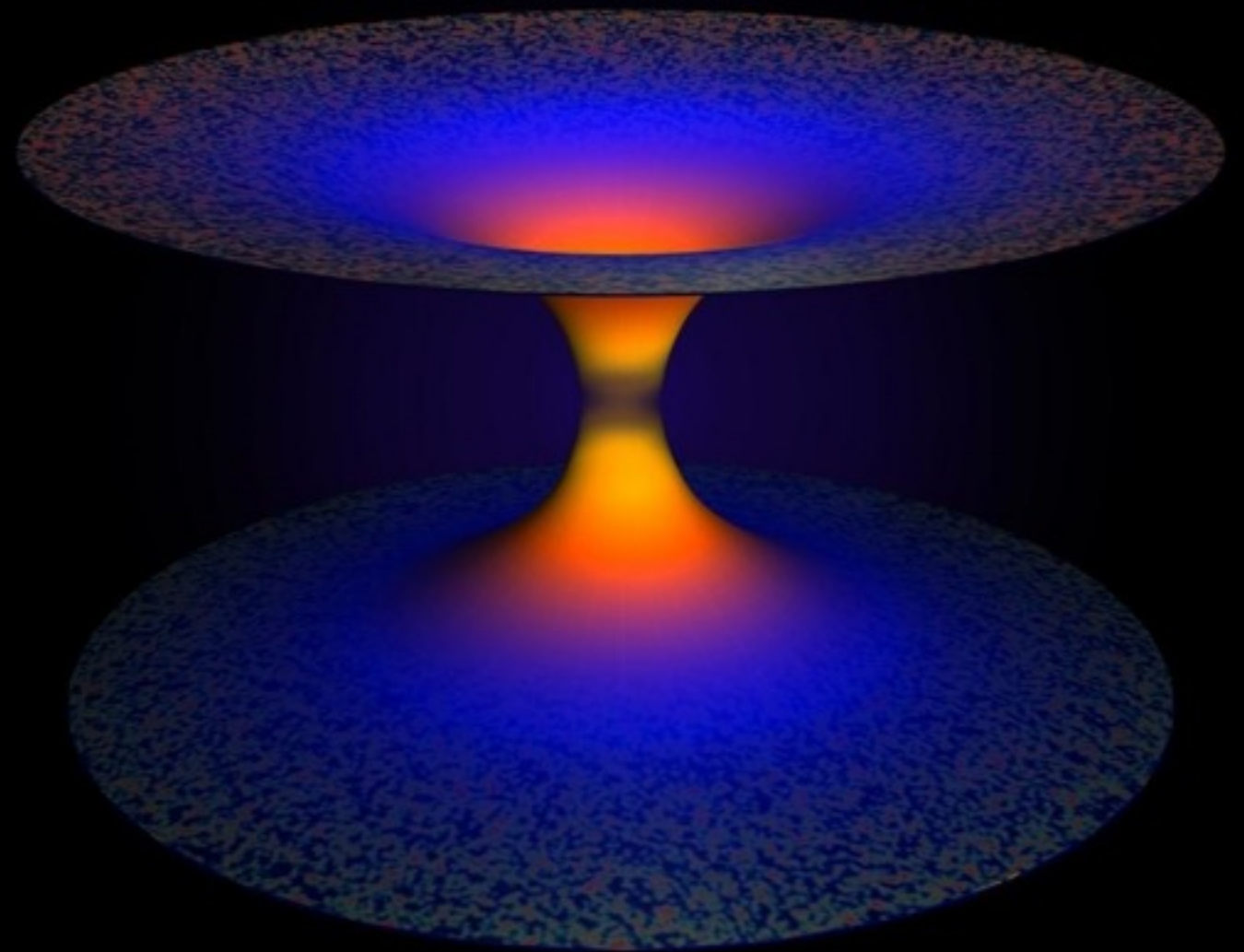


What can we learn from Loop Quantum Cosmology?

the case of **Planck Stars**

Francesca Vidotto

569.WE-Heraeus-Seminar: “Quantum Cosmology”
31 July 2014 Physikzentrum Bad Honnef (Germany)



Netherlands Organisation for Scientific Research

Radboud University Nijmegen



In this talk:

- Quantum cosmology from the full theory:

Spinfoam Cosmology

- A new look at singularity resolution:

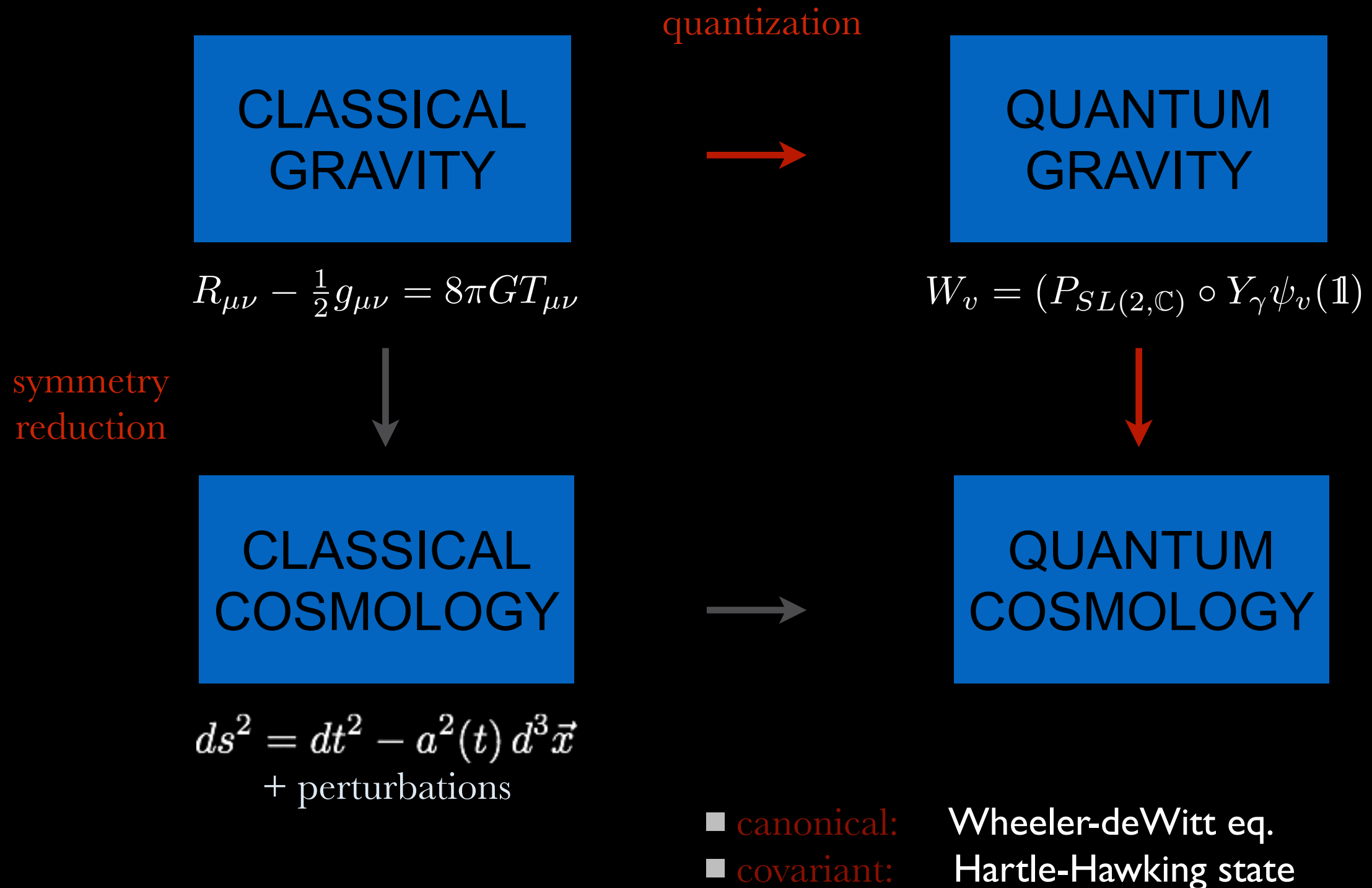
maximal acceleration

- Black holes tunnels into white holes:

