QUANTUM COSMOLOGY CONFERENCE

Bad Honnef, Germany, july 2014

QUANTUM COSMOLOGY FROM THE DE BROGLIE-BOHM PERSPECTIVE

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CBPF ICRA

PLAN OF THE TALK

1) A short review of the de Broglie-Bohm quantum theory.

2) Applications: the quantum-to-classical transition of quantum cosmological perturbations in a classical background.

3) Discrepancies in quantum cosmology

4) Quantum cosmological perturbations in quantum backgrounds: possible observational consequences of Bohmian trajectories

5) Conclusions

1) THE DE BROGLIE-BOHM QUANTUMTHEORY

“The kinematics of the world, in this ortodox picture, is given by a wave function for the quantum part, and classical variables -variables which *have* values - for the classical part: ($\Psi(t,q \ldots), X(t) \ldots$). The Xs are somehow macroscopic. This is not spelled out very explicitly. The dynamics is not very precisely formulated either. It includes a Schrödinger equation for the quantum part, and some sort of classical mechanics for the classical part, and `collapse’ recipes for their interaction.

It seems to me that the only hope of precision with the dual (Ψ,x) kinematics is to omit completely the shifty split, and let both Ψ and x refer to the world as a whole. Then the xs must not be confined to some vague macroscopic scale, but must extend to all scales.”

John Stewart Bell.

The de Broglie-Bohm interpretation

$$i\hbar\frac{\partial\Psi(x,t)}{\partial t} = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(x)\right]\Psi(x,t)$$

$$p = m\dot{x} = \nabla S(x,t)$$

The guidance relation allows the determination of the trajectories (different from the classical)

$$\Psi = A\exp(iS/\hbar) \qquad \frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V - \frac{\hbar^2}{2m}\frac{\nabla^2 A}{A} = 0.$$

$$Q \equiv -\frac{\hbar^2}{2m}\frac{\nabla^2 A}{A}.$$

$$\frac{\partial A^2}{\partial t} + \nabla\cdot\left(A^2\frac{\nabla S}{m}\right) = 0,$$

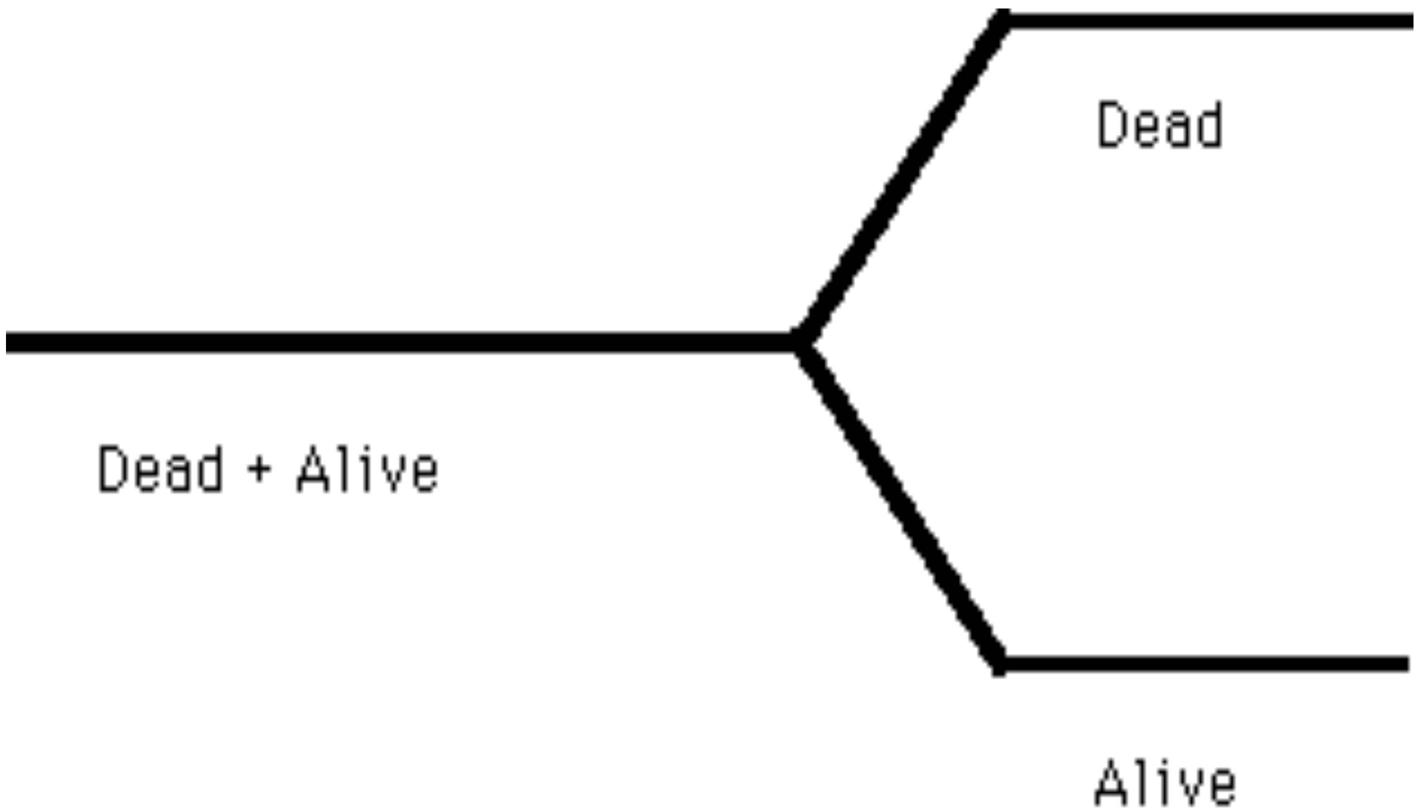
If $P(x,t=0) = A^2(x, t=0)$, all the statistical predictions of quantum mechanics are recovered.

