Parameters of inflation in the Einstein and modified gravity with new CMB data

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Present status of the inflationary scenario

Generation of scalar and tensor perturbations during inflation

Predictions for post-inflationary metric perturbations

From metric perturbations to CMB observations

What CMB observations tell us about inflation

Four epochs of the history of the Universe

$H \equiv \frac{\dot{a}}{a}$ where $a(t)$ is a scale factor of an isotropic homogeneous spatially flat universe (a Friedmann-Lemaître-Robertson-Walker background):

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2) + small\ perturbations$$

The history of the Universe in one line according to the present paradigm:

$$? \longrightarrow DS \Longrightarrow FLRWRD \Longrightarrow FLRWMD \Longrightarrow \overline{DS} \longrightarrow ?$$

$$|\dot{H}| << H^2 \Longrightarrow H = \frac{1}{2t} \Longrightarrow H = \frac{2}{3t} \Longrightarrow |\dot{H}| << H^2$$

$$p \approx -\rho \Longrightarrow p = \rho/3 \Longrightarrow p \ll \rho \Longrightarrow p \approx -\rho$$

Main advantages of inflation

1. Aesthetic elegance

   Inflation – hypothesis about an almost maximally symmetric (quasi-de Sitter) stage of the evolution of our Universe in the past, before the hot Big Bang. If so, preferred initial conditions for (quantum) inhomogeneities with sufficiently short wavelengths exist – the adiabatic in-vacuum ones. In addition, these initial conditions represent an attractor for a much larger compact open set of initial conditions having a non-zero measure in the space of all initial conditions.

2. Predictability, proof and/or falsification

   Given equations, this gives a possibility to calculate all subsequent evolution of the Universe up to the present time and even further to the future. Thus, any concrete inflationary model can be proved or disproved by observational data.

3. Naturalness of the hypothesis

Remarkable qualitative similarity between primordial and present dark energy.

4. Relates quantum gravity and quantum cosmology to astronomical observations

Makes quantum gravity effects observable at the present time and at very large – cosmological – scales.

Present status of inflation

From "proving" inflation to using it as a tool

Present status of inflation: transition from "proving" it in general and testing some of its simplest models to applying the inflationary paradigm to investigate particle physics at super-high energies and the actual history of the Universe in the remote past using real observational data on $n_s(k) - 1$ and $r(k)$.

Generation of scalar and tensor perturbations during inflation

A genuine quantum-gravitational effect: a particular case of the effect of particle-antiparticle creation by an external gravitational field. Requires quantization of a space-time metric. Similar to electron-positron creation by an electric field. From the diagrammatic point of view: an imaginary part of a one-loop correction to the propagator of a gravitational field from all quantum matter fields including the gravitational field itself, too.

The effect can be understood from the behaviour of a light scalar field in the de Sitter space-time.

De Sitter space-time

Constant curvature space-time.

$$R_{\alpha\beta\gamma\delta} = H_0^2 (g_{\alpha\gamma} g_{\beta\delta} - g_{\alpha\delta} g_{\beta_\gamma})$$

4 most popular forms of its space-time metric (only the first metric covers the whole space-time):

$$ds^2 = dt_c^2 - H_0^{-2} \cosh^2(H_0 t_c) \left(d\chi_c^2 + \sin^2 \chi_c d\Omega^2\right)$$

$$ds^2 = dt^2 - a_1^2 e^{2H_0 t} \left(dr^2 + r^2 d\Omega^2\right), \;\; a_1 = const$$

$$ds^2 = dt_o^2 - H_0^{-2} \sinh^2(H_0 t_o) \left(d\chi_o^2 + \sinh^2 \chi_o d\Omega^2\right)$$

$$ds^2 = (1 - H_0^2 R^2) d\tau^2 - (1 - H_0^2 R^2)^{-1} dR^2 - R^2 d\Omega^2$$

$$d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$$

Anomalous growth of light scalar fields in the de Sitter space-time

Background - fixed - de Sitter or, more interestingly, quasi-de Sitter space-time (slow roll inflation).
Occurs for $0 \leq m^2 \ll H^2$ where $H \equiv \frac{\dot{a}}{a}$, $a(t)$ is a FRW scale factor. The simplest and textbook example:
$m = 0$, $H = H_0 = const$ for $t \geq t_0$ and the initial quantum state of the scalar field at $t = t_0$ is the adiabatic vacuum for modes with $k/a(t_0) \gg H_0$ and some infrared finite state otherwise.