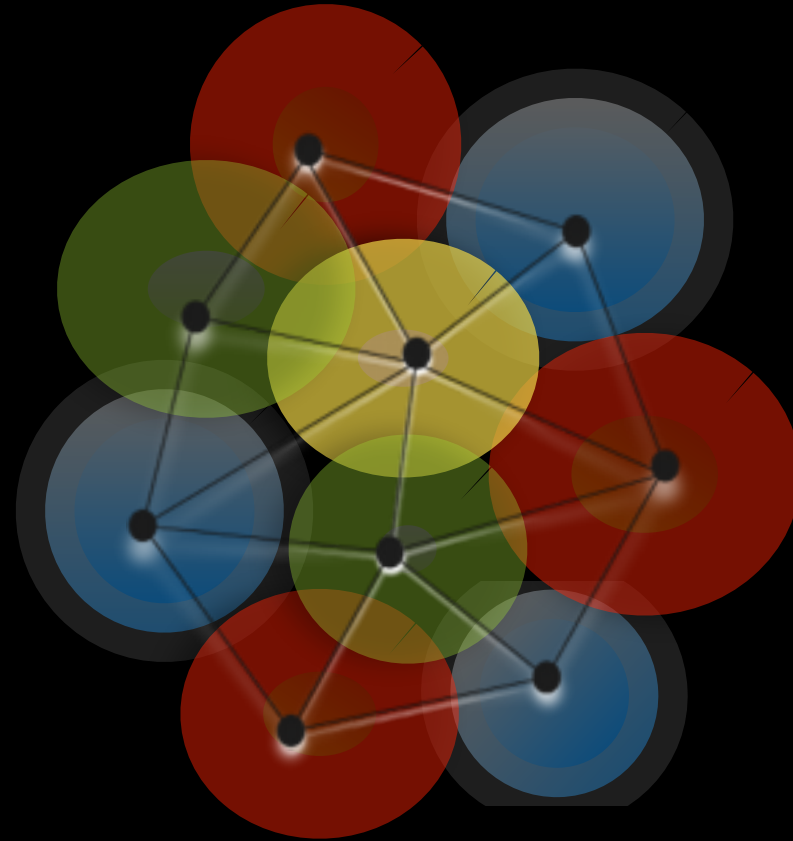
Planck Stars

Quantum Cosmology from the full theory

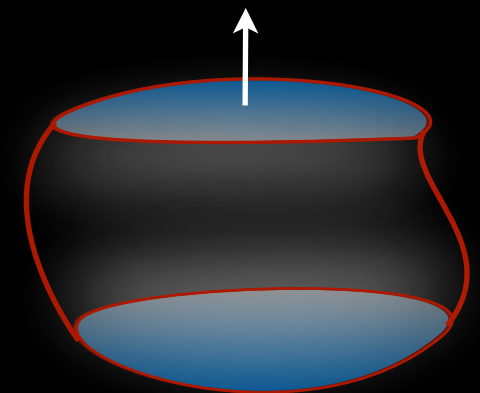
(Ryan's diagram)



Loop Quantum Gravity



- It is a theory about quanta of spacetime
- Local Lorentz invariance
- The states are boundary states at fixed time
- The physical phase space is spanned by $SU(2)$ group variables



$$SL(2, \mathbb{C}) \rightarrow SU(2)$$

Hilbert space and operator algebra

- Group variables: $\begin{cases} h_l \in SU(2) \\ \vec{L}_l \in su(2) \end{cases}$ ■ Adjacency: $\Gamma = \{N, L\}$

- Graph Hilbert space: $\mathcal{H}_\Gamma = L_2[SU(2)^L / SU(2)^N]$

- The space \mathcal{H}_Γ admits a basis $|\Gamma, j_\ell, v_n\rangle$

- Gauge invariant operator $G_{ll'} = \vec{L}_l \cdot \vec{L}_{l'}$ with $\sum_{l \in n} G_{ll'} = 0$
Penrose's **spin-geometry theorem** (1971)

- h_l “Holonomy of the Ashtekar-Barbero connection along the link”

- $\vec{L}_l = \{L_l^i\}, i = 1, 2, 3$ SU(2) generators $L^i \psi(h) \equiv \left. \frac{d}{dt} \psi(h e^{t\tau_i}) \right|_{t=0}$ (tetrad)

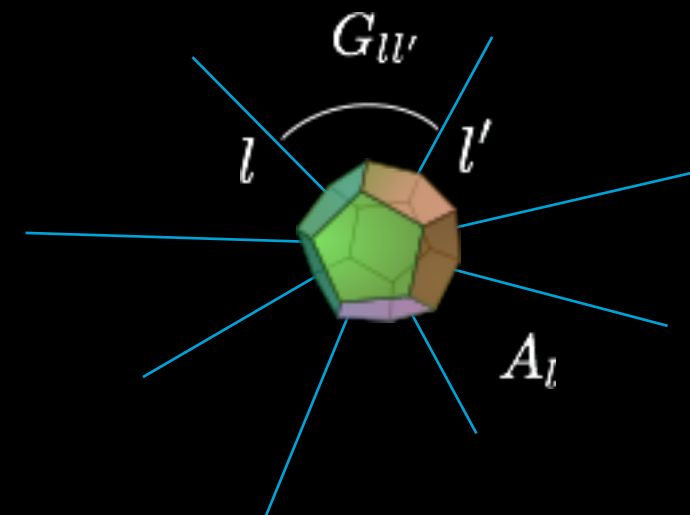
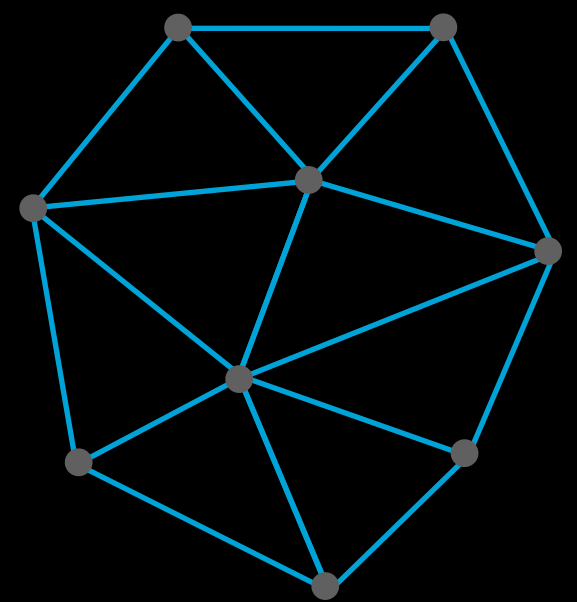
- Area $A_\Sigma = \sum_{l \in \Sigma} \sqrt{L_l^i L_l^i}$

- Volume $V_R = \sum_{n \in R} V_n, \quad V_n^2 = \frac{2}{9} |\epsilon_{ijk} L_l^i L_{l'}^j L_{l''}^k|$

- Angles $L_l^i L_{l'}^i$

- eigenvalues are discrete
- the operators do not commute
- quantum superposition

↪ coherent states



Spinfoam amplitudes

Engle, Pereira, Livine, Rovelli 0711.0146

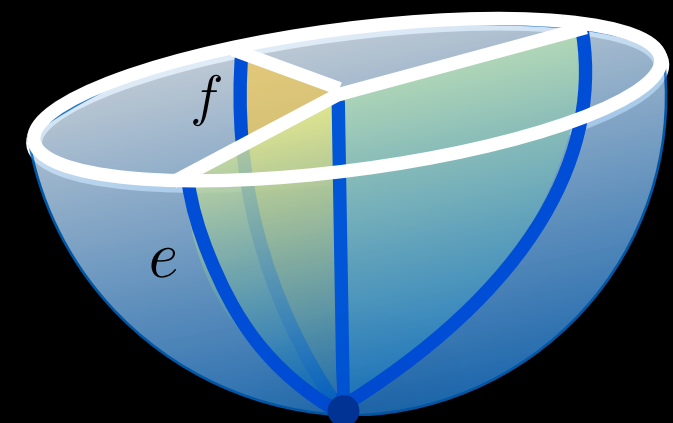
Probability amplitude $P(\psi) = |\langle W | \psi \rangle|^2$
for a state ψ associated to the boundary of a 4d region

$$W(q) \sim \int_{\partial g=q} Dq e^{iS[q]}$$

- Superposition principle $\langle W | \psi \rangle = \sum_{\sigma} W(\sigma)$
- Locality: vertex amplitude $W(\sigma) \sim \prod_v W_v$.
- Lorentz covariance $W_v = (P_{SL(2,\mathbb{C})} \circ Y_{\gamma} \psi_v)(\mathbf{I})$
- UV and IR finite (with Λ)
- Classical limit: GR (with Λ)
(via Regge discretisation)

