19 May 2005, Yuval-Fest, Tel Aviv University

Violation of Lorentz invariance, nonmetricity, and metric-affine gravity (MAG)*†

Friedrich W. Hehl‡
Univ. of Cologne, Germany, and
Univ. of Missouri-Columbia, USA

*Dedicated to Yuval Ne'eman with gratitude on the occasion of his 80th birthday.

†Thanks for the invitation to Aharon Levy and the organizers. For literature see Gronwald & Hehl, gr-qc/9602013 and refs. given. *file YuvalFest2.tex*

‡Collaboration with Yuri Obukhov (Cologne/Moscow) and Yakov Itin (Jerusalem).

1. Introduction — Yuval Ne'eman ($\rightarrow$ Ruffini & Verbin)
2. "Weak" Newton-Einstein gravity and translational gauge theory
3. Building blocks of the spacetime of a general relativ. field theory: Coframe, connection, metric
4. Teleparallel equivalent $\mathrm{GR}_{||}$ of general relativity GR
5. Poincaré gauge theory PGT: "Strong" Yang-Mills type gravity
6. Einstein-Cartan theory: GR plus an additional spin-spin contact interaction
7. Measuring torsion

—— up to here, established results ——

8. Quadratic models: PGT and MAG
9. A generic exact solution of MAG
10. Test matter and nonmetricity
11. Discussion

2. “Weak” Newton-Einstein gravity and translational gauge theory

...gravity is that field which corresponds to a gauge invariance with respect to displacement transformations...
R. Feynman (1963)

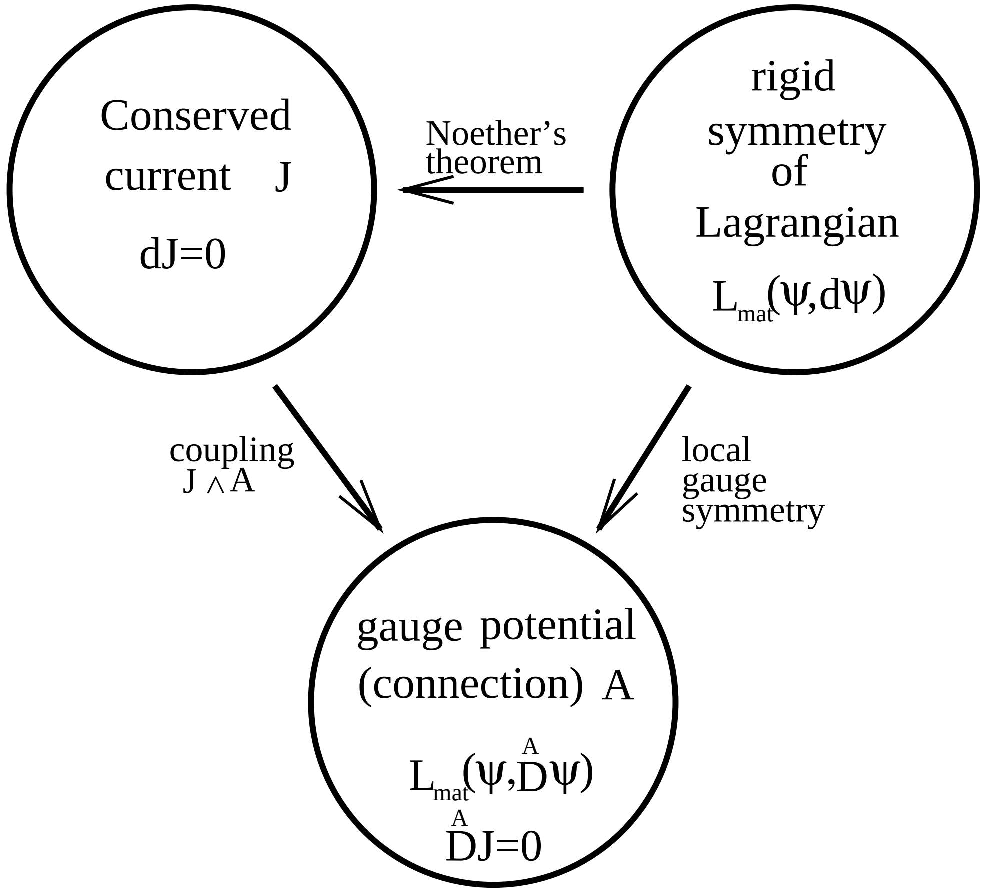
• *Gravity:* Mass $\xrightarrow{\text{spec.rel.}}$ energy-momentum is the source of gravity: “weak” *Newton-Einstein gravity.*

Conserved energy-momentum current because of translational invariance in SR, 4 conserved vector currents, $\alpha = 0, 1, 2, 3$, spacetime index $i = 0, 1, 2, 3$:

$$\partial_i \, \Sigma_\alpha{}^i = 0 \,, \quad \text{Gauge the translations!}$$

Conserv. energy-mom. current and transl. invariance.

• *Definition of a gauge theory* (see Yang & Mills 1954, “Conservation of isotopic spin and isotopic gauge invariance”): Formalism that starts with a conserved current and a corresponding rigid symmetry. The rigid symmetry is postulated to be *locally* valid; in order to achieve this, so-called gauge field potentials (covectors carrying spin 1) have to be newly introduced (“intermediate vector bosons”) which couple to the conserved current and are suitable to describe elementary interactions.