However, $P(x,t=0) \neq A^2(x, t=0)$, relaxes rapidly to $P(x,t) = A^2(x, t)$ (quantum H theorem -- Valentini)

Born rule deduced, not postulated

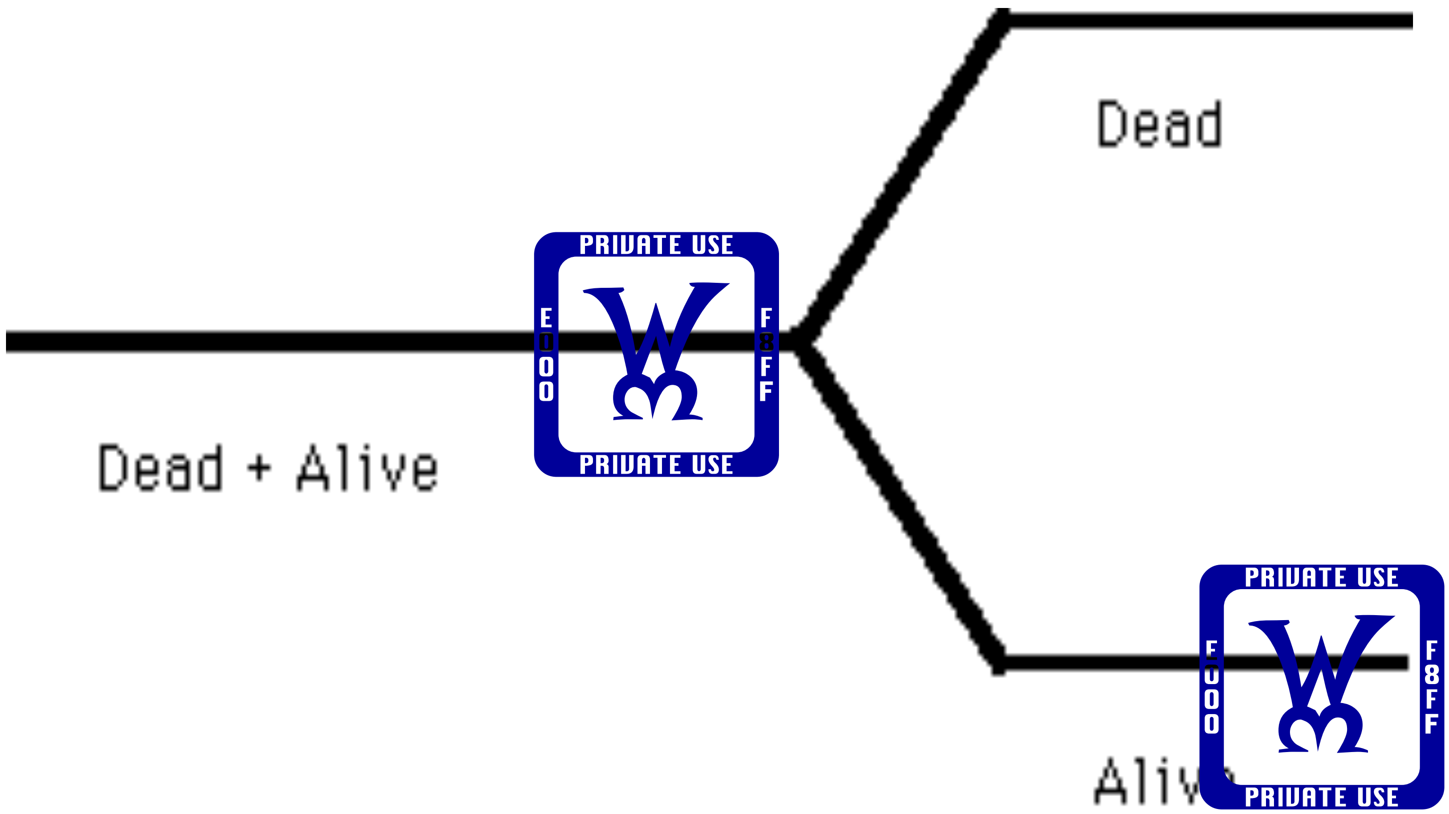
MEASUREMENT PROBLEM → Decoherence: explain why we do not see macroscopic interference, but...

IT DOES NOT EXPLAIN THE UNICITY OF FACTS!



Solving the measurement problem:

position in configuration space determines chosen branch (depends on X_0)



SOLUTION OF THE MEASUREMENT PROBLEM:

Bohm-de Broglie: particles and fields have real trajectories, independently of any observation (ontology). One trajectory enter in one branch and singularize it with respect to the others.

Some remarks

a) Q is highly non-local and context dependent!
(Bell´s inequalities are violated, like in usual QM)

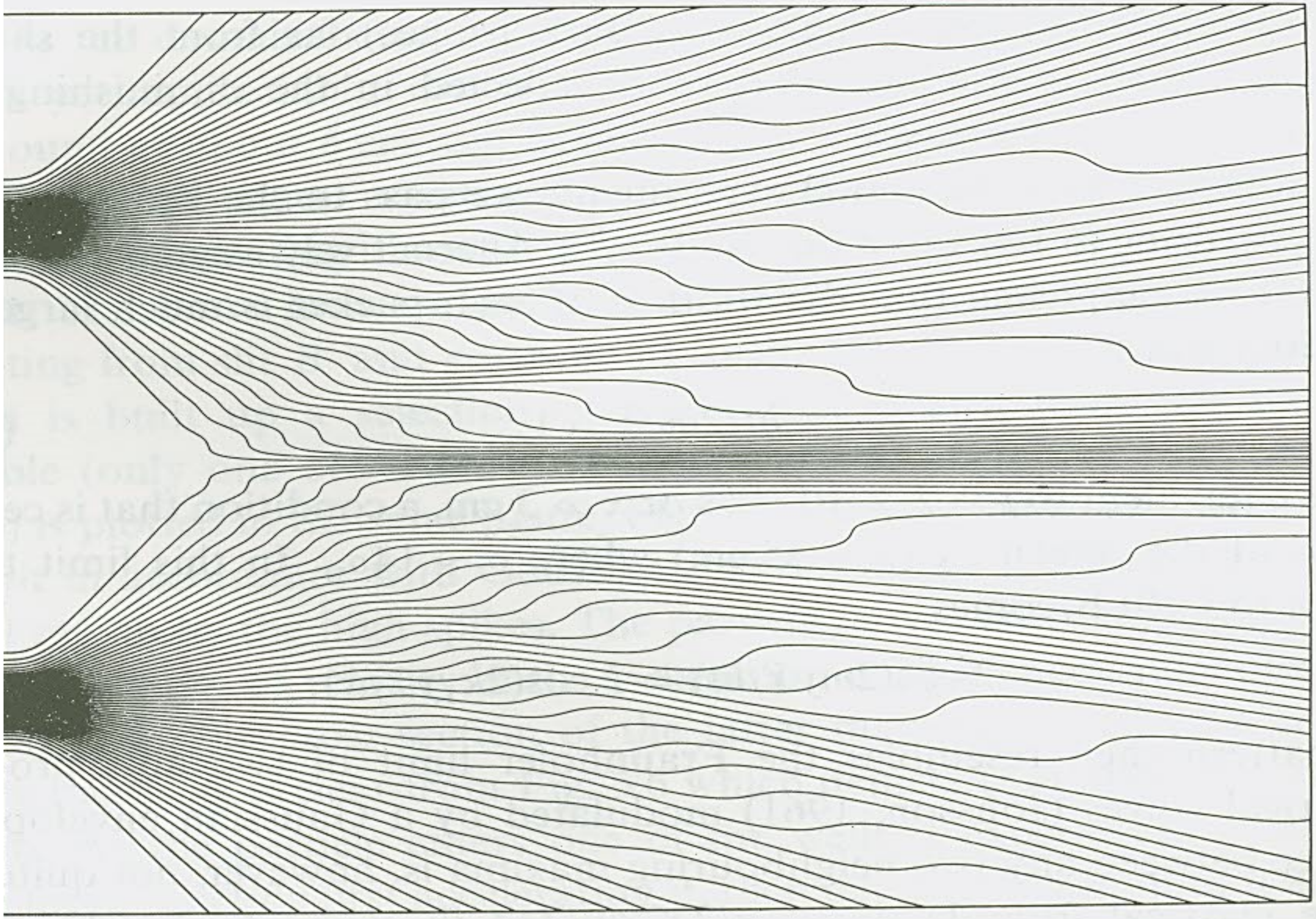
It offers a simple characterization of the classical limit:

<u>Q=0</u>

b) Probabilities are derived in this theory. The unknown variable is the initial position.

c) With objective reality but with the same statistical interpretation of standard quantum theory.

d) One postulate more (existence of a particle trajectory) and two postulates less (collapse and Born rule) than standard quantum theory: 1-2 = -1 postulate



Bell in *Speakable and unspeakable in quantum mechanics*

"In 1952 I saw the impossible done. It was in papers by David Bohm. … the subjectivity of the orthodox version, the necessary reference to the 'observer,' could be eliminated. . . . But why then had Born not told me of this 'pilot wave'? If only to point out what was wrong with it? Why did von Neumann not consider it? . . . Why is the pilot wave picture ignored in text books? Should it not be taught, not as the only way, but as an antidote to the prevailing complacency? To show us that vagueness, subjectivity, and indeterminism, are not forced on us by experimental facts, but by deliberate theoretical choice?" (Bell, page 160)

2) The quantum-to-classical transition of primordial cosmological perturbations.

with Grasiele Santos and Ward Struyve

Phys. Rev. D 85, 083506 (2012)
Phys. Rev. D 89, 023517 (2014)

Evolution of scalar perturbations:

$$ds^2 = a^2(\eta)\left\{[1+2\phi(\eta,\mathrm{x})]\,d\eta^2 - [1-2\phi(\eta,\mathrm{x})]\,\delta_{ij}dx^i dx^j\right\},$$

Φ(x) is the inhomogeneous perturbation, related to theNewtonian potential in the nonrelativistic limit, δφ is the scalar field perturbation.

Mukhanov-Sasaki variable →

$$y(\eta,\mathrm{x}) \equiv a\left[\delta\varphi(\eta,\mathrm{x}) + \frac{\varphi'}{\mathcal{H}}\phi(\eta,\mathrm{x})\right], \qquad (2)$$

where $\mathcal{H} = a'/a$, and a prime denotes derivative with respect to η.

The Hamiltonian describing the Mukhanov-Sasaki variable dynamics coming from General Relativity reads

Hamiltonian for the perturbations from GR →

$$H = \frac{1}{2}\int d^3x \left(p^2 + y^{,i}y_{,i} + 2\frac{z'}{z}yp\right), \qquad (3)$$

where $z \equiv 2\sqrt{\pi}a\varphi'/(m_{Pl}\mathcal{H})$, and the symbol $,i$ corresponds to derivative with respect to the ith component of x. The corresponding equation of motion is

Equations of motion →

$$y'' - y_{,i}^{\,,i} - \frac{z''}{z}y = 0. \qquad (4)$$

IN TERMS OF THE FOURIER MODES

In terms of the Fourier modes, defined through

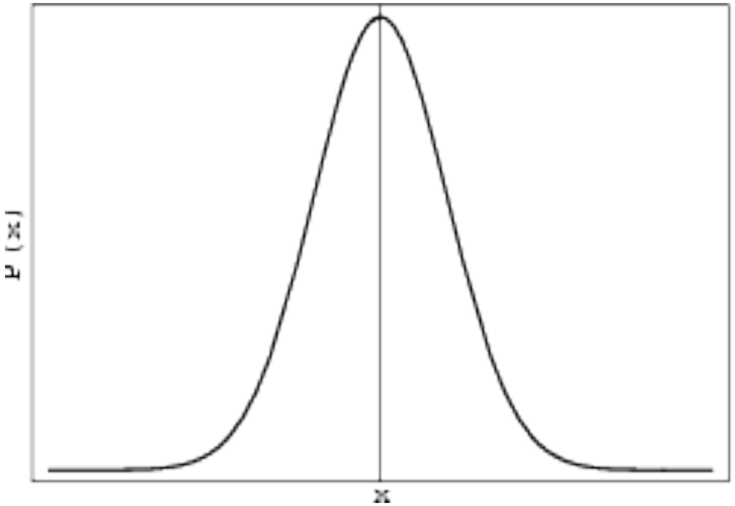
$$y(\eta, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} y_{\mathbf{k}}(\eta) \exp(\mathrm{i}\mathbf{k} \cdot \mathbf{x}), \tag{5}$$

where $y^*_{\mathbf{k}} = y_{-\mathbf{k}}$ (due to the reality of $y(\eta, \mathbf{x})$), the Hamiltonian reads

$$H = \int_{\mathbb{R}^{3+}} d^3k \left[p_{\mathbf{k}} p^*_{\mathbf{k}} + k^2 y_{\mathbf{k}} y^*_{\mathbf{k}} + \frac{z'}{z} (p_{\mathbf{k}} y^*_{\mathbf{k}} + y_{\mathbf{k}} p^*_{\mathbf{k}}) \right], \tag{6}$$

yielding the equation for the modes,

$$y''_{\mathbf{k}} + \left(k^2 - \frac{z''}{z} \right) y_{\mathbf{k}} = 0, \tag{7}$$



The classical solutions

at early times ($\eta \to \eta_i$, where η_i is some initial time, with $|\eta_i| \gg 1$), is given by

$$y_{\mathbf{k}}(\eta) \sim \mathrm{e}^{-ik\eta} \left(1 + \frac{A_k}{\eta} + \dots \right). \tag{8}$$

The physical modes will grow larger during inflation and will eventually obtain wavelengths much bigger than the curvature scale, i.e., $k^2 \ll z''/z$. At that stage, the modes are in general approximately given by

$$y_{\mathbf{k}}(\eta) \sim A^d_k \eta^{\alpha_d} + A^g_k \eta^{\alpha_g} \approx A^g_k \eta^{\alpha_g}, \tag{9}$$

where $\alpha_d > 0$ and $\alpha_g < 0$. The first term represents the

QUANTIZATION

In the Schroedinger picture:

Ψ(y,η) = <y|0,η>

$$\Psi(y, y^*, \eta), \quad = \quad \Pi_{\mathbf{k}\in\mathbb{R}^{3+}} \Psi_{\mathbf{k}}(y_{\mathbf{k}}, y^*_{\mathbf{k}}, \eta)$$

$$i\frac{\partial \Psi_{\mathbf{k}}}{\partial \eta} = \left[-\frac{\partial^2}{\partial y^*_{\mathbf{k}} \partial y_{\mathbf{k}}} + k^2 y^*_{\mathbf{k}} y_{\mathbf{k}} - i\frac{z'}{z}\left(\frac{\partial}{\partial y^*_{\mathbf{k}}} y^*_{\mathbf{k}} + y_{\mathbf{k}} \frac{\partial}{\partial y_{\mathbf{k}}}\right)\right]\Psi_{\mathbf{k}}. \tag{10}$$

The ground state mode wave function reads (see Ref. [3])

$$\Psi_{\mathbf{k}} = \frac{1}{\sqrt{2\pi}|f_k(\eta)|} \exp\left\{-\frac{1}{2|f_k(\eta)|^2}|y_{\mathbf{k}}|^2 + \mathrm{i}\left[\left(\frac{|f_k(\eta)|'}{|f_k(\eta)|} - \frac{z'}{z}\right)|y_{\mathbf{k}}|^2 - \int^{\eta} \frac{d\tilde{\eta}}{2|f_k(\tilde{\eta})|^2}\right]\right\}, \tag{11}$$

with f_k a solution to the classical field equations (7), and $f_k(\eta_i) = 1/\sqrt{2k}$, where $|\eta_i| \gg 1$. This state is homogenous and isotropic.

fk'' + (k^2 – z''/z) fk = 0

$$\Psi_{\vec{k}}(y_{\vec{k}}, y^*_{\vec{k}}, \eta) \propto \exp\left(-k|y_{\vec{k}}(\eta_i)|^2\right)\exp\left(-i\eta k\right), \tag{22}$$

where the phase corresponds to the ground state energy of the mode k, $E_k = k$.

h=1
m=2
k=w/2

THE PROBLEM

|0> is homogeneous and isotropic,
and so is <0|y(x)y(x)|0>
(= <0|T* y(x)T T* y(x)T|0> = <0|y(x+δ)y(x+δ)|0>)

$$\frac{\delta T}{T_0}(\theta,\varphi) = \sum_{lm} \alpha_{lm} Y_{lm}(\theta,\varphi),$$

$$\alpha_{lm} = \frac{4\pi i^l}{3} \int \frac{d^3k}{(2\pi)^3} j_l(kR_D) Y^*_{lm}(\hat{k}) \Delta(k) \Psi_{\vec{k}}(\eta_R),$$

$$\Psi = \Phi$$

What do we put in the place of φ?
The mean value (zero!), a realization?

Attempts for solving the problem:

squeezing → positive Wigner distribution in phase space
→quantum distribution looks like classical stochastic distribution of realizations of the Universe with different inhomogeneous configurations.

decoherence: avoids interference among realizations.

Criticized by Lyth, Liddle, Mukhanov, Sudarsky, Weinberg, ...

1) The state is still homogemeous and isotropic;

2) What is the environment of the perturbations in the Universe?

3) In the standard interpretation, different potentialities are not realities: how ONE of the potentialities become our real Universe?;

4) What makes the role of a measurement in the early Universe?
(we cannot collapse the wave function: we could not exist without stars!)

QUANTIZATION

In the Schroedinger picture:

$\Psi(y,\eta) = \langle y|0,\eta\rangle$

$$\Psi(y, y^*, \eta) \quad = \quad \Pi_{\mathbf{k}\in\mathbb{R}^{3+}} \Psi_{\mathbf{k}}(y_{\mathbf{k}}, y^*_{\mathbf{k}}, \eta)$$

$$i\frac{\partial \Psi_{\mathbf{k}}}{\partial \eta} = \left[-\frac{\partial^2}{\partial y^*_{\mathbf{k}} \partial y_{\mathbf{k}}} + k^2 y^*_{\mathbf{k}} y_{\mathbf{k}} - i\frac{z'}{z}\left(\frac{\partial}{\partial y^*_{\mathbf{k}}} y^*_{\mathbf{k}} + y_{\mathbf{k}} \frac{\partial}{\partial y_{\mathbf{k}}}\right)\right] \Psi_{\mathbf{k}}. \tag{10}$$

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where the phase corresponds to the ground state energy of the mode k, $E_k = k$.

The de Broglie-Bohm solution

→The existence of an actual field configuration breaks translational and rotational invariance.

→It obeys guidance equations.

→ Its initial condition satisfies Born rule at initial time.

$$\frac{\partial S_{\mathbf{k}}}{\partial \eta} + \frac{\partial S_{\mathbf{k}}}{\partial y^*_{\mathbf{k}}}\frac{\partial S_{\mathbf{k}}}{\partial y_{\mathbf{k}}} + k^2 y^*_{\mathbf{k}} y_{\mathbf{k}} + \frac{z'}{z}\left(\frac{\partial S_{\mathbf{k}}}{\partial y^*_{\mathbf{k}}} y^*_{\mathbf{k}} + y_{\mathbf{k}}\frac{\partial S_{\mathbf{k}}}{\partial y_{\mathbf{k}}}\right) - \frac{1}{R_{\mathbf{k}}}\frac{\partial^2 R_{\mathbf{k}}}{\partial y^*_{\mathbf{k}}\partial y_{\mathbf{k}}} = 0,$$

$$\frac{\partial R^2_{\mathbf{k}}}{\partial \eta} + \frac{\partial}{\partial y^*_{\mathbf{k}}}\left[R^2_{\mathbf{k}}\left(\frac{\partial S_{\mathbf{k}}}{\partial y} + \frac{z'}{z} y^*_{\mathbf{k}}\right)\right] + \frac{\partial}{\partial y_{\mathbf{k}}}\left[R^2_{\mathbf{k}}\left(\frac{\partial S}{\partial y^*_{\mathbf{k}}} + \frac{z'}{z} y_{\mathbf{k}}\right)\right] = 0,$$

$$y^{*\prime}_{\mathbf{k}} = \frac{\partial S_{\mathbf{k}}}{\partial y_{\mathbf{k}}} + \frac{z'}{z} y^*_{\mathbf{k}}, \quad y'_{\mathbf{k}} = \frac{\partial S_{\mathbf{k}}}{\partial y^*_{\mathbf{k}}} + \frac{z'}{z} y_{\mathbf{k}}.$$

$$y_{\mathbf{k}}(\eta) = y_{\mathbf{k}}(\eta_i)\frac{|f_k(\eta)|}{|f_k(\eta_i)|}.$$

f(η) classical, y(η) quantum

$$y''_{\mathbf{k}} + \left(k^2 - \frac{z''}{z}\right) y_{\mathbf{k}} = -\frac{\partial Q_{\mathbf{k}}}{\partial y^*_{\mathbf{k}}}, \qquad Q_{\mathbf{k}} \equiv -\frac{1}{R_{\mathbf{k}}}\frac{\partial^2 R_{\mathbf{k}}}{\partial y^*_{\mathbf{k}}\partial y_{\mathbf{k}}}$$