Barrett et al. 0907.2440
Han, Zhang 1109.0499

Spinfoam Hartle-Hawking state



Spinfoam amplitudes

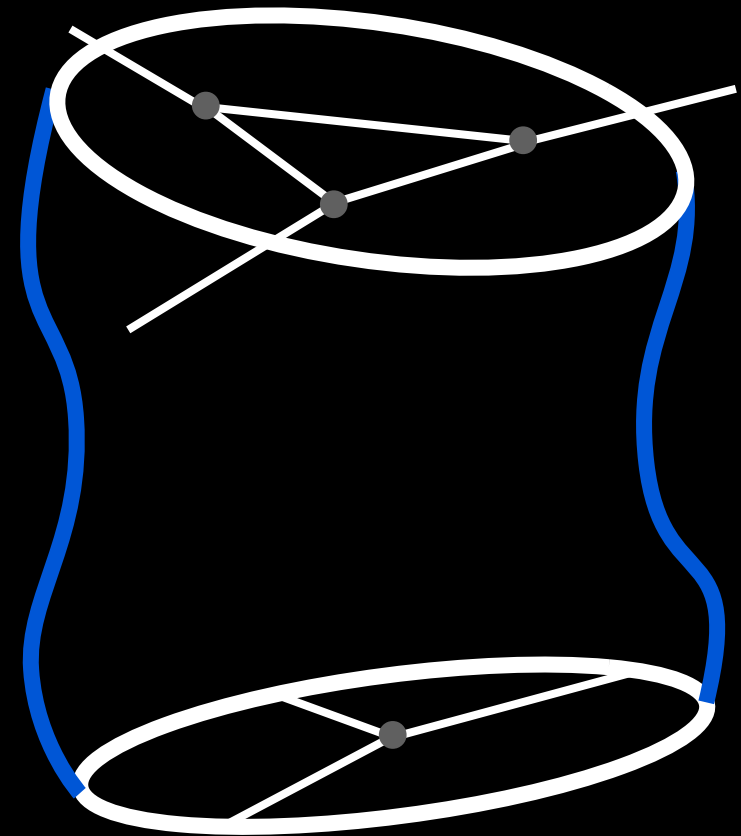
Engle, Pereira, Livine, Rovelli 0711.0146

Probability amplitude $P(\psi) = |\langle W | \psi \rangle|^2$
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$$W(q'_{ij}, q_{ij}) \sim \int_{\partial g = q', q} Dq e^{iS}$$

- Superposition principle $\langle W | \psi \rangle = \sum_{\sigma} W(\sigma)$
- Locality: vertex amplitude $W(\sigma) \sim \prod_v W_v$.
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For more details see: Rovelli, Vidotto “Introduction to Covariant Loop Quantum Gravity” CUP 2014

Cosmological transition amplitudes

Vidotto 1011.4705

Philosophy:

- Fixed graph with N nodes: approx kinematics of the universe
- The graph captures large scale dof
- The full theory can be regarded as an expansion for growing N

Rovelli, Vidotto 0805.4585

Results:

- **Coherent States** peaked on Homogeneous and Isotropic geometry
- Friedmann Equation recovered in the classical limit: Minkowski, de Sitter, Bianchi I
Bianchi, Rovelli, Vidotto 1003.3483
Bianchi, Krajeski, Rovelli, Vidotto 1101.4049
Rennert, Sloan 1308.0687
- The result holds for:
 - every regular graph in the boundary
 - considering radiative corrections in the bulk

Vidotto 1107.2633

Puchta 1307.4747
Riello 1310.2174

Hope:

- Understanding the quantum state at the bounce


(similar results in: Calcagni, Gielen, Oriti 1201.4151)


Spinfoam Dynamics

Unitary irr reps of $SU(2)$ $|j; m\rangle \in \mathcal{H}_j$ and $SL(2, \mathbb{C})$ $|k, \nu; j, m\rangle \in \mathcal{H}_{k, \nu} = \bigoplus_{j=k, \infty} \mathcal{H}_{k, \nu}^j$

- γ -simple representations: $\nu = \gamma(k + 1)$
- $SU(2) \rightarrow SL(2, \mathbb{C})$ map: $Y_\gamma : \mathcal{H}_j \rightarrow \mathcal{H}_{j, \gamma j}$ with image s.t. $j = k$
 $|j; m\rangle \mapsto |(j, \gamma(j + 1)); j, m\rangle$

- **Simplicity constraint** $\vec{K} + \gamma \vec{L} = 0$ satisfied weakly on the image of Y_γ

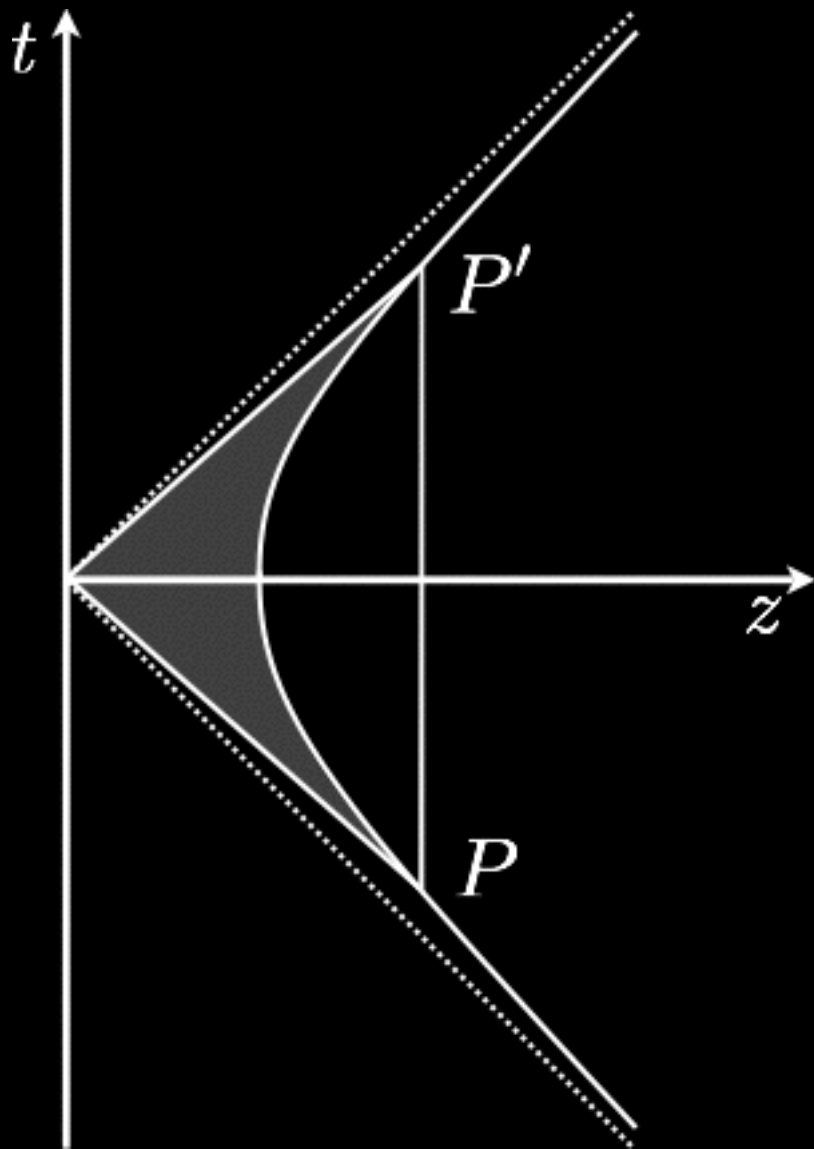

 Boost generator


 Rotation generator

- L^i is the area operator: the Lorentzian area $A = \int_{\mathcal{R}} L^i$ has a minimal value!

simplicity constant: $A = \int_{\mathcal{R}} \gamma K^i$ has also a minimal value!