The logical pattern of a Yang-Mills theory (adapted from Mills)

Example: Dirac-Maxwell. One conserved vector current (the electric current), $i^2 = -1$, Ψ electron wave function (matter), γ^k Dirac matrices:

$$\partial_k \mathfrak{J}^k = 0\,, \quad \mathfrak{J}^k \sim \overline{\Psi}\gamma^k\Psi$$

$$\Psi \longrightarrow e^{i\phi}\Psi\,, \quad \phi = \text{const}\,, \quad L_{\text{mat}} \to L_{\text{mat}}$$

$$\Psi \longrightarrow e^{i\phi(x)}\Psi\,, \quad \phi = \phi(x)\,, \quad L_{\text{mat}} \neq L_{\text{mat}}$$

• Introduce gauge pot. $A = A_i\, dx^i$ and min. coupling:

$$\partial_k \to \overset{A}{D}_i := \partial_i + ieA_i\,, \quad L_{\mathsf{mat}}(\Psi, d\Psi) \to L_{\mathsf{mat}}(\Psi, \overset{A}{D}\Psi)\,.$$

$$A_i \to A_i + \partial_i \Phi \qquad \text{(covector with 4 ind. comp.)}$$

A_i is non-trivial, iff its curl in non-vanishing. Field strength $F \sim curl\, A$, in components (6 indep. ones)

$$F_{ij} = \partial_i A_j - \partial_j A_i\,, \qquad F_{ij} = -F_{ji}\,.$$

Add $\sim F^2$ to the matter Lagrangian$\Rightarrow$ recover Maxwell.

• Back to gravity: Make transl. invariance **local**. This yields 4 translational gauge potentials, i.e., 4 (co-) vector currents $\vartheta_i{}^\alpha$ $(= \delta_i^\alpha + A_i{}^\alpha)$ that couple to $\Sigma_\alpha{}^i$.

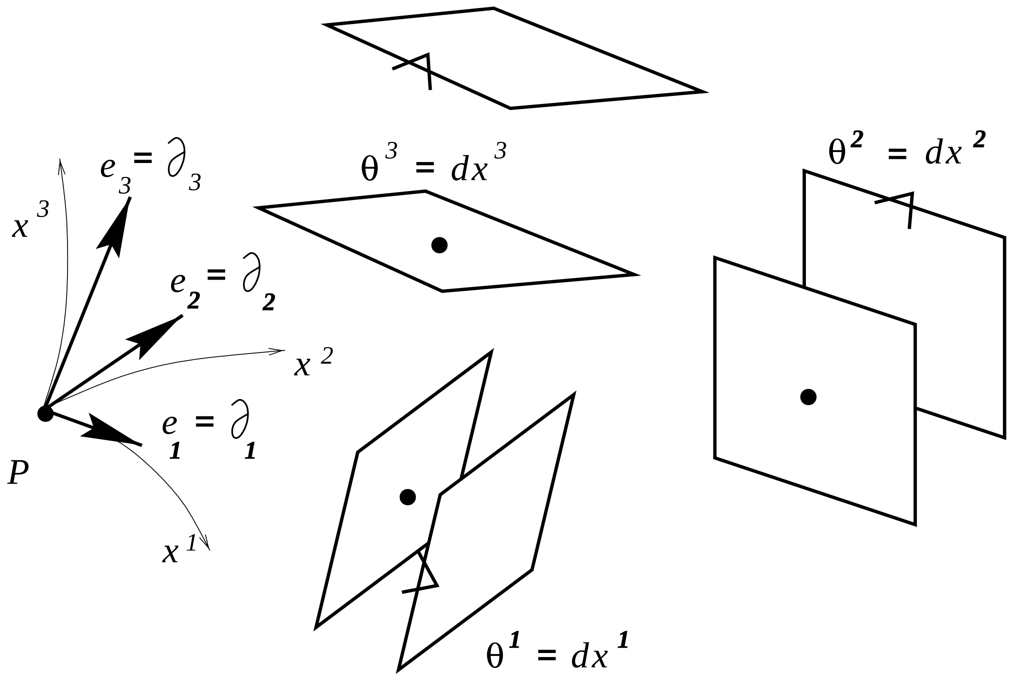
Potentials interpreted geometrically as (dual to the) tetrad field of spacetime ("vierbein" field). Is defined at each point. Will be illustrated below.

• Translational potential is ϑ, then the field strength (Cartan's torsion) should read $T \sim curl\, \vartheta$ (6×4 independent components):

$$T_{ij}{}^\alpha \sim \partial_i \vartheta_j{}^\alpha - \partial_j \vartheta_i{}^\alpha + \text{suppl. term}\,, \; T_{ij}{}^\alpha = -T_{ji}{}^\alpha\,.$$

3. Building blocks of the spacetime of a general relativ. field theory: Coframe, connection, metric

a) Coframe



Local coord. (x^1, x^2, x^3) at a point P of a 3D manifold and the basis vectors (e_1, e_2, e_3). The basis 1-forms $\vartheta^a = dx^a$, $a = 1, 2, 3$, are supposed to be also at P. Note that $\vartheta^1(e_1) = 1$, $\vartheta^1(e_2) = 0$, etc., i.e., ϑ^a is dual to e_b acc. to $\vartheta^a(e_b) = \delta^a_b$.

Perform linear comb. of the ϑ in order to find an arbitarary frame. In 4D, $\vartheta^\alpha = \vartheta_i{}^\alpha \, dx^i$. These are the 4 transl. potentials, num. disussions: Hayashi & Nakano, Cho, Ne'eman, Nitsch, Lord & Goswami, Mielke, Tresguerres & Tiemblo, ... Itin & Kaniel, Obukhov & Pereira, Delphenich, Ortín...

b) Connection

"...the essential achievement of general relativity, namely to overcome 'rigid' space (ie the inertial frame), is *only indirectly* connected with the introduction of a Riemannian metric. The directly relevant conceptual element is the 'displacement field' (Γ^l_{ik}), which expresses the infinitesimal displacement of vectors. It is this which replaces the parallelism of spatially arbitrarily separated vectors fixed by the inertial frame (ie the equality of corresponding components) by an infinitesimal operation. This makes it possible to construct tensors by differentiation and hence to dispense with the introduction of 'rigid' space (the inertial frame). In the face of this, it seems to be of secondary importance in some sense that some particular Γ field can be deduced from a Riemannian metric..."