The quantum-to-classical transition

$\boxed{k^2 \gg z''/z}$ **f(η)** $\sim e^{-ik\eta}\left(1+\frac{A_k}{\eta}+\ldots\right)$

For small wave lengths and $\eta \to \eta_i$, the behaviour of $f_k(\eta)$ is given by Eq. (8). As such,

$$y_{\mathbf{k}}(\eta) \sim \left[1+\frac{\mathrm{Re}A_k}{\eta}+\frac{|A_k|^2-2(\mathrm{Re}A_k)^2}{\eta^2}+\ldots\right] \quad (18)$$

$\boxed{k^2 \ll z''/z}$

f(η) classical, y(η) quantum

f(η) $\sim A_k^d\eta^{\alpha_d}+A_k^g\eta^{\alpha_g} \approx A_k^g\eta^{\alpha_g},$ **y(η) α |f(η)| α f(η)**

In terms of the quantum potential

$$Q_{\mathbf{k}} = \frac{1}{4|f_k|^4}(2|f_k|^2 - |y_{\mathbf{k}}|^2), \tag{19}$$

and gives rise to the quantum force

$$F_{Q,\mathbf{k}} \equiv -\frac{\partial Q_{\mathbf{k}}}{\partial y_{\mathbf{k}}^*} = \frac{y_{\mathbf{k}}}{4|f_k|^4} \tag{20}$$

for the mode $\mathbf{k}$. The classical force can be read from Eq. (7) and is given by

$$F_{C,\mathbf{k}} = -\left(k^2 - \frac{z''}{z}\right) y_{\mathbf{k}}. \tag{21}$$

Their ratio is

$$\frac{F_{C,\mathbf{k}}}{F_{Q,\mathbf{k}}} = -4|f_k|^4 \left(k^2 - \frac{z''}{z}\right). \tag{22}$$

Statistical predictions: the two point correlation function

$$
\begin{aligned}
&\langle y(\eta,\mathbf{x})y(\eta,\mathbf{x}+\mathbf{r})\rangle_{\rm dBB} && (23)\\
&= \int \mathcal{D}y_i |\Psi(y_i,\eta_i)|^2 y(\eta,\mathbf{x};y_i)y(\eta,\mathbf{x}+\mathbf{r};y_i) && (24)\\
&= \int \mathcal{D}y |\Psi(y,\eta)|^2 y(\mathbf{x})y(\mathbf{x}+\mathbf{r}) && (25)
\end{aligned}
$$

which is the usual expression for the correlation function, and can be calculated to yield

$$
\langle y(\eta,\mathbf{x})y(\eta,\mathbf{x}+\mathbf{r})\rangle_{\rm dBB} = \frac{1}{2\pi^2}\int dk \frac{\sin kr}{r} k |f_k(\eta)|^2, \tag{26}
$$

3) DISCREPANCIES IN QUANTUM COSMOLOGY

Physical Review D 86, 063504 (2012).

Massless free scalar field φ

$$ds^2 = N^2(t)\, dt^2 - a^2(t)\, (dx^2 + dy^2 + dz^2)$$

$$\alpha = \ln a$$

$$H = NH_0 = N[\, -(P_\alpha)^2 + (P_\varphi)^2\,]$$

Defining $v\_r = \alpha - \varphi$ and $v\_l = \alpha + \varphi$, the classical solutions are $v\_r = \text{const.}$ and $v\_l = \text{const.}$

They represent universes expanding to infinity from a singularity and universes contracting from infinity to a singularity, respectively.

$H_0 \Psi = 0 \rightarrow$ Klein-Gordon equation

The Wheeler-DeWitt equation is:

$$\left(\partial_\phi^2 - \frac{4\pi G}{3}\partial_\alpha^2\right)\Psi(\alpha,\phi) = 0,$$

Put it in a Scroedinger form in order to define a positive measure by taking its square-root

$$\pm i\partial_\phi\Psi(\alpha.\phi) = \sqrt{\mathbf{\Theta}}\ \Psi(\alpha,\phi),$$

with

$$\mathbf{\Theta} := -\frac{4\pi G}{3}\partial_\alpha^2.$$

and choose one sign (single frequency approach).

The physical scalar product is given by

$$\langle\Phi|\Psi\rangle := \int_{\phi=\phi_0} d\alpha\,\bar{\Phi}(\alpha,\phi)\Psi(\alpha,\phi),$$

Craig and Singh, Phys. Rev. D 82, 123526 (2010).

- Single frequency with allowing superpositions of left and right-handed sectors.

- Consistent histories approach

Family of two-time histories, the two times taken to be the infinity past and the infinity future → the family is consistent and the probability that in one of these times the universe is singular and in the other it is spatially infinity is ONE!

Conclusion:

No bounce, singularities are always present.

HOWEVER

What happens if one add a third moment of time in between and construct the family with 3 times?

Then one can show that the this family is consistent if and only if the following integral is null:

$$\int_{\alpha*-\phi}^{\infty} dv_r \int_{-\infty}^{\alpha*-\phi} dv_r'' \left[\frac{\Psi(v_r)\Psi^*(v_r'')}{v_r''-v_r}\right] \qquad \textbf{where} \qquad v_r := \alpha - \phi$$

This is not null in general. As the domains of intregation are disjoint, this integral can be made null if the wave function is highly concentrated in $v_r = \alpha - \varphi$, which is a semi-classical state, which is of course singular.

Hence, when we have a third intermediate time, the consistent histories interpretation cannot assign probabilities for non-classical states, and hence cannot decide whether there are singularities or not in this case.

The two-frequencies approach

Halliwell et al were able to define a positive measure without restricting to a single square-root of the Klein-Gordon equation.