Lorentzian Area



$$A = \int_{\mathcal{R}} e^o \wedge e^i = \int_{\mathcal{R}} \gamma K^i = \int_{\mathcal{R}} L^i$$

$$\ell = 1/a$$

$$A = \frac{\ell^2}{2} \eta = \frac{1}{2a^2} \eta$$

η is the boost parameter along the trajectory from P to P'

■ Lorentzian area $A_{min} = 4\pi G\hbar$

■ Max acceleration $a_{max} = \sqrt{\frac{1}{8\pi G\hbar}}$

■ Min length $\ell_{max} = \sqrt{8\pi G\hbar}$

[Cainiello '81]

[Cainiello, Gasperini, Scarpetta '91]

[Bozza, Feoli, Lambiase, Papini, Scarpetta]

Spinfoam “wedge” amplitudes

- motion of an accelerated observer in spacetime
- evolution of spacetime seen by an observer

$$W(g, g', h) = \sum_j (2j + 1) \operatorname{Tr}_j [Y^\dagger \underbrace{g' g^{-1}} Y h]$$

$$g, g' \in SL(2, C) \quad h \in SU(2)$$

- $g' g^{-1}$ is a boost:

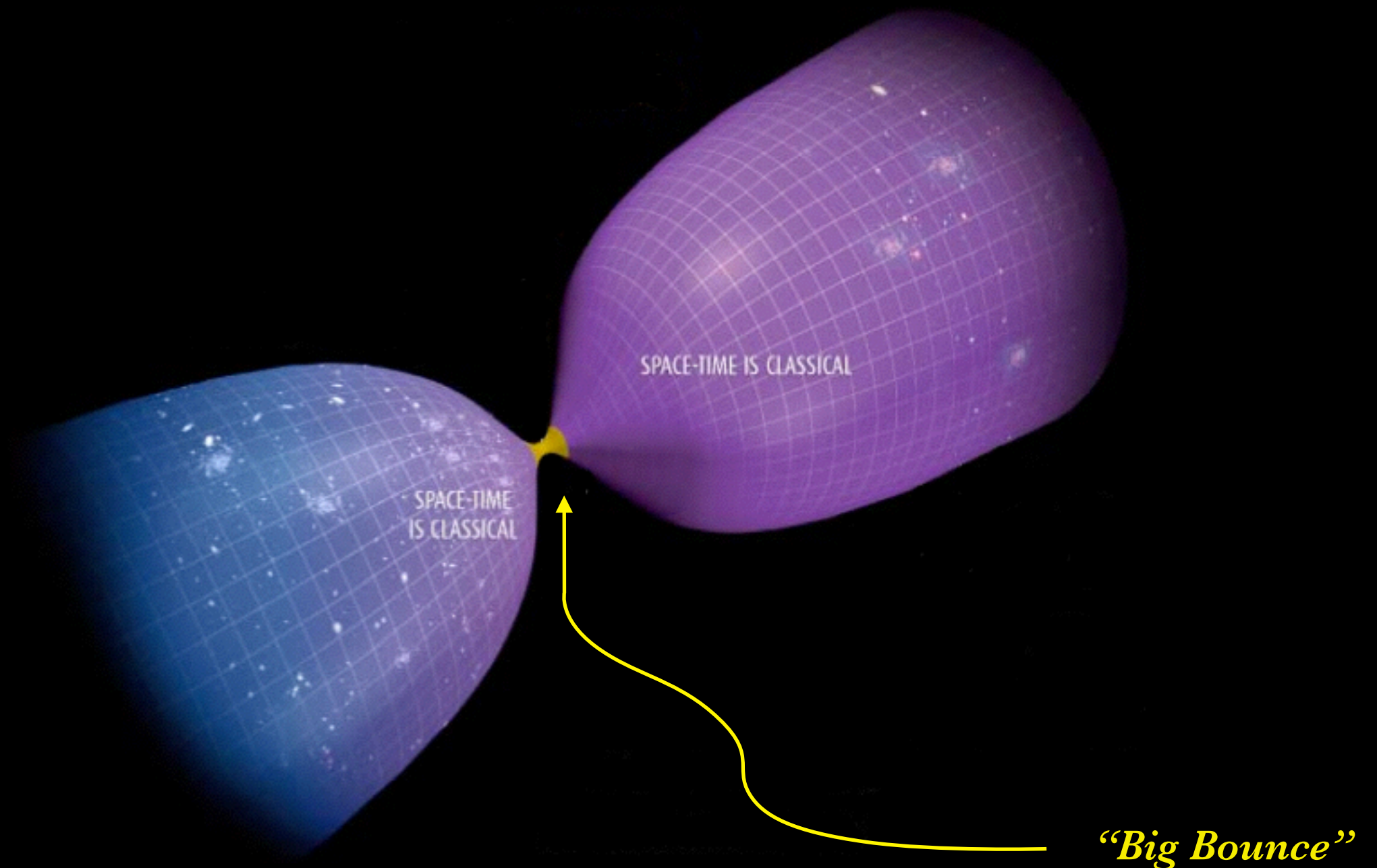
$$W(\eta, h) = \sum_j (2j + 1) \operatorname{Tr}_j [Y^\dagger e^{i\eta K_z} Y h]$$

- Fourier transform:

$$W(\eta, j, m, m') = \langle j, m | Y^\dagger e^{i\eta K_z} Y | j, m' \rangle \xrightarrow{m = m' = j} W(\eta, j) = \langle j, j | Y^\dagger e^{i\eta K_z} Y | j, j \rangle$$

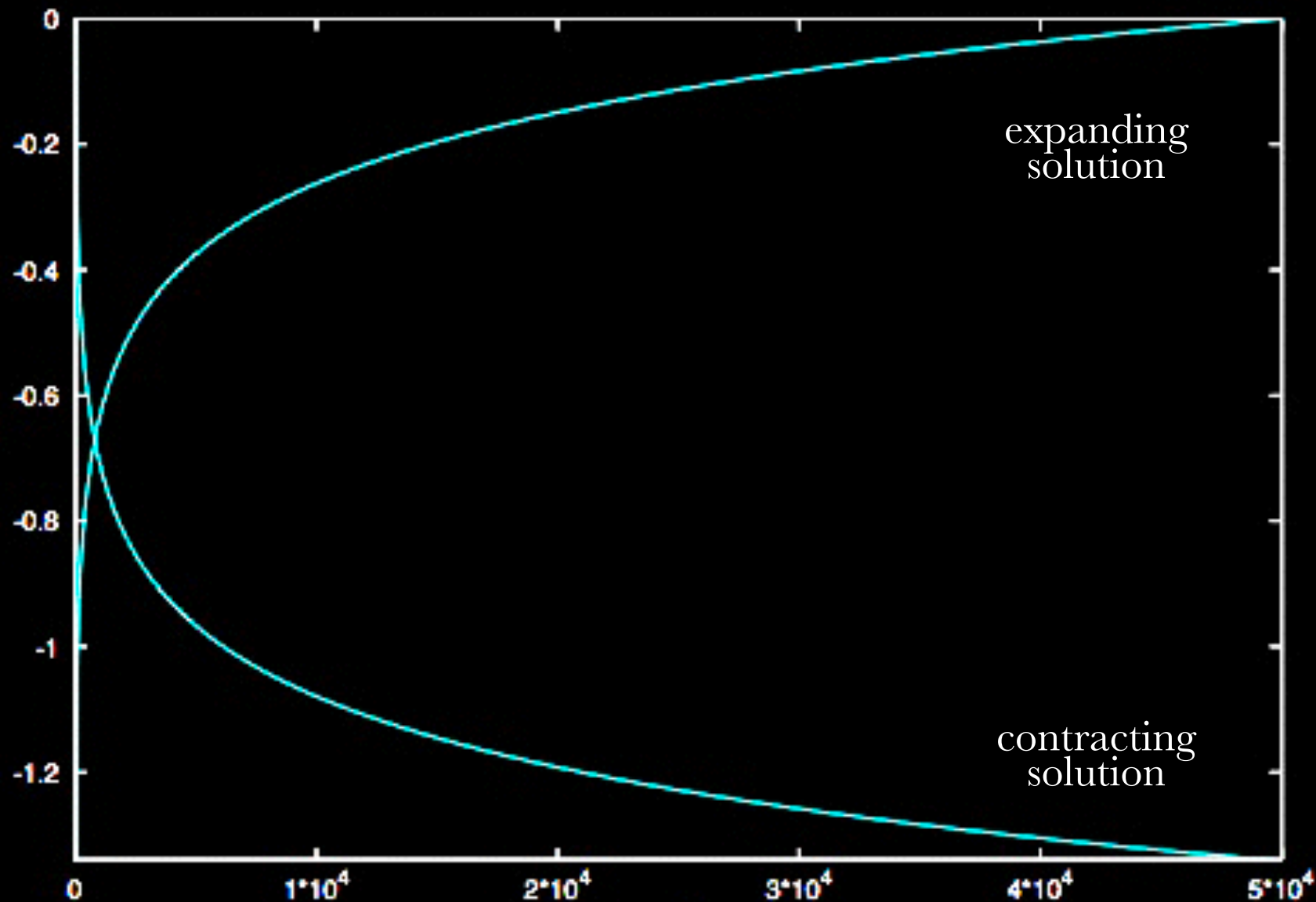
Singularity resolution

Bojowald 1999



See also talk by Piechocki

What have we learnt from Loop Quantum Cosmology?



