Highways and Byways of Quantum Cosmology in Ancient Days

1968-~1974

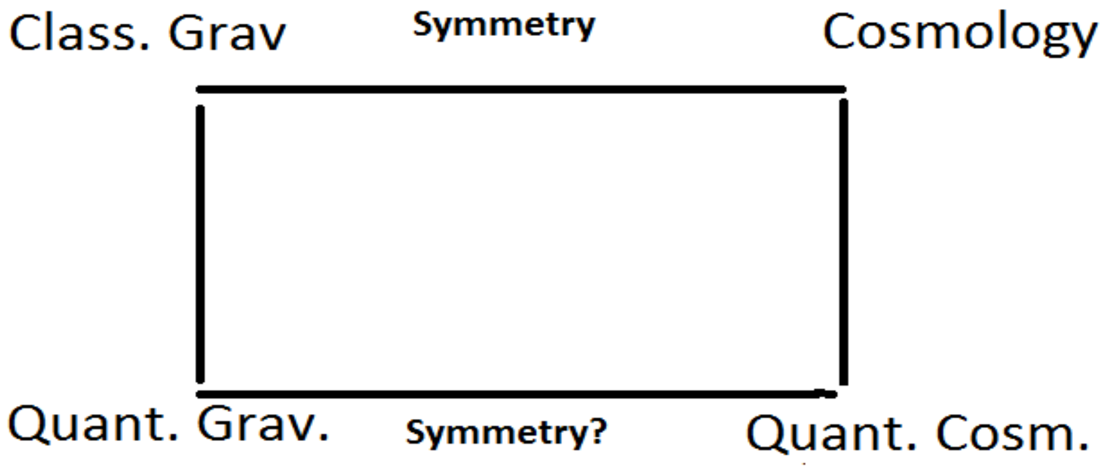
~ 45 yrs. ago

- When did quantum cosmology begin?
- Bryce DeWitt 1967
- Quantization of an FRW model
- A toy model of quantum gravity

Misner group 1968- ~1974

- Why study quantum cosmology?
- Again, a toy model of quantum gravity to study solutions to technical problems in canonical quantum gravity
- e.g. time choice
- No physics necessarily (but very seductive)

- Later, quantum cosmology as physics became popular
- Does quantum cosmology really have anything to do with physics?
- Question: Does the following diagram commute?


Class. Grav
Symmetry
Cosmology
Quant. Grav.
Symmetry?
Quant. Cosm.

- Answer: Maybe

- Toy model test: Cosmologies of higher symmetry imbedded in those of less symmetry (microsuperspace in minisuperspace)

- Result: Sometimes yes, sometimes no.
- Deciding factor: ~ $\delta^2({}^3R)/\delta g_{ij}\delta g_{kl}$ near track of cosm. in superspace

  (i.e. perturbations to second order)

Byways: Forgotten or unknown highways

I) Exponential parametrization

Very old attempt to maintain $g_{ij} > 0$

(to avoid “singularities”)

Doesn’t always work, e.g. logarithmic solns. for exponents in Schwarzschild.

Bianchi Cosmologies

- Misner parametrization:

Bianchi I: $ds^2 = -N^2 dt^2 + a(t)^2 dx^2 + b(t)^2 dy^2 + c(t)^2 dz^2$

$$a = e^{\alpha + \beta_+ + \sqrt{3}\beta_-}$$

$$b = e^{\alpha + \beta_+ - \sqrt{3}\beta_-}$$

$$c = e^{\alpha - 2\beta_+}$$

- Momenta conjugate to

$$\alpha, \beta_+, \beta_- = p_\alpha, p_+, p_-$$

Hamiltonian constraint α, β_+, β_-

H = $$\frac{e^{-3\alpha}}{24}\left(-p_\alpha{}^2 + p_+{}^2 + p_-{}^2\right)$$

Hamiltonian constraint, a, b, c

$$H = \frac{1}{abc}(a^2\pi_a{}^2 + b^2\pi_b{}^2 + c^2\pi_c{}^2 + 2ab\pi_a\pi_b + 2ac\pi_a\pi_c + 2bc\pi_a\pi_c)$$

(?)