A. Einstein (04 April 1955)

If a linear (or affine) connection is given, the parallel transfer of a vector $C = C^\alpha e_\alpha$, can be defined, e.g.:

$$\delta^{||} C^\beta = -\Gamma_\alpha{}^\beta \, C^\alpha \,, \qquad \text{with} \qquad \Gamma_\alpha{}^\beta = \Gamma_{i\alpha}{}^\beta dx^i$$

$\Gamma_\alpha{}^\beta$ represents 4×4 potentials of the 4D group of general linear transformations $GL(4, R)$. Very similar to the Yang-Mills potential of the $SU(3)$, say, which Yuval discussed in 1961. Field strength is called curvature $R \sim curl\,\Gamma$ or (16×6 indep. comp.)

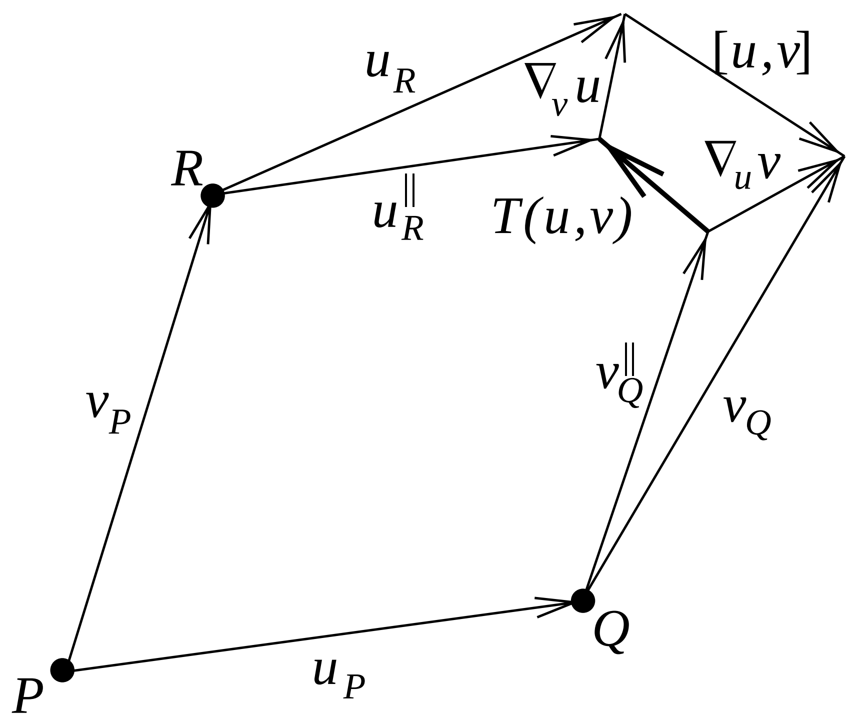
$$R_{ij\alpha}{}^\beta \sim \partial_i \Gamma_{j\alpha}{}^\beta - \partial_j \Gamma_{i\alpha}{}^\beta + \text{nonl. term}\,, \;\; R_{ij\alpha}{}^\beta = -R_{ji\alpha}{}^\beta \,.$$

Torsion $T^\alpha = T_{ij}{}^\alpha dx^i \wedge dx^j/2$ and curvature $R_\alpha{}^\beta = R_{ij\alpha}{}^\beta dx^i \wedge dx^j/2$ as field strengths. Symbolically,

$$
\begin{aligned}
T^\alpha &= d\vartheta^\alpha + \Gamma_\beta{}^\alpha \wedge \vartheta^\beta \\
R_\alpha{}^\beta &= d\Gamma_\alpha{}^\beta - \Gamma_\alpha{}^\gamma \wedge \Gamma_\gamma{}^\beta
\end{aligned}
$$

Field strengths of the gauge theory of the affine group

$$A(4,R) = \mathrm{R}^4 \rtimes GL(4,R)$$



On the geometrical interpretation of torsion: It represents the closure failure of an infinitesimal parallelogram.

c) Metric $\mathbf{g}(x)$

Experience points to more structure. Time and space intervals and angles should be measurable $\Rightarrow$ pseudo-Riemannian (or Lorentzian) metric $g_{ij}(x) = g_{ji}(x)$. 10 indep. comp. If $g_{\alpha\beta}$ are comp. w.r.t. coframe, then $g_{ij} = \vartheta_i{}^\alpha \vartheta_j{}^\beta g_{\alpha\beta}$ and $\mathbf{g} = g_{\alpha\beta}\, \vartheta^\alpha \otimes \vartheta^\beta$. In the 4D spacetime of SR and GR w.r.t. an orthonorm. basis:

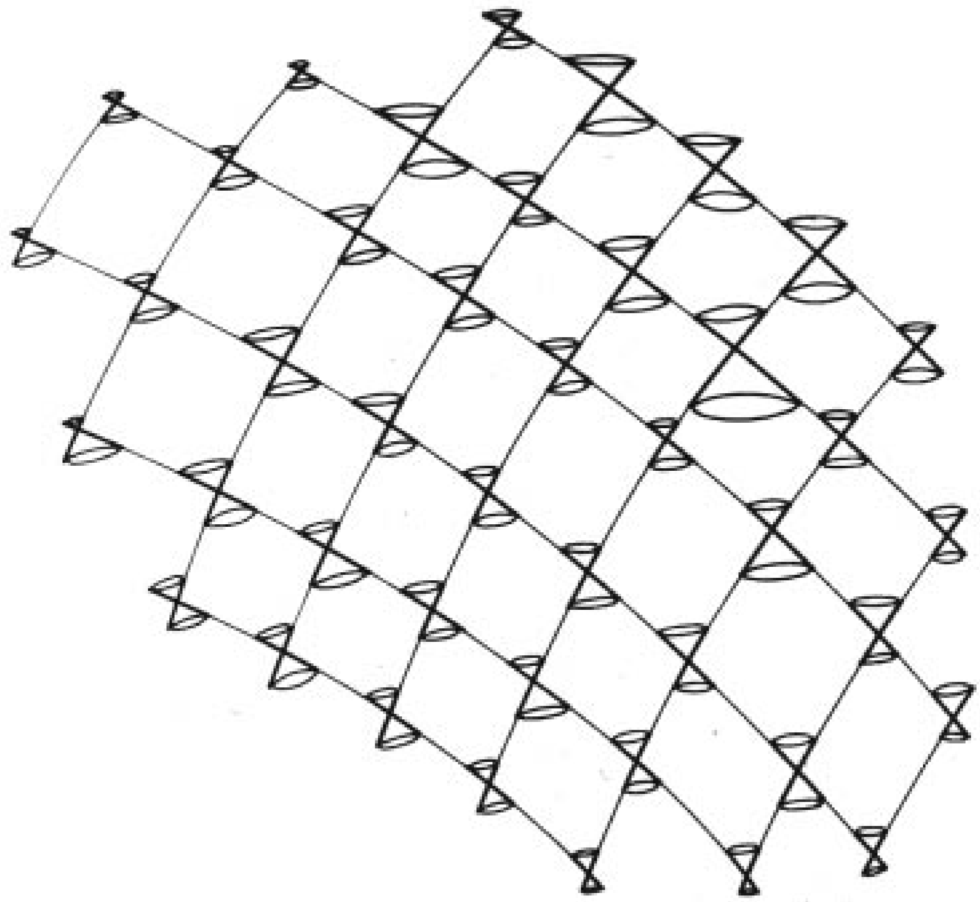
$$g_{\alpha\beta} = o_{\alpha\beta} := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

More recent results (Peres, Toupin, M.Schönberg..., Obukhov, Rubilar, H): Metric $\mathbf{g}(x)$ is an electromagnetic 'animal', it is *not* a fundamental field.

Premetric electrodynamics*: $dH = J$, $dF = 0$ with $H = (\mathcal{H}, \mathcal{D})$ and $F = (E, B)$. Formulation free of metric. Assume local and linear spacetime relation $H = \kappa(F)$. Here κ carries 36 components.

*Hehl & Yu.Obukhov: Foundations of Class. Electrodynamics. Birkhäuser,Boston (2003), see also Post, Kovetz, and Lindell

Study propag. of elmg. waves in the geom. optics limit by using $dH = J$, $dF = 0$ and $H = \kappa(F)$. Forbid birefringence in vacuum (Lämmerzahl & H) $\Rightarrow$ light-cone. Recover conformally invariant part of metric:



Light cones are defined at each point of the 4D spacetime manifold (see Pirani & Schild). A light cone, if parallelly transfered, is deformed by the nonmetricity $\Rightarrow$ violation of Lorentz invariance.

The ‘gravitational’ potentials are

$$\begin{array}{rc} g_{\alpha\beta} & \text{metric} \\ \vartheta^{\alpha} & \text{coframe} \\ \Gamma_{\alpha}{}^{\beta} & \text{connection} \end{array}$$

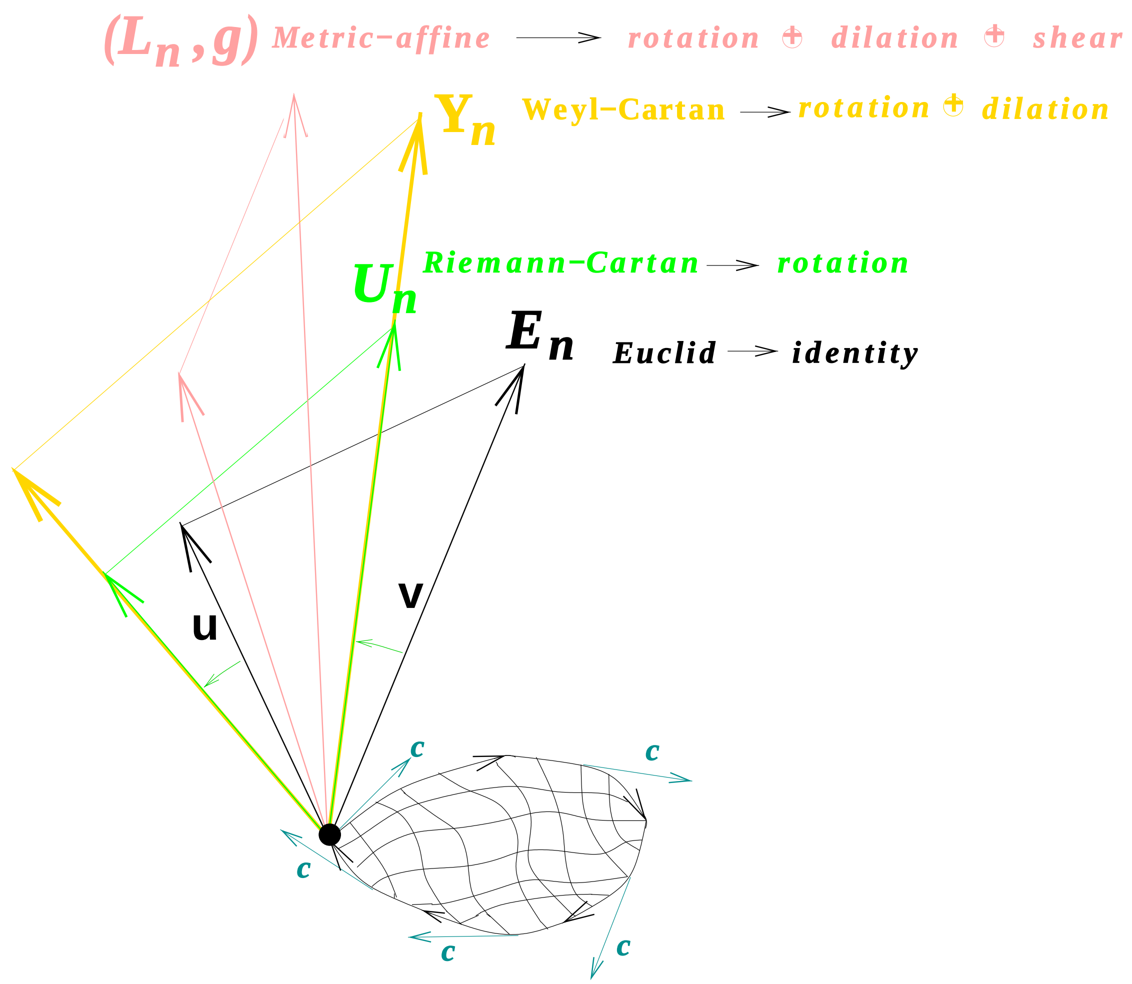
By differentiation, we find the field strengths