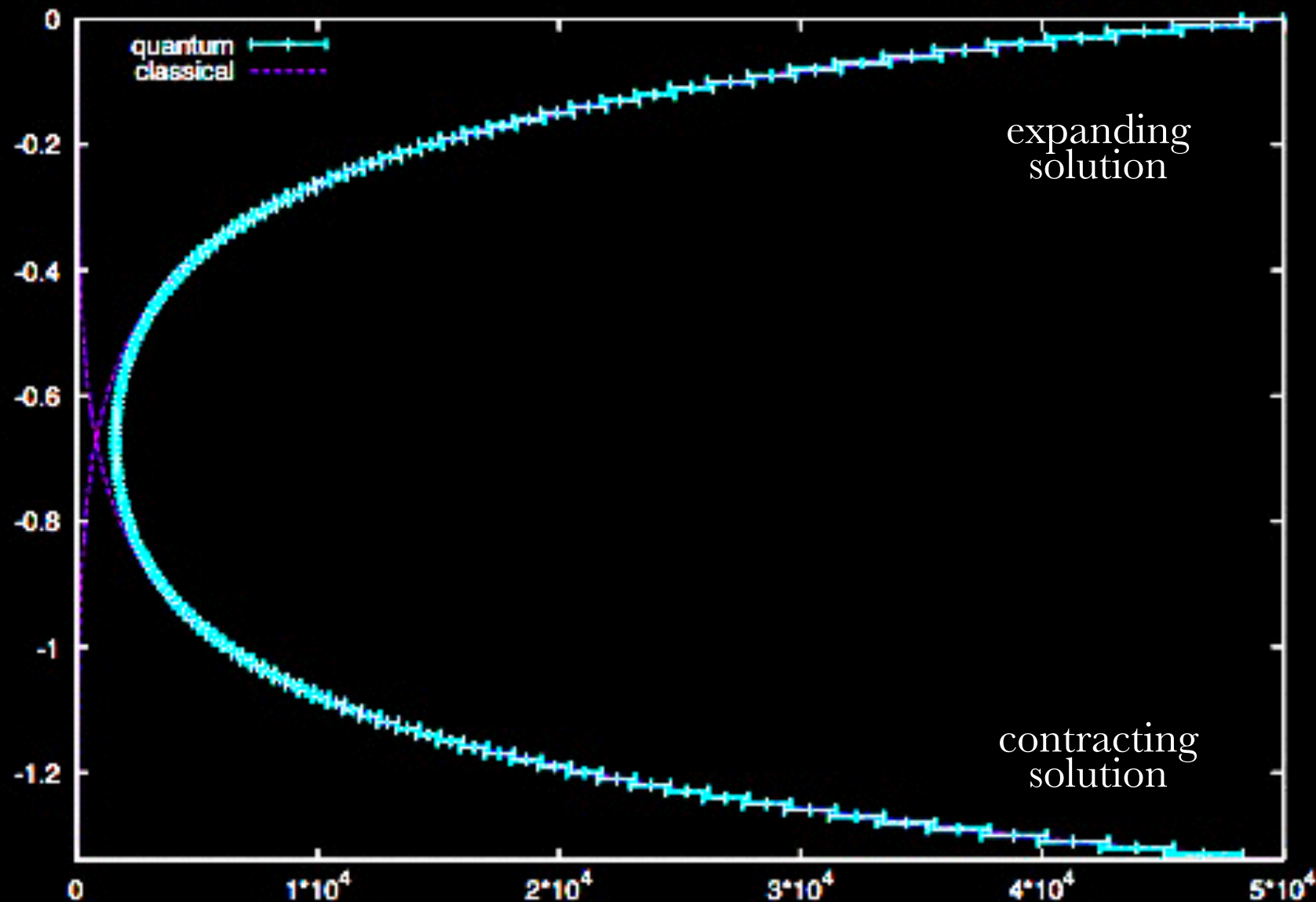
Big Bounce

- Quantum Tunneling
 - superposition
- Effective repulsive force
 - Planck density
- Size \gg Planck length
$$V_b \sim \frac{m}{m_P} \ell_P^3 \approx 10^{24} cm^3$$

Ashtekar, Pawłowski, Singh,
Vandersloot 0612104

See talks by Bojowalk and Mielczarek

What have we learnt from Loop Quantum Cosmology?



Big Bounce

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Ashtekar, Pawłowski, Singh,
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Where does matter falling into a Black Hole go?



Planck Star

- Quantum Tunneling
 - superposition
- Effective repulsive force
 - Planck density
- Size \gg Planck length

$$r_b \sim \left(\frac{m}{m_P} \right)^{\frac{1}{3}} \ell_P$$

Rovelli, Vidotto 1401.6562

Barrau, De Lorenzo, Haggard, Pacillo, Speziale...

See also recent work by Bianchi, Perez, Pullin...

(similar ideas in: Mersini-Houghton 1406.1525)

