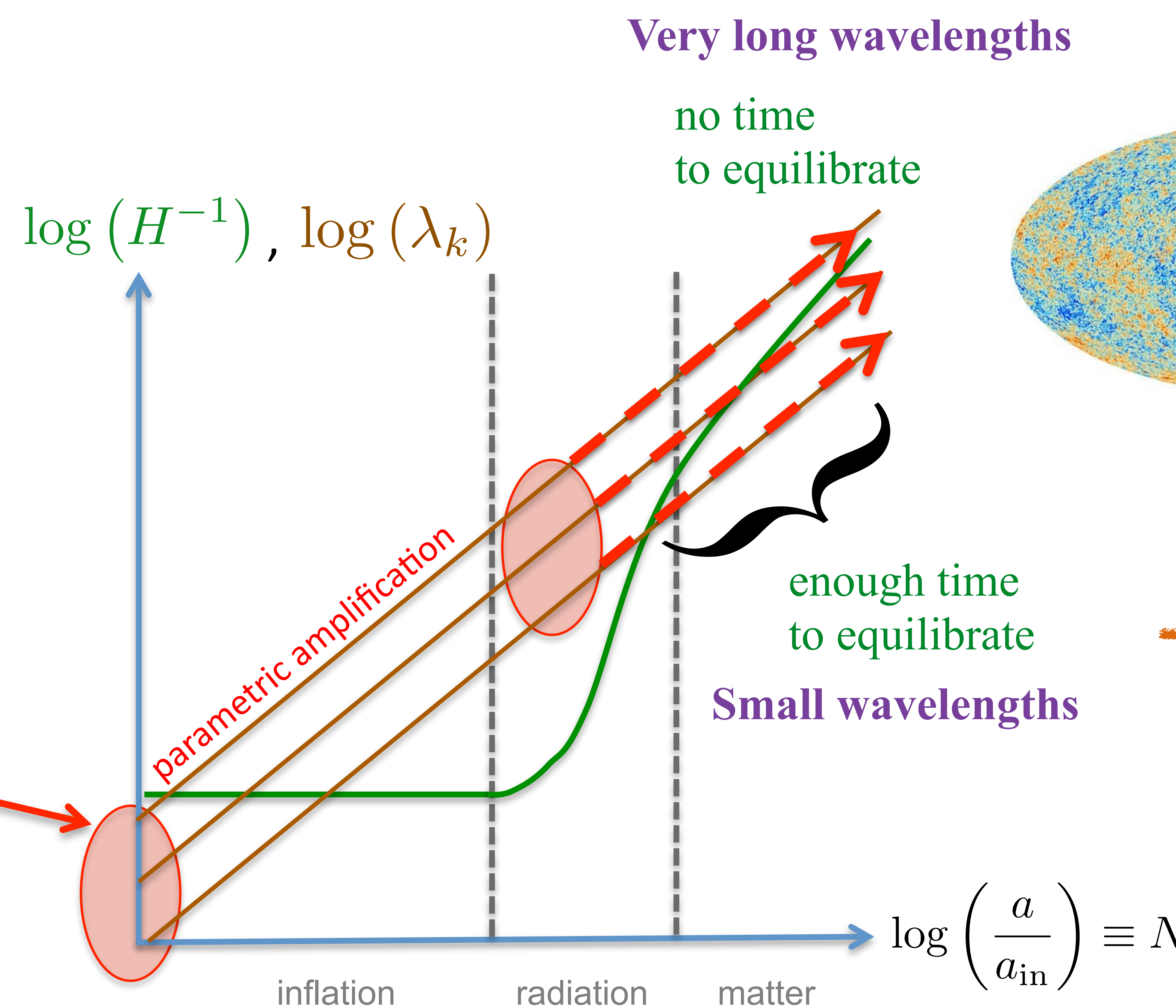
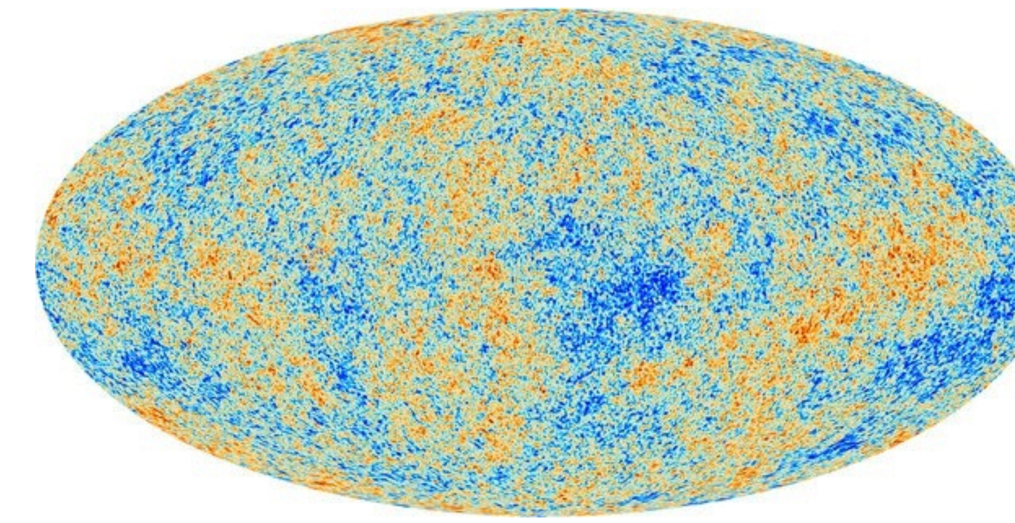
Out-of-Equilibrium
initial density:
less quantum noise

Harmonic oscillator fundamental state

$$\Psi_{\boldsymbol{k}} = \left(\frac{k}{\pi}\right)^{\frac{1}{4}} \mathrm{e}^{-\frac{k}{2}v_{\boldsymbol{k}}^2}$$

