What can we learn from Loop Quantum Cosmology?

the case of

Planck Stars

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In this talk:

- Quantum cosmology from the full theory:
  
  **Spinfoam Cosmology**

- A new look at singularity resolution:
  
  **maximal acceleration**

- Black holes tunnels into white holes:
  
  **Planck Stars**
Quantum Cosmology from the full theory

(CLASSICAL GRAVITY)

\( R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} = 8\pi G T_{\mu\nu} \)

(CLASSICAL COSMOLOGY)

\( ds^2 = dt^2 - a^2(t) d^3\vec{x} \)

+ perturbations

quantization

(QUANTUM GRAVITY)

\( W_\nu = (P_{SL(2,\mathbb{C})} \circ Y_{\gamma} \psi_\nu(\mathbb{1}) \)

(QUANTUM COSMOLOGY)

\begin{itemize}
  \item canonical: Wheeler-deWitt eq.
  \item covariant: Hartle-Hawking state
\end{itemize}
Loop Quantum Gravity

- It is a theory about quanta of spacetime
- Local Lorentz invariance
- The states are boundary states at fixed time
- The physical phase space is spanned by SU(2) group variables

$SL(2, \mathbb{C}) \rightarrow SU(2)$
**Hilbert space and operator algebra**

- Group variables: \( \begin{cases} h_l \in SU(2) \\ \vec{L}_l \in su(2) \end{cases} \)  
- Adjacency: \( \Gamma = \{N,L\} \)

- Graph Hilbert space: \( \mathcal{H}_\Gamma = L_2[SU(2)^L / SU(2)^N] \)

- The space \( \mathcal{H}_\Gamma \) admits a basis \( |\Gamma, j_l, v_n \rangle \)

- Gauge invariant operator \( G_{l\ell} = \vec{L}_l \cdot \vec{L}_{l'} \) with \( \sum_{l\in n} G_{l\ell} = 0 \)  
  Penrose’s spin-geometry theorem (1971)

- \( h_l \) “Holonomy of the Ashtekar-Barbero connection along the link”

- \( \vec{L}_l = \{L_l^i\}, i = 1, 2, 3 \) \( SU(2) \) generators  
  \( L^i \psi(h) \equiv \frac{d}{dt} \psi(h e^{t \tau_i}) \bigg|_{t=0} \) (tetrad)

- Area  
  \( A_\Sigma = \sum_{l \in \Sigma} \sqrt{L_l^i L_l^i} \)

- Volume  
  \( V_R = \sum_{n \in R} V_n, \quad V_n^2 = \frac{2}{9} |\epsilon_{ijk} L_i^k L_j^i L_l^k| \)

- Angles  
  \( L_l^i L_{l'}^i \)

- eigenvalues are discrete  
- the operators do not commute  
- quantum superposition  
- coherent states
Spinfoam amplitudes

Probability amplitude \( P(\psi) = |\langle W | \psi \rangle|^2 \) for a state \( \psi \) associated to the boundary of a 4d region

- Superposition principle \( \langle W | \psi \rangle = \sum_{\sigma} W(\sigma) \)
- Locality: vertex amplitude \( W(\sigma) \sim \prod_v W_v. \)
- Lorentz covariance \( W_v = (P_{SL(2,\mathbb{C})} \circ Y_{\gamma} \psi_v)(1) \)
- UV and IR finite (with \( \Lambda \))
- Classical limit: GR (with \( \Lambda \)) (via Regge discretisation)

\[
W(q) \sim \int_{\partial g=q} Dq \ e^{iS[q]}
\]

Spinfoam Hartle-Hawking state

Barrett et al. 0907.2440
Han, Zhang 1109.0499

www.cpt.univ-mrs.fr/~rovelli/IntroductionLQG.pdf
Spinfoam amplitudes

Probability amplitude $P(\psi) = |\langle W | \psi \rangle|^2$
for a state $\psi$ associated to the boundary of a 4d region

- Superposition principle $\langle W | \psi \rangle = \sum_\sigma W(\sigma)$
- Locality: vertex amplitude $W(\sigma) \sim \prod_v W_v$.
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- UV and IR finite (with $\Lambda$)
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$W(q_{ij}', q_{ij}) \sim \int_{\partial g = q', q} Dq \, e^{iS}$

For more details see: Rovelli, Vidotto “Introduction to Covariant Loop Quantum Gravity” CUP 2014
Cosmological transition amplitudes

Philosophy:
- Fixed graph with N nodes: approx kinematics of the universe
- The graph captures large scale dof
- The full theory can be regarded as an expansion for growing N

Results:
- **Coherent States** peaked on Homogeneous and Isotropic geometry
- Friedmann Equation recovered in the classical limit: Minkowski, de Sitter, Bianchi I
- The result holds for:
  - every regular graph in the boundary
  - considering radiative corrections in the bulk

Hope:
- Understanding the quantum state at the bounce

(similar results in: Calcagni, Gielen, Oriti 1201.4151)
Spin foam dynamics

Unitary irr reps of $SU(2)$ $|j; m\rangle \in \mathcal{H}_j$ and $SL(2, \mathbb{C})$ $|k, \nu; j, m\rangle \in \mathcal{H}_{k, \nu} = \bigoplus_{j=k, \infty} \mathcal{H}_{k, \nu}^j$

- $\gamma$-simple representations: $\nu = \gamma(k + 1)$

- $SU(2) \to SL(2, \mathbb{C})$ map: $Y_\gamma : \mathcal{H}_j \to \mathcal{H}_{j, \gamma j}$ with image s.t. $j = k$

  $|j; m\rangle \mapsto |(j, \gamma(j + 1)) ; j, m\rangle$

- Simplicity constraint $\vec{K} + \gamma \vec{L} = 0$ satisfied weakly on the image of $Y_\gamma$

