Rapid mixing and clustering of correlations in open quantum systems

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Outline

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Setting

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- Markovian Dynamics

$$\partial_t \rho = \mathcal{L}^*(\rho) = i[H, \rho] + \sum_k L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\}_+$$

Typically, we will assume that L_k and H are bounded (there exists a $K < \infty$ s.t. $||L_k|| \leq K$ for all k) and geometrically local on a d-dimensional cubic lattice of side length L.

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  Typically, we will assume that L_k and H are bounded (there exists a $K < \infty$ s.t. $||L_k|| \leq K$ for all k) and geometrically local on a d-dimensional cubic lattice of side length L.
- We say that $\mathcal{L}$ is *primitive* if it has has a unique full-rank stationary state $\sigma > 0$.
- We say $\mathcal{L}$ is *reversible* (detailed balance) if

$$\mathcal{L}^*(\sqrt{\sigma} g \sqrt{\sigma})) = \sqrt{\sigma}\mathcal{L}(g)\sqrt{\sigma}.$$

Mixing times:

There exist constant $A, b > 0$ such that:

$$\|e^{t\mathcal{L}^*}(\rho_0) - \sigma\|_1 \leq Ae^{-bt}.$$

Mixing $\Leftrightarrow$ Clustering

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Clustering of correlations:

There exist constants $C, \xi > 0$ such that for any subsets of the lattice A, B we get

$$\mathrm{Corr}_\sigma(A : B) \leq C \,\mathrm{poly}(|A|, |B|) e^{-d(A:B)/\xi},$$

where $d(A : B)$ is the distance separating regions A, B.

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The goal of this talk is to explain to what extent these two statements are equivalent.

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Why are these bounds useful?

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4. (Runtimes of dissipative algorithms and state preparation (Nature Phys. **5**, 633 (2009)).)
5. (Bounds on the thermalization times of quantum systems, i.e. efficient Gibbs samplers?)

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χ^2 bound:

Let $\mathcal{L}$ be a primitive reversible Liouvillian with stationary state $\sigma > 0$, then

$$\|e^{t\mathcal{L}^*}(\rho_0) - \sigma\|_1 \leq \sqrt{\|\sigma^{-1}\|}e^{-\lambda t},$$

for any initial state ρ_0.

Rapid mixing: χ^2 bound

χ^2 bound:

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for any initial state ρ_0.

Proof sketch:
write $\rho_t = e^{t\mathcal{L}^*}(\rho_0)$, then

$$\|\rho_t - \sigma\|_1^2 \leq \chi^2(\rho_t, \sigma) \leq \chi^2(\rho_0, \sigma) e^{-2t\lambda},$$

where $\chi^2(\rho, \sigma) = \mathrm{tr}\left[(\rho - \sigma)\sigma^{1/2}(\rho - \sigma)\sigma^{1/2}\right]$ is the χ^2 divergence, and it satisfies $\chi^2(\rho, \sigma) \leq \|\sigma^{-1}\|$. □

Note that if $\mathcal{L}$ is reversible, then λ is just the spectral gap of $\mathcal{L}$. For a system of N spins (qubits) $\|\sigma^{-1}\| \geq 2^N$.

Log-Sobolev bound:

Let $\mathcal{L}$ be a primitive reversible Liouvillian with stationary state $\sigma > 0$, then

$$\|e^{t\mathcal{L}}(\rho_0) - \sigma\|_1 \leq \sqrt{2\log\left(\|\sigma^{-1}\|\right)}e^{-2\alpha t},$$

for any initial state ρ_0.

Ultra-rapid mixing: Log-Sobolev bound

Log-Sobolev bound:

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$$\|e^{t\mathcal{L}}(\rho_0) - \sigma\|_1 \leq \sqrt{2\log\left(\|\sigma^{-1}\|\right)}e^{-2\alpha t},$$

for any initial state ρ_0.

- Same proof but with $\chi^2(\rho, \sigma)$ replaced by $S(\rho\|\sigma) = \mathrm{tr}\,[\rho(\log\rho - \log\sigma)]$.
- The *Log-Sobolev* constant α can only be obtained by a complicated variational formula $\Rightarrow$ equivalent to Hypercontractivity of the semigroup.
- The bound provides an exponentially improved pre-factor! Importantly, $\alpha \leq \lambda$
- See *J. Math. Phys. 54, 052202 (2013)* for more details.