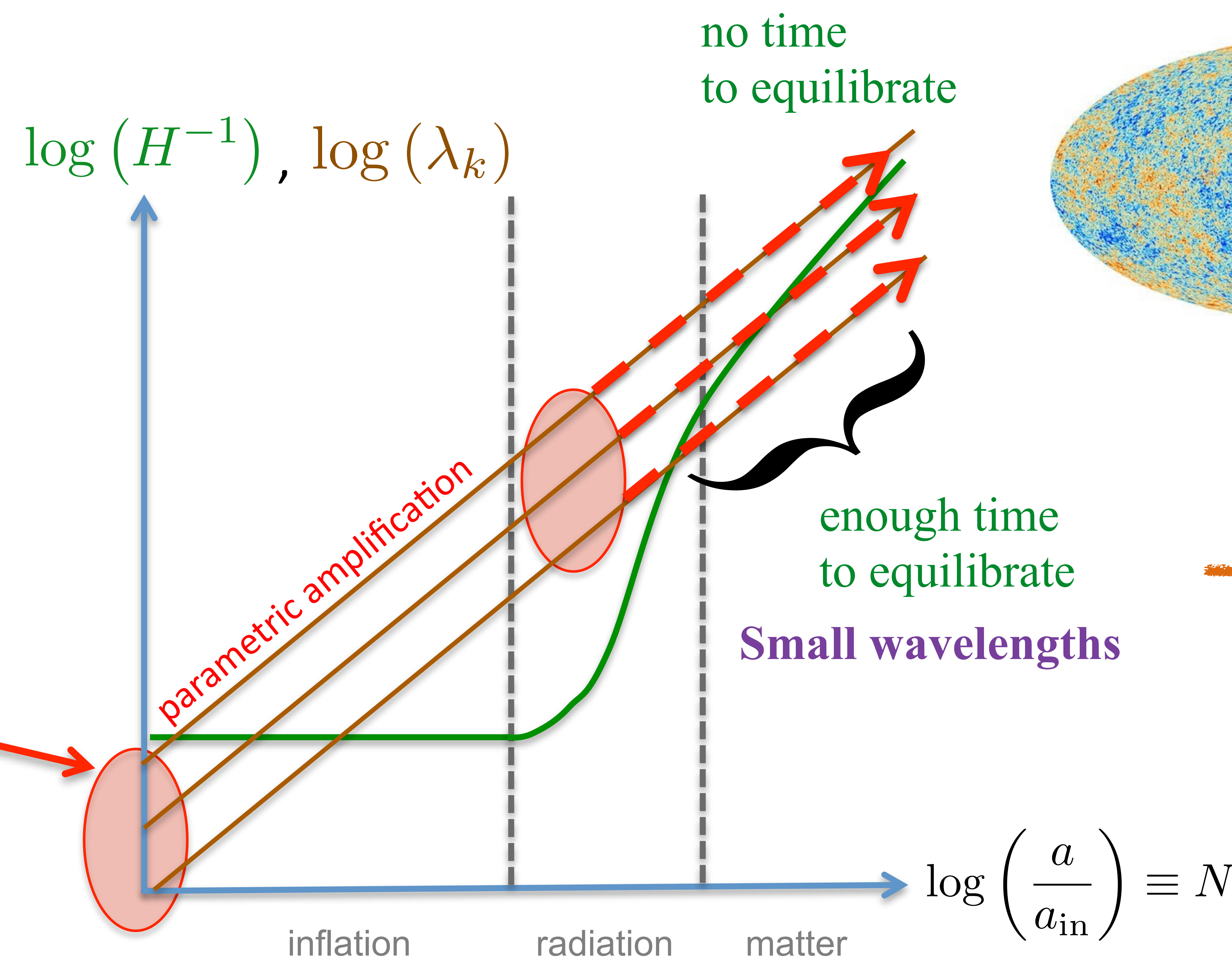


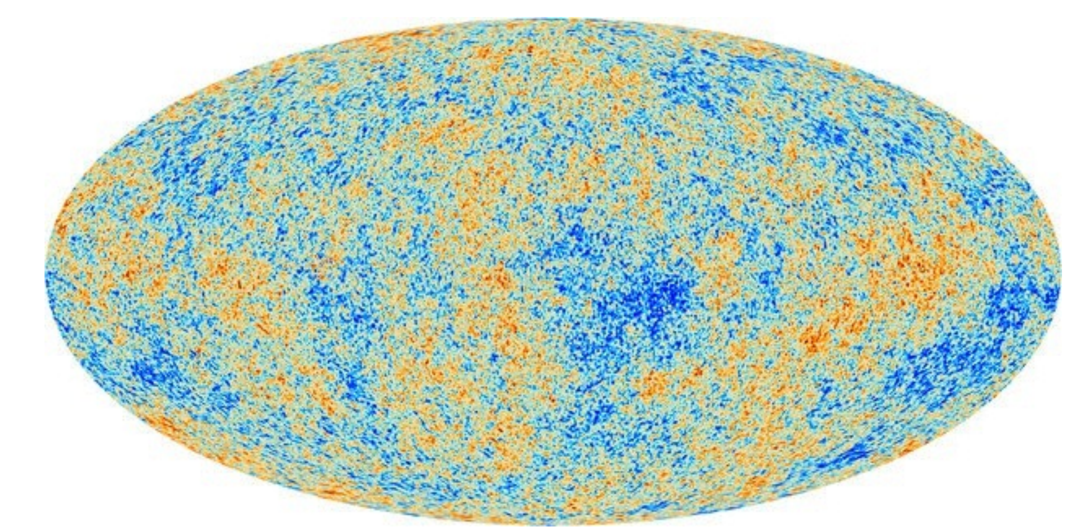


usual spectrum

Out-of-Equilibrium
initial density:
less quantum noise

less power ← **Very long wavelengths**



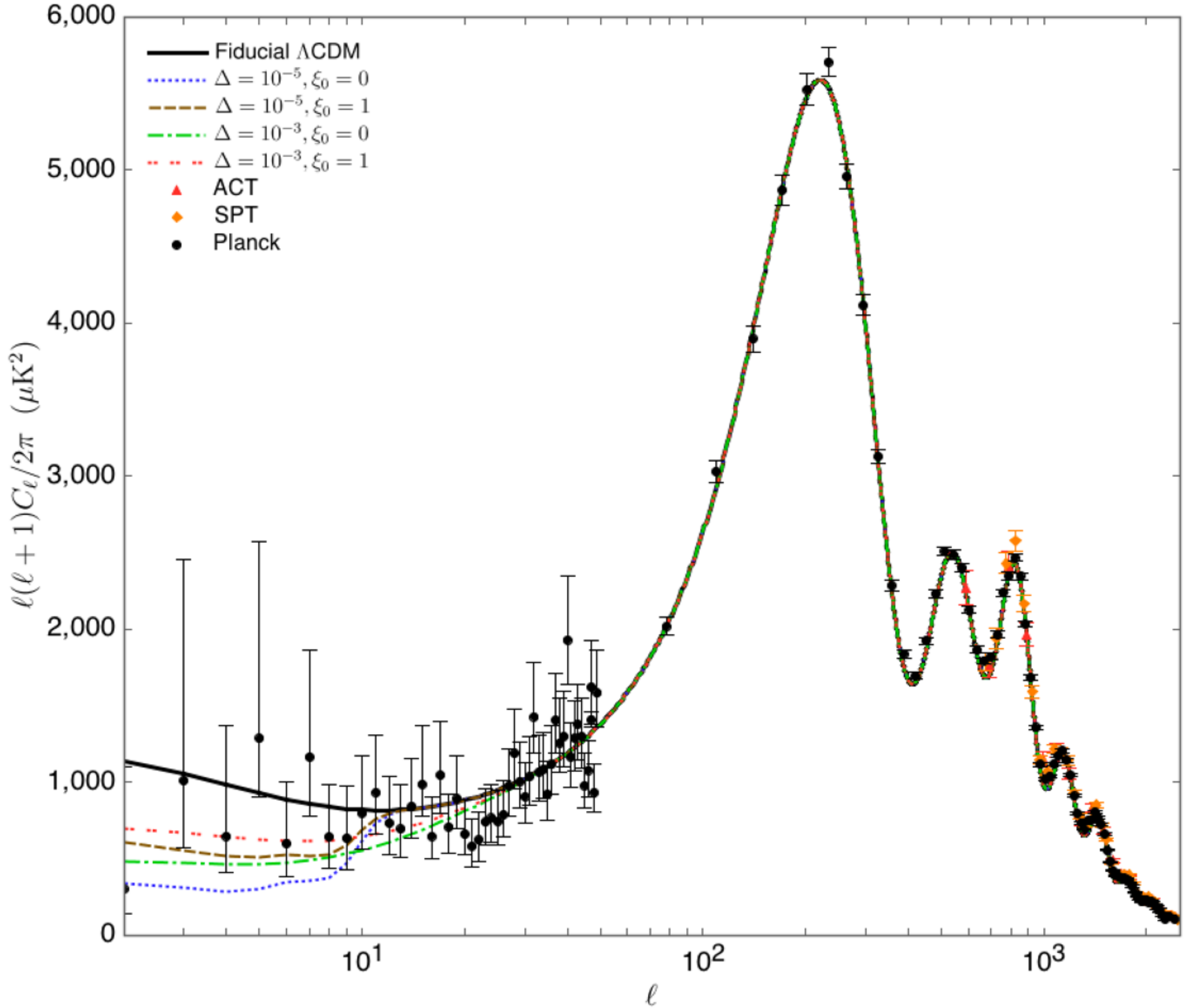


→ usual spectrum

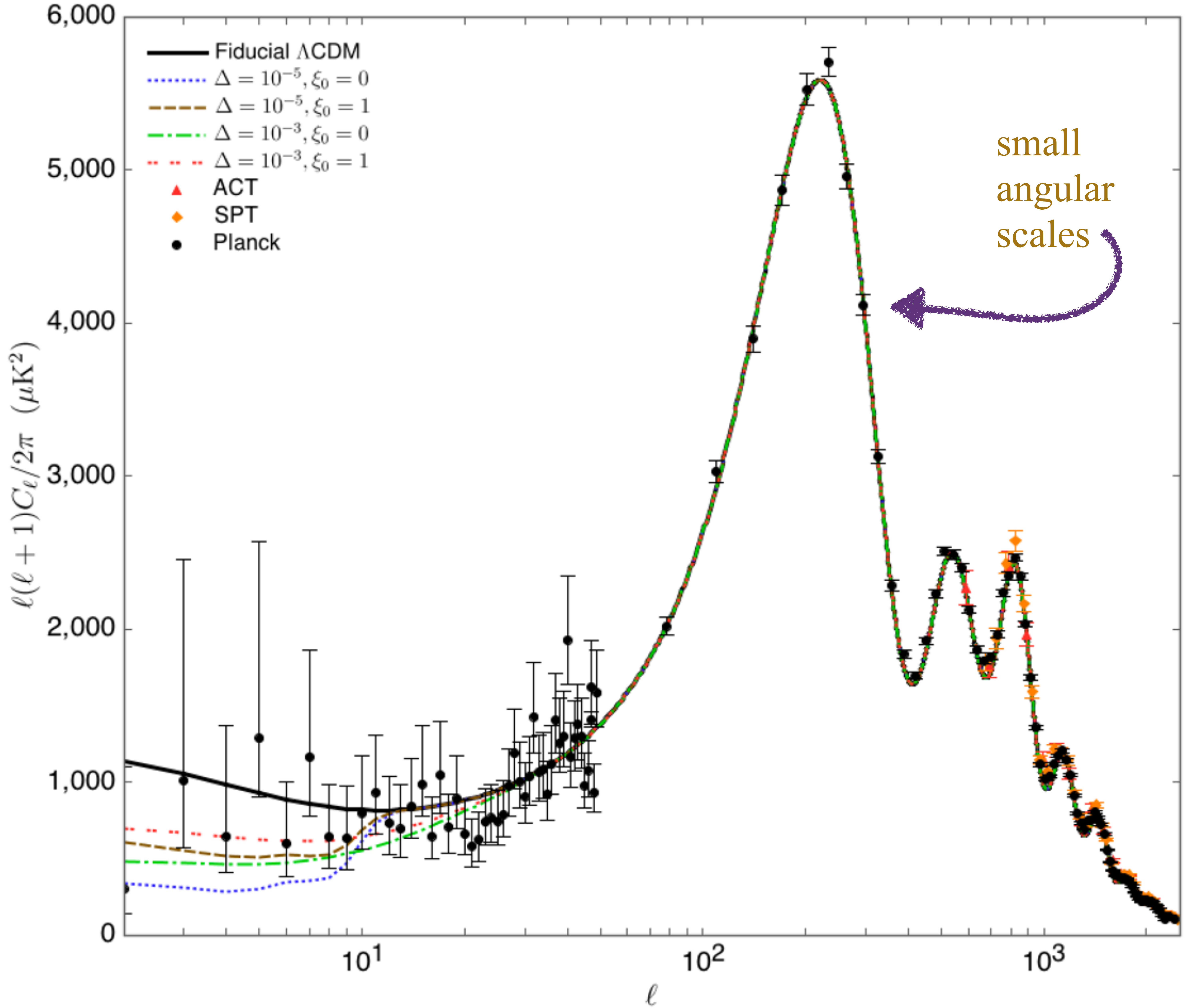
Harmonic oscillator fundamental state

$$\Psi_{\boldsymbol{k}} = \left(\frac{k}{\pi}\right)^{\frac{1}{4}} \mathrm{e}^{-\frac{k}{2}v_{\boldsymbol{k}}^2}$$

Out-of-Equilibrium
initial density:
less quantum noise



6,000
5,000
4,000
3,000
2,000
1,000
0
Fiducial ΛCDM
$\Delta = 10^{-5}, \xi_0 = 0$
$\Delta = 10^{-5}, \xi_0 = 1$
$\Delta = 10^{-3}, \xi_0 = 0$
$\Delta = 10^{-3}, \xi_0 = 1$
ACT
SPT
Planck
$\ell(\ell+1)C_\ell/2\pi \ (\mu K^2)$
10^1
10^2
10^3
ℓ



