Classical Mechanics

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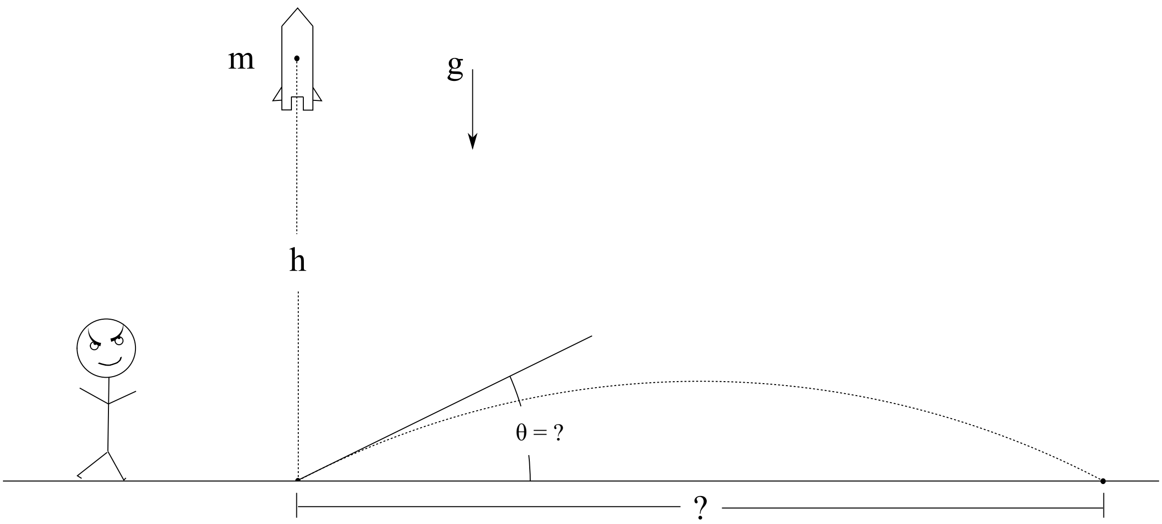
Exercise sheet 1 Due: October, 19 at 12:00

1 Dictator as career choice: Rocket science

Presumably you have been thinking of what to do after your physics studies. One career choice would be to become dictator of some random country. With solid knowledge in classical mechanics you can threaten the entire world with your powerful rockets.



Suppose that you want to test the nice new rocket that your minions have built. It is a smart move to direct the rocket vertically[1]. In that way you look threatening, but do not directly provoke your neighbours. To simplify things we assume that all energy is released at the initial moment when the rocket is fired, so that after the initial burst there is no additional propulsion[2]. Moreover, we assume that there is no friction, that gravity is independent of vertical position, and that we can ignore the curvature (as well as rotation) of Earth[3].

Suppose that the rocket has mass m and reaches the distance h above ground when fired vertically. How far could it reach horizontally if it instead is aimed at an angle? What is the optimal angle?

Hint: For the vertical launch, what is the initial speed required to reach h? Use Newton's second law to find the trajectory when the rocket instead is fired at an angle. How far can it reach as a function of θ?

(7 points)

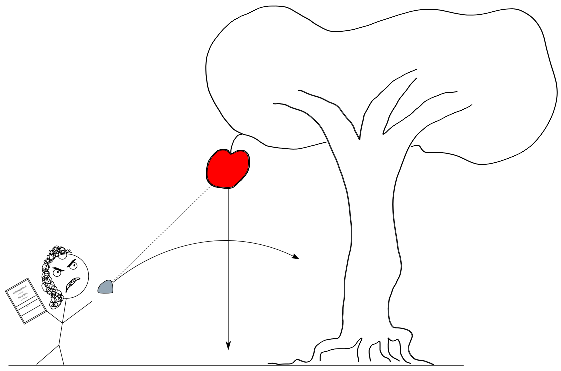
[1] *Dictatorship for dummies*, 2nd Edition, Wiley (2011).

[2] To be honest, we are here modeling a cannon rather than a rocket, but cannons are much less fashionable among dictators these days.

[3] These are of course not very good assumptions if you want your rocket to reach some big country on the other side of Earth.

2 Newton hates apples

Sir Isaac Newton is terribly tired of apples falling on his head all the time, so he wants them all down from the trees. In his fury he totally forgets about the laws of gravity and throws a stone aiming directly at an apple. As it so happens, the apple falls out of the tree at the very same moment as the stone leaves his hand.[4] Assuming that we can ignore friction, show that the stone actually hits the apple.[5]

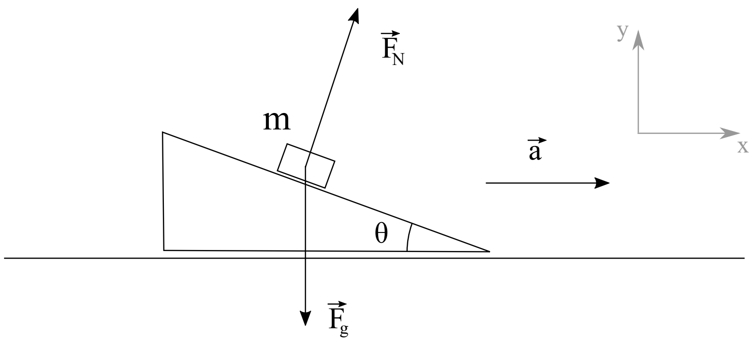


Hint: There is not much information given, but we know that the initial velocity vector of the stone is directed straight towards the apple. This means that it is parallel to the vector that points from the stone to the apple. (This does for example give a relation between the horizontal and vertical components of these vectors.) Use the equations of motion for the stone and the apple. What are the relations between the initial conditions? Given the solutions of the equations of motion, what is the condition for a collision?

(7 points)

3 Block on a wedge

A block of mass m can slide without friction on a wedge that is pulled with constant acceleration $\vec{a} = a\hat{x}$, for $a > 0$. The angle of the wedge is $0 < \theta < \pi/2$, and the block is affected by gravity $\vec{F}_g = -mg\hat{y}$. Here $\hat{x}$ and $\hat{y}$ are the unit-vectors in the x- and y-direction, respectively. What does a have to be in order for the block not to slide along the wedge? What is the magnitude of the normal force that the wedge exerts on the block?



Hint: Focus on the forces acting on the block. These are the gravity and the normal force. (The normal force is orthogonal to the surface, and prevents the block from sinking through the surface of the wedge.) In what way do we want the block to accelerate? What force (regarded as a vector) is needed in order to achieve that?

(6 points)

[4] The apple is initially at rest.

[5] Strictly speaking we should also assume that he throws the stone fast enough, such that it actually collides with the apple before it hits the ground, but you can ignore that for the calculations. Alternatively we could make the very realistic assumption that there is an infinitely deep hole (with constant gravitational field) into which the stone and apple can fall for an arbitrarily long time.