Quantenmechanik

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Sheet 12 **Due: 02.07 um 12 Uhr**

This is the last exercise sheet, and it is a bit smaller than normal. Please use the extra time to barbecue and prepare yourselves for the exam.

1 Perturbation theory and the Zeeman effect (for simplicity without spin) (10 P)

The Hamiltonian for a particle (charge $-e$, mass M) in an electromagnetic field given by a vector potential $\vec{A}$ is

$$H = \frac{1}{2M}\left(\vec{P} + e\vec{A}(\vec{X},t)\right)^2 - \frac{e^2}{r}.$$

A hydrogen atom is exposed to a homogeneous, time-independent magnetic field $\vec{B}$.

a) **(1,5 P)** Show that the vector potential for this $\vec{B}$ is given by $\vec{A} = \frac{1}{2}\vec{B} \times \vec{X}$. From now on we assume that the magnetic field is parallel to the e_z direction: $\vec{B} = Be_z$.

b) **(2,5 P)** Show that the Hamiltonian can be written as $H = H_0 + H_1 + H_2$, where

$$H_0 = \frac{1}{2M}\vec{P}^2 - \frac{e^2}{r}, \qquad H_1 = \frac{e}{2M}\vec{B}.\vec{L}, \qquad H_2 = \frac{e^2}{8M}\vec{B}^2\vec{R}_\perp^2,$$

and $\vec{R}_\perp^2 = x^2 + y^2$.

c) **(1,5 P)** We ignore for now the term H_2. Show that the eigenfunction $\psi_{n,l,m}$ of $H_0 + H_1$ is the same as the eigenfunction of the free hydrogen atom (known from the lecture), but that the energies are shifted as follows:

$$E'_{n,l,m} = E_n + m\mu_B B.$$

Here E_n are the energies of the unperturbed atom and $\mu_B = \frac{e\hbar}{2M}$.

Hint: Show that $[H_0, H_1] = 0$. What happens with the eigenvalues and eigenvectors of H_0, when one adds a commuting term H_1? Do *not* use perturbation theory here.

d) **(1,5 P)** In a strong magnetic field B the term H_2 becomes relevant. Show that

$$H_2 = \frac{e^2B^2}{8M}r^2\sin^2\vartheta.$$

e) **(3 P)** Compute the first order correction to the ground state $\psi_{1,0,0}$.

Hint: The correct normalised expression for the ground state wavefunction is $\psi_{1,0,0} = 2(a_0)^{-3/2}e^{-\frac{r}{2a_0}}$.