$$\begin{array}{rcll} Q_{\alpha\beta} & = & -Dg_{\alpha\beta} & \qquad\qquad \text{nonmetricity} \\ T_{\alpha} & = & D\vartheta^{\alpha} & \qquad\qquad \text{torsion} \\ R_{\alpha}{}^{\beta} & = & d\Gamma_{\alpha}{}^{\beta} - \Gamma_{\alpha}{}^{\gamma} \wedge \Gamma_{\gamma}{}^{\beta} & \qquad\qquad \text{curvature} \end{array}$$

The material currents coupled to the potentials ($g_{\alpha\beta}, \vartheta^{\alpha}, \Gamma_{\alpha}{}^{\beta}$) are energy-momentum and hypermomentum ($\sigma_{\alpha\beta}, \Sigma_{\alpha}, \Delta^{\alpha}{}_{\beta}$). The hypermomentum splits into spin current $\oplus$ dilat(at)ion current $\oplus$ shear current (add. sources of gravity):

$$\Delta_{\alpha\beta} = \tau_{\alpha\beta} + \frac{1}{4} g_{\alpha\beta}\, \Delta^{\gamma}{}_{\gamma} + \widehat{\not\Delta}_{\alpha\beta}\,, \qquad \tau_{\alpha\beta} = -\tau_{\beta\alpha}$$

The 3 potentials span the geometry of spacetime: It is the metric-affine space (L_4, g). The corr. first order Lagrangian gauge field theory is called MAG. It is a framework for gravitational gauge field theories. We developed mainly the *bosonic*, Yuval Ne’man, together with Šijački, its *fermionic* version.



Two vectors at a point P span a triangle. If we parallelly transfer both vectors around a closed loop back to P, then in the course of the round trip the triangle gets linearly transformed.

After this excursion to geometry, we come back to gravity.

4. Teleparallel equivalent GR$_{||}$ of general relativity GR

GR$_{||}$ in gauge $\Gamma \stackrel{*}{=} 0$, Weitzenböck spacetime, field equation Maxwell like

$$D_k T^{ki}{}_\alpha + \text{nonlin. terms} \;\sim\; \kappa \times \Sigma_\alpha{}^i$$

$$\Box\, \vartheta^i{}_\alpha + \text{nonlin. terms} \;\sim\; \kappa \times \Sigma_\alpha{}^i$$

