Femtoprojects, Classical Mechanics 2018

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[M]: math-oriented, [C]: includes coding

1. Influencing sound texture in string instruments

String instruments are very versatile, they allow you to vary from very "bright" to very "warm" sound by choosing where to pluck the string and how hard. In this project you will investigate this influence on a measure of sound texture called the *unitless centroid.*

References:

J Billingham, AC King. *Wave Motion.* Publisher: Cambridge texts in applied mathematics. Sections 2.1-2.2

Project by Jason Pelc in Stanford, web: **http://large.stanford.edu/courses/2007/ph210/pelc2/** Also in the link above there are nice software tools to convert signals to sound.

E Schubert, J Wolfe. *Does timbral brightness scale with frequency and spectroid?* Acta acustica, 2006. —> Just the introduction is strictly necessary, for the definition of a centroid but students are encouraged to read the whole article.

2. The physics of the trumpet

There are many subtleties in the functioning of brass wind instruments, from acoustic impedances and their influence in wave propagation, to the working of the reed of the instrument and how different mouthpieces influence the sound. Here we will concentrate on studying the effect that the tube shape and air temperature have on the acoustic impedance of the trumpet.

References:

Bostjan Berkopec. *The physics of the trumpet.* Seminar at University of Ljubljana. 2013.

Billingham, AC King. *Wave Motion.* Publisher: Cambridge texts in applied mathematics. Section 3.4 (although it would be good to go over 3.1-3.3 lightheartedly).

3. The sound of a spherical drum [optionally M]

The vibrations of a spherical drum are described by the mathematics of spherical harmonics (which I guess is fitting). We will study spherical harmonics and possibly produce movies of how these vibration modes *look like*. Importantly, given a certain momentary deformation of the sphere which would simulate "hitting it", we will simulate the sound that it produces. Optionally, mathematically-inclined students may also link these modes to the representation theory of the symmetries of a sphere.

References:

In any introductory text to quantum mechanics (yes :D), the chapter for hydrogen atoms will cover all the essential ingredients. For example:

DJ Griffiths. *Introduction to quantum mechanics.* 3rd edition, Sections 4.1 to 4.3.
Arfken, Webber. *Mathematical methods for physicists.* 7th edition, Chapter 16: Angular Momentum.
RL Herman. *A first course in partial differential equations.* Sections 6.5, 6.6.

For the representation theory part:

Heine. *Group Theory in Quantum Mechanics.* Chapter II.

4. The sound of a triangular drum —is it nicer than squares?

For those who don't love spheres, we can study triangular and square drums. What's more, we can compare the sounds produced in both cases and have a vote of which sounds worse. We can also produce movies or graphics showing how these vibrations *look like*.

References:

Mathews, Walker. *Mathematical methods of physics.* Second edition, pp. 237-239.
DJ Griffiths. *Introduction to quantum mechanics.* 3rd edition, Sections 2.1, 2.2.
RL Herman. *A first course in partial differential equations.* Sections 1.8, 6.1, 6.3.

Spicy: M Kac. *Can one ear the shape of a drum?* The American Mathematical Monthly, Vol 73, No. 4 Part 2.

5. The geometry of the Foucault pendulum [M]

The Foucault pendulum —like the one in HS I— is a surprising consequence of the fact that the Earth rotates. Equivalently, it's the result that the reference frame of the Earth is not inertial. (This makes Newton's contributions all the more impressive, since he was sitting on a *non-inertial reference frame* and formulating mechanics *in inertial reference frames.*)

The Foucault pendulum can be also understood in a *geometrical* way: it a consequence of the geometry of parallel transport in spheres —usually called *holonomy*. I should note that holonomy plays a key role in many areas of theoretical physics today, from solid state to particle physics. In this project we will study this formulation of the Foucault pendulum.

References:

J Oprea. *Geometry and the Foucault pendulum.*

MI Munk-Nielsen. *Geometric phases in classical mechanics.* Bachelor's thesis, University of Copenhagen.

6. The control theory of geo-stationary satellites

Geo-stationary satellites play an important role in many areas of modern life, from television broadcasting to weather forecasting. Geo-stationary satellites are in orbits that are synchronised with the Earth —so they appear to be above a single point of the earth all the time. Keeping geo-stationary satellites in these orbits is a miracle of control theory. Luckily for us, however, the mathematics is understandable for bachelor's level physics students. In this project we will study the process of stabilising a geo-stationary satellite.