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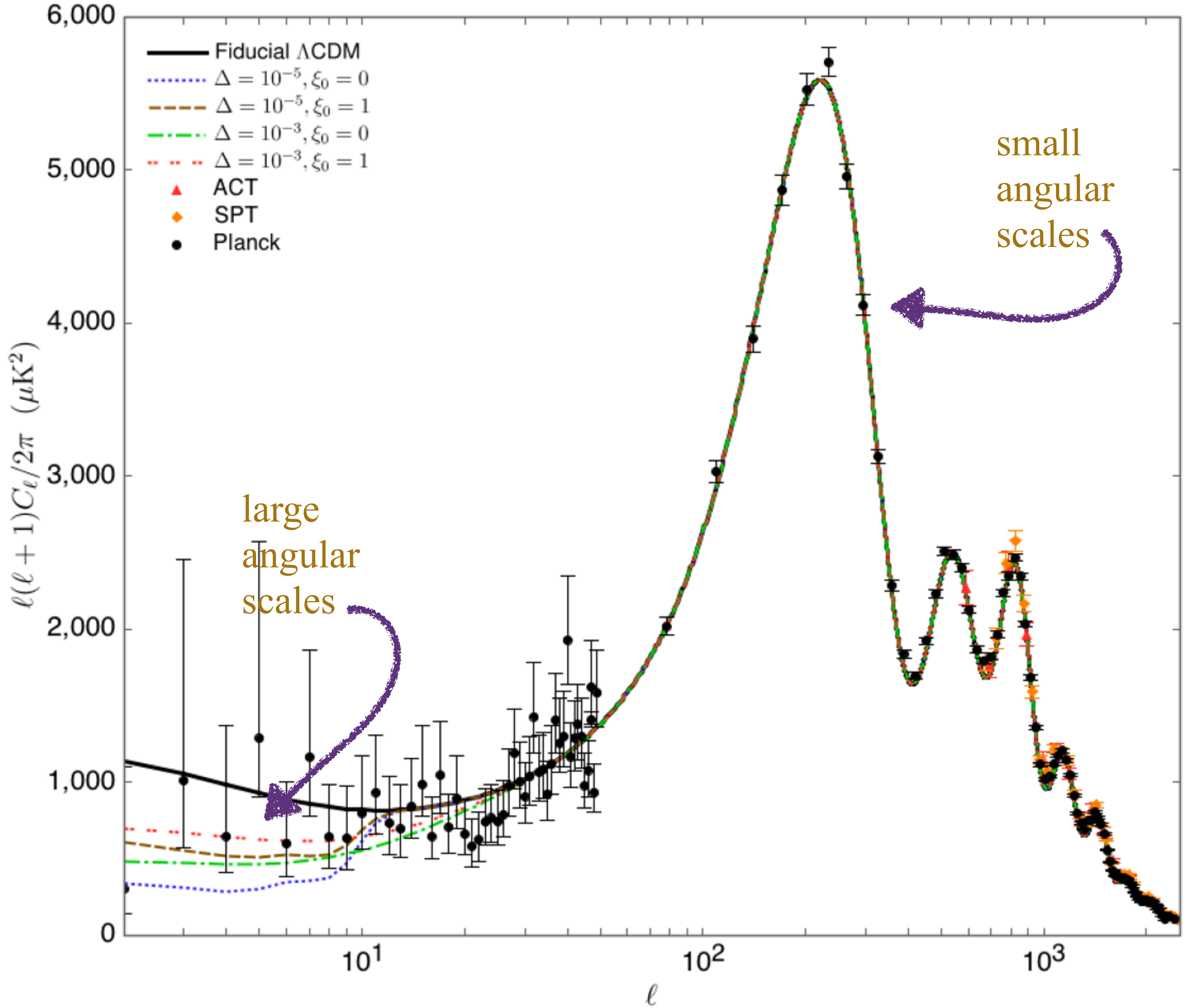
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small angular scales
large angular scales
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6,000
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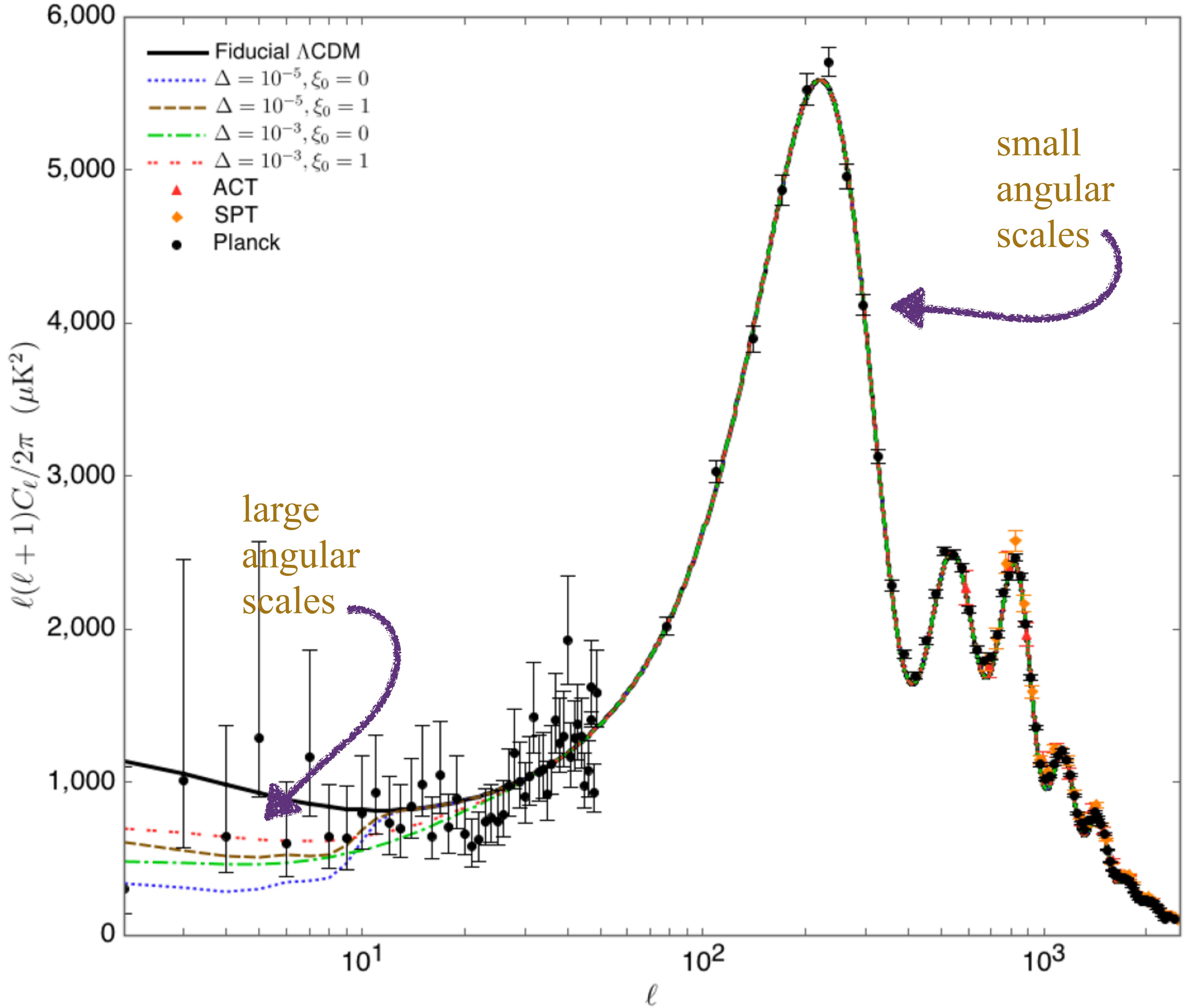

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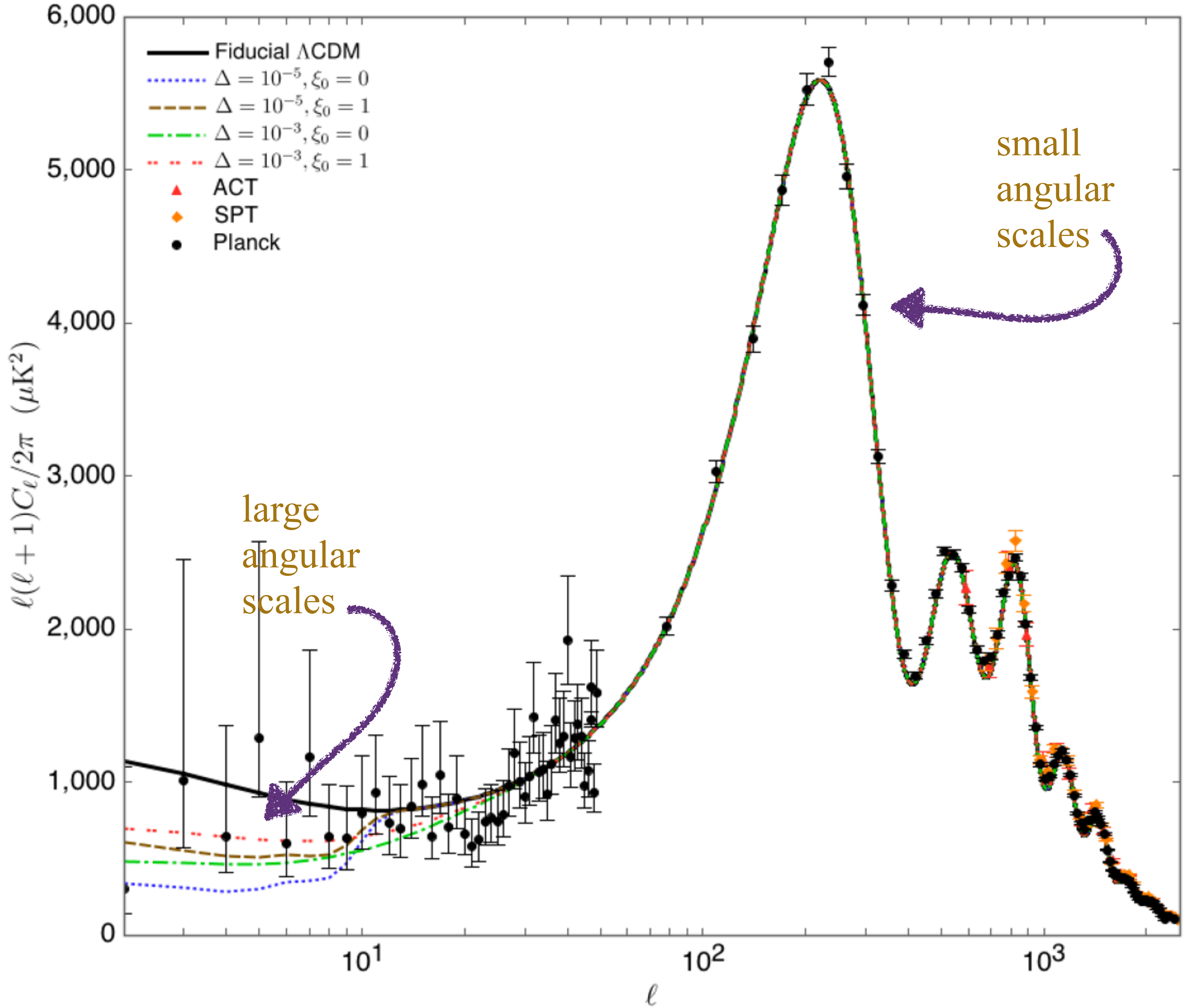
Better fit???



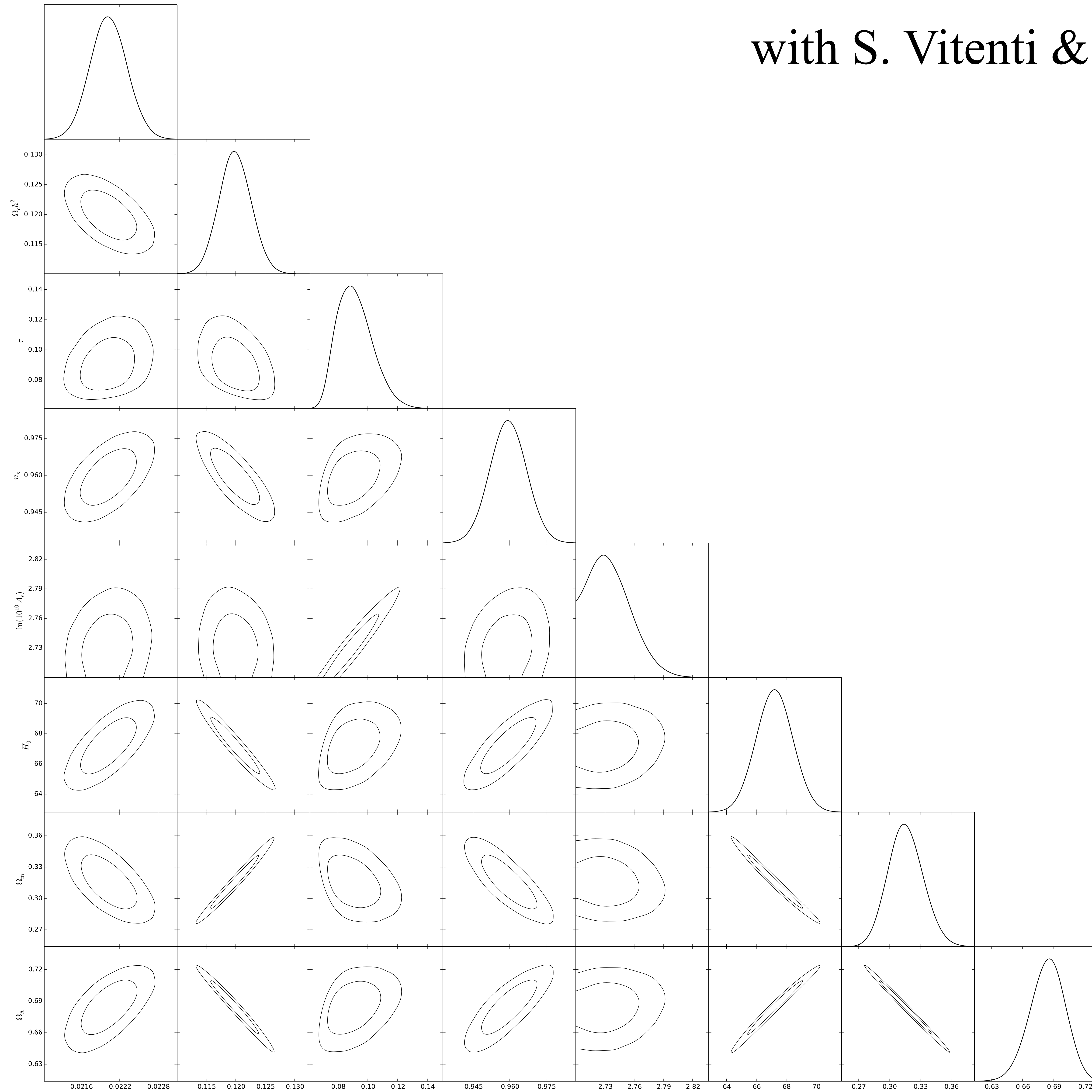
Better fit???

