

PROBABILISTIC STRUCTURES
IN EVOLUTION

DFG SPP 1590

COLLABORATIVE RESEARCH CENTER | SFB 680

Molecular Basis of
Evolutionary Innovations

Mutational pathways in complex fitness landscapes

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joint work with Arjan de Visser & group (Wageningen)

Modeling Tumor Evolution: Initiation, Growth and Progression

ZiF Bielefeld, September 14, 2016

Cancer as an evolutionary process

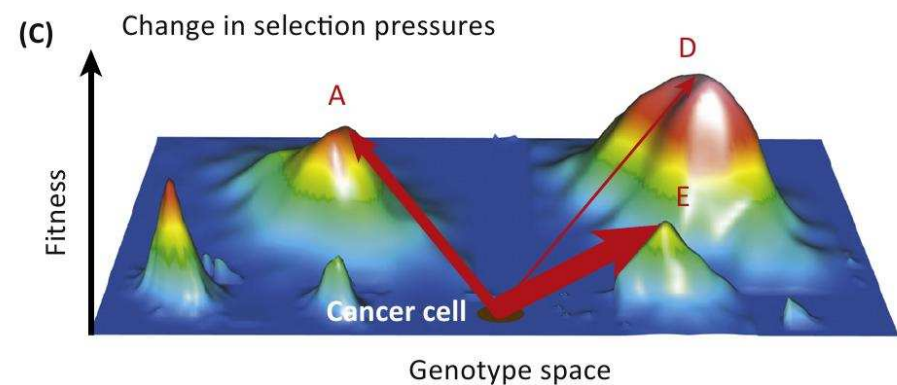
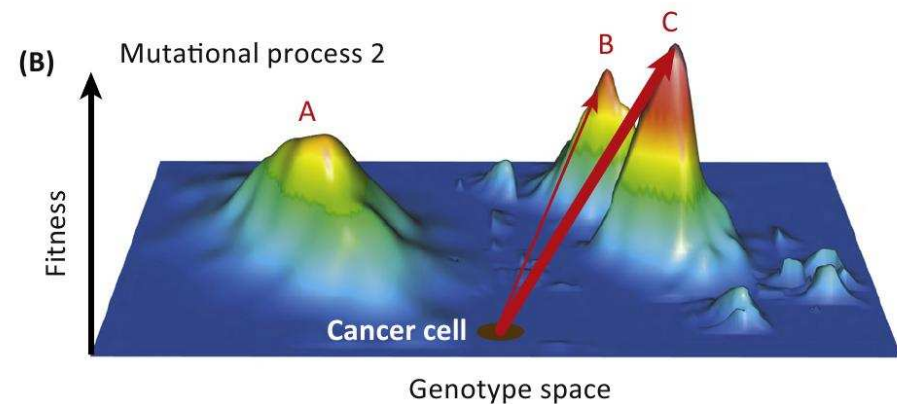
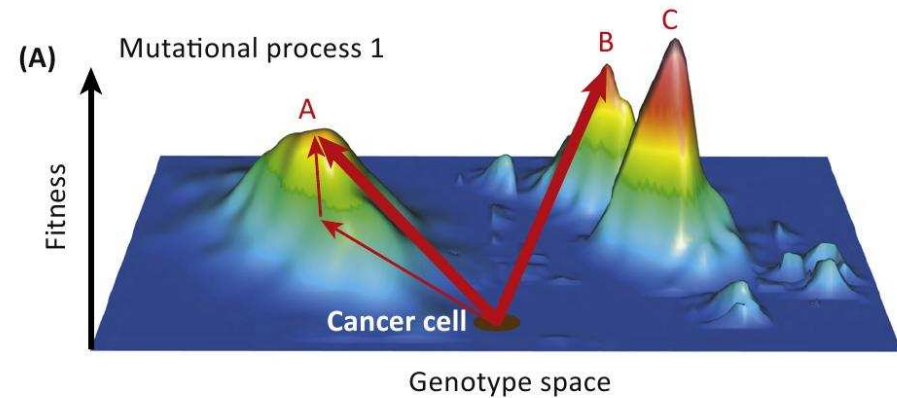
- Cancer initiation and resistance evolution typically requires multiple mutational steps
- These mutations can interact
 - **directly** in terms of their phenotypic effects, or
 - **indirectly** through the clonal dynamics
- Direct interactions are called epistatic and can be encoded in a multidimensional **fitness landscape** [de Visser & Krug, Nat. Rev. Gen. 2014](#)

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- Direct interactions are called epistatic and can be encoded in a multidimensional **fitness landscape** [de Visser & Krug, Nat. Rev. Gen. 2014](#)
- The focus of this talk will be on epistatic interactions and their consequences for **predictability**
- Empirical examples will be drawn from antibiotic resistance evolution in bacteria

The fitness landscape metaphor

- Multiple fitness peaks shape the evolutionary process [S. Wright 1932](#)
- Only a subset of peaks are accessible by mutational pathways
- How to turn this picture into a quantitative, predictive tool?