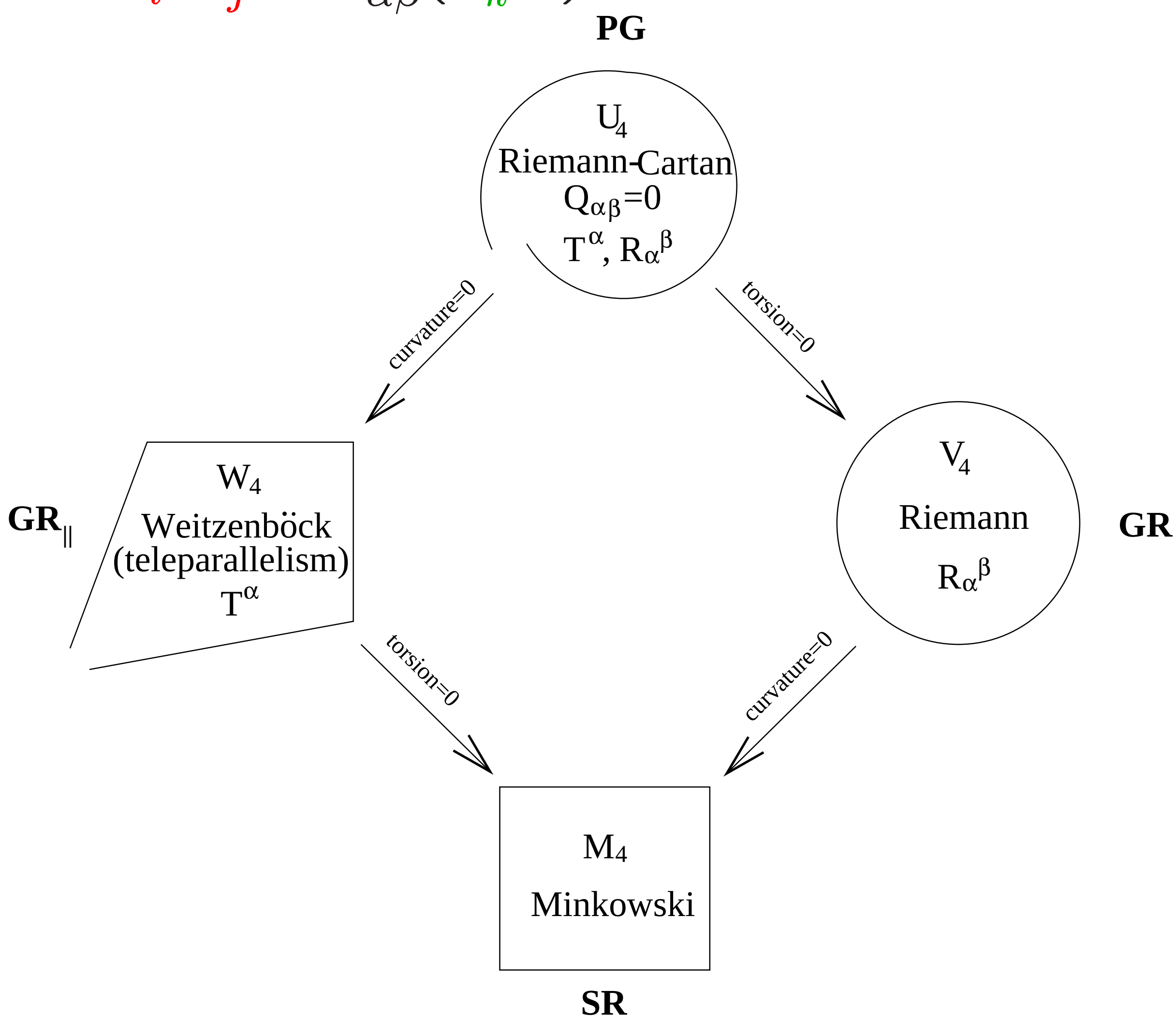
Compare Einstein's equation ($g_{ij} = g_{ji}$):

$$\Box\, g_{ij} + \text{nonlin. terms} \sim \kappa \times \sigma_{ij}$$

For scalar and for Maxwell matter, that is, for $\Sigma_{ij} = \sigma_{ij}$, it can be shown that GR$_{||}$ and GR are equivalent. That T^2-Lagrangian which is locally Lorentz invariant is equivalent to the Hilbert-Einstein Lagrangian.

Mini-Review: Mass as source of gravity $\rightarrow$ energy-momentum $\rightarrow$ conservation of it $\rightarrow$ *rigid* translational invariance $\rightarrow$ *local* translational invariance $\rightarrow$ coframe ϑ^α as potential $\rightarrow$ torsion T^α as field strength $\rightarrow$ Maxwell type field equations $\rightarrow$ equivalence to GR for scalar and Maxwell matter $\Rightarrow$ *...gravity is that field which corresponds to a gauge invariance with respect to displacement transformations...*

Riemann-Cartan spacetime. Simplest Lagrangian is $L \sim \vartheta_i{}^\alpha \vartheta_j{}^\beta \, R^{ij}{}_{\alpha\beta}(\Gamma_k{}^{\gamma\delta})$



Nonmetricity=0: A space with a metric and a metric compatible connection is called a Riemann-Cartan space U_4. It can either become a Weitzenböck space W_4, if its curvature vanishes, or a Riemann space V_4, if the torsion happens to vanish.

5. Poincaré gauge theory PGT: "Strong" Yang-Mills type gravity

$GR_{||}$ is somewhat degenerate. Extend translations to Poincaré transformations. Additional conserved angular momentum current (spin + orbital), 6 components of the spin vector current ${\tau_{\alpha\beta}}^i = -{\tau_{\beta\alpha}}^i$,

$$\partial_i \left({\tau_{\alpha\beta}}^i + x_\alpha {\Sigma_\beta}^i - x_\beta {\Sigma_\alpha}^i \right) = \partial_i {\tau_{\alpha\beta}}^i + \Sigma_{\beta\alpha} - \Sigma_{\alpha\beta} = 0$$

Spin is intrinsic part of angular momentum, we need 6 Lorentz gauge potentials ${\Gamma_i}^{\alpha\beta} = -{\Gamma_i}^{\beta\alpha}$. **PGT** with potentials $(\vartheta^\alpha, \Gamma^{\alpha\beta})$ and currents $(\Sigma_\alpha, \tau_{\alpha\beta})$ and 16+24 second order field equations. Simplest Lagrangian: Einstein-Cartan Lagrangian $\Rightarrow$ **ECT**, see below.

Can be generalized to 16 potentials $\Gamma^{\alpha\beta}$, if antisymmetry is dropped $\rightarrow$ metric-affine gravity **MAG**. In both cases, new *hypothetical* "strong" gravity à la Yang-Mills with dimensionless coupling constant [Yang himself proposed such a theory, PRL 33 (1974) 445]. Recall earlier strong gravity, fg-gravity (tensor dominance model), etc. by Isham-Salam-Strathdee, Wess-Zumino, Renner..., chromogravity by Ne'eman & Šijački. Hadronic stress tensor and gravity type fields are involved. Strong gravity can be (very) *massive* or *massless,* depending on the Lagrangian.