CosmoMC chains

Results…

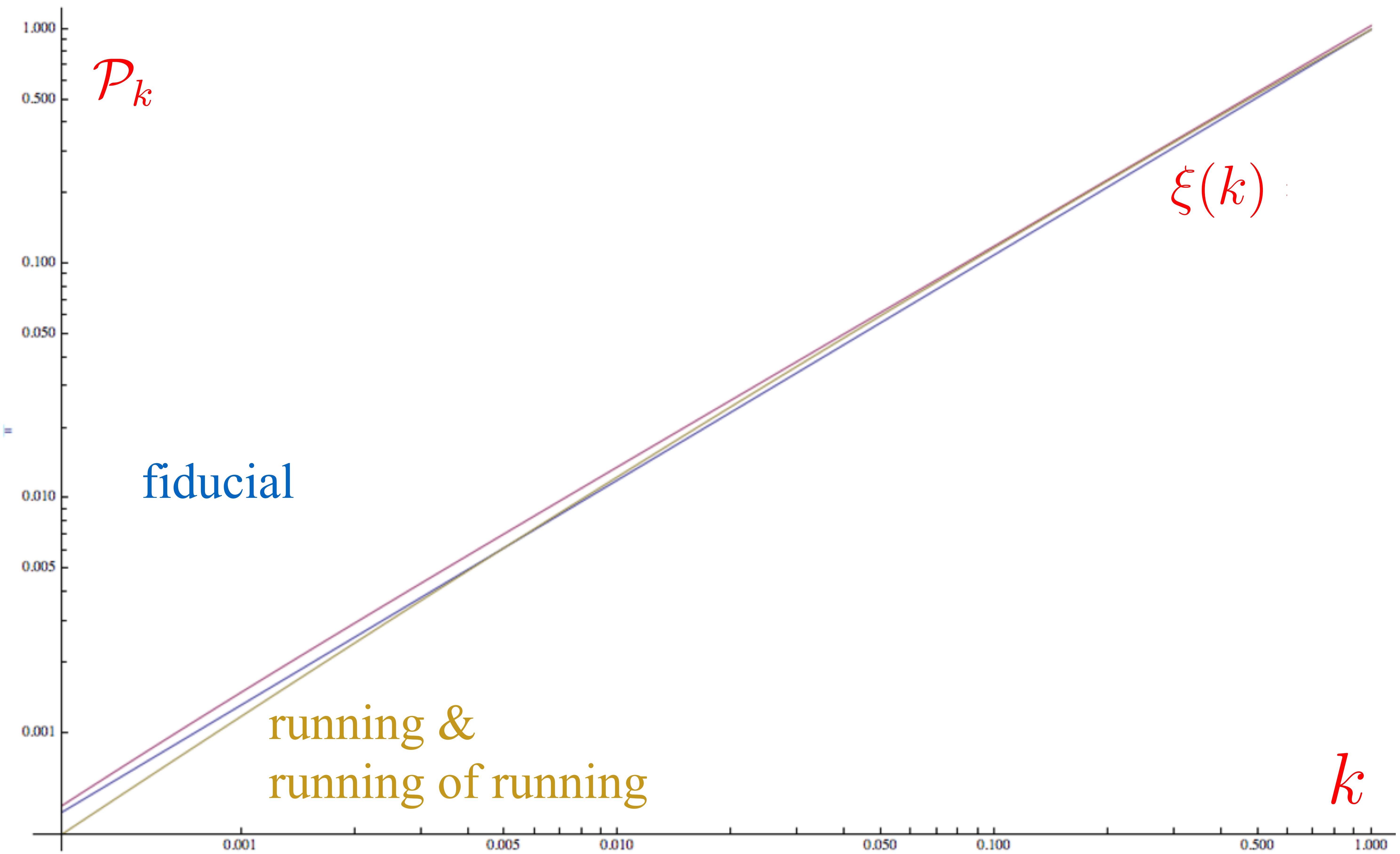
work in progress!

with S. Vitenti & A. Valentini



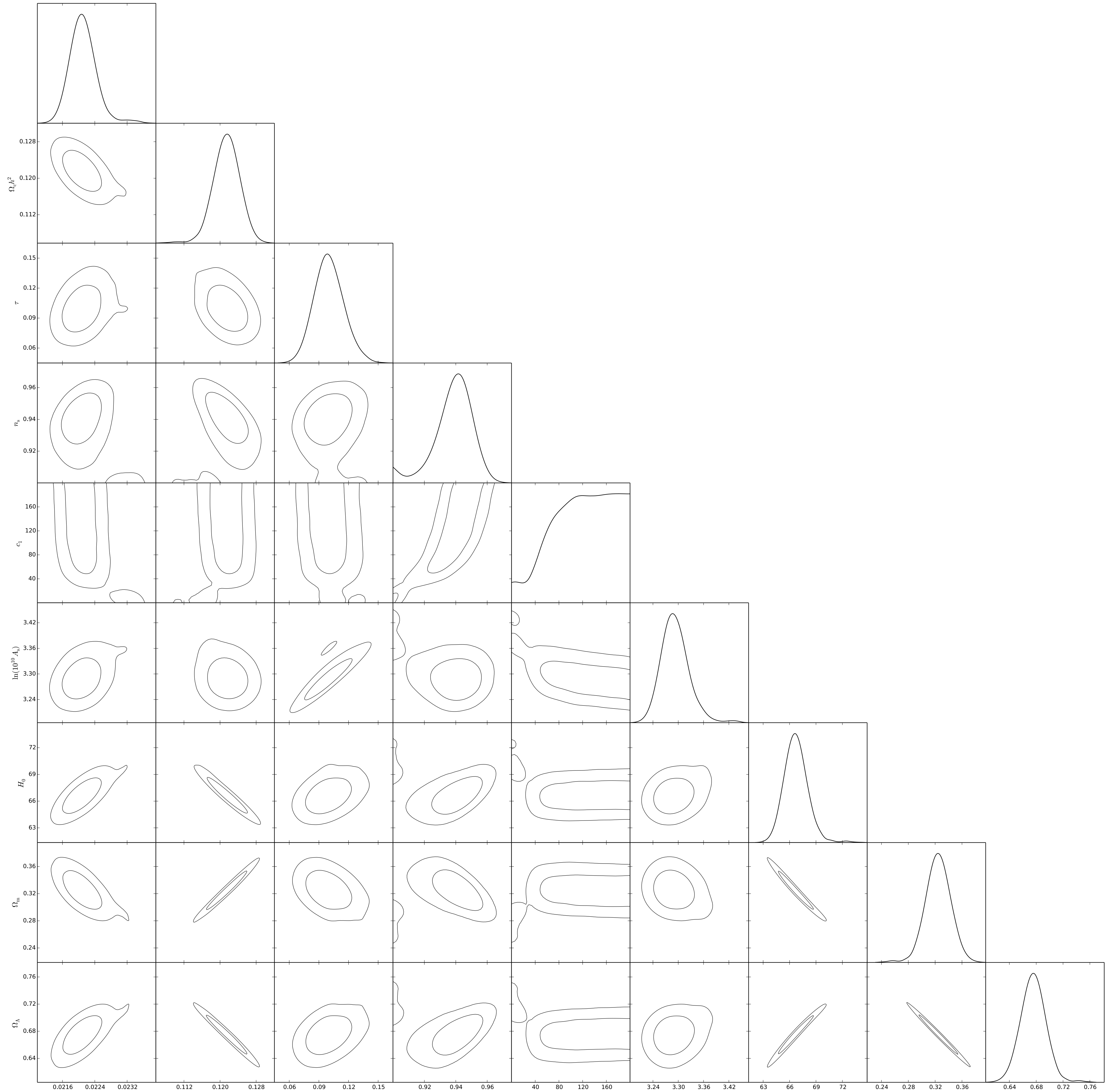
Usual Planck best-fit

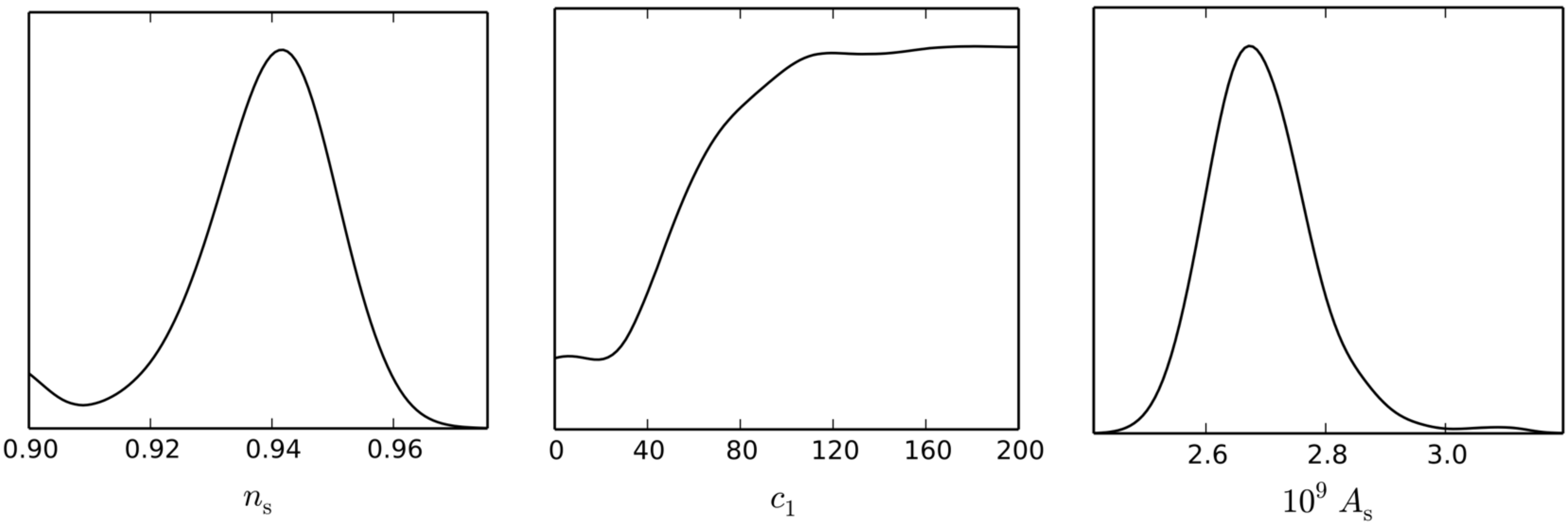
with only one parameter added, others held fixed: $\xi(k) = \tan^{-1}\left[c_1\left(\frac{k}{k_\star}\right) + \underbrace{c_2}_{3.0}\right] + \underbrace{c_3}_{0.85} - \frac{\pi}{2}$



with only one parameter added, others held fixed: $\xi(k) = \tan^{-1}\left[c_1\left(\frac{k}{k_\star}\right) + c_2\right] + c_3 - \frac{\pi}{2}$