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Correlation measures

We consider a cubic lattice Λ and denote subsets of the lattice $A \subset \Lambda$.
Assume $A \cap B = \emptyset$,

Correlation measures

- The covariance correlation:

$$C_\rho(A:B) := \sup_{\|f\|=\|g\|=1} |\mathrm{tr}\,[(f \otimes g)(\rho_{AB} - \rho_A \otimes \rho_B)]\,|, \tag{1}$$

  where f is supported on region A, and g is supported on region B.
- The trace norm correlation:

$$T_\rho(A:B) := \|\rho_{AB} - \rho_A \otimes \rho_B\|_1 . \tag{2}$$

- The mutual information correlation:

$$I_\rho(A:B) := S(\rho_{AB}\|\rho_A \otimes \rho_B), \tag{3}$$

  where $S(\rho\|\sigma) = \mathrm{tr}\,[\rho(\log\rho - \log\sigma)]$ is the relative entropy.

Correlation measures: Theorem

The different correlation measures can be easily related:

Theorem

Let ρ be a full rank state of the lattice Λ, and let $A, B \subset \Lambda$ be non-overlapping subsets. Let d_{AB} be the dimension of the subsystem defined on AB, then the following inequalities hold,

$$\begin{aligned} \frac{1}{2d_{AB}^2} T_\rho(A:B) &\leq C_\rho(A:B) \leq T_\rho(A:B), \\ \frac{1}{2} T_\rho^2(A:B) &\leq I_\rho(A:B) \leq \log(\|\rho_{AB}^{-1}\|) T_\rho(A:B). \end{aligned}$$

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There is also an exponential separation between correlation measures. Is there a connection to the exponential separation in rapid mixing regimes? YES

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χ^2 clustering

Theorem

- $A, B \subset \Lambda$ are subsets of the $\mathcal{D}$-dimensional cubic lattice Λ.
- $\mathcal{L} = \sum_{Z \subset \Lambda} \mathcal{L}_Z$ is a local, bounded, reversible Liouvillian with stationary state σ
- λ is the gap, v is the Lieb-Robinson velocity v

Then there exists a constant $c > 0$ such that

$$C_\sigma(A : B) \leq c\, d(A : B)^{\mathcal{D}-1} e^{-\frac{\lambda d(A:B)}{v+2\lambda}}.$$

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Weak rapid mixing implies weak clustering

χ^2 clustering: Proof sketch

- Define $\mathrm{Cov}(f, g) = \mathrm{tr}\,[\sigma fg] - \mathrm{tr}\,[\sigma f]\,\mathrm{tr}\,[\sigma g]$, write $f_t := e^{t\mathcal{L}}(f)$ and consider

$$|\mathrm{Cov}(f, g)| \leq |\mathrm{Cov}(f_t, g_t)| + |\mathrm{Cov}(f_t, g_t) - \mathrm{Cov}(f, g)|$$

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$$|\mathrm{Cov}(f,g)| \leq |\mathrm{Cov}(f_t,g_t)| + |\mathrm{Cov}(f_t,g_t) - \mathrm{Cov}(f,g)|$$

- The first term is bounded using a mixing argument

$$\begin{aligned} |\mathrm{Cov}(f_t,g_t)| &\leq \sqrt{\mathrm{Var}(f_t)\mathrm{Var}(g_t)} \\ &\leq \sqrt{\mathrm{Var}(f)\mathrm{Var}(g)}e^{-t\lambda} \leq \|f\|\,\|g\|e^{-t\lambda} \end{aligned}$$

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- The second term is bounded using quasi-locality of the dynamics

$$\begin{aligned} |\mathrm{Cov}(f_t,g_t) - \mathrm{Cov}(f,g)| &\leq |\mathrm{tr}\,[\sigma((fg)_t - f_t g_t]\,| \\ &\leq c\,\|f\|\,\|g\|e^{tv-d(A:B)/2} \end{aligned}$$

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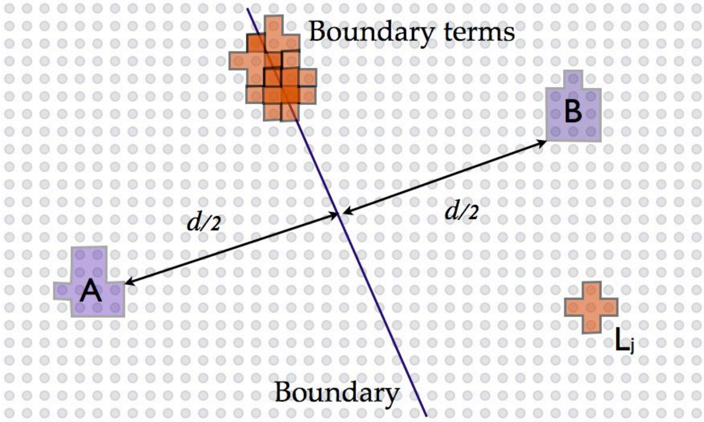
- Finally, chose the t which minimizes the sum of both expressions.

Removal of boundary terms



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Log-Sobolev clustering

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- $A, B \subset \Lambda$ are subsets of the $\mathcal{D}$-dimensional cubic lattice Λ.
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- α is the Log-Sobolev constant, v is the Lieb-Robinson velocity.