The wave equation:

$$\phi_{;\mu}^{;\mu} = 0$$

Equation for the time dependent part of tensor perturbations on a FLRW background supported by ideal fluids or minimally coupled scalar fields has the same form.

Quantization with the adiabatic vacuum initial condition:

$$\phi = (2\pi)^{-3/2} \int \left[\hat{a}_{\mathbf{k}}\, \phi_k(\eta)\, e^{-i\mathbf{kr}} + \hat{a}_{\mathbf{k}}^{\dagger}\, \phi_k^*\, e^{i\mathbf{kr}}\right] d^3k$$

$$\phi_k(\eta) = \frac{H_0 e^{-ik\eta}}{\sqrt{2k}} \left(\eta - \frac{i}{k}\right),\ a(\eta) = \frac{1}{H_0\eta},\ \eta_0 < \eta < 0,\ k = |\mathbf{k}|$$

Then

$$<\phi^2> = \frac{H_0^2 N}{4\pi^2} + const$$

Here $N = \ln \frac{a}{a(t_0)} \gg 1$ is the number of e-folds from the beginning of inflation and the constant depends on the initial quantum state (Linde, 1982; AS, 1982; Vilenkin and Ford, 1982).

Straightforward generalization to the slow-roll case $|\dot{H}| \ll H^2$.

For $0 < m^2 \ll H^2$, the Bunch-Davies equilibrium value

$$< \phi^2 >= \frac{3H_0^4}{8\pi m^2} \gg H_0^2$$

is reached after a large number of e-folds $N \gg \frac{H_0^2}{m^2}$.
Purely infrared effect - creation of real field fluctuations; renormalization is not important and does not affect it.

For the de Sitter inflation (gravitons only) (AS, 1979):

$$P_g(k) = \frac{16GH_0^2}{\pi}; \quad < h_{ik}h^{ik} >= \frac{16GH_0^2 N}{\pi} .$$

The assumption of small perturbations breaks down for $N \gtrsim 1/GH_0^2$. Still ongoing discussion on the final outcome of this effect. My opinion - no screening of the background cosmological constant, instead - stochastic drift through an infinite number of locally de Sitter, but globally non-equivalent vacua.

Reason: the de Sitter space-time is not the generic late-time asymptote of classical solutions of GR with a cosmological constant Λ both without and with hydrodynamic matter. The generic late-time (expanding) asymptote is (AS, 1983):

$$ds^2 = dt^2 - \gamma_{ik} dx^i dx^k$$

$$\gamma_{ik} = e^{2H_0 t} a_{ik} + b_{ik} + e^{-H_0 t} c_{ik} + \ldots$$

where $H_0^2 = \Lambda/3$ and the matrices a_{ik}, b_{ik}, c_{ik} are functions of spatial coordinates. a_{ik} contains two independent physical functions (after 3 spatial rotations and 1 shift in time + spatial dilatation) and can be made unimodular, in particular.

Generation of metric perturbations

One spatial Fourier mode $\propto e^{i\mathbf{kr}}$ is considered.

For scales of astronomical and cosmological interest, the effect of creation of metric perturbations occurs at the primordial de Sitter (inflationary) stage when $k \sim a(t)H(t)$ where $k \equiv |\mathbf{k}|$ (the first Hubble radius crossing).

After that, for a very long period when $k \ll aH$ until the second Hubble radius crossing (which occurs rather recently at the radiation or matter dominated stages), there exist one mode of scalar (adiabatic, density) perturbations and two modes of tensor perturbations (primordial gravitational waves) for which metric perturbations are constant (in some gauge) and independent of (unknown) local microphysics due to the causality principle.