6. Einstein-Cartan theory: GR plus an additional spin-spin contact interaction $\quad \mathrm{Ric}_{ij} := R_{kij}{}^{k}$

$$\begin{aligned} \mathrm{Ric} - \frac{1}{2}\, tr(\mathrm{Ric}) \quad &\sim \quad \kappa \times \Sigma \sim \kappa \times \text{energy-mom.}\,, \\ \mathrm{Tor} + 2\, tr(\mathrm{Tor}) \quad &\sim \quad \kappa \times \tau \sim \quad \kappa \times \text{spin} \end{aligned}$$

Here is κ Einstein's gravitational constant $8\pi G/c^4$. If spin $\tau \;\rightarrow$ 0, then EC-theory $\rightarrow$ GR, and RC-spacetime $\rightarrow$ Riem. spacetime.

With $\tau \neq 0$, modified source of Einstein's equation: $\rho \rightarrow \rho + \kappa\tau^2 \Rightarrow$ at suff. high densities $\kappa\tau^2 \sim \rho \Rightarrow$ $\rho_{\text{crit}} \sim m/\left(\lambda_{\text{Co}}\ell^2_{\text{Pl}}\right)$, more than $10^{52} g/cm^3$ or $10^{24}\,K$ for electrons, see H. et al., RMP 1976. This is valid up to $10^{-34}\,s$ after the big bang, cf. Raychaudhuri's book on cosmology ($10^{-43}\,s$ corr. to Planck era). For Dirac spins, the contact interaction is repulsive (O'Connell). The EC-theory is a viable gravitational theory.

Contact interactions in particle physics were searched for by Ellerbrock, Ph.D. thesis DESY 2004, see review by Goy (2004) on HERA, LEP, Tevatron. Nothing found so far.

7. Measuring torsion

• Precession of elementary particle spin (of electron or neutron, e.g.) in torsion field. Rumpf (1979) (polarisation vector $\mathbf{w}$ of the spin) found

$$\dot{\mathbf{w}} = 3\mathbf{t} \times \mathbf{w}\,, \quad t^{\alpha} := -\epsilon^{\alpha\beta\gamma\delta}\, T_{\beta\gamma\delta}/3!$$

Independent of particular model, check $\mathrm{GR}_{||}$ against GR. On the Earth, using $\mathrm{GR}_{||}$, find only $|\mathbf{T}| \sim 10^{-15}\,\frac{1}{s}$, see Lämmerzahl (1997). He determined experimental limits of admissible torsion by using Hughes & Drever tpye experiments.

• Gravity Probe B, instead of the quartz balls, spin polarized balls: Spin gyroscope (see Ni `gr-qc/0407113`, also Kornack et al. `physics/0505089`). GR plus torsion induced spin precession, eqs. of motion by Yasskin and Stoeger (1980), Leclerc CQG (2005).

• Papini et al. (2004): measuring a spin flip of a neutrino induced by torsion, Lambiase calculated cross section for corr. process.

• Preuss, Solanki, Haugan: birefringence caused by torsion if *non-minimally* coupled to electromag. field.

8. Quadratic models: PGT and MAG

- Make contact interaction propagating $\Rightarrow$ intermediate GL(4,R) [or Lorentz SO(1,3)] gauge bosons with spins 0,1,2, massive or massless. Quadratic Lagrangian

$$V_{\mathrm{MAG}} \sim \frac{1}{\kappa}(R_{\mathrm{sc}}+\lambda_0+T^2+TQ+Q^2)+\frac{1}{\rho}(W^2+Z^2)\,.$$

$Q_{\alpha\beta} := -Dg_{\alpha\beta}$ (nonmetricity) $\neq 0$, if $\Gamma_i{}^{\alpha\beta} \neq \Gamma_i{}^{\beta\alpha}$ (see figure), "strong gravity" with dimensionless coupling constant ρ. Rotational curvature $W_{\alpha\beta} := (R_{\alpha\beta} - R_{\beta\alpha})/2$, strain curvature $Z_{\alpha\beta} := (R_{\alpha\beta} + R_{\beta\alpha})/2$

- If $Q_{\alpha\beta} = 0$, PGT; otherwise MAG. For PGT ghost-free Lagrangians exist which can be quantized: Sezgin+van Nieuwenhuizen ('80), Kuhfuss+Nitsch ('86). Limits of PGT, Pascual-Sánchez ('85), Leclerc ('05)

- MAG: Generalization of Dirac spinors to world spinors of Ne'eman, see Kirsch and Sijacki ('02). "Fermionic" hyperfluid of Obukhov and Tresguerres ('93). Post-Riemannian cosmologies: Puetzfeld ('04,'05), Cheng, Harko, Wang ('05), Preuss et al., loc.cit.

- Exact solutions with $Q \sim 1/r^d$, see Obukhov et al.('97), suggest existence of massless modes.

9. A generic exact solution of MAG

• Generic exact spherically symmetric solutions of the field equations belonging to the Lagrangian [Heinicke, Baekler, H. `gr-qc/0504005`, $\eta_{\alpha\beta} := {}^\star(\vartheta_\alpha \wedge \vartheta_\beta)$, $\eta =$ volume element]

$$V = \frac{1}{2\kappa}\left[-R^{\alpha\beta}\wedge\eta_{\alpha\beta} - 2\lambda_0\,\eta + Q_{\alpha\beta}\wedge{}^\star\!\left(\frac{1}{4}\,{}^{(1)}Q^{\alpha\beta} - \frac{1}{2}\,{}^{(3)}Q^{\alpha\beta}\right)\right] - \frac{z_3}{2\rho}\,{}^{(3)}Z^{\alpha\beta}\wedge{}^{\star(3)}Z_{\alpha\beta}\,.$$