Lipinski et al.,
[Trends in Cancer 2016](#)

Predictability of a single mutational step

The probability of parallel evolution

H.A. Orr, *Evolution* 2005

- n beneficial single step mutations are available from the initial genotype
- Each mutant is characterized by its selective advantage $s_i > 0$
- The **fixation probability** for the i 'th mutant is $2s_i$ (Haldane 1927), hence the probability that the i 'th mutant is the first to fix is given by

$$\pi_i = \frac{s_i}{\sum_{j=1}^n s_j}$$

and the same mutation is fixed in two replicate populations with probability

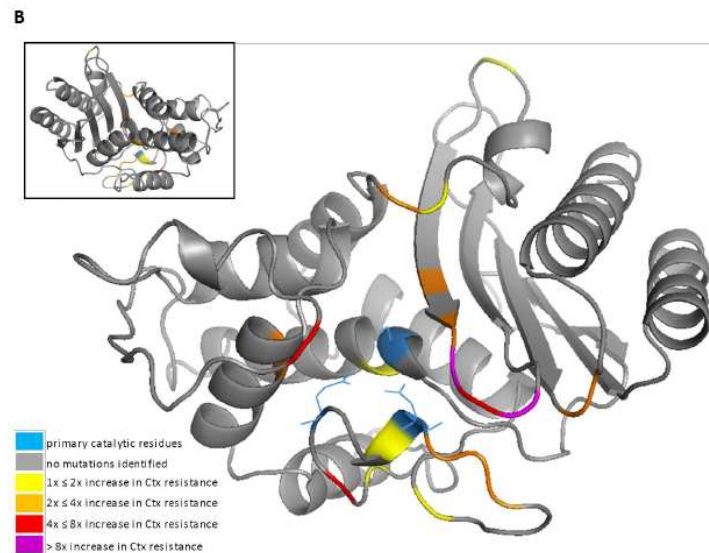
$$P_2 = \sum_{i=1}^n \pi_i^2$$

- This quantity is determined by the distribution of beneficial fitness effects

The TEM-1 β -lactamase enzyme

M.F. Schenk, I.G. Szendro, JK, J.A.G.M. de Visser, PLoS Genet. 2012

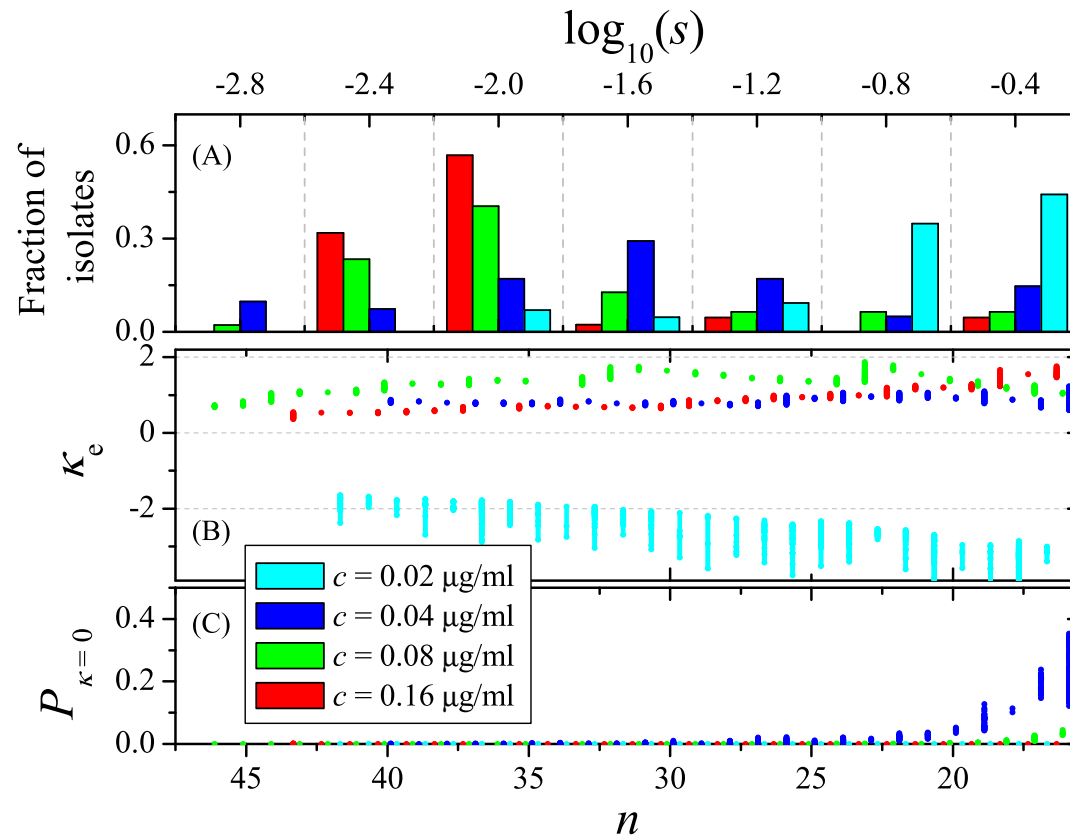
- β -lactamase confers resistance against penicillin to *E. coli*