Classical-to-quantum transition for the leading modes of perturbations

In the superhorizon regime in the coordinate representation:

$$ds^2 = dt^2 - a^2(t)(\delta_{lm} + h_{lm})dx^l dx^m, \quad l, m = 1, 2, 3$$

$$h_{lm} = 2\zeta(\mathbf{r})\delta_{lm} + \sum_{a=1}^{2} g^{(a)}(\mathbf{r})\, e_{lm}^{(a)}$$

$$e_l^{l(a)} = 0, \;\; g^{(a)}_{,l}\, e_m^{l(a)} = 0, \;\; e_{lm}^{(a)}\, e^{lm(a)} = 1$$

ζ describes primordial scalar perturbations, g – primordial tensor perturbations (primordial gravitational waves (GW)).

Quantum-to-classical transition: in fact, metric perturbations h_{lm} are quantum (operators in the Heisenberg representation) and remain quantum up to the present time. But, after omitting of a very small part, decaying with time, they become commuting and, thus, equivalent to classical (c-number) stochastic quantities with the Gaussian statistics (up to small terms quadratic in $\zeta, \; g$).

Remaining quantum coherence: deterministic correlation between $\mathbf{k}$ and $-\mathbf{k}$ modes - shows itself in the appearance of acoustic oscillations (primordial oscillations in case of GW).

δN formalism for scalar perturbations

Local duration of inflation in terms of $N_{tot} = \ln\left(\frac{a(t_{fin})}{a(t_{in})}\right)$ is different is different point of space: $N_{tot} = N_{tot}(\mathbf{r})$. Then

$$\zeta(\mathbf{r}) = \delta N_{tot}(\mathbf{r})$$

Correct generalization to the non-linear case: the space-time metric after the end of inflation at super-Hubble scales

$$ds^2 = dt^2 - a^2(t)e^{2N_{tot}(\mathbf{r})}(dx^2 + dy^2 + dz^2)$$

First derived in A. A. Starobinsky, Phys. Lett. B **117**, 175 (1982) in the case of one-field inflation.

FLRW dynamics with a scalar field

The simplest case: GR and a minimally coupled scalar field with some potential. However, scalar-tensor gravity and its particular case – $F(R)$ gravity – can work as well.
In the absence of spatial curvature and other matter:

$$H^2 = \frac{\kappa^2}{3}\left(\frac{\dot{\phi}^2}{2} + V(\phi)\right)$$

$$\dot{H} = -\frac{\kappa^2}{2}\dot{\phi}^2$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

where $\kappa^2 = 8\pi G$ ($\hbar = c = 1$).

Inflationary slow-roll dynamics

Slow-roll occurs if: $|\ddot{\phi}| \ll H|\dot{\phi}|, \;\; \dot{\phi}^2 \ll V$, and then $|\dot{H}| \ll H^2$.

Necessary conditions: $|V'| \ll \kappa V, \;\; |V''| \ll \kappa^2 V$. Then

$$H^2 \approx \frac{\kappa^2 V}{3}, \;\; \dot{\phi} \approx -\frac{V'}{3H}, \;\; N \equiv \ln \frac{a_f}{a} \approx \kappa^2 \int_{\phi_f}^{\phi} \frac{V}{V'}\, d\phi$$

First obtained in A. A. Starobinsky, Sov. Astron. Lett. 4, 82 (1978) in the $V = \frac{m^2\phi^2}{2}$ case and for a bouncing model.

Spectral predictions of the one-field inflationary scenario in GR

Scalar (adiabatic) perturbations:

$$P_\zeta(k) = \frac{H_k^4}{4\pi^2\dot{\phi}^2} = \frac{GH_k^4}{\pi|\dot{H}|_k} = \frac{128\pi G^3 V_k^3}{3V_k'^2}$$

where the index k means that the quantity is taken at the moment $t = t_k$ of the Hubble radius crossing during inflation for each spatial Fourier mode $k = a(t_k)H(t_k)$. Through this relation, the number of e-folds from the end of inflation back in time $N(t)$ transforms to $N(k) = \ln\frac{k_f}{k}$ where $k_f = a(t_f)H(t_f)$, t_f denotes the end of inflation.
The spectral slope

$$n_s(k) - 1 \equiv \frac{d\ln P_\zeta(k)}{d\ln k} = \frac{1}{\kappa^2}\left(2\frac{V_k''}{V_k} - 3\left(\frac{V_k'}{V_k}\right)^2\right)$$

Generically $n_s \neq 1$, though $|n_s - 1| \ll 1$ – deviation from the Harrison-Zeldovich spectrum is expected!