  Boost generator Rotation generator

- $L^i$ is the area operator: the Lorentzian area $A = \int_{\mathcal{R}} L^i$ has a minimal value!

  simplicity constant: $A = \int_{\mathcal{R}} \gamma K^i$ has also a minimal value!
Lorentzian Area

\[ A = \int_{\mathcal{R}} e^o \wedge e^i = \int_{\mathcal{R}} \gamma K^i = \int_{\mathcal{R}} L^i \]

\[ \ell = 1/a \]

\[ A = \frac{\ell^2}{2} \eta = \frac{1}{2a^2} \eta \]

\[ \eta \text{ is the boost parameter along the trajectory from } P \text{ to } P' \]

- Lorentzian area \[ A_{\text{min}} = 4\pi G\hbar \]
- Max acceleration \[ a_{\text{max}} = \sqrt{\frac{1}{8\pi G\hbar}} \]
- Min length \[ \ell_{\text{max}} = \sqrt{8\pi G\hbar} \]

[Cainiello '81]
[Cainiello, Gasperini, Scarpetta '91]
[Bozza, Feoli, Lambiase, Papini, Scarpetta]
Spinfoam “wedge” amplitudes

- motion of an accelerated observer in spacetime
- evolution of spacetime seen by an observer

\[ W(g, g', h) = \sum_j (2j + 1) \text{Tr}_j[Y^\dagger g' g^{-1} Y h] \]

\[ g, g' \in SL(2, C) \quad h \in SU(2) \]

- \( g' g^{-1} \) is a boost:

\[ W(\eta, h) = \sum_j (2j + 1) \text{Tr}_j[Y^\dagger e^{i\eta K_z} Y h] \]

- Fourier transform:

\[ W(\eta, j, m, m') = \langle j, m|Y^\dagger e^{i\eta K_z} Y |j, m'\rangle \quad m = m' = j \]

\[ W(\eta, j) = \langle j, j|Y^\dagger e^{i\eta K_z} Y |j, j\rangle \]

Bianchi 1204.5122
Singularity resolution

See also talk by Piechocki
What have we learnt from Loop Quantum Cosmology?

**Big Bounce**

- **Quantum Tunneling**
  - superposition
- **Effective repulsive force**
  - Planck density
- **Size \( \gg \) Planck length**
  \[
  V_b \sim \frac{m}{m_P} \varphi^3 \approx 10^{24} \text{cm}^3
  \]

Ashtekar, Pawlowski, Singh, Vandersloot 0612104

See talks by Bojowalk and Mielczarek
What have we learnt from Loop Quantum Cosmology?

Big Bounce

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- Size $\geq$ Planck length

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Ashtekar, Pawlowski, Singh, Vandersloot 0612104

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Where does matter falling into a Black Hole go?

**Planck Star**

- Quantum Tunneling
  - superposition

- Effective repulsive force
  - Planck density

- Size $\gg$ Planck length
  \[ r_b \sim \left(\frac{m}{m_P}\right)^{\frac{1}{3}} \ell_P \]

Rovelli, Vidotto 1401.6562
Barrau, De Lorenzo, Haggard, Pacillo, Speziale...
See also recent work by Bianchi, Perez, Pullin...

(similar ideas in: Mersini-Houghton 1406.1525)
Eddington-Finkelstein coordinates

\[ ds^2 = r^2 d\omega^2 + 2d\nu dr - F(r)du^2 \]

\[ F(r) = (1 - 2m/r) \]
Eddington-Finkelstein coordinates

\[ ds^2 = r^2 d\omega^2 + 2dv dr - F(r)du^2 \]

\[ F(r) = 1 - \frac{2mr^2}{r^3 + 2\alpha^2 m} \]

Vidotto, Rovelli 1401.6562
Hayward 0506126
Koch, Saueressig 1401.4452
Penrose diagram

Hájíček, Kiefer 0107102

a collapsing quantum light-like shell bounces and re-expands

time-symmetric process: **white hole**

Black hole $\rightarrow$ white hole
quantum tunneling
Metric that describe the process vacuum solution of Einstein equations

\[ ds^2 = -F(u, v)du dv + r^2(u, v)(d\theta^2 + \sin^2 \theta d\phi^2) \]

I. Minkowski

II. Schwarzschild

III. Quantum Gravity

Haggard, Rovelli 1407.0989
Planck Star: Black Hole $\rightarrow$ White Hole

- **Quantum pressure**

- Planck density object radius $\gg$ Planck length $r_b \sim \left(\frac{m}{m_P}\right)^{\frac{1}{3}} \ell_P$

- quantum effects appear at $r \sim \frac{7}{6} 2m$

- asymptotic proper time $\tau \sim \frac{m^2}{\ell_P}$

**BH are:** Shortcut to the future / Bounce seen in slow motion

- emission at $E_{\text{burst}} = \frac{hc}{2 r_f} \approx 3.9 \text{ Gev}$

$r=\text{const}$
space-like in the trapped region

quantum tunneling
Energy spectrum of photons

\[ E_{\text{burst}} = \frac{hc}{2 r_f} \approx 3.9 \text{ GeV} \]

Detectable?

- ~10 MeV
- One event per day
- Isotropic
- Short gamma-ray burts
- From ~200 light years
Caveat

Death Star Cat

Still under construction
Summary:

- Quantum cosmology from the full theory:
  
  **Spinfoam Cosmology**

- A new look at singularity resolution:
  
  **maximal acceleration**

- Black holes tunnels into white holes:
  
  **Planck Stars**

- Quantum Tunneling
- Effective repulsive force
- Size $\gg$ Planck length

- Metric for Black-to-White process
- BH are bounce in slow motion
- Quantum Gravity Phenomenology!