Then there exists a constant $c > 0$ such that

$$I_\rho(A:B) \leq c\, d(A:B)^{\mathcal{D}-1} (\log(\|\rho^{-1}\|))^{3/2} e^{-\frac{\alpha d(A:B)}{2(v+\alpha)}},$$

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Strong rapid mixing implies strong clustering

Corollary: Local perturbations perturb locally

- $A, B \subset \Lambda$ are subsets of the $\mathcal{D}$-dimensional cubic lattice Λ.
- $\mathcal{L} = \sum_{Z \subset \Lambda} \mathcal{L}_Z$ is a local, bounded, reversible Liouvillian with stationary state ρ
- $\mathcal{Q}_A$ is a local Liouvillian perturbation, acting trivially outside of A. Let σ be the stationary state of $\mathcal{L} + \mathcal{Q}_A$.
- α is the Log-Sobolev constant and v is the Lieb-Robinson velocity of $\mathcal{L}$

Then,

$$\|\rho_B - \sigma_B\|_1 \leq c\, d(A:B)^{\mathcal{D}-1} (\log(\|\rho^{-1}\|))^{1/2} e^{-\frac{\alpha d(A:B)}{v+\alpha}},$$

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Mutual information Area Law

Mutual information Area Law

Let $\mathcal{L}$ be a regular, reversible Liouvillian with stationary state ρ and Log-Sobolev constant α. Let $A \subset \Lambda$, then for any $\epsilon > 0$, there exist constants $\gamma_1, \gamma_2 > 0$ such that

$$I_\rho(A, A^c) \leq (\gamma_1 + \gamma_2 \log\log \|\rho^{-1}\|)|\partial_A| + \epsilon,$$

where $|\partial_A|$ is the boundary of A.

Note: it is not know whether one can get rid of the $\log\log \|\rho^{-1}\|$ factor?

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The main theorem

- Let $H = \sum_j H_j$ be a bounded, local, commuting Hamiltonian (i.e. $[H_j, H_k]$).
- Let $\rho = e^{-\beta H}/\mathrm{tr}\left[e^{-\beta H}\right]$ be the Gibbs state of H.
- Suppose that there exist constants $c, \xi > 0$ such that for all observables f, g,

$$\mathrm{Cov}_\rho(f, g) \leq c\sqrt{\mathrm{Var}_\rho(f)\mathrm{Var}_\rho(g)}e^{-d(\Lambda_f,\Lambda_g)/\xi}$$

- $\mathrm{Cov}_\rho(f, g) = \mathrm{tr}\left[\sqrt{\rho}f^\dagger\sqrt{\rho}g\right] - \mathrm{tr}\left[\rho f\right]\mathrm{tr}\left[\rho g\right]$, $\mathrm{Var}_\rho(f) = \mathrm{Cov}_\rho(f, f)$, and $d(\Lambda_f, \Lambda_g)$ is the minimum distance separation the supports of f, g.

Then, there exists a local, bounded parent Liouvillian $\mathcal{L}^p$ such that ρ is its unique stationary state, and the spectral gap of $\mathcal{L}^p$ is independent of the size of the lattice.

The parent Liouvillian

The parent Liouvillian

$$\mathcal{L}^{\rho}_{\Lambda}(f) = \sum_{j\in\Lambda}(\mathbb{E}^{\rho}_{j}(f) - f),$$

where

$$\mathbb{E}^{\rho}_{j}(f) = \mathrm{tr}_{j}[\gamma_j f \gamma_j^{\dagger}]$$

and

$$\gamma_j = (\mathrm{tr}_j[\rho])^{-1/2}\rho^{1/2}$$

$\mathbb{E}^{\rho}_{j}$ should be interpreted as a conditional expectation value of ρ on site j which minimally disturbs the sites around j.

Note: if H has locally commuting terms, then γ_i has support on a ball of radius r, where r is the range of the Hamiltonian. Then $\mathcal{L}^{\rho}_{\Lambda}$ is local.

Proof sketch I

We will show that the gap of a lattice Λ is approximately the same as the gap on half the lattice size: $\lambda(\Lambda) \approx \lambda(\Lambda/2)$.

- The variational expression of the gap. Let $A \subset \Lambda$,

$$\lambda(A) = \sup_{f=f^\dagger} \frac{\mathcal{E}_A(f)}{\mathrm{Var}_A(f)}$$

where $\mathcal{E}_A(f) = \langle f, -\mathcal{L}_A(f) \rangle_\rho$ and $\mathrm{Var}_A(f) = \langle f - \mathbb{E}_A(f), f - \mathbb{E}_A(f) \rangle_\rho$, and $\langle f, g \rangle_\rho = \mathrm{tr}\left[\sqrt{\rho} f^\dagger \sqrt{\rho} g\right]$ is an $\mathbb{L}_2$ inner product.