$c_2 \to 3.0$ $\quad c_3 \to 0.85$

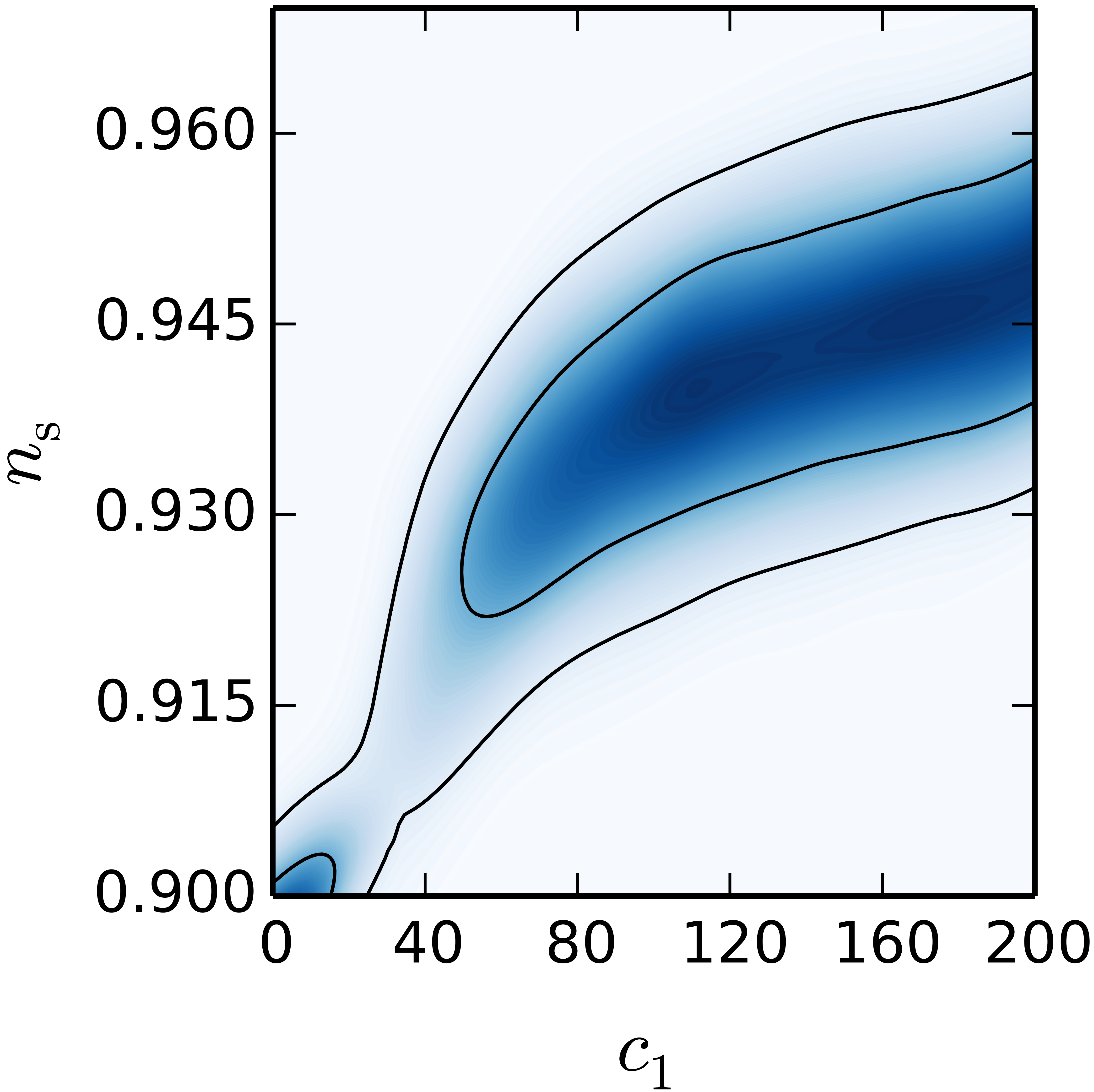




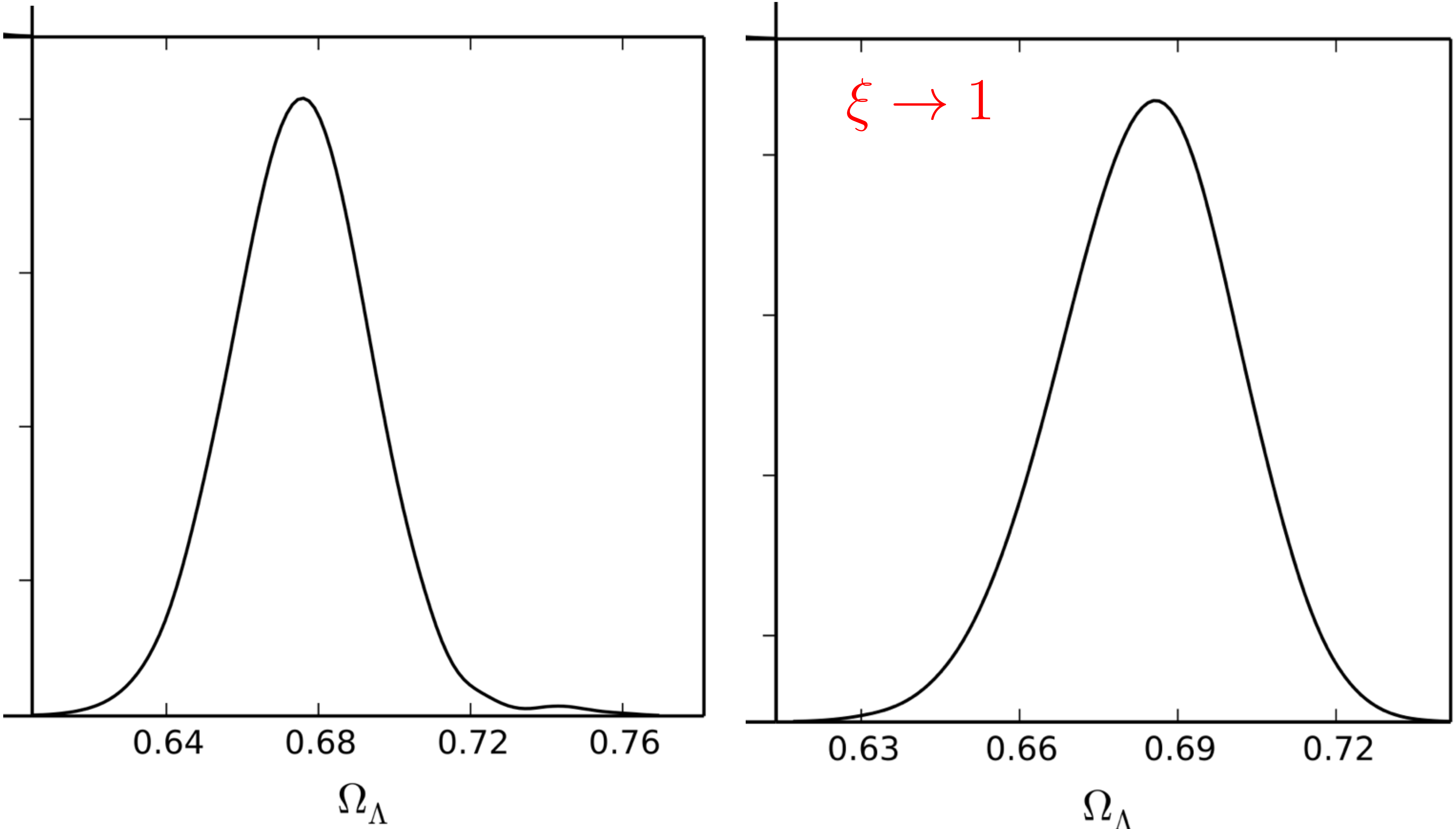
redder…

not conclusive

degeneracy

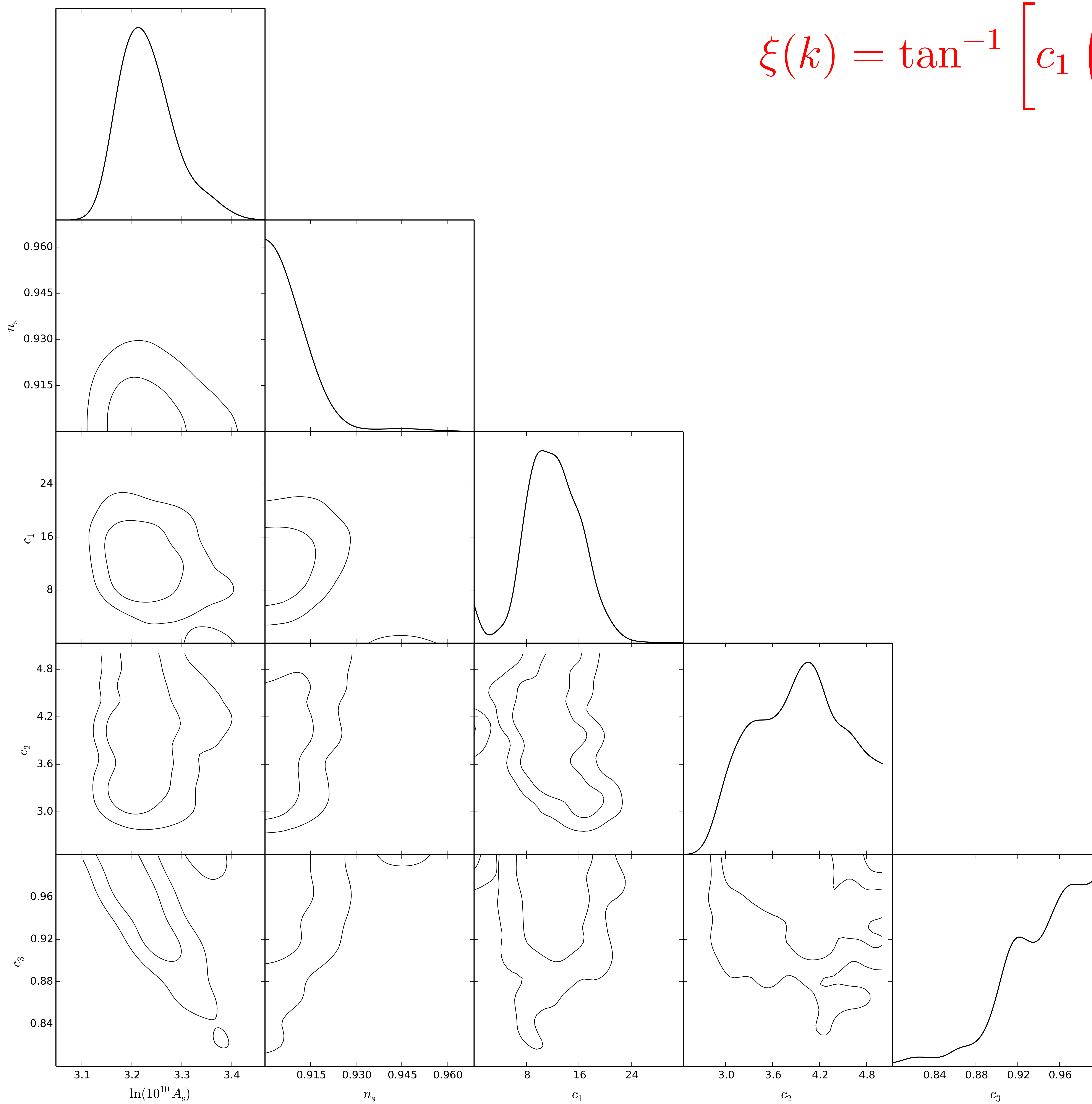


convergence???



compatibility with SN Ia data?