Reference:

HL Trentelman, AA Stoorvogel, M Haatus. *Control theory for linear systems*. Chapter 1, especially Sections 1.6 to 1.8.

7. The control theory of a moon lander

There is a spaceship landing on the moon and you have to tell the engines what to do so that it doesn't just crash against the moon and ruins everybody's day. This is another elegant application of control theory to solve a real world problem. We will first get a glimpse into general control theory to get the needed mathematical tools and then dedicate to understanding the solution of the moon lander problem.

References:

LC Evans. *An introduction to mathematical optimal control theory.* Berkeley. Chapter 1 and 4, especially Section 4.4.4.

A "down to Earth" formulation of the problem:

PA Hokayem, E Gallestey. *Lecture notes on non-linear systems and control.* Exercise 1 of page 151.

8. Limit cycles in non-linear dynamics

Limit cycles are especially useful as simplified models of certain biological processes such as circadian rhythms, the dynamics of the heart and even chemical reactions.

Here we will study the concept of a limit cycle, focusing on the case of the van der Pol oscillator. We will try to see the problem from different points of view: from the phase space picture, from position space, and also viewing limit cycles as a particular kind of *self-oscillation.*

References:

SH Strogatz. *Non-linear dynamics and chaos.* Chapter 7, especially Sections 7.4 and 7.5.
A Jenkins. *Self-oscillation.* arXiv:1109.6640. Section IV C.
These nice lecture notes: http://www.phys.uconn.edu/~rozman/Courses/P2200_13F/downloads/vanderpol/vanderpol-oscillator-draft.pdf

9. The three crabs situation

This one is for those who don't want to read stuff, but just full-on tackle a problem. Three crabs —which we, being good physicists, model by points— sit on the vertices of an equilateral triangle. Let's call them C1, C2, C3. They will all start walking at a constant speed v in the following way: at any point of time t

1. C1 walks in the direction where C2 is at that t
2. C2 walks in the direction where C3 is at that t
3. C3 walks in the direction where C3 is at that t.

Will they ever meet? If so, where will they meet?

Symmetry is everything here.

10. Potential surprises in Newtonian gravity

Suppose that you are sitting by your telescope, somehow watching a interstellar cloud move under the gravitational influence of dark matter. Suppose that somehow you are able to determine from this observation the gravitational field in the region surrounding the cloud. With this observation, will you be able to pin down the dark matter distribution uniquely?

The answer is no, you won't, and the way there is the truly interesting part of this.
Reference:

T Padmanabhan. *Sleeping beauties in theoretical physics.* Chapter 5.

11. Hodographs and why planet orbits are circles after all

Despite what you might have heard in the lecture, one *can* reinstall the Aristotelean ideal of describing heavenly bodies with circles. The price you pay for this is introducing the concept of a *Hodograph* to describe the orbit. In this project you have one clearly defined objective: convincing the rest of us that circles should be used instead of ellipses to describe planetary motion.

Reference:

T Padmanabhan. *Sleeping beauties in theoretical physics.* Chapter 3.

12. Chaos in the Lorenz equations [C]

Chaos theory started the day Edward Lorenz tried to model atmospheric convection with three innocent-looking equations. Much to his surprise, the solutions of these equations where *ludicrously* sensitive to the initial conditions: a tiny perturbation would blow up to a completely different dynamics after a certain time. The chaotic dynamics of the Lorenz equations is due to the presence of what is called a *strange attractor.* We will numerically integrate Lorenz's equations to study the appearance of this attractor.

Reference:

SH Strogatz. *Non-linear dynamics and chaos.* Chapter 9.
CTM Clack. *Dynamics of the Lorenz equations.* University of Manchester 2006.

13. Shrimps in Hénon's sea of Chaos [C]

Hénon's map is a chaotic model similar to the logistic map. It depends on two parameters, (a,b) which in the original formulation of the model had fixed values of a=1.4 and b=0.3. If you instead allow (a,b) to vary an interesting thing happens: there is a sea of chaos in parameter space and some very symmetric non-chaotic islands —these islands are called "shrimps" because of their shape. We will study the emergence of chaos in this system: we will vary the parameters from an initial point inside of a shrimp to a final point outside of it.