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- Decomposition of the conditional variance: If $\langle \mathbb{E}_A(f), \mathbb{E}_B(f)\rangle_\rho \leq \epsilon$, then for $A \cup B = \Lambda$ and $A \cap B \neq \emptyset$, then

$$\mathrm{Var}_\Lambda(f) \leq (1 - 2\epsilon)^{-1} (\mathrm{Var}_A(f) + \mathrm{Var}_B(f))$$

Proof sketch II

- Let $A\cap B \approx \sqrt{L}\times L$ (in 2D). If ρ is clustering, then $\langle \mathbb{E}_A(f), \mathbb{E}_B(f)\rangle_\rho \leq ce^{-\sqrt{L}/\xi}$. Then

$$\begin{aligned}
\mathrm{Var}_\Lambda(f) &\leq (1-ce^{-\sqrt{L}/\xi})^{-1}(\mathrm{Var}_A(f)+\mathrm{Var}_B(f)) \\
&\leq (1-ce^{-\sqrt{L}/\xi})^{-1}\left(\frac{\mathcal{E}_A(f)}{\lambda(A)}+\frac{\mathcal{E}_B(f)}{\lambda(B)}\right) \\
&\leq (1-ce^{-\sqrt{L}/\xi})^{-1}\max\left\{\frac{1}{\lambda(A)},\frac{1}{\lambda(B)}\right\}(\mathcal{E}_\Lambda(f)+\mathcal{E}_{A\cap B}(f))
\end{aligned}$$

Proof sketch II

- Let $A \cap B \approx \sqrt{L} \times L$ (in 2D). If ρ is clustering, then $\langle \mathbb{E}_A(f), \mathbb{E}_B(f) \rangle_\rho \leq ce^{-\sqrt{L}/\xi}$. Then

$$\begin{aligned}
\mathrm{Var}_\Lambda(f) &\leq (1 - ce^{-\sqrt{L}/\xi})^{-1}(\mathrm{Var}_A(f) + \mathrm{Var}_B(f)) \\
&\leq (1 - ce^{-\sqrt{L}/\xi})^{-1}(\frac{\mathcal{E}_A(f)}{\lambda(A)} + \frac{\mathcal{E}_B(f)}{\lambda(B)}) \\
&\leq (1 - ce^{-\sqrt{L}/\xi})^{-1} \max\{\frac{1}{\lambda(A)}, \frac{1}{\lambda(B)}\}(\mathcal{E}_\Lambda(f) + \mathcal{E}_{A\cap B}(f))
\end{aligned}$$

- By an averaging trick over $L^{1/3}$ different overlaps, we can upper bound the following upper bound:

$$\begin{aligned}
\mathrm{Var}_\Lambda(f) &\leq (1 - ce^{-\sqrt{L}/\xi})^{-1}(1 + \frac{1}{L^{1/3}}) \max\{\frac{1}{\lambda(A)}, \frac{1}{\lambda(B)}\}\mathcal{E}_\Lambda(f) \\
&\leq (1 + \frac{1}{\sqrt{L}}) \max\{\frac{1}{\lambda(A)}, \frac{1}{\lambda(B)}\}\mathcal{E}_\Lambda(f)
\end{aligned}$$

If $L \geq L_0$ for some L_0 independent of the systems size.

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Important Corollary

- Let $H = \sum_j S_j$ stabilizer Hamiltonian.
- Let $\rho = e^{-\beta H}/\mathrm{tr}\left[e^{-\beta H}\right]$ be the Gibbs state of H.
- Suppose that there exist constants $c, \xi > 0$ such that for all observables f, g,

$$\mathrm{Cov}_\rho(f, g) \le c\sqrt{\mathrm{Var}_\rho(f)\mathrm{Var}_\rho(g)}e^{-d(\Lambda_f,\Lambda_g)/\xi}$$

Then, there Davies generator $\mathcal{L}^D$ has a spectral gap which is independent of the size of the lattice.

Note: the Davies generator is obtained by a canonical weak system bath coupling, where the bath is in a thermal state.

A partial extension

1D non-commuting Hamiltonians

- Let $H = \sum_j H_j$ be a local bounded Hamiltonian in 1D.
- Let $\rho = e^{-\beta H}/\mathrm{tr}\left[e^{-\beta H}\right]$ be the Gibbs state of H.

Then, there exists a local, bounded parent Liouvillian $\mathcal{L}^p$ such that ρ is its unique stationary state, and the spectral gap of $\mathcal{L}^p$ is independent of the size of the lattice.

Collaborators

F. Brandao

J. Eisert

F. Pastawski

K. Temme

Alexander von Humboldt
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Thank you for your attention!

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MJK and F.G.S.L Brandao

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in preparation

MJK, F. Pastawski, K. Temme

Comparison theorems for thermal quantum semigroups.

in preparation