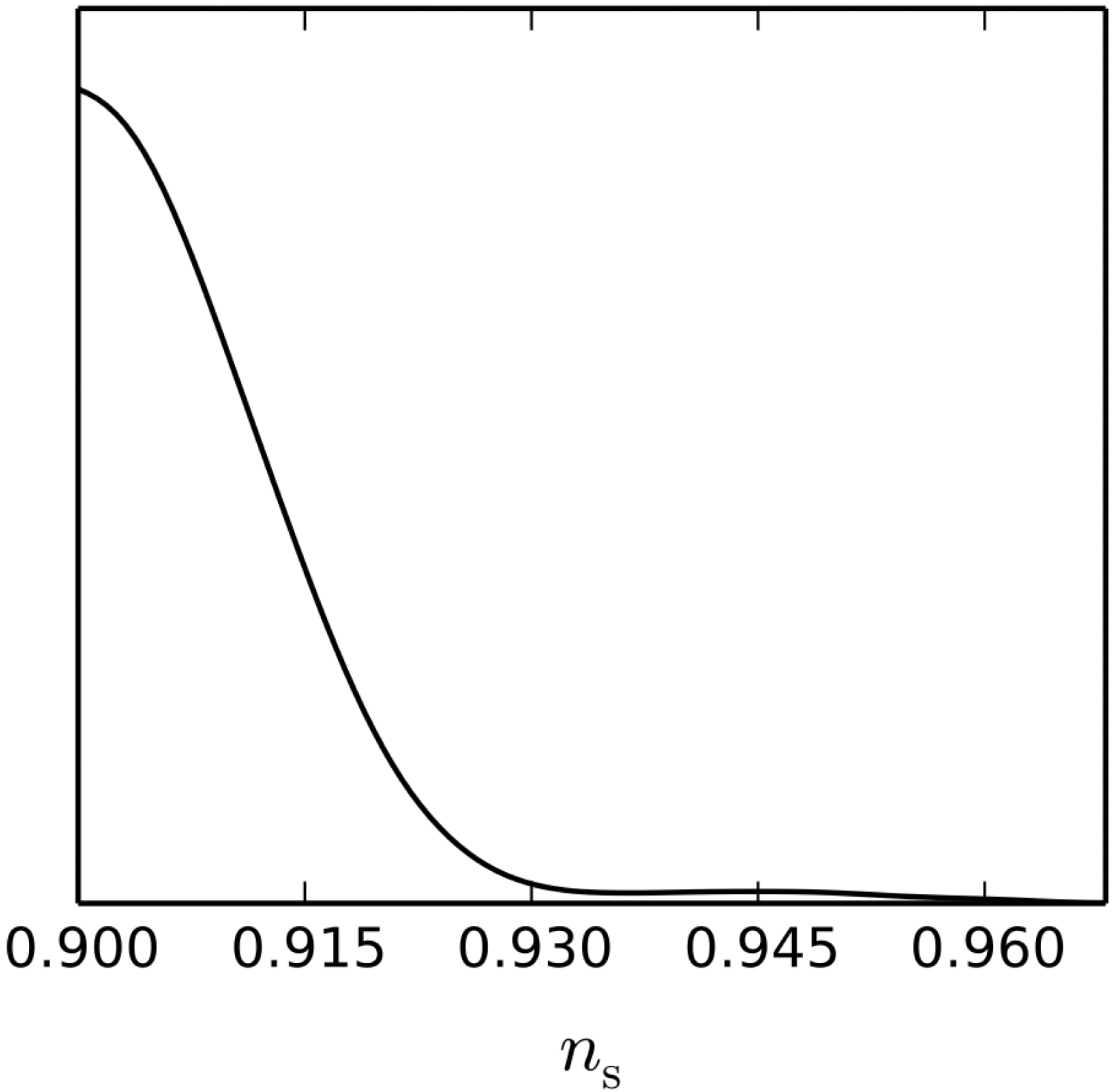
Constrained model



$$\xi(k) = \tan^{-1}\left[c_1\left(\frac{k}{k_\star}\right) + c_2\right] + c_3 - \frac{\pi}{2}$$

prior

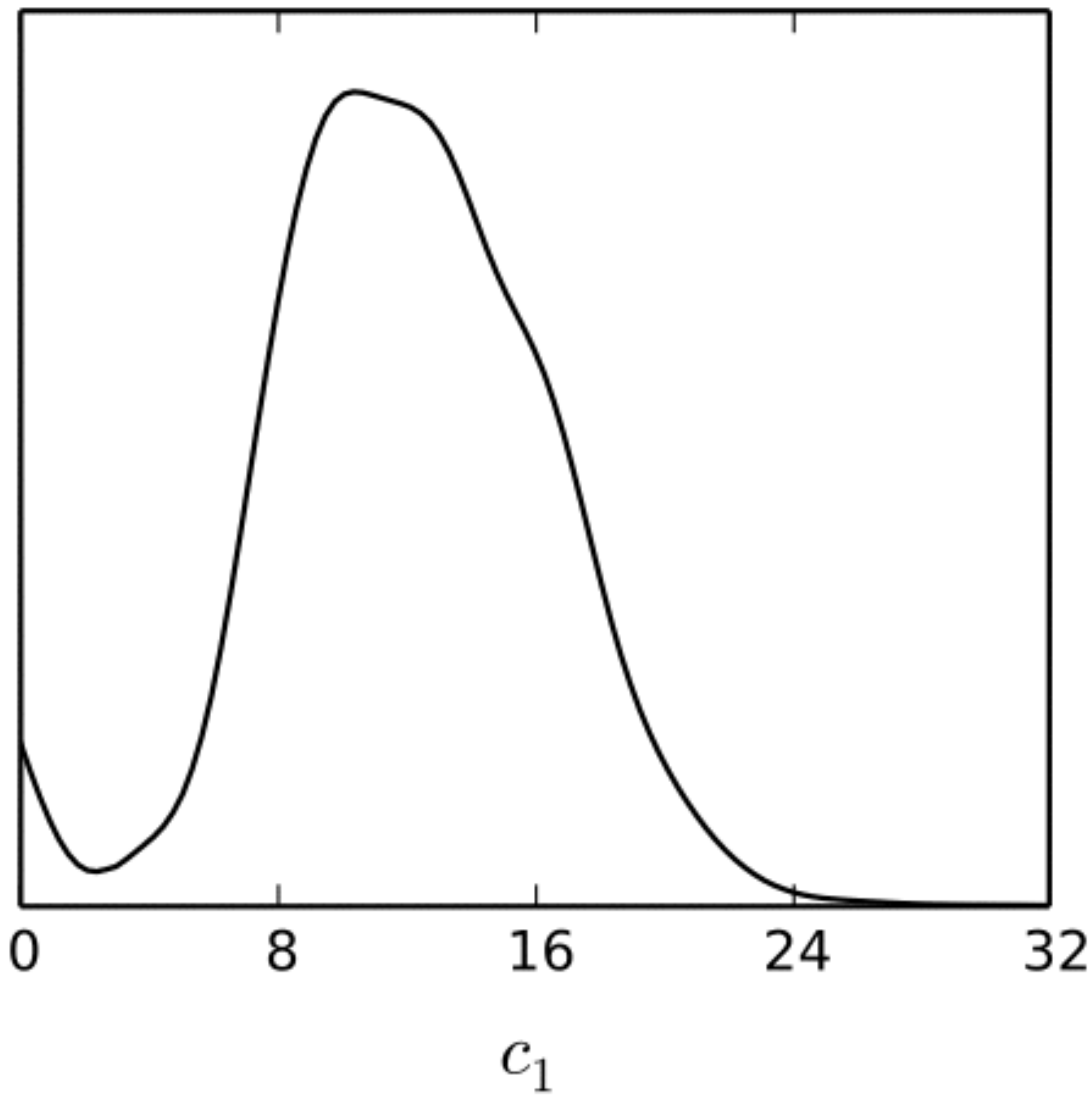
$c_3 < 1$

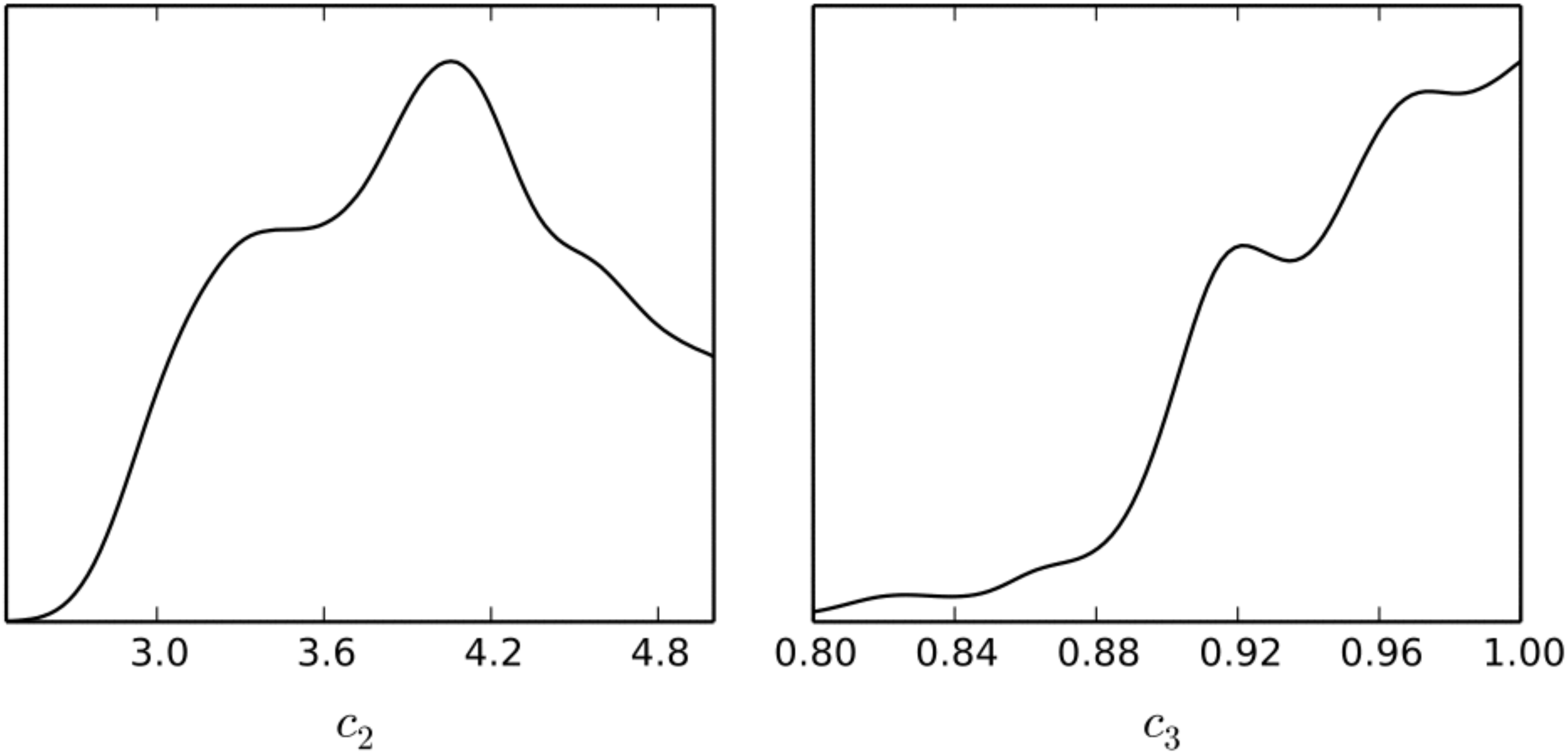


demands a very
red primordial spectrum

$$\xi(k) = \tan^{-1}\left[c_1\left(\frac{k}{k_\star}\right) + c_2\right] + c_3 - \frac{\pi}{2}$$