They define the inner product,

$$(\Psi, \Phi) := i \int d\alpha \left(\Psi_+^* \overleftrightarrow{\partial_\phi} \Phi_+ - \Psi_-^* \overleftrightarrow{\partial_\phi} \Phi_- \right) \longrightarrow$$

$$\begin{aligned}(\Psi, \Psi) &= i \int d\alpha \left(\Psi_+^* \overleftrightarrow{\partial_\phi} \Psi_+ - \Psi_-^* \overleftrightarrow{\partial_\phi} \Psi_- \right) \\ &= 2 \int_{-\infty}^{\infty} dk |k| \left(\left|\Psi_+(k)\right|^2 + \left|\Psi_-(k)\right|^2 \right)\end{aligned}$$

from where they obtain the off-diagonal term of the decoherence functional

$$D(\Delta, \bar{\Delta}) = \int_\Delta d\alpha \int_{\bar{\Delta}} d\alpha' \Big[\Psi_+^*(\alpha', \phi') \overleftrightarrow{\partial_{\phi'}} G^{(+)}(\alpha', \phi'; \alpha, \phi) \overleftrightarrow{\partial_\phi} \Psi_+(\alpha, \phi) +$$

We have shown that such off-diagonal terms cannot be made null, **even with only two instants of time!**

The de Broglie-Bohm approach

Whatever is the probability density defined, one can construct a velocity field satisfying the continuity equation:

$$\frac{d\alpha}{d\phi} = v_2(\alpha, \phi) = -\frac{1}{\rho(\alpha, \phi)} \int_{-\infty}^{\alpha} d\bar{\alpha} \partial_\phi \rho(\bar{\alpha}, \phi) \, .$$

and $$\partial_\phi \rho + \partial_\alpha \left(v_2 \rho \right) = 0 \, ,$$

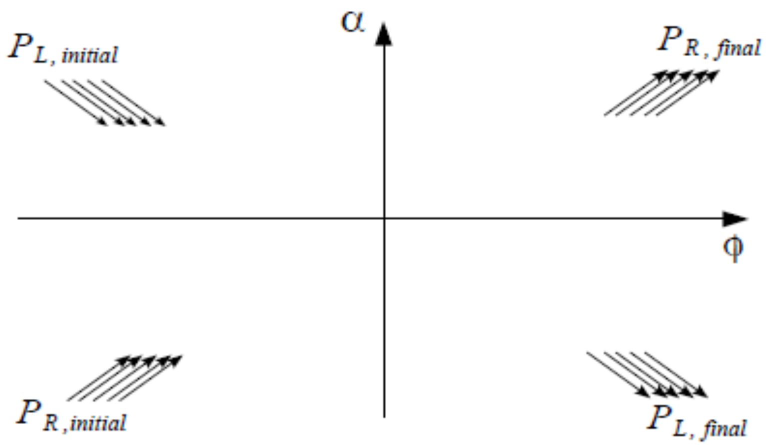
General two-frequencies solution

$$\Psi(v_l, v_r) = \int_{-\infty}^{\infty} dk U(k) \, e^{ikv_l} + \int_{-\infty}^{\infty} dk V(k) \, e^{ikv_r}$$

General results:

a) In the infinity past and infinite future the classical limit is always valid!
b) Knowing that, and using the figure below, we get:'

$$P_{R,i} + P_{L,i} = 1$$



We can summarize the situation for an arbitrary state Ψ as

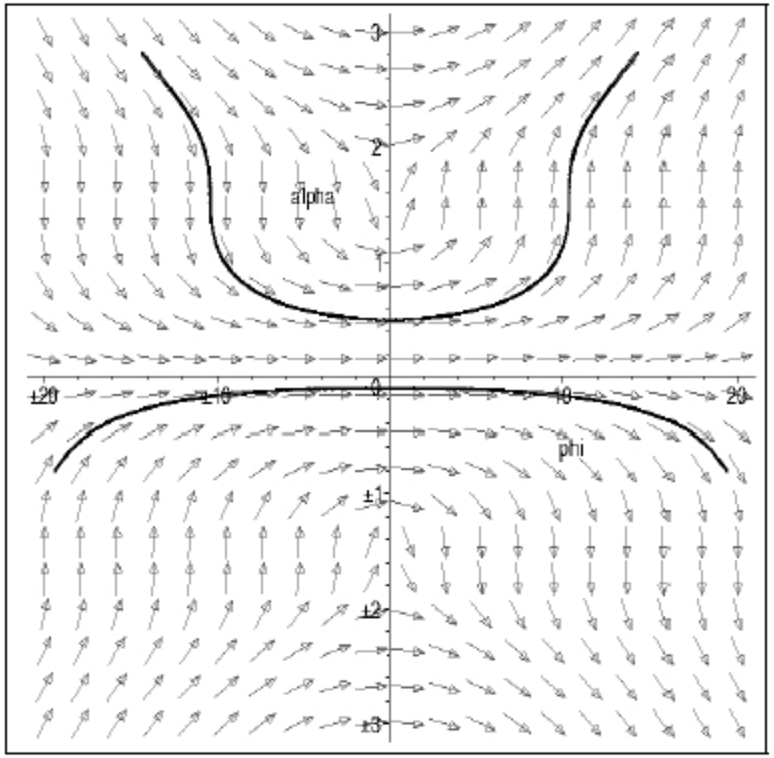
$$\begin{aligned} P_{\text{bounce}} &= P_{\text{recollapsing}} = \min(P_{R,i}, P_{L,i})\,, \\ P_{\text{expanding}} &= \max(P_{R,i} - P_{L,i}, 0)\,, \\ P_{\text{contracting}} &= \max(P_{L,i} - P_{R,i}, 0)\,. \end{aligned} \tag{1}$$

This implies that the probability $P_{\text{singularity}} = 1 - P_{\text{bounce}}$ to run into a singularity satisfies

$$\frac{1}{2} \leqslant P_{\text{singularity}} \leqslant 1\,, \tag{2}$$

If $P_{R,i} = P_{L,f}$ then there is only recollapse and bounce

In this case, P bounce = 1/2



OPPOSITE CONCLUSIONS FROM THE PREVIOUS APPROACH !

Discrepant conclusions coming from the de Broglie-Bohm and consistent histories perspectives.

Furthermore, consistent histories approach does not make predictions for histories with more than two instants of time

In the two-slit experiment this already happens: from de Broglie-Bohm one can say from where the particle comes while in the consistent histories approach one cannot.

Can such kind of discrepancy be tested?
Maybe in cosmology, when we put perturbations!

4) Quantum cosmological perturbations in quantum backgrounds.