The special case when $n_s \equiv 1$: $V(\phi) \propto \phi^{-2}$ in the slow-roll approximation.

Omitting the slow-roll assumption:

let $x = \sqrt{4\pi G}\phi, \quad y = B\sqrt{4\pi G}H, \quad v(x) = \frac{32\pi^2 G^2 B^2}{3} V(\phi)$.

Then (A. A. Starobinsky, JETP Lett. 82, 169 (2005)):

$$y = e^{x^2/2}\left(\int_x^\infty e^{-\tilde{x}^2/2}\, d\tilde{x} + C\right)$$

$$v = y^2 - \frac{1}{3}\left(\frac{dy}{dx}\right)^2$$

Tensor perturbations (A. A. Starobinsky, JETP Lett. 50, 844 (1979)):

$$P_g(k) = \frac{16GH_k^2}{\pi}; \quad n_g(k) \equiv \frac{d \ln P_g(k)}{d \ln k} = -\frac{1}{\kappa^2}\left(\frac{V_k'}{V_k}\right)^2$$

The consistency relation:

$$r(k) \equiv \frac{P_g}{P_\zeta} = \frac{16|\dot{H}_k|}{H_k^2} = 8|n_g(k)|$$

Tensor perturbations are always suppressed by at least the factor $\sim 8/N(k)$ compared to scalar ones. For the present Hubble scale, $N(k_H) = (50 - 60)$.

Potential reconstruction from scalar power spectrum

In the slow-roll approximation:

$$\frac{V^3}{V'^2} = C P_\zeta(k(t(\phi))), \quad C = const$$

Changing variables for ϕ to $N(\phi)$ and integrating, we get:

$$\frac{1}{V(N)} = -\frac{\kappa^2}{C} \int \frac{dN}{P_\zeta(N)}$$

$$\kappa\phi = \int dN \sqrt{\frac{d \ln V}{dN}}$$

An ambiguity in the form of $V(\phi)$ because of an integration constant in the first equation. Information about $P_g(k)$ helps to remove this ambiguity.

In particular, if primordial GW are not discovered in the order $n_s - 1$:

$$r \ll 8|n_s - 1| \approx 0.3 \ ,$$

then $\left(\frac{V'}{V}\right)^2 \ll \left|\frac{V''}{V}\right|$, $|n_g| = \frac{r}{8} \ll |n_s - 1|$, $|n_g|N \ll 1$.

This is possible only if $V = V_0 + \delta V$, $|\delta V| \ll V_0$ – a plateau-like potential. Then

$$\delta V(N) = \frac{\kappa^2 V_0^2}{C} \int \frac{dN}{P_\zeta(N)}$$

$$\kappa\phi = \int \frac{dN}{\sqrt{V_0}} \sqrt{\frac{d(\delta V(N))}{dN}}$$

Here, integration constants renormalize V_0 and shift ϕ. Thus, the unambiguous determination of the form of $V(\phi)$ without knowledge of $P_g(k)$ becomes possible.

CMB temperature anisotropy

$$\Delta T(\theta, \phi) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\theta, \phi)$$

$$< a_{\ell m} a_{\ell' m'} >= C_\ell \delta_{ll'} \delta_{mm'}$$

Theory: averaging over realizations.
Observations: averaging over the sky for a fixed ℓ.

For scalar perturbations, generated mainly at the last scattering surface (the surface or recombination) at $z_{LSS} \approx 1090$ (the Sachs-Wolfe, Silk and Doppler effects), but also after it (the integrated Sachs-Wolfe effect).
For GW – only the ISW works.