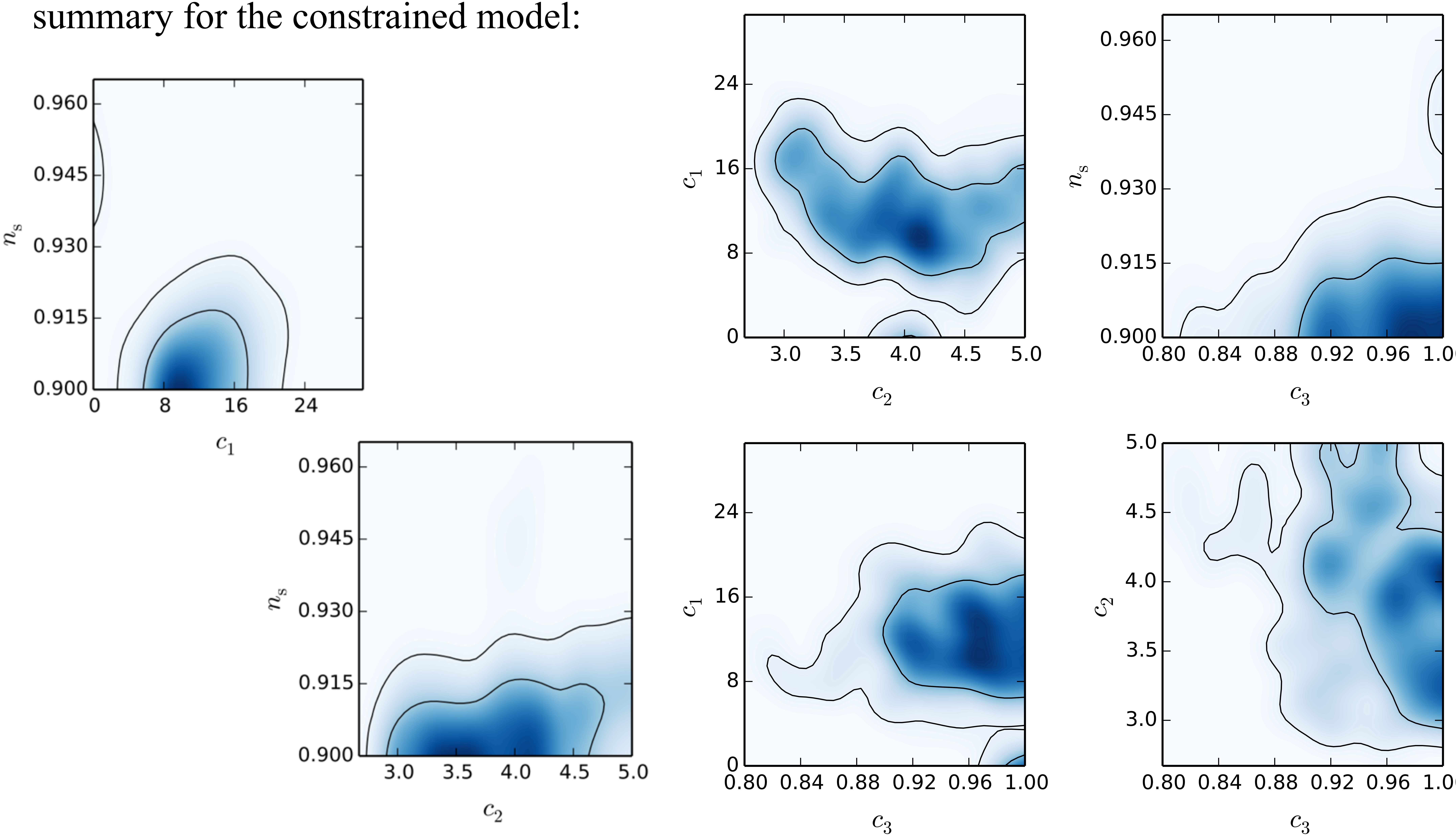
much smaller quantum scale





not very conclusive, but seems to favor $c_3 \geq 1$

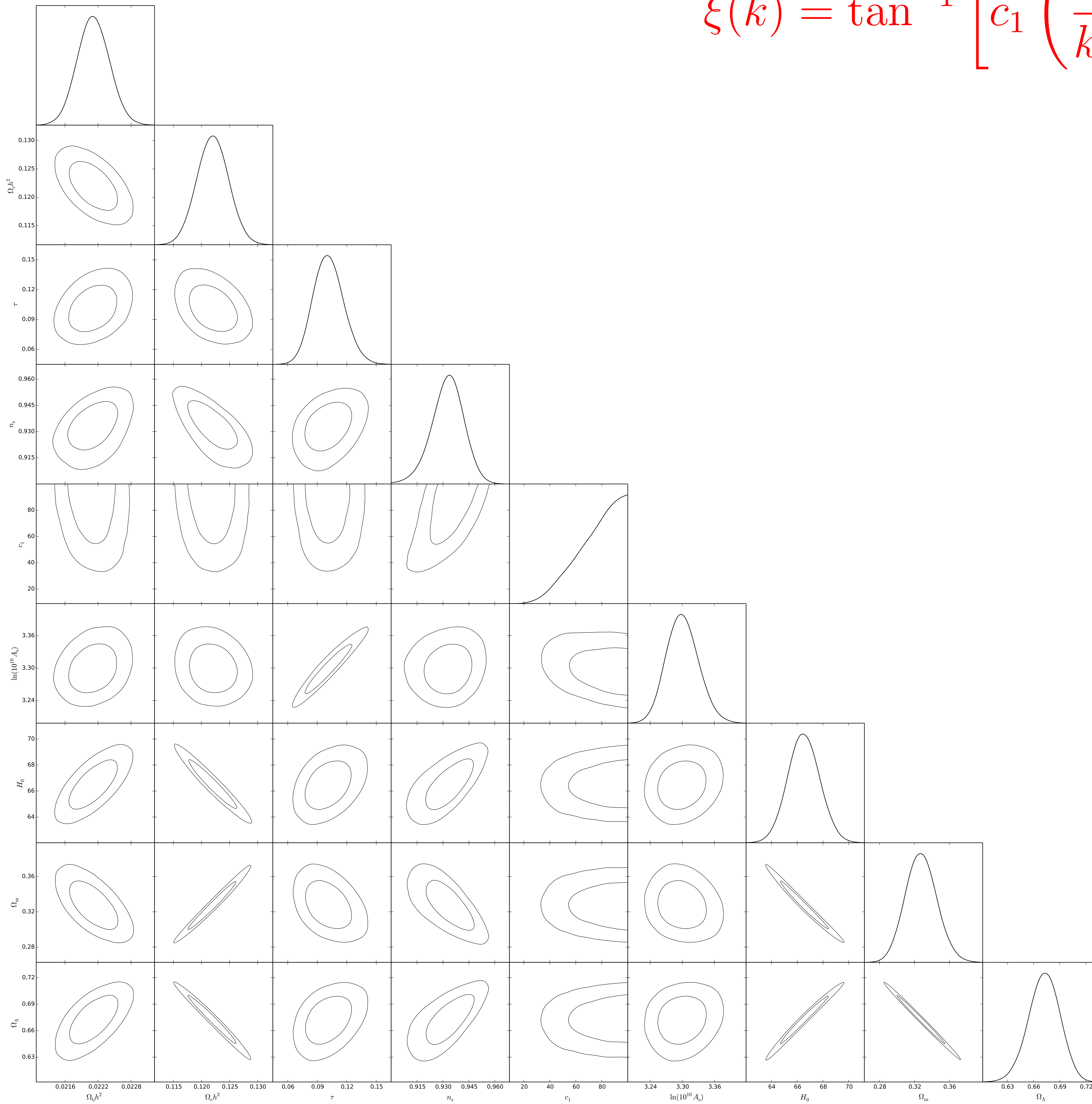
summary for the constrained model:

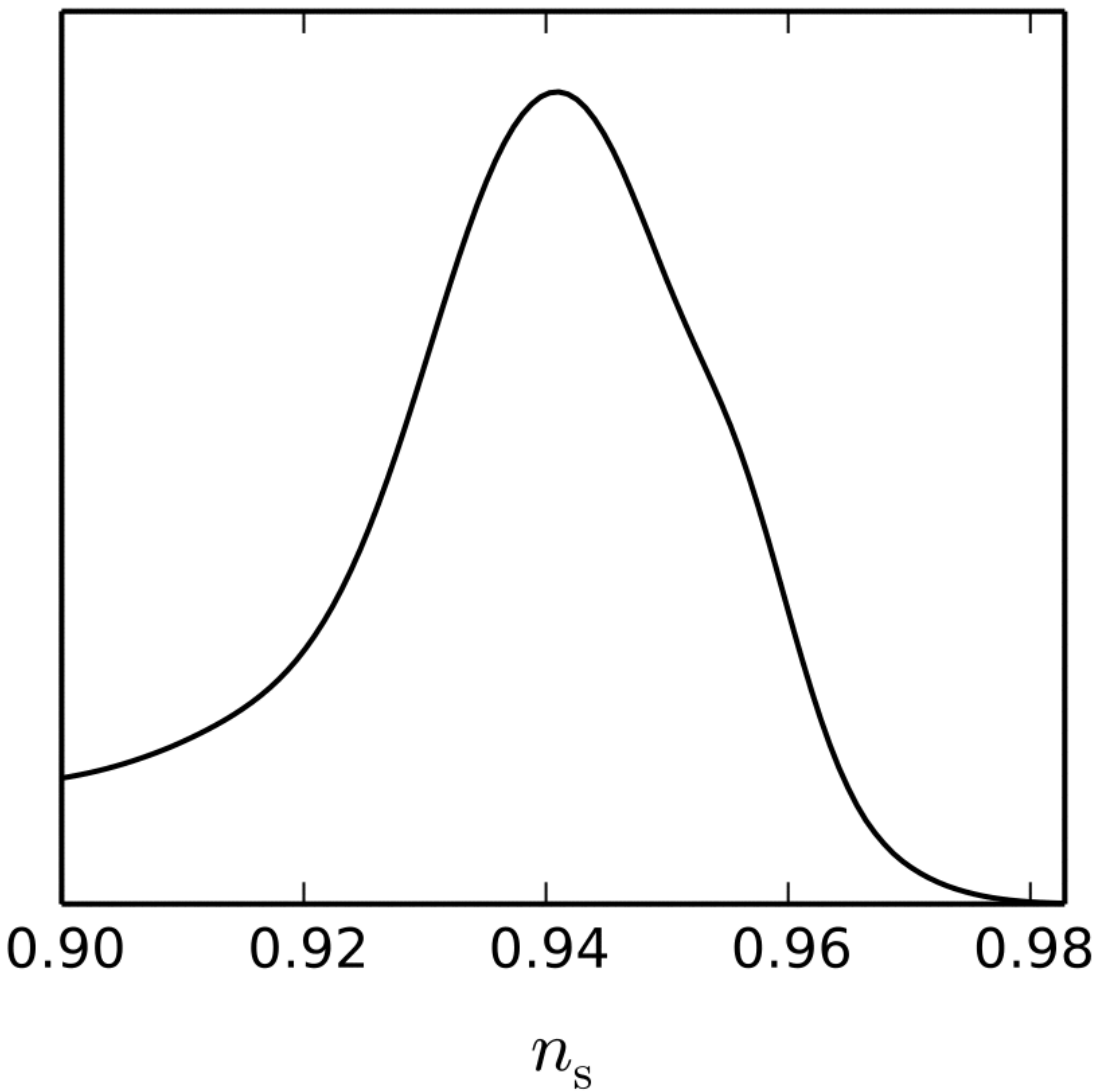


Full model

$$\xi(k) = \tan^{-1}\left[c_1\left(\frac{k}{k_\star}\right) + c_2\right] + c_3 - \frac{\pi}{2}$$