Eddington-Finkelstein coordinates

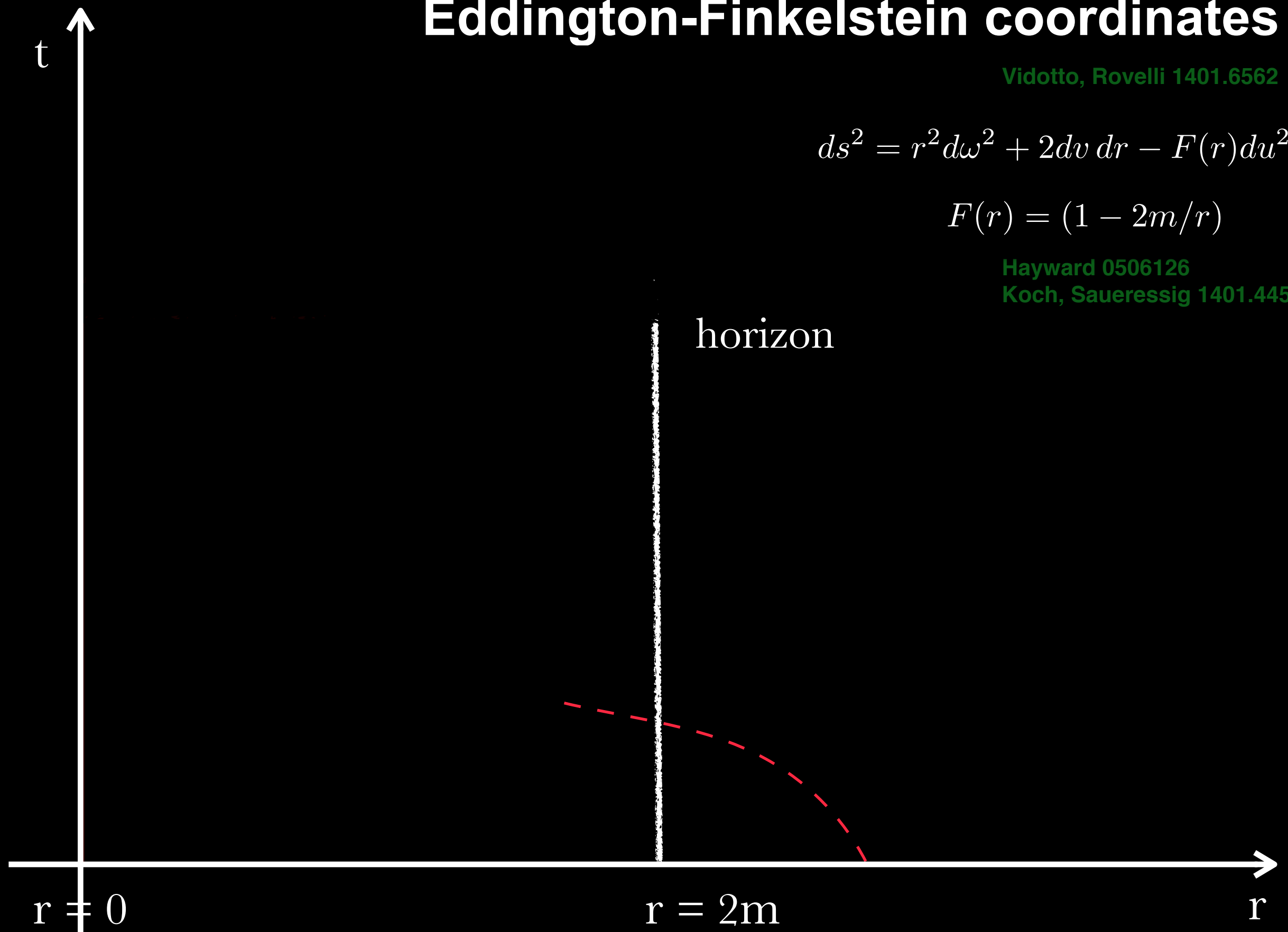
Vidotto, Rovelli 1401.6562

$$ds^2 = r^2 d\omega^2 + 2dv dr - F(r) du^2$$

$$F(r) = (1 - 2m/r)$$

Hayward 0506126

Koch, Saueressig 1401.4452



Eddington-Finkelstein coordinates

Vidotto, Rovelli 1401.6562

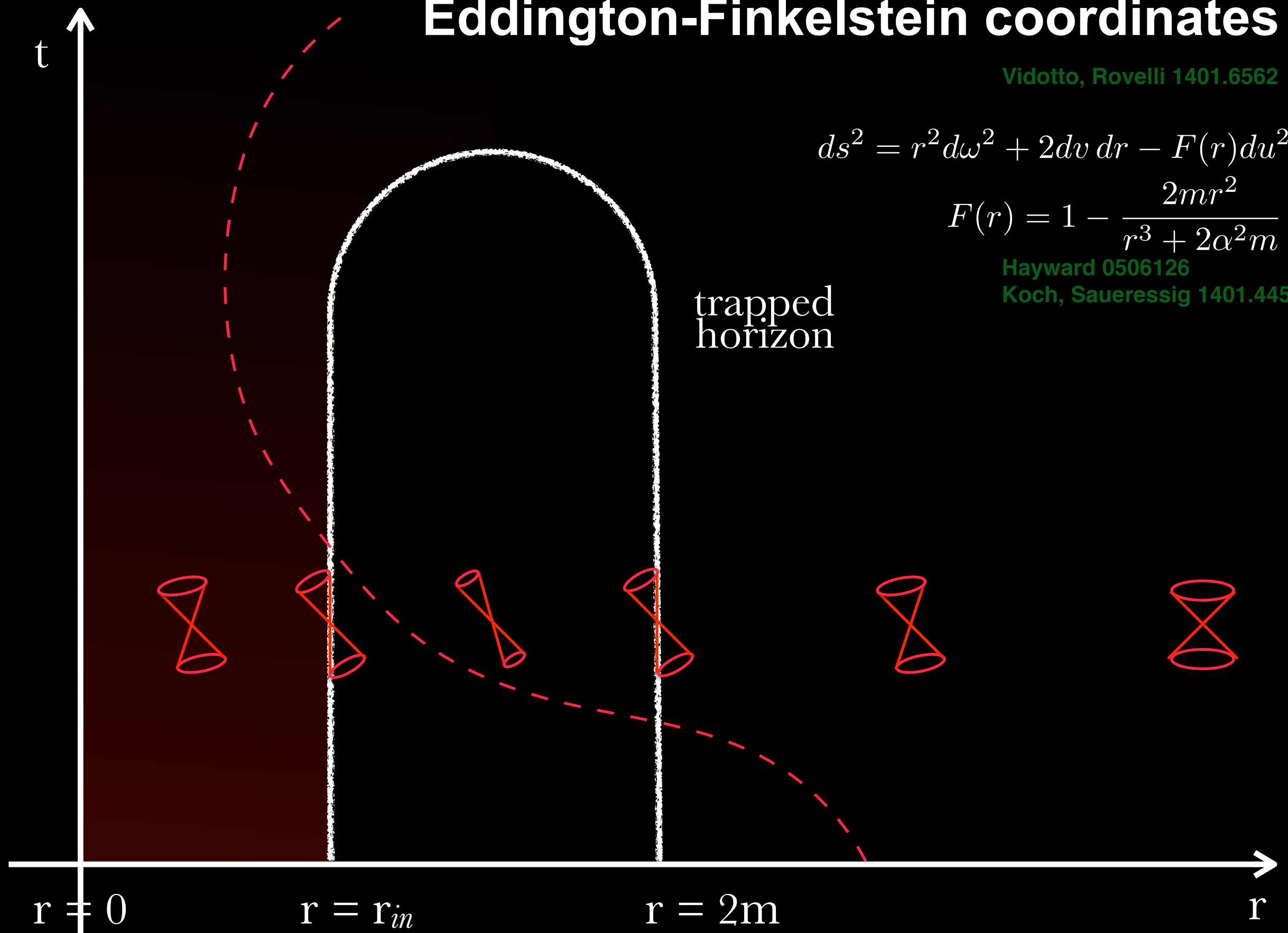
$$ds^2 = r^2 d\omega^2 + 2dv dr - F(r) du^2$$