For $\ell \lesssim 50$, neglecting the Silk and Doppler effects, as well as the ISW effect due the presence of dark energy,

$$\frac{\Delta T(\theta, \phi)}{T_\gamma} = -\frac{1}{5}\zeta(r_{LSS}, \theta, \phi) = -\frac{1}{5}\delta N_{tot}(r_{LSS}, \theta, \phi)$$

For $n_s = 1$,

$$\ell(\ell+1)C_{\ell,s} = \frac{2\pi}{25}P_\zeta$$

For $1 \ll \ell \lesssim 50$, the Sachs-Wolfe plateau occurs for the contribution from GW, too:

$$\ell(\ell+1)C_{\ell,g} = \frac{\pi}{36}\left(1 + \frac{48\pi^2}{385}\right) P_g$$

assuming $n_t = 1$ (A. A. Starobinsky, Sov. Astron. Lett. 11, 133 (1985)). So,

$$C_\ell = C_{\ell,s} + C_{\ell,g} = (1 + 0.775r)\,C_{\ell,s}$$

.

For larger $\ell > 50$, $\ell(\ell+1)C_{\ell,s}$ grows and the first acoustic peak forms at $\ell \approx 200$, while $\ell(\ell+1)C_{\ell,g}$ decreases quickly. Thus, the presence of GW should lead to a step-like enhancement of $\ell(\ell+1)C_\ell$ for $\ell \lesssim 50$.

CMB polarization

Produced at the last scattering surface only due to the Thomson scattering of photons on electrons, suppressed by the factor $\Delta z_{LSS}/z_{LSS} \sim 0.1$ compared to a temperature anisotropy.

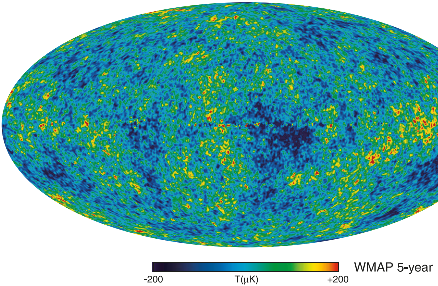
No circular polarization, only linear one.

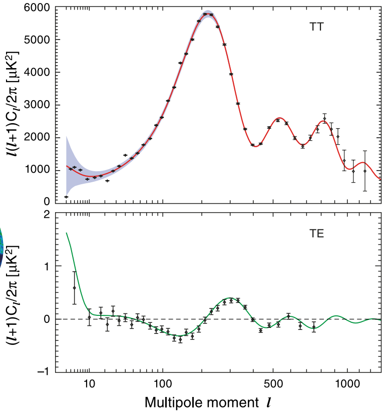
Linear polarization on the sky (2-sphere) can be decomposed into the E-mode (scalar) and the B-mode (pseudoscalar).

1. Expand the $Q \pm iU$ combinations of the Stokes parameters into spin-weighted spherical harmonics ${}_{\pm 2}Y_{\ell m}$.
2. Then

$$a_{E,\ell m} = -\left(a_{2,\ell m} + a_{-2,\ell m}\right)/2, \;\; a_{B,\ell m} = \left(a_{2,\ell m} - a_{-2,\ell m}\right)/2$$

In the first order, the E-mode is produced both by scalar perturbations and GW, the B-mode is produced by GW only. The most important second order effect through which scalar perturbations produce B-mode: gravitational lensing of CMB fluctuations, screens the first order effect for multipoles $\ell > 150$.


WMAP 5-year
-200
T(μK)
+200


TT
TE
6000
5000
4000
3000
2000
1000
0
2
1
0
−1
10
100
500
1000
l(l+1)C_l/2π [μK²]
(l+1)C_l/2π [μK²]
Multipole moment l

Outcome of recent CMB observations

I. More than a year ago

The most important for the history of the early Universe are:

1. The primordial spectrum of scalar perturbations has been measured and its deviation from the flat spectrum $n_s = 1$ in the first order in $|n_s - 1| \sim N^{-1}$ has been discovered (using the multipole range $\ell > 40$):

$$P_\zeta(k) = \int \frac{\Delta_\zeta^2(k)}{k} dk, \quad \Delta_\zeta^2 = \left(2.20^{+0.05}_{-0.06}\right) 10^{-9} \left(\frac{k}{k_0}\right)^{n_s - 1}$$

$$k_0 = 0.05\mathrm{Mpc}^{-1}, \quad n_s - 1 = -0.040 \pm 0.007$$

N.B.: The value is obtained under some natural assumptions, the most critical of them is $N_\nu = 3$, for $N_\nu = 4$ many things have to be reconsidered and $n_s \approx 1$ is not excluded.