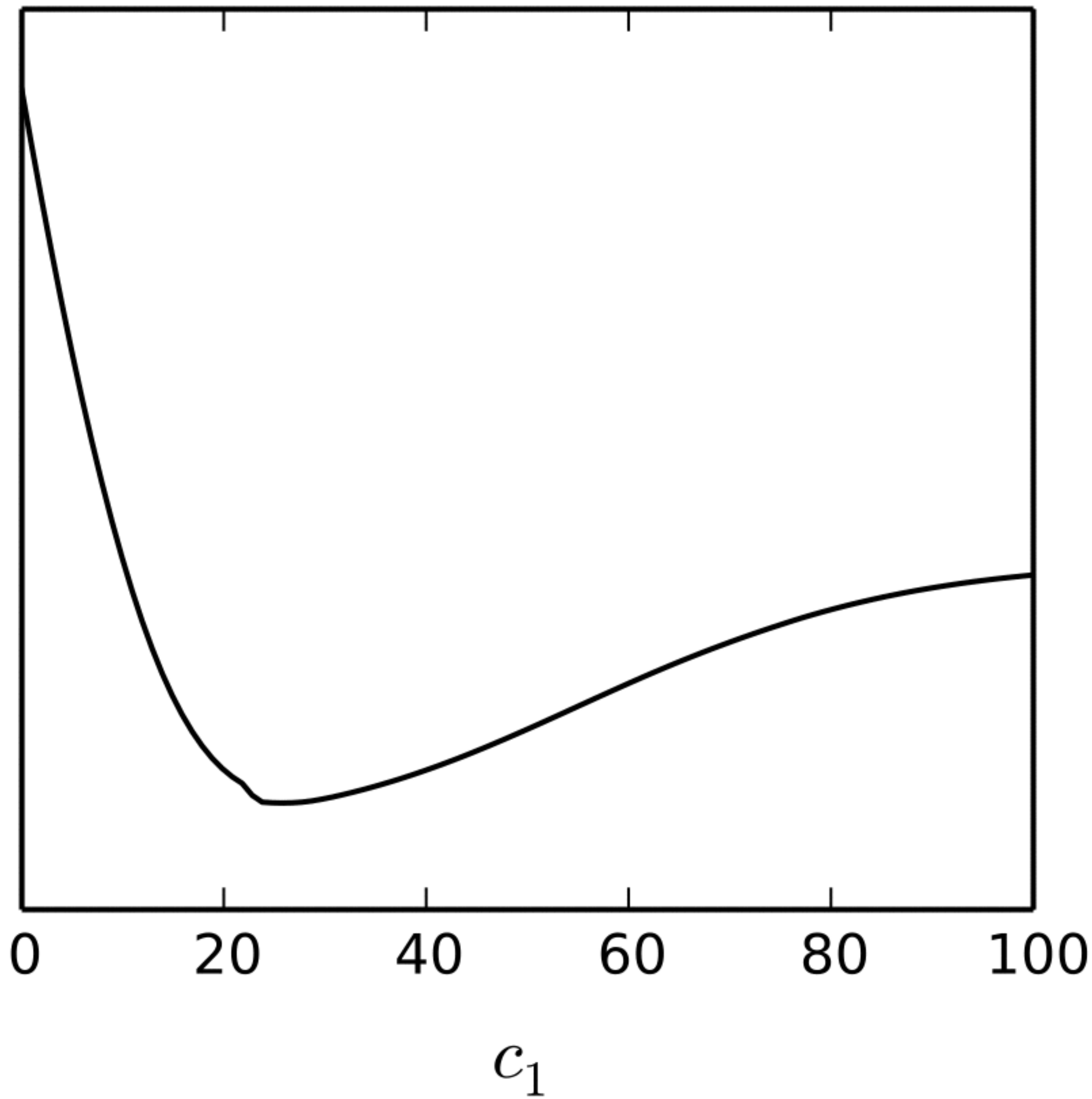
no prior



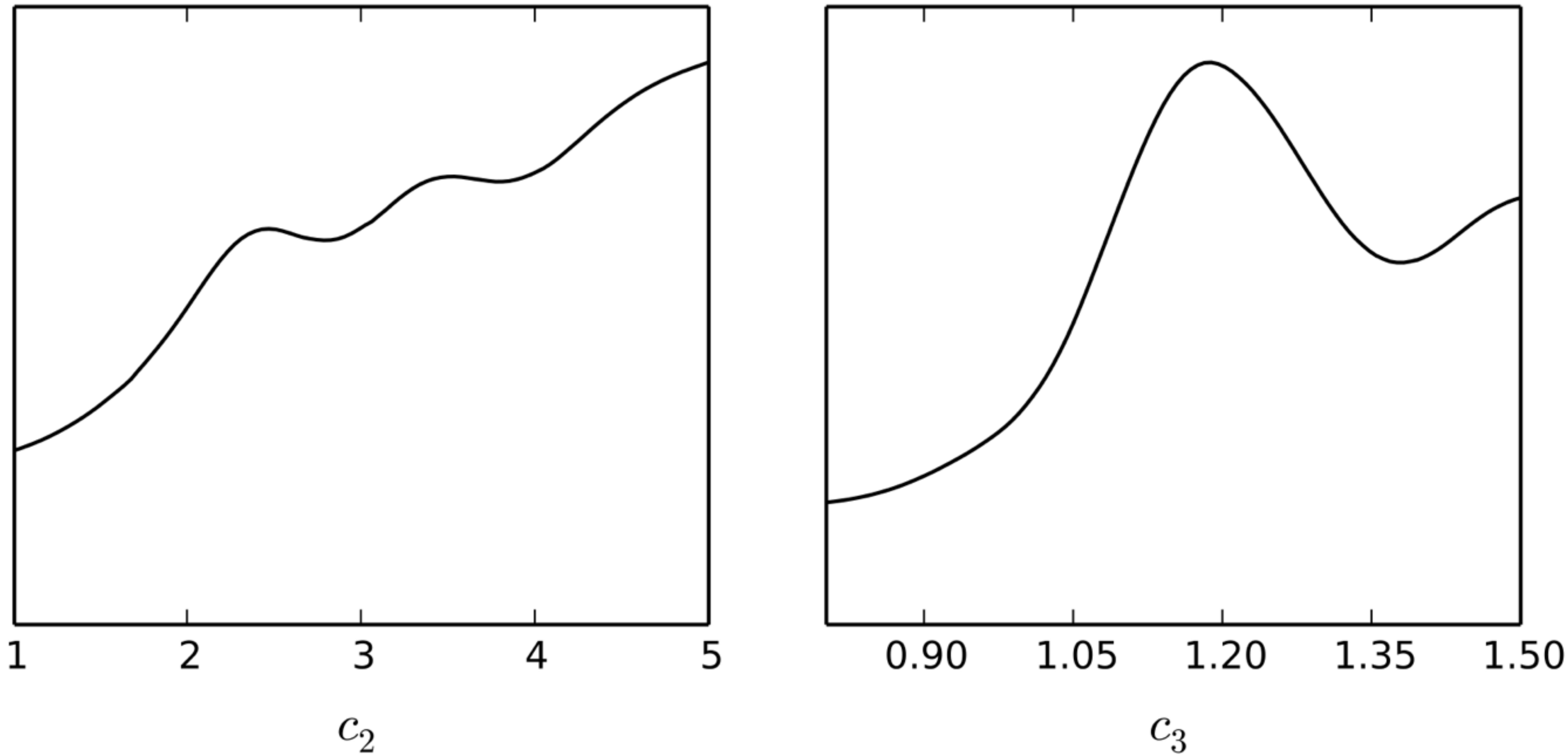


still redder primordial spectrum, but converging!

$$\xi(k) = \tan^{-1}\left[c_1\left(\frac{k}{k_\star}\right) + c_2\right] + c_3 - \frac{\pi}{2}$$

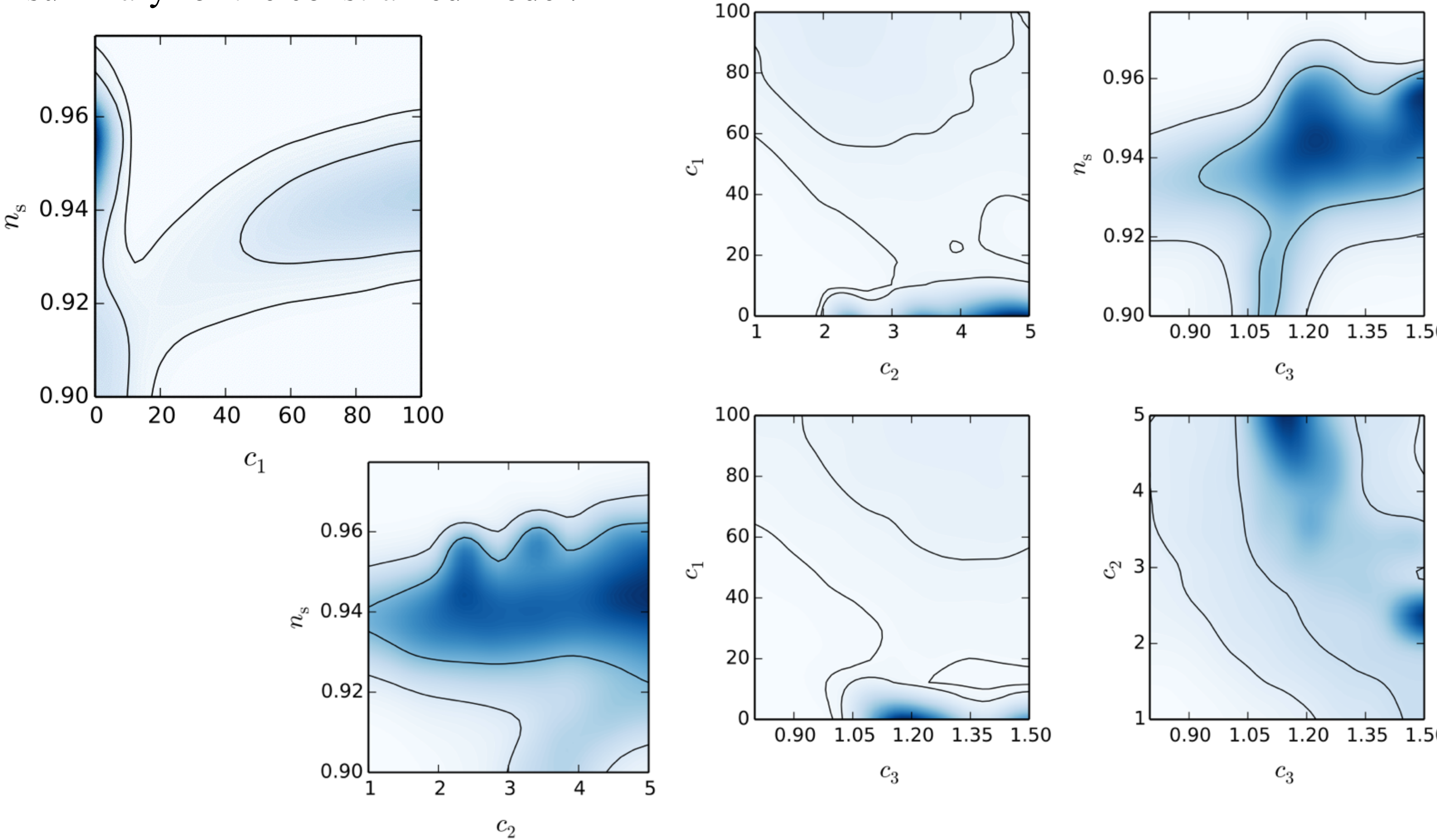


2 possible options:
very large & small
quantum scale



still not very conclusive, but definitely favors $c_3 \geq 1$

summary for the constrained model:



Conclusions

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(1) dBB = testable formulation of QM

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