Scalar perturbations:

$$ds^2 = a^2(\eta)\left[(1+2\Phi)\mathrm{d}\eta^2 - (1-2\Phi)\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j\right],$$

Φ(x) is the inhomogeneous perturbation, related to the Newtonian potential in the nonrelativistic limit. dt = a dη

NOW PERTURBATIONS AND BACKGROUND SHOULD BE QUANTIZED!

MORE GENERAL THAN MINISUPERSPACE AND SEMICLASSICAL THEORY OF COSMOLOGICAL PERTURBATIONS

No use of background equations of motion.
The first to try: Halliwell and Hawking.

Hamiltonian → Quantization

$H_0\ \psi = 0$: too complicated

Is it possible to obtain a simplification without using the background classical equations?

YES → HAMILTONIAN FROM GR

(without using background equations)

$$\mathrm{H} = \mathrm{N}\,\mathrm{H} = \mathrm{N}\,[\mathrm{H}_0(a, p_a, \varphi, p_\varphi) + \mathrm{H}_2(v(x), \pi_v(x), a, \varphi)]$$

$$\mathbf{H} = \frac{\sqrt{2V}}{2\ell_{Pl} e^{3\alpha}}\left[-P_\alpha^2 + P_\varphi^2 + \int d^3x\left(\frac{\pi^2}{\sqrt{\gamma}} + \sqrt{\gamma} e^{4\alpha} v^{,i} v_{,i}\right)\right].$$

v (=y) is the Mukhanov-Sasaki variable, which is a linear combination of Φ and δφ.

DIRAC QUANTIZATION

$$\mathrm{H}\,\Psi = 0$$

Proceeding with the Dirac quantization

$$H\Psi = (\hat{H}_0 + \hat{H}_2)\Psi = 0$$

$$\Psi = \Psi_0(a,\varphi)\ \Psi_2(a,\varphi,v(x))$$

1) At zero order:

$$\hat{H}_0\ \Psi_0 = 0$$

Solution yields a phase and Bohmian trajectories $a(\eta)$, $\varphi(\eta)$, as we have shown, where BOUNCES MAY OCCUR.

Connection with observations

Are there observational consequences of a primordial contracting phase in our Universe?

Cosmological perturbations → structures → anisotropies of CMBR

What happens with the perturbations in the case of a bounce with a preceding contracting phase?

In the case of quantum cosmological bounces, can we calculate the evolution of perturbations when the background is also quantized?

2) At second order:

$$\square_0\square_0 \cdot \square_0\Psi_2 + \hat{H}_0\,\Psi_2 + \hat{H}_2\,\Psi_2 = 0$$

Choosing a Ψ_2 suitable for a semi-classical limit, which predicts classical behaviour when the universe is big;

Using the background Bohmian trajectories in order to define a conformal time derivative;

$$\square_0\,\square_0 \cdot \square_0\,\Psi_2\,\square$$

$$(-\,\square S/\square a\;\square/\square a \;+\; \square S/\square\square\;\square/\square\square)\square_2 = (a'\,\square/\square a \;+\; \square'\,\square/\square\square)\square_2 = \square\square_2/\square\square$$

Making a unitary time-dependent transformation;

$$U[a(\square),v,\square] = e^{iA}e^{-iB} \quad \text{with } A = a'/a\square\, v^2 \text{ and } B=\ln(a)\square(v\square + \square v)$$

QUANTUM EQUATIONS FOR PERTURBATIONS

$$i\frac{\partial\Psi_{(2)}[v,\eta]}{\partial\eta} = \int \mathrm{d}^3x \left(-\frac{1}{2}\frac{\delta^2}{\delta v^2} + \frac{\lambda}{2}v_{,i}v^{,i} - \frac{a''}{2a}v^2\right)\Psi_{(2)}[v,\eta].$$

$p = \lambda\rho$

For the modes we have:

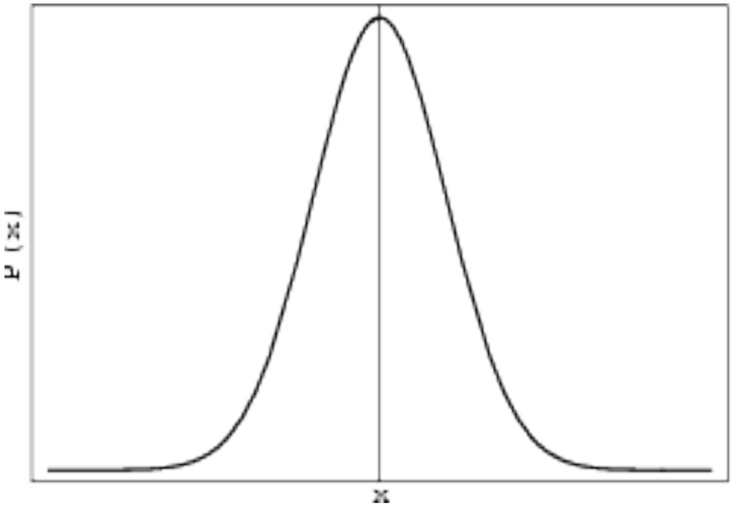
$$\boxed{v''_k + \left(\lambda k^2 - \frac{a''}{a}\right)v_k = 0.}$$

where now a is the Bohmian trajectory.

$$v''_k + \left(\lambda k^2 - \frac{a''}{a}\right) v_k = 0.$$

$\lambda = c_s^2$

Before the bounce, in the contracting phase



After the bounce, In the expanding phase.

$\lambda k^2 > a''/a \Leftrightarrow l_{phys} < c_s R$

Sound horizon: $c_s R$ (R-Hubble radius) Physical wavelength: $l_{phys} = a/k$

Point of crossing: $\lambda k^2 = a''/a \Leftrightarrow l_{phys} = c_s R$

THE POWER SPECTRUM

$$k^3 \mathcal{P}_s \equiv \frac{2k^3}{\pi^2} |\Phi|^2$$

$$n_{\rm s} = 1 + \frac{d\ln(\mathcal{P})}{d\ln(k)}.$$

$$\boxed{n_{\rm s} = 1 + \frac{12\lambda}{1+3\lambda}}$$

- Non relativistic fluid (dark matter?): scale invariant.

It is not necessary to have ordinary matter dominating all along; just at the moment when perturbation scale becomes comparable with the sound horizon.

Another fluid or field may dominate at the bounce: radiation.