2. Neither the B-mode of CMB polarization, nor primordial GW were discovered: $r < 0.11$ at the 95% CL.

NB: The assumption: $n_s - 1 = -\frac{2}{N} \approx -0.04$ for all $N = 1 - 60$ implies a lower bound on r. In particular, if $r \ll 8|n_s - 1|$, then

$$V(\phi) = V_0 \left(1 - \exp(-\alpha\kappa\phi)\right)$$

with $\alpha\kappa\phi \gg 1$ but α not very small, and

$$r = \frac{8}{\alpha^2 N^2}$$

Combined results from WMAP and Planck

P. A. R. Ade et al., arXiv:1303.5082

Model	Parameter	*Planck*+WP	*Planck*+WP+lensing	*Planck* + WP+high-ℓ	*Planck*+WP+BAO
ΛCDM + tensor	n_s	0.9624 ± 0.0075	0.9653 ± 0.0069	0.9600 ± 0.0071	$0.9643 + 0.0059$
	$r_{0.002}$	< 0.12	< 0.13	< 0.11	< 0.12
	$-2\Delta \ln \mathcal{L}_{max}$	0	0	0	-0.31

Table 4. Constraints on the primordial perturbation parameters in the ΛCDM+r model from *Planck* combined with other data sets. The constraints are given at the pivot scale $k_* = 0.002\ \mathrm{Mpc}^{-1}$.

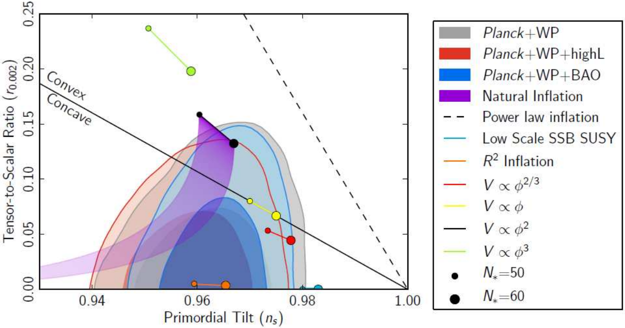


Fig. 1. Marginalized joint 68% and 95% CL regions for n_s and $r_{0.002}$ from *Planck* in combination with other data sets compared to the theoretical predictions of selected inflationary models.

The simplest models producing the observed scalar slope

I. In the Einstein gravity:

$$V(\phi) = \frac{m^2\phi^2}{2}$$

$$m \approx 1.8 \times 10^{-6} \left(\frac{55}{N}\right) M_{Pl} \approx 2 \times 10^{13}\,\mathrm{GeV}$$

$$n_s - 1 = -\frac{2}{N} \approx -0.036, \quad r = \frac{8}{N} \approx 0.15$$

$$H_{dS}(N = 55) = 1.4 \times 10^{14}\mathrm{GeV}$$

II. In the modified, scalar-tensor gravity:

$$f(R) = R + \frac{R^2}{6M^2}$$

$$M = 2.6 \times 10^{-6} \left(\frac{55}{N}\right) M_{Pl} \approx 3 \times 10^{13}\,\mathrm{GeV}$$

$$n_s - 1 = -\frac{2}{N} \approx -0.036, \quad r = \frac{12}{N^2} \approx 0.004$$

$$H_{dS}(N = 55) = 1.4 \times 10^{14}\mathrm{GeV}$$

The same prediction from a scalar field model with $V(\phi) = \frac{\lambda\phi^4}{4}$ at large ϕ and strong non-minimal coupling to gravity $\xi R\phi^2$ with $\xi < 0, \ |\xi| \gg 1$, including the Brout-Englert-Higgs inflationary model.

Note similar predictions for inflaton masses and essentially the same prediction for H_{dS}.