• Coframe ϑ^α is Schwarzschild-deSitter,

$$\vartheta^0 = e^{\mu(r)}dt,\, \vartheta^1 = e^{-\mu(r)}dr,\, \vartheta^2 = r\,d\theta,\, \vartheta^3 = r\,\sin\theta\,d\phi,$$

with Schwarzschild coordinates $x^i = (t, r, \theta, \phi)$ and with the function

$$e^{2\mu(r)} = 1 - 2\frac{m}{r} - \frac{\lambda_0}{3}r^2\,.$$

• The coframe is *orthonormal.* The metric reads

$$g = -\vartheta^0\otimes\vartheta^0 + \vartheta^1\otimes\vartheta^1 + \vartheta^2\otimes\vartheta^2 + \vartheta^3\otimes\vartheta^3\,.$$

Nonmetricity 1-form $Q^{\alpha\beta} = Dg^{\alpha\beta}$:

$$Q^{\alpha\beta} = \frac{e^{-\mu(r)}}{2r^2} \times$$

$$\begin{pmatrix} 3\ell_1\vartheta^0 + \ell_0\vartheta^1 & 0 & -\ell_1\vartheta^2 & -\ell_1\vartheta^3 \\ 0 & -\ell_1(\vartheta^0 - 3\vartheta^1) & \ell_0\vartheta^2 & \ell_0\vartheta^3 \\ -\ell_1\vartheta^2 & \ell_0\vartheta^2 & 0 & 0 \\ -\ell_1\vartheta^3 & \ell_0\vartheta^3 & 0 & 0 \end{pmatrix}.$$

Integration constants ℓ_0 and ℓ_1 can be interpreted as a measure for the violation of Lorentz invariance.

Torsion 2-form $T^\alpha = D\vartheta^\alpha$:

$$T^\alpha = \frac{e^{-\mu(r)}}{4r^2} \begin{pmatrix} \ell_0\vartheta^{01} \\ -\ell_1\vartheta^{01} \\ -\ell_1\vartheta^{02} - \ell_0\vartheta^{12} \\ -\ell_1\vartheta^{03} - \ell_0\vartheta^{13} \end{pmatrix}.$$

We have a Coulomb-like behavior for torsion and for nonmetricity!

The curvature can be decomposed into antisymmetric and symmetric pieces:

$$\begin{aligned} W_{\alpha\beta} &:= (R_{\alpha\beta} - R_{\beta\alpha})/2 \quad \text{(rotational curvature)}\,, \\ Z_{\alpha\beta} &:= (R_{\alpha\beta} + R_{\beta\alpha})/2 \quad \text{(strain curvature)}\,. \end{aligned}$$

We find with the help of Reduce-Excalc, for example,

$$^{(1)}W_{\alpha\beta} = \Big(\underbrace{\frac{m}{r^3}}_{Riem.} \underbrace{- \frac{(\ell_0 + \ell_1)(4\ell_1 - \ell_0)}{96 r^4\, e^{2\mu(r)}}}_{post-Riemannian} \Big) \times$$

$$\begin{pmatrix} 0 & 2\vartheta^{01} & -\vartheta^{02} & -\vartheta^{03} \\ \diamond & 0 & \vartheta^{12} & \vartheta^{13} \\ \diamond & \diamond & 0 & -2\vartheta^{23} \\ \diamond & \diamond & \diamond & 0 \end{pmatrix} \quad (\texttt{weyl})\,,$$

$$^{(4)}Z_{\alpha\beta} = \underbrace{-\frac{\ell_1}{2r^3}\, g_{\alpha\beta}\, \vartheta^{01}}_{post-Riemannian} \quad (\texttt{dilcurv})\,.$$

The exact expressions for torsion, nonmetricity, and curvature can be taken in order to evaluate possible effects on equations of motion etc.

10. Test matter and nonmetricity

Equation of motion of a matter field in MAG: Derive it with the help of the corresponding Bianchi identities (quantities with tilde denote the Riemannian parts),

$$\widetilde{D}\left[\Sigma_\alpha + \Delta^{\beta\gamma}\left(e_\alpha\rfloor\diamondsuit\Gamma_{\beta\gamma}\right)\right] + \Delta^{\beta\gamma}\wedge\left(\pounds_{e_\alpha}\diamondsuit\Gamma_{\beta\gamma}\right)$$
$$= \tau^{\beta\gamma}\wedge\left(e_\alpha\rfloor\widetilde{R}_{\gamma\beta}\right) \qquad \text{(Obukhov '96, Ne'eman \& H '96),}$$

where $\diamondsuit\Gamma_{\beta\gamma} := \Gamma_{\beta\gamma} - \widetilde{\Gamma}_{\beta\gamma}$ denotes the post-Riemannian part of the connection. On the r.h.s., Mathisson-Papapetrou force density of GR for matter with spin $\tau^{\beta\gamma} := \Delta^{[\beta\gamma]}$. For $\Delta^{\beta\gamma} = 0$, we have $\widetilde{D}\Sigma_\alpha = 0$. Without dilation, shear, and spin 'charges' the particle follows a *Riem. geodesic*, irresp. of the form of V_{MAG}.

Thus, test matter has to carry dilation, shear or spin charges, whether macroscopic or at the quantum particle level. At the latter, the hadron Regge trajectories provide adequate test matter, as Ne'eman's world spinors with shear.

Detailed discussions show that torsion can be measured by precession and nonmetricity by pulsations (mass quadrulole excitations) of test matter.

11. Discussion

It is possible to compute the conserved *charges* of our exact solution, mass, spin $\oplus$ orbital angular momentum, dilation charge, shear charge. The shear charge is a measure for violating Lorentz invariance. MAG provides straightforwardly consistent models for violating local Lorentz invariance by attributing the violation of Lorentz invariance to a geometrical property of spacetime, namely the nonmetricity $Q_{\alpha\beta}$.

Soli Deo Gloria