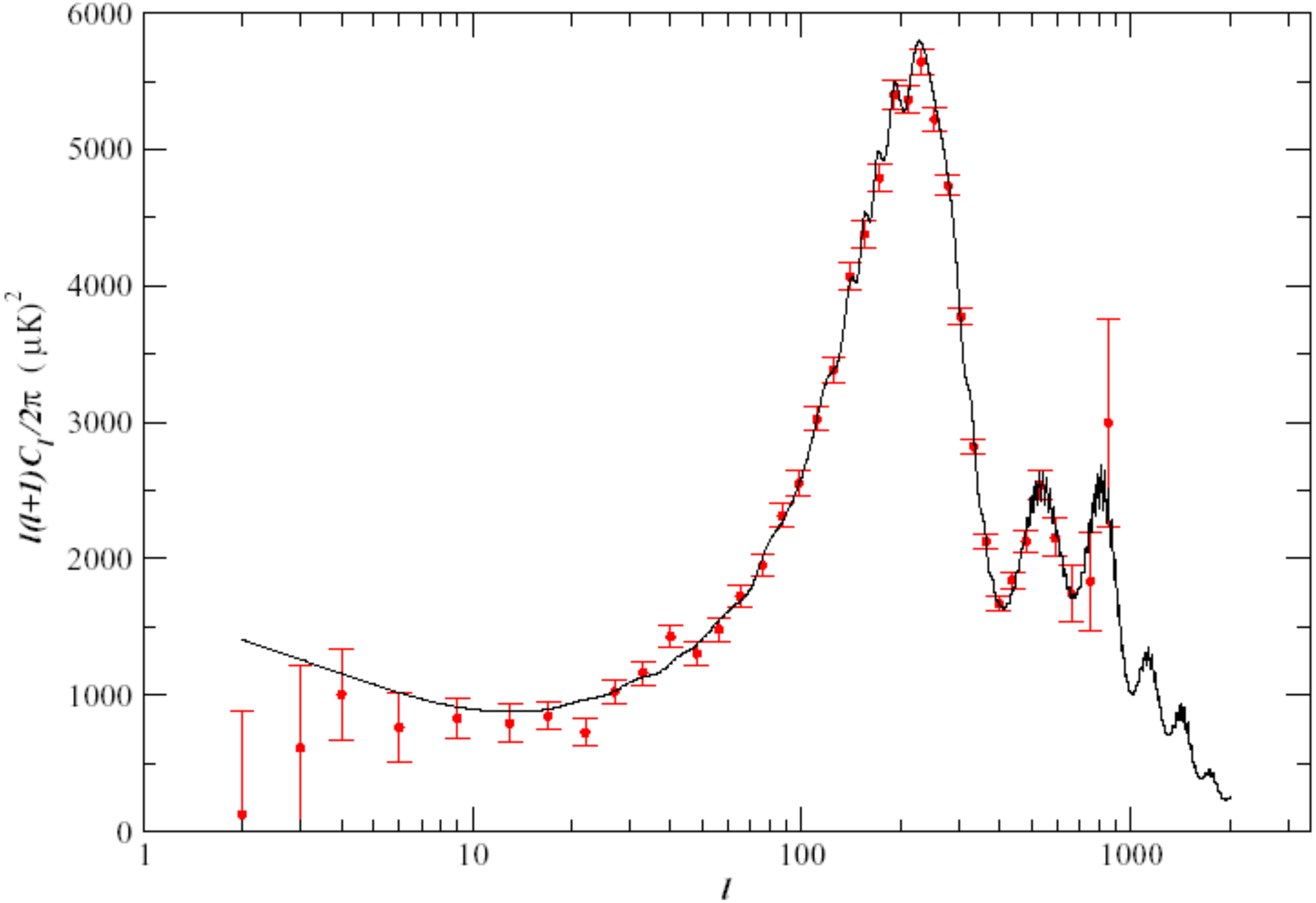
One fluid: **three free parameters:** η_0 **(cuvature scale at the bounce).**
a_0 **(scale factor at the bounce).**
λ_{nr} **(equation of state parameter).**

$\eta_0 \sim 10^3 (\lambda_{nr})^{-1/4} l_{pl}$

Large range of values for a_0: avoid transplanckian problems.

Two fluids: **calculation of C_l with radiation at the bounce and a non relativistic fluid at sound horizon crossing in order to find the best fit parameters.**

- **n_s close to one.**
- **reasonable amplitudes for bounces between nucleosynthesis and Planck scale.**
- **superimposed oscillations and running due to a cosmological constant.**
- **non gaussianities**
- **more than one fluid: entropy perturbations**
- **gravitational waves**
- **dark energy**



Maybe a solution of the missing power problem.

FEATURES OF THE MODEL

1) No singularity.
2) Perturbations of quantum mechanical origin.
3) Enhancement of perturbations at the bounce.
4) No horizon problem.
5) Flatness problem: if the contraction phase is much longer then the expansion phase, then the Universe is almost flat because it has not expanded enough!
6) One fundamental parameter: the curvature radius L_0 at the bounce, which must have the reasonable value $10^3\, l_{pl}$.
7) Transplanckian problem can be solved.
8) Homogeneity problem may be less severe.

$$\Omega_T \equiv \epsilon_T / \epsilon_c \approx 1$$

$$|\Omega_T \dot{} - 1| = -2\frac{\ddot{a}}{\dot{a}^3}$$

Quantum theory helping cosmology ...

cosmology helping quantum theory:

Consequences for quantum theory:

1) One instance where one quantum theory (BDB) may yield observational results which are not known how to be obtained in others.

2) Observational effects of a quantum trajectory $a_q(t)$!

3) Valentini → early freeze out of some particle may suppress quantum relaxation: dark matter, long wavelength perturbations originated from vacuum state, RELIC GRAVITONS.

4) Corrections to Schrödinger equation for the perturbations in the quantum background regime: departure from quantum equilibrium.

V - CONCLUSION

de Broglie-Bohm quantum theory is very suitable for quantum aspects of cosmology!

It explains in a very simple way a very old controversy concerning cosmological perturbations of quantum mechanical origin.

It can go beyond other quantum theories!

Basic General Relativity and de Broglie-Bohm QuantumTheory yield a sensible bouncing model which can explain the origin of cosmological perturbations.

-- There are no observational reasons for a beginning of the Universe, so why not exploring the consequences of bouncing models? (In such models inflation can be present but it is not necessary: another perspective concerning initial conditions).

Allows calculations of potentially observational effects.

What about the other quantum theories?

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Contemporary quantum theory … constitutes an optimum formulation of [certain] connections … [but] offers no useful point of departure for future developments.

Albert Einstein.

**"To try to stop all attempts to pass beyond the present viewpoint of quantum physics could be very dangerous for the progress of science and would furthermore be contrary to the lessons we may learn from the history of science.
This teaches us, in effect, that the actual state of our knowledge is always provisional
and that there must be, beyond what is actually known, immense new regions to discover."**

Louis de Broglie