II. Four months ago

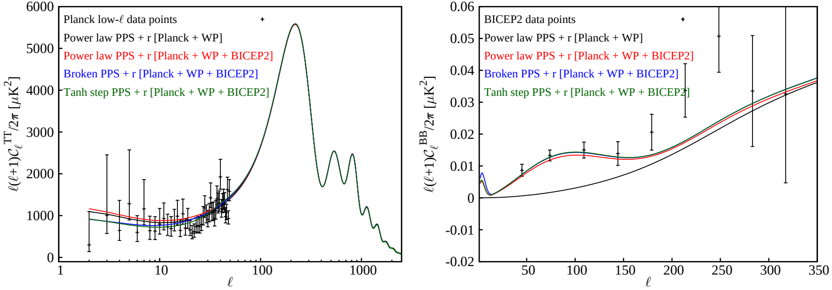
BISEP2 collaboration: P. A. R. Ade et al., arXiv:1403.3985: discovery of the B-mode in the multipole range $30 < l < 150$ (for larger l it was discovered earlier this year with the amount in agreement from gravitational lensing of scalar perturbations) with

$$r = 0.20^{+0.07}_{-0.05}$$

The unsubtracted result – contains an unknown foreground non-thermal part.

Consequence – assuming the Einstein gravity:

$$\sqrt{G}H_{dS} = 0.99 \times 10^{-5} \left(\frac{r_{0.002}}{0.2}\right)^{1/2} 5^{0.96-n_s}$$


Planck low-ℓ data points
Power law PPS + r [Planck + WP]
Power law PPS + r [Planck + WP + BICEP2]
Broken PPS + r [Planck + WP + BICEP2]
Tanh step PPS + r [Planck + WP + BICEP2]
$\ell(\ell+1)C_\ell^{TT}/2\pi$ [μK^2]
ℓ
BICEP2 data points
Power law PPS + r [Planck + WP]
Power law PPS + r [Planck + WP + BICEP2]
Broken PPS + r [Planck + WP + BICEP2]
Tanh step PPS + r [Planck + WP + BICEP2]
$\ell(\ell+1)C_\ell^{BB}/2\pi$ [μK^2]
ℓ

Consequences of the would be discovery primordial GW

If confirmed by an independent measurement:

1. Discovery of a real physical singularity – a state of the Universe in the past with a very high curvature (with H only 5 orders of magnitude less than the Planck mass).

2. Discovery of a new class of gravitational waves – primordial ones.

3. Decisive argument for the necessity of quantization of gravitational waves.

4. Decisive test of the inflationary paradigm as a whole.

5. Discovery of $\sim 20\%$ deviation of the power spectrum of scalar perturbations from a scale-free one – new physics during inflation!

The most intriguing discordance between WMAP and Planck results from one side and the BISEP2 ones from the other: no sign of GW in the CMB temperature anisotropy power spectrum.

Instead of the $\sim 10\%$ increase of $\ell(\ell+1)C_\ell$ over the multipole range $2 \ll l < 50$, a $\sim 10\%$ depression is seen for $20 \lesssim l \lesssim 40$ (see e.g. Fig. 39 of arXiv:1303.5076).

The feature exists even if $r \ll N^{-1}$ but the presence of $r \sim 0.1$ makes it larger.

More detailed analysis in D. K. Hazra, A. Shafieloo, G. F. Smoot and A. A. Starobinsky, JCAP 1406, 061 (2014), arXiv:1403.7786 :
the power-law form of $P_\zeta(k)$ is excluded at more than 3σ CL.

Broken scale models describing both WMAP-Planck and BISEP2 data

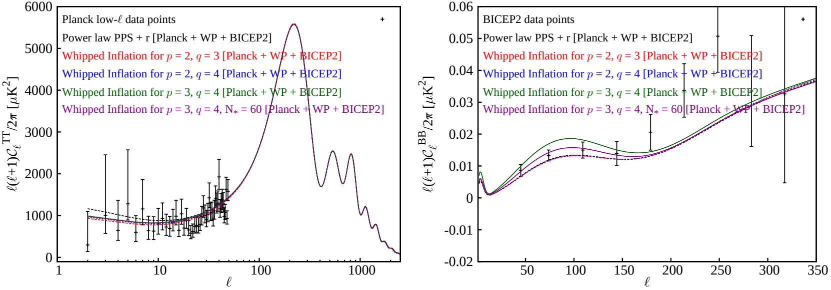
Next step: "whipped inflation" D. K. Hazra, A. Shafieloo, G. F. Smoot and A. A. Starobinsky, arXiv:1404.0360.