- 48 out of 2583 point mutations increase resistance against cefotaxime
- Colony survival translated into fitness using branching process simulations

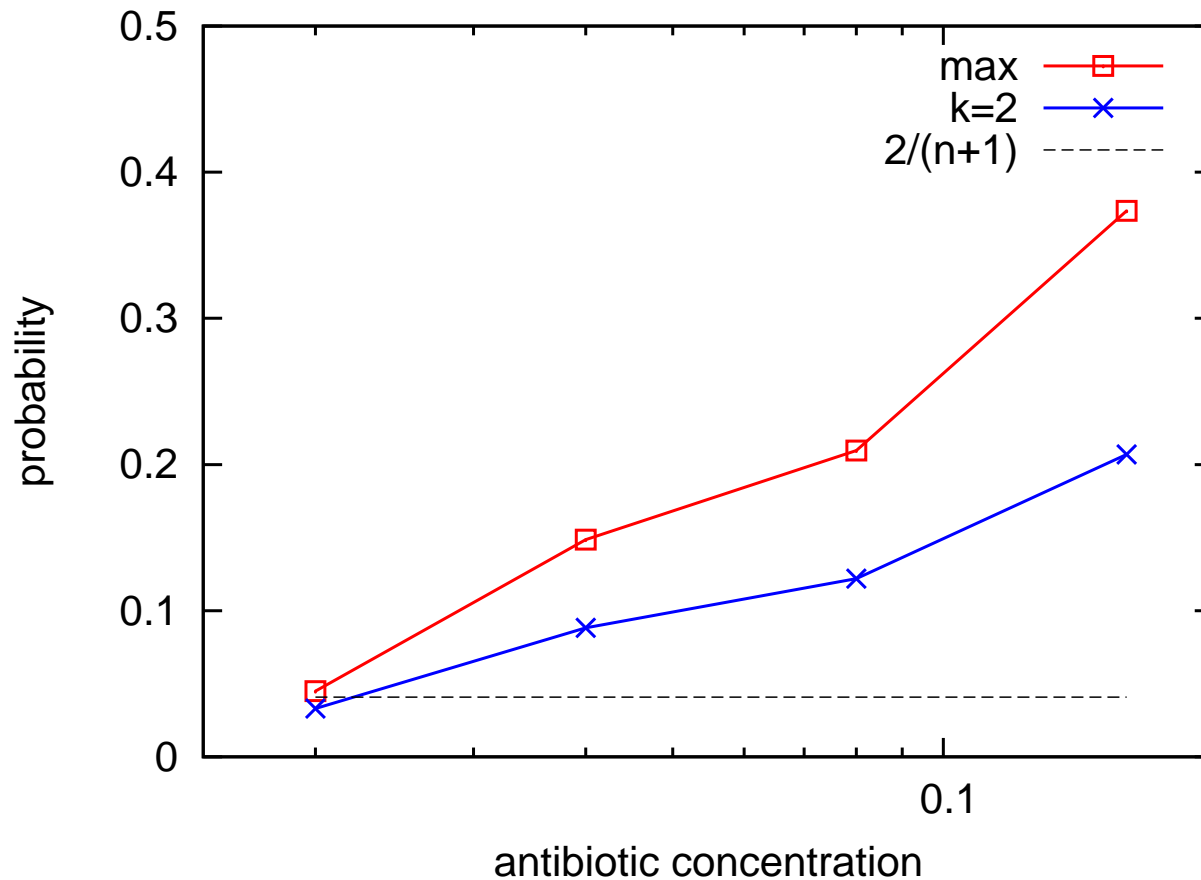
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- Data analysis based on extreme value theory reveals a heavy-tailed distribution of effect sizes with extreme value index $\kappa \sim 1$