$$F(r) = 1 - \frac{2mr^2}{r^3 + 2\alpha^2 m}$$

Hayward 0506126

Koch, Saueressig 1401.4452

trapped
horizon



Penrose diagram

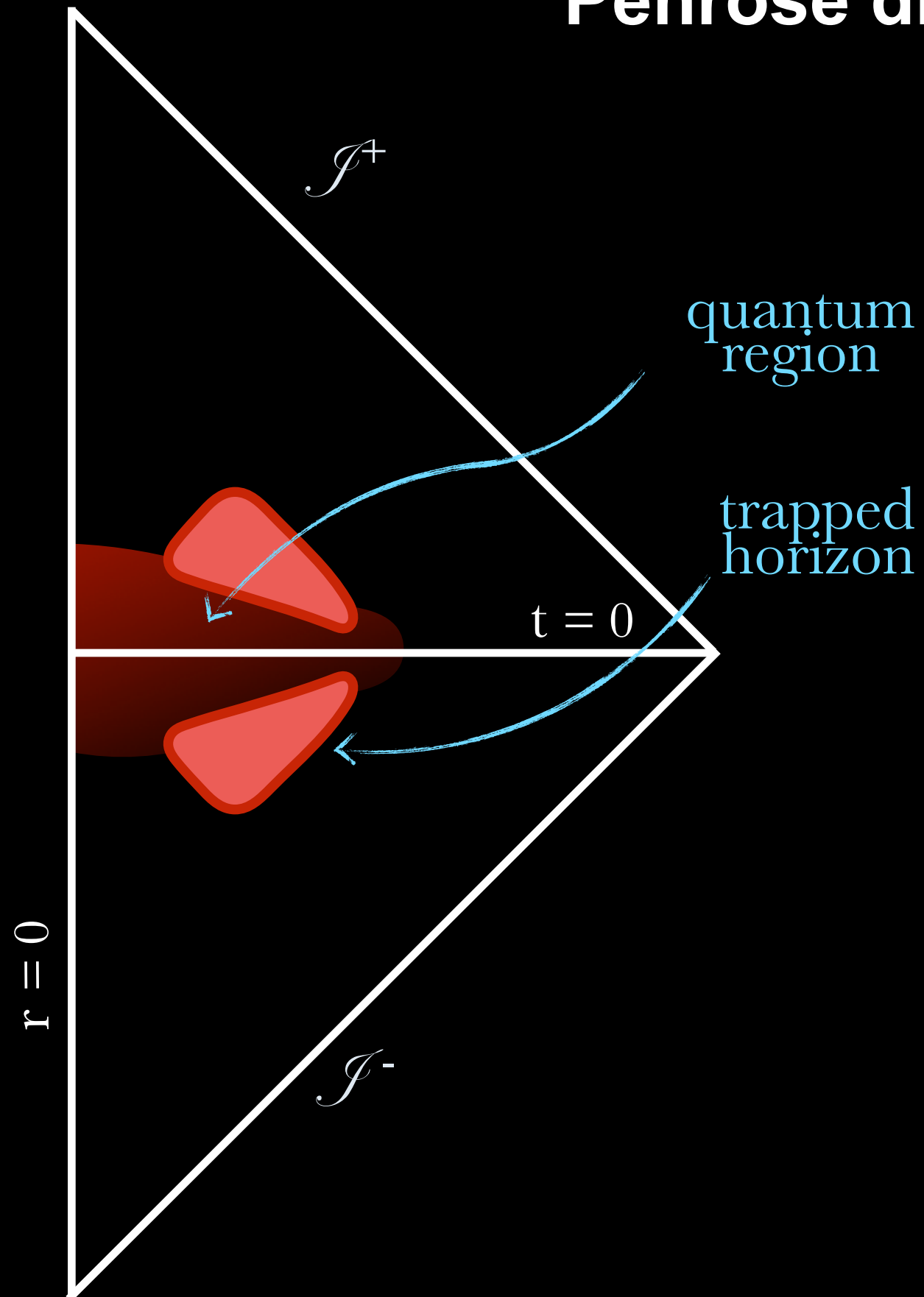
Hájíček, Kiefer 0107102

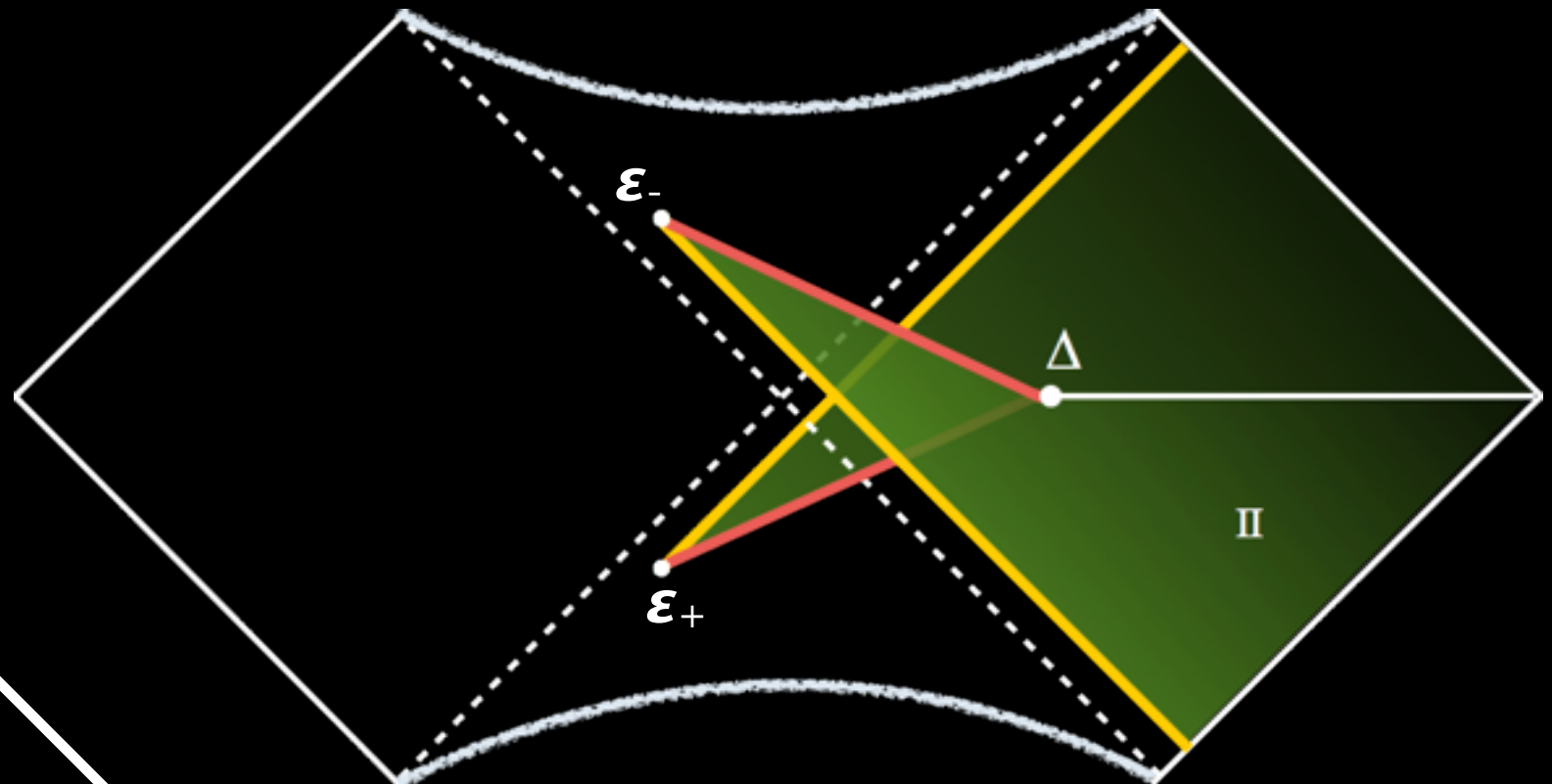
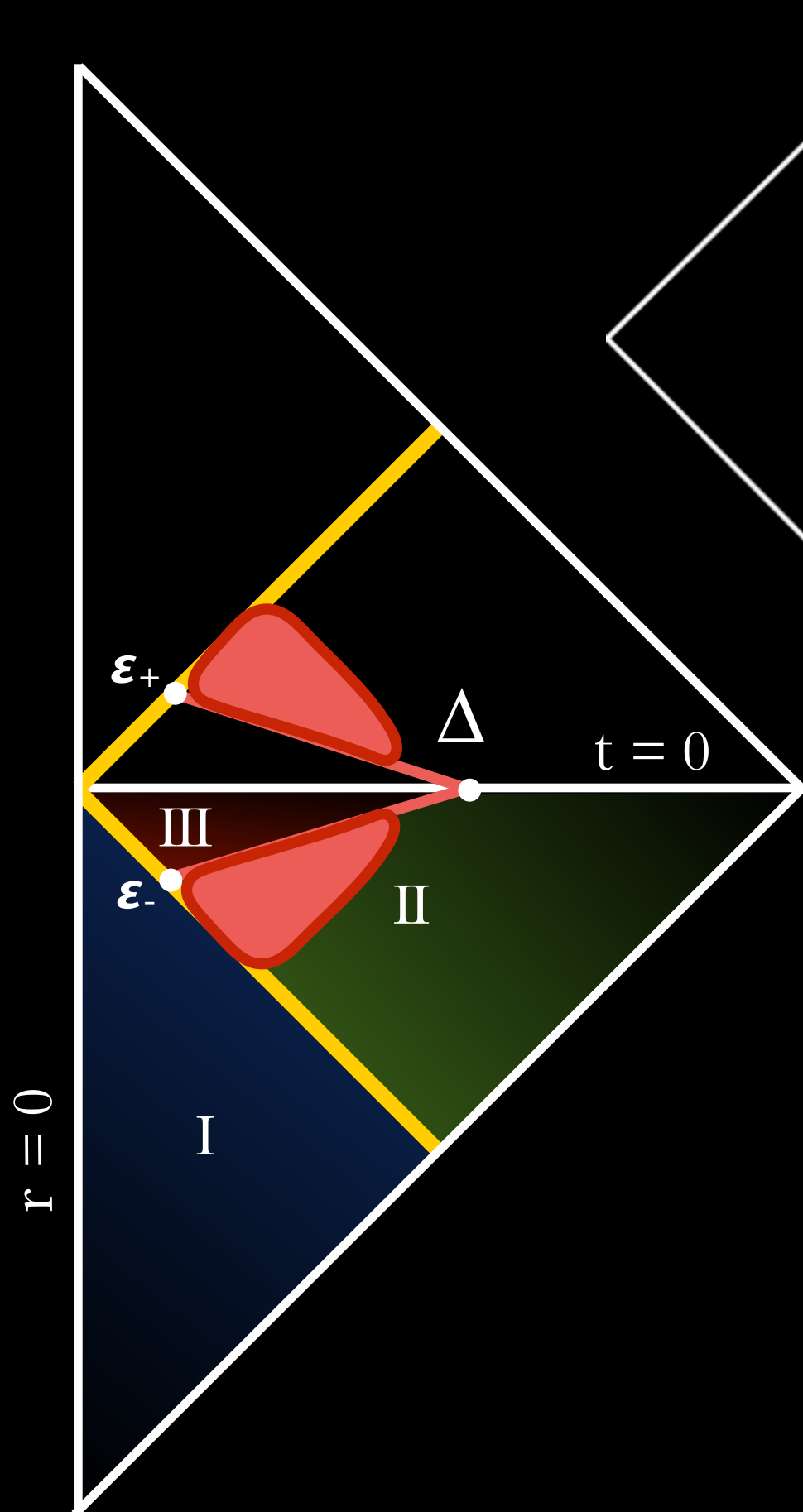
a collapsing quantum
light-like shell
bounces and re-expands