References:

JAC Gallas. *Structure of the parameter space of the Hénon map.* Physical review letters, 1993.
H Wen. *A review of the Hénon map and its physical interpretation.* Georgia Tech 2014. Mainly Sections I and II, although the rest of the report is interesting as well.

14. Space elevators

A space elevator is a tower that would stretch from the equator and up past the geostationary orbit. Such a tower could provide a cheaper method to transport material into space. How should one build such a thing? How strong materials would we need?

Reference:

P. K. Aravind, *The physics of the space elevator*, American Journal of Physics, 75, 125 (2007).

15. Maximum overhang

How far can one stack of n identical rectangular blocks reach over the edge of a table without the whole thing collapsing? Straight forward to ask, but very interesting to answer.

References:

J. F. Hall, *Fun with stacking blocks*, American Journal of Physics, 73, 1107 (2005).

M. Paterson, Y. Peres, M. Thorp, P. Winkler, and U. Zwick, *Maximum Overhang*, arXiv:0707.0093 (2007).

16. The physical origin of tides

People say tides are caused by the Moon's gravitational attraction. If this were true, there would only be one high tide and one low tide per day —this prediction is downright false: if you've paid attention on the beach, you'll have noticed there are two high tides and two low tides per day!

In reality, the physics explaining the origin of tides is a bit more complicated, and much more interesting. In this project we will study this physics.

References:

JR Taylor. *Classical Mechanics.* University Science Books (2005), Section 9.2.

T Franc. *Tides in the Earth-Moon system.* WDS'12 Proceedings of Contributed Papers, Part III, 98–104, 2012.

Spicy: M Hendershott. *Lecture 1: Introduction to ocean tides.* http://www.whoi.edu/cms/files/lecture01_21372.pdf

17. Anti-Newtonian Mechanics

Imagine an alternative world where Newton's first and second law hold, but the third law is *replaced* with the rule that two bodies affect each other with a force of equal magnitude, but in the same direction. In this project we will investigate how this world would look like. We could potentially run some simulations to visualise the dynamics.

Reference:

J. C. Sprott, *Anti-Newtonian dynamics*, American Journal of Physics, 77, 783 (2009).

18. Eighteenth Century treatment of a classical mechanical problem

Physics is a dynamical field, the tools we use to solve problems and the intuitions we have about systems have changed dramatically through history. Here you will expose these changes through a case study: the problem of describing a cylinder that slides on the ground. You will compare how people treated this problem in the eighteenth century with a contemporary treatment.

Reference:

L. Stefanini, *Eighteenth century treatment of a a classical mechanic problem*, American journal of physics 80, 47 (2012).

19. Bertrand's theorem [M]

Bertrand's theorem singles out mathematically two potentials that are conceptually very important. More precisely, it tells us that the only central force potentials for which all bound orbits are closed, are the harmonic potential and the 1/r-potential (like gravity and the electrostatic potential). This would be a more theoretical project, where the task would be to understand the proof of Bertrand's theorem

References:

S. A. Chin, *A truly elementary proof of Bertrand's theorem*, American journal of physics, 83, 320 (2015).

Y. Grandati, A. B´erard, F. M´enas, *Inverse problem and Bertrand's theorem*, American journal of physics, 76, 782 (2008).

20. Catastrophe theory

Non-linear systems that relax to equilibrium can sometimes respond very dramatically on small changes of external parameters —these are called catastrophes. Catastrophes are studied by, well, *catastrophe theory.* In this project you can explore the basic ideas of catastrophe theory, maybe you will even build a catastrophe machine!

Reference:

E. C. Zeeman, *Catastrophe Theory*, Scientific American, April, 65 (1976)

21. Backwards equilibrium: the Kapitza pendulum

A pendulum has a stable equilibrium when it is pointing down, but also an unstable equilibrium when it is pointing upwards. However, if you oscillate the pivot of the pendulum up and down rapidly, it turns out that you can make the upwards position the stable one. You could potentially build a Kapitza pendulum in lego.

References:

E. I. Butikov, *On the dynamic stabilisation of an inverted pendulum*, American journal of physics, 69, 755 (2001).

J. A. Blackburn, H. J. T. Smith, N. Grobbech-Jensen, *Stability and Hopf bifurcation in an inverted pendulum*, American journal of physics, 60, 903 (1992).