Of course, this is a simple change of variables.

What about quantization?

Consider the hydrogen atom (s-states, $\hbar = e = 1$):

Classically

$$S = \int (p_r \frac{dr}{dt} - H) dt, \qquad H = \frac{1}{2} p_r^{\,2} - \frac{1}{r}$$

$r > 0$, so try exponential parametrization, $r = e^{\alpha}$

$$S = \int (p_r \frac{d\alpha}{dt} e^{\alpha} - H) dt = \int (p_\alpha \frac{d\alpha}{dt} - H) dt$$

$$\Longrightarrow p_r = e^{-\alpha} p_\alpha$$

and $$H = e^{-\alpha} (\frac{1}{2} e^{-\alpha} p_\alpha^2 - 1)$$

Quantization:

- Time-independent Schrödinger eqn.

1) "Misner" factor ordering:

$$-\frac{1}{2}e^{-2\alpha}\frac{d^2\psi(\alpha)}{d\alpha^2}-e^{-\alpha}\psi(\alpha)=E\psi(\alpha)$$

$$\Rightarrow\frac{d^2\psi}{d\alpha^2}+2(e^{2\alpha}E+e^{\alpha})\psi=0$$

Taking $\psi = e^{-\kappa e^{\alpha}/2} F(\kappa e^{\alpha})$,

$$E = -\frac{1}{2(n+\frac{1}{2})}, n = 0, \ldots$$

Wrong!

(by experiment)

2) "DeWitt" factor ordering:

Laplace-Beltrami operator with $\sqrt{g} = e^{\alpha} \sin\theta$

$$\Rightarrow -\frac{1}{2} e^{-3\alpha} \frac{d}{d\alpha}\left(e^{\alpha} \frac{d\psi}{d\alpha} \right) - e^{-\alpha}\psi = E\psi$$

Right answer

Side remark:

3) “Hawking” factor ordering

$$-\frac{1}{2}e^{-2\alpha-B\alpha}\frac{d}{d\alpha}\left(e^{B\alpha}\frac{d\psi}{d\alpha}\right)-e^{-\alpha}\psi=E\psi$$

B arbitrary constant

Wrong for $B \neq 1$

(In QC the “Hawking” factor ordering leads to interesting exact solutions for Bianchi IX)

Hydrogen atom: this change of variables is OK up to factor ordering.

In QC, lacking experiment, take your choice.

No one can prove you wrong!

II) Superspace

- Space of **geometries** NOT space of metrics

True gravitational variables in vacuum = 2

As for any massless field.

$6g_{ij}$,4 coordinate choices, 4 constraints

$\Longrightarrow$ 2 true dynamical degrees of freedom (TDDF) and their momenta

Gauge choices as in E + M

Gravitation: Gauge choice = coordinate choice, leading to “evil” “internal time”

Contemporary choice: scalar field ϕ allows

3 variables,

$\boldsymbol{\phi(t)} \longrightarrow \boldsymbol{T}$ and a, b, c or α, β_+, β_-

Basically the true spirit of the “ADM method” is based on superspace (or vice versa) not on the space of metrics.

Coordinate choices determine the TDDF’s, often transverse traceless, TT, metric components

Origin of an “already parametrized” view of gravity.