time-symmetric process:
white hole

Black hole \rightarrow white hole

quantum tunneling



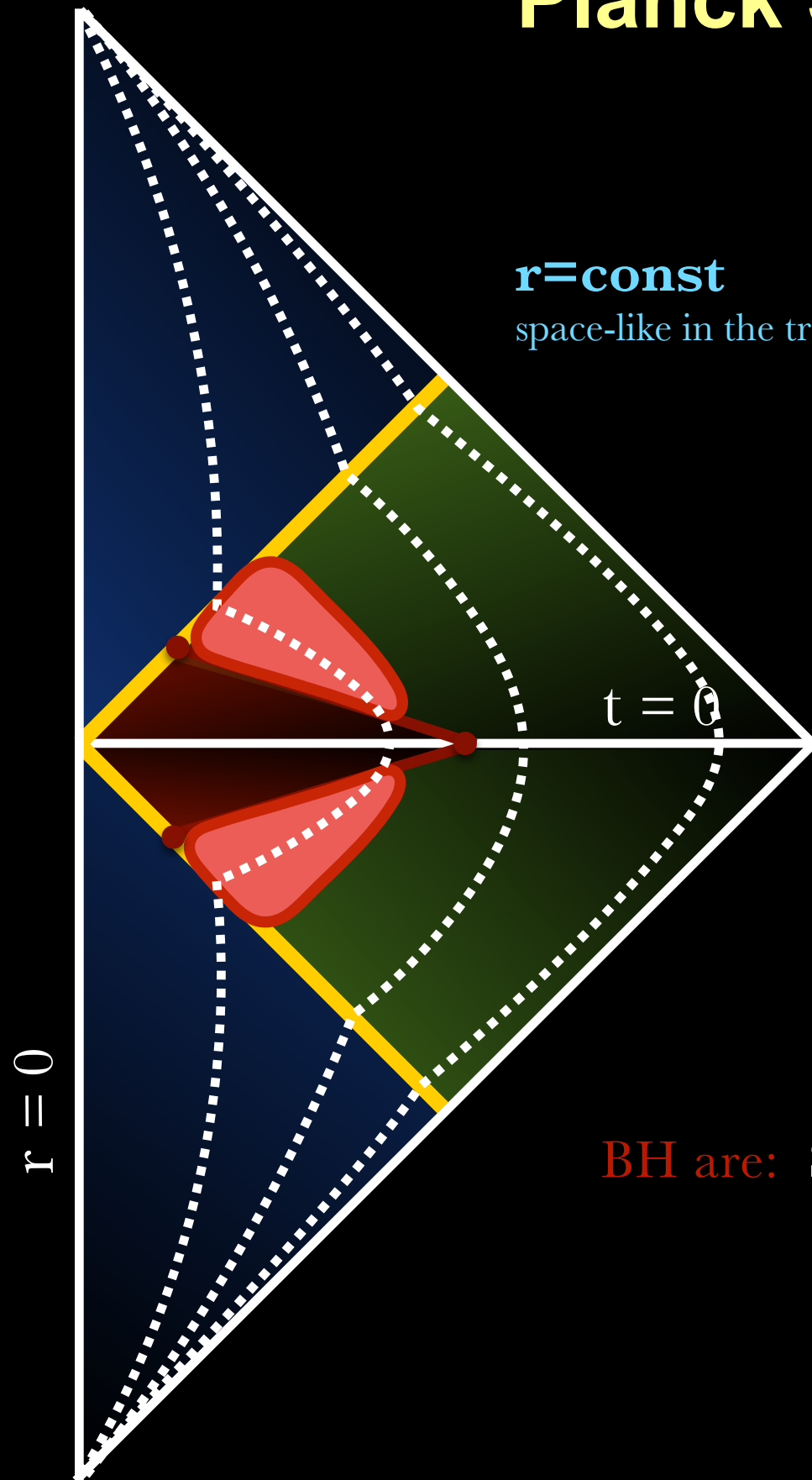


**Metric that describe the process
vacuum solution of Einstein equations**

$$ds^2 = -F(u, v)dudv + r^2(u, v)(d\theta^2 + \sin^2 \theta d\phi^2)$$

- I. Minkowski**
- II. Schwarzschild**
- III. Quantum Gravity**

Planck Star: Black Hole → White Hole



quantum tunneling

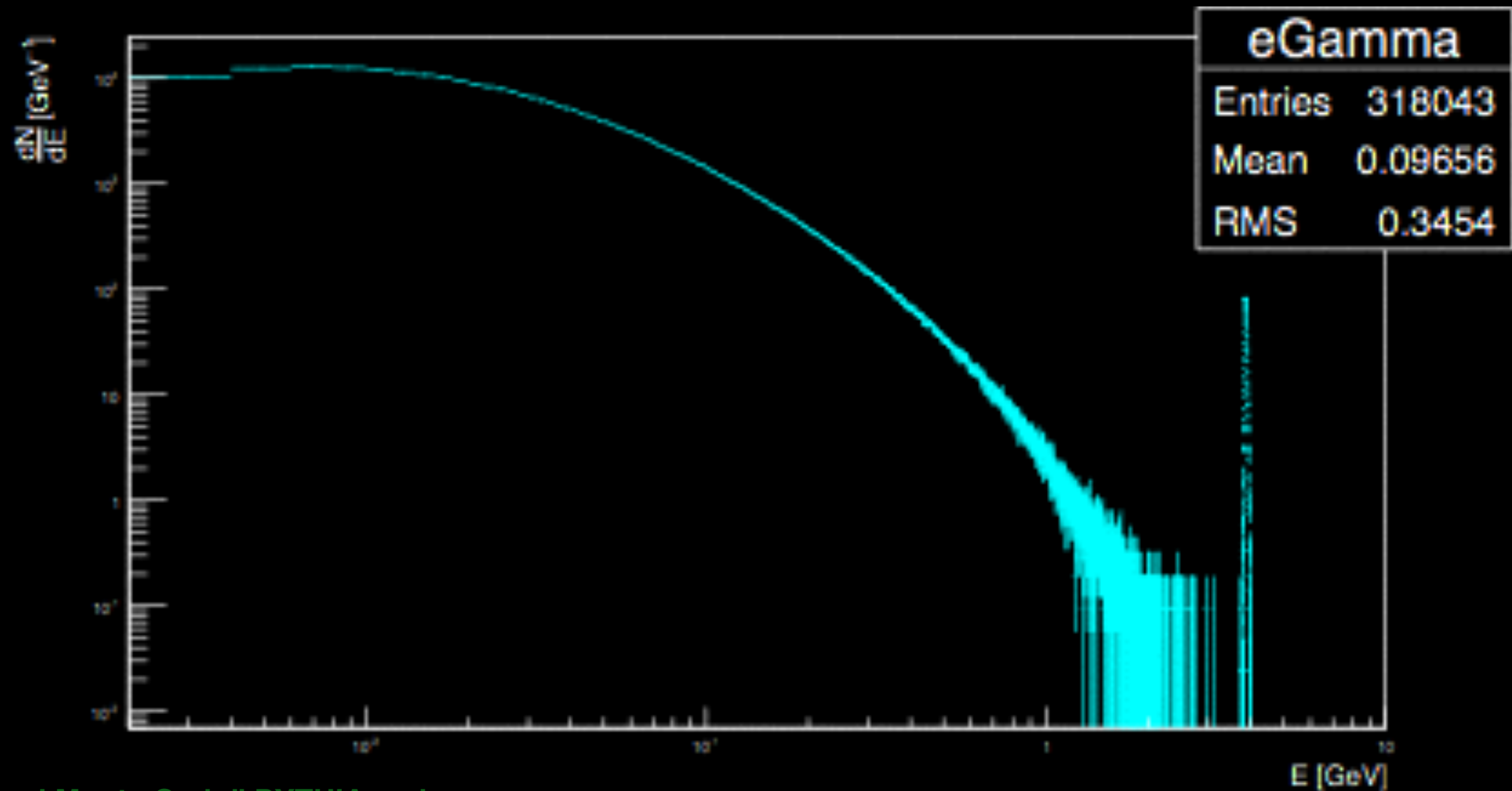
- Quantum pressure
- Planck density object
radius \gg Planck length $r_b \sim \left(\frac{m}{m_P}\right)^{\frac{1}{3}} \ell_P$
- quantum effects appear at $r \sim \frac{7}{6} 2m$
- asymptotic proper time $\tau \sim \frac{m^2}{\ell_P}$

BH are: Shortcut to the future / Bounce seen in slow motion

- emission at $E_{burst} = \frac{hc}{2r_f} \approx 3.9 \text{ Gev}$

Energy spectrum of photons

Barrau, Rovelli 1404.5821



"Lund Monte Carlo" PYTHIA code

$$E_{burst} = \frac{hc}{2r_f} \approx 3.9 \text{ Gev}$$

Detectable?

- $\sim 10 \text{ MeV}$
- One event per day
- Isotropic
- Short gamma-ray bursts
- From ~ 200 light years

Caveat



Summary:

- Quantum cosmology from the full theory:

Spinfoam Cosmology

- A new look at singularity resolution:

maximal acceleration

- Black holes tunnels into white holes:

Planck Stars

- | | | |
|-----------------------------|---|-------------------------------------|
| ■ Quantum Tunneling | | ■ Metric for Black-to-White process |
| ■ Effective repulsive force | → | ■ BH are bounce in slow motion |
| ■ Size \gg Planck length | | ■ Quantum Gravity Phenomenology! |