Exercise Sheet 6

Kastoryano: Quantum Error Correction

November 15, 2018

1 Exercise 1: Lower bound on the threshold

Exercise 1.1 : Argue (as I did in class) that the logical failure of a toric code of length L with physical error rate p is upper bounded by the expression:

$$P_{\text{fail}}(n,p) \leq (1-p)^n \sum_{l=d}^{n} N_{\text{con}}(l) \sum_{u=l/2}^{l} \sum_{v=0}^{n-l} \binom{l}{u} \binom{n-l}{v} \left(\frac{p}{1-p}\right)^{u+v}, \tag{1}$$

where N_{con} is the number of length-l non-contractible closed loops on the torus, constrained by the requirement that they can have no self-intersections.

Recall: $\binom{a}{b} = \frac{a!}{b!(a-b)!}$ is the binomial coefficient.

Exercise 1.2 Show that this expression can be upper bounded as

$$\begin{aligned} P_{\text{fail}} &\leq \sum_{l=d}^{n} N_{\text{con}}(l) 2^l p^{l/2} (1-p)^{l/2} && (2) \\ &\leq \sum_{l=d}^{n} N_0 (2c\sqrt{p(1-p)})^l, && (3) \end{aligned}$$

where we use that $N_{\text{con}}(l) < N_0 c^l$, for some constant N_0 and the the lattice expansion coefficient $c \approx 2.638$. What value does this lower bound give on the threshold?