Simple example:

Harmonic oscillator:

$$S = \int (p_x \frac{dx}{dt} - \frac{1}{2}[p_x^2 + x^2])dt$$

Now expand variable set, adding $\tau(t)$

$\Longrightarrow$ constrained action

$$S \to \int [p_x \frac{dx}{dt} + p_\tau \frac{d\tau}{dt} - N(p_\tau - \frac{p_x^2}{2} - \frac{x^2}{2})]dt$$

$$\tau = t \Rightarrow p_\tau = H, H = \frac{p_x^2}{2} + \frac{x^2}{2}$$

Relativity is already in this form

Coordinate (time) choices in QC:

The Misner time choice was the "naughty"

$$\alpha = t$$

Problems:

1) Pushes singularity to $t = -\infty$

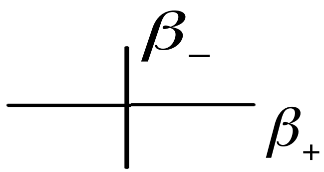
2) $\alpha = \frac{1}{3}\ln(V), V =$ Volume of universe

(Makes it hard to avoid singularity, i.e have a "bounce", "time" would have to run backward, or new coord. patch)

Coordinate patches in quantum gravity?

Extreme superspace

Consider β_+, β_- space:



Change of variables:
$$\beta_+ = \beta \cos \chi$$
$$\beta_- = \beta \sin \chi$$

$$ds^2 = -N^2 dt^2 + a^2 dx^2 + b^2 dy^2 + c^2 dz^2 \quad \Longrightarrow$$

$$a = e^{\alpha + 2\beta \cos(\chi + \pi/3)} \quad b = e^{\alpha + 2\beta \cos(\chi - \pi/3)} \quad c = e^{\alpha - 2\beta \cos \chi}$$

Notice that: 1) $\chi \to \chi - 2\pi/3$

$$\Rightarrow a \to c,\ b \to a,\ c \to b$$

and 2) $\chi \to \chi + 2\pi/3$

$$\Rightarrow\ a \to b,\ b \to c,\ c \to a$$

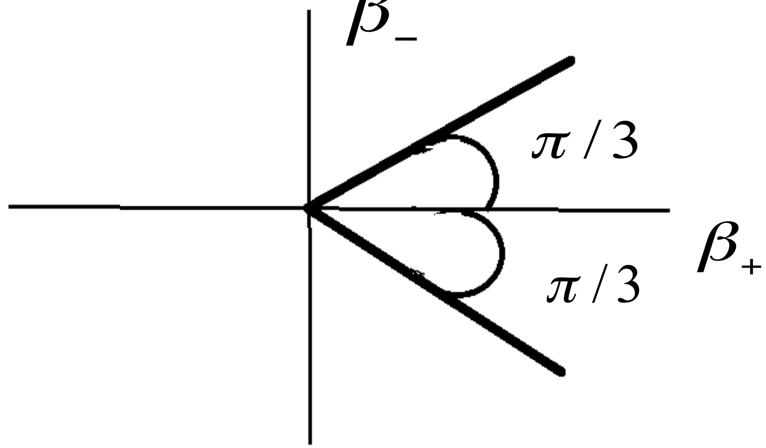
1) Is equivalent to $x' = y, y' = x, z' = y$

2) Is equivalent to $z' = x,\ x' = y,\ y' = z$

Extreme superspace would try to gauge out these diffeos

These transformations can be used to map

β_+, β_- space to the wedge:



Identifying top and bottom edges gives a conical minisuperspace

Hamiltonian constraint for the α, β, χ variables:

$$\mathrm{H} = \frac{e^{-3\alpha}}{24}\left(-p_\alpha^2 + p_\beta^2 + \frac{p_\chi^2}{\beta^2}\right)$$

with

$$S = \int \left[p_\alpha \frac{d\alpha}{dt} + p_\beta \frac{d\beta}{dt} + p_\chi \frac{d\chi}{dt} - N \, \mathrm{H} \right] dt$$

Invariant under $\chi \to \chi \pm 2\pi / 3$

This space has a conical singularity at $\beta_+ = \beta_- = 0$ (FRW $k = 0$).

Not surprising: Superspace can be singular at metrics of higher symmetry.

Quantize on this minisuperspace?