The model contains a new scale at which the effective inflaton potential has a feature which the inflaton crosses about 50 e-folds before the end of inflation. The existence of such a feature, in turn, requires some new physics (e.g. fast phase transition in a second field coupled to the inflaton).

$$V(\phi) = V_S(\phi) + V_R(\phi)$$

$$V_S(\phi) = \gamma\phi^p, \quad V_R(\phi) = \lambda(\phi - \phi_0)^q \Theta(\phi - \phi_0)$$

Best results for $(p, q) = (2, 3)$.


Planck low-ℓ data points
Power law PPS + r [Planck + WP + BICEP2]
Whipped Inflation for p = 2, q = 3 [Planck + WP + BICEP2]
Whipped Inflation for p = 2, q = 4 [Planck + WP + BICEP2]
Whipped Inflation for p = 3, q = 4 [Planck + WP + BICEP2]
Whipped Inflation for p = 3, q = 4, N* = 60 [Planck + WP + BICEP2]
ℓ(ℓ+1)C_ℓ^TT/2π [μK²]
ℓ
BICEP2 data points
Power law PPS + r [Planck + WP + BICEP2]
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ℓ(ℓ+1)C_ℓ^BB/2π [μK²]
ℓ

Wiggles in the power spectrum

The effect of the same order: an upward wiggle at $\ell \approx 40$ and a downward one at $\ell \approx 22$.

Lesson: irrespective of a future analysis of foreground contamination in the BISEP2 result, features in the anisotropy spectrum for $20 \lesssim \ell \lesssim 40$ confirmed by WMAP and Planck should be taken into account and studied seriously.

A more elaborated class of model suggested by previous studies of sharp features in the inflaton potential caused, e.g. by a fast phase transition occurred in another field coupled to the inflaton during inflation:
WWI (Wiggly Whipped inflation)
D. K. Hazra, A. Shafieloo, G. F. Smoot and A. A. Starobinsky, arXiv:1405.2012

In particular, the potential with a sudden change of its first derivative:

$$V(\phi) = \gamma\phi^2 + \lambda\phi^p(\phi - \phi_0)\,\theta(\phi - \phi_0)$$

which generalizes the exactly soluble model considered in A. A. Starobinsky, JETP Lett. **55**, 489 (1992) produces $-2\Delta \ln \mathcal{L} = -11.8$ compared to the best-fitted power law scalar spectrum, partly due to the better description of wiggles at both $\ell \approx 40$ and $\ell \approx 22$.

A sharp feature in the potential leads to a rapid increase of the effective inflaton mass, $m^2 = V''(\phi)$, in the vicinity of $\phi = \phi_0$. While $m \approx 2 \times 10^{13}$ GeV for $\phi < \phi_0$, it becomes of the order of 10^{14} GeV and larger at earlier times when $\phi \geq \phi_0$ (but still much less than the energy density scale of the inflaton potential $\sim 3 \times 10^{16}$ GeV).

Conclusions

- Inflation is being transformed into a normal physical theory, based on some natural assumptions confirmed by observations and used to obtain new theoretical knowledge from them.
- First quantitative observational evidence for small quantities of the first order in the slow-roll parameters: $n_s(k) - 1$ and $r(k)$.
- The quantitative theoretical prediction of these quantities is based on gravity (space-time metric) quantization and requires very large space-time curvature in the past of our Universe with a characteristic length only five orders of magnitude larger than the Planck one. Thus, quantum gravity and physical singularity become observable.

- The BISEP2 result by itself is the confirmation of the general prediction (made in 1979) of the early Universe scenario with the de Sitter (inflationary) stage preceding the radiation dominated stage (the hot Big Bang).
- However, would the BISEP2 result be confirmed, inflation is not so simple: the scalar primordial power spectrum deviates from a scale-free one that implies the existence of some scale (i.e. new physics) during inflation.
- Though the Einstein gravity plus a minimally coupled inflaton remains sufficient for description of inflation with existing observational data, modified (in particular, scalar-tensor or $f(R)$) gravity can do it as well.
- The conceptual change in utilizing CMB and other observational data from "proving inflation" to using them to determine the spectrum of particle masses in the energy range $(10^{13} - 10^{14})$ GeV by making a "tomographic" study of inflation.