Repeatability measures



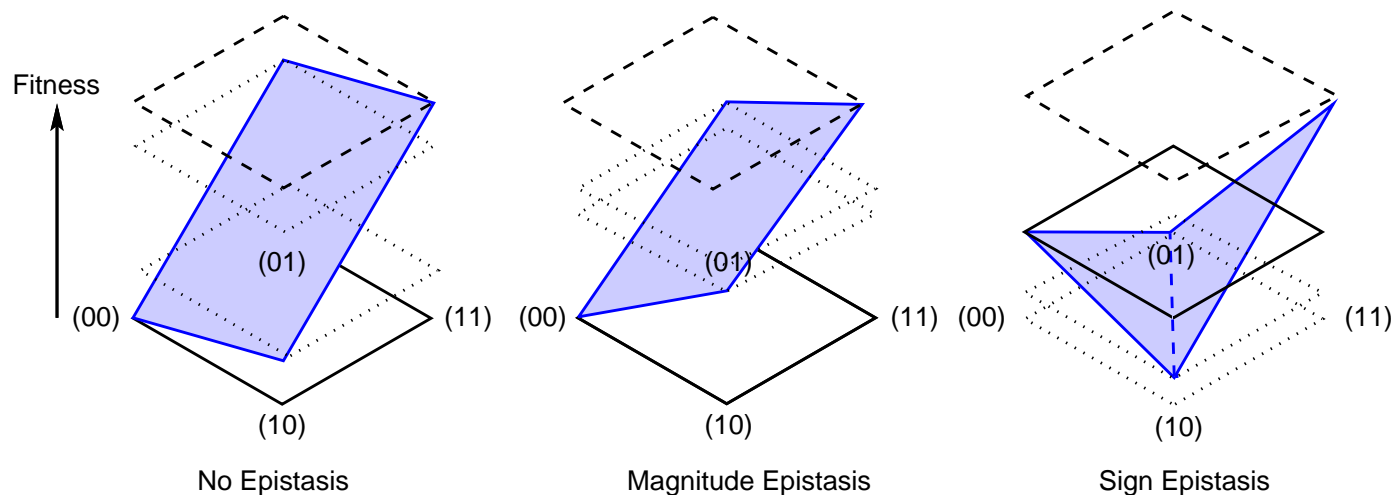
- P_2 and $P_{\max} = \max_i \pi_i > P_2$ increase with antibiotic concentration
- $P_2 = \frac{2}{n+1}$ predicted for effect size distributions with exponential-like tails

Mutational pathways

Epistasis and sign epistasis

- **General setting:** L diallelic haploid loci τ_i at which a mutation can be present ($\tau_i = 1$) or absent ($\tau_i = 0$).
- A **genotypic fitness landscape** is a function on the set of 2^L genotypes
- **Epistasis** implies interactions between the effects of different mutations
- **Sign epistasis:** Mutation at a given locus is beneficial or deleterious depending on the state of other loci

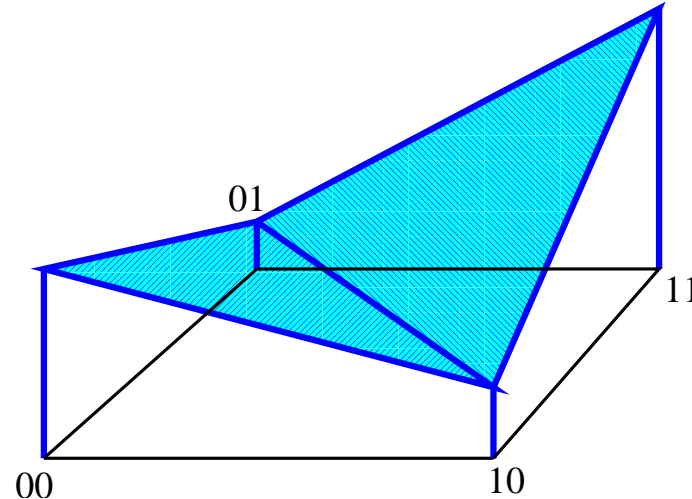
Weinreich, Watson & Chao 2005



Complex fitness landscapes

- A genotypic fitness landscape is complex/rugged if it has multiple fitness maxima
- The existence of **reciprocal sign epistasis** is a necessary condition for the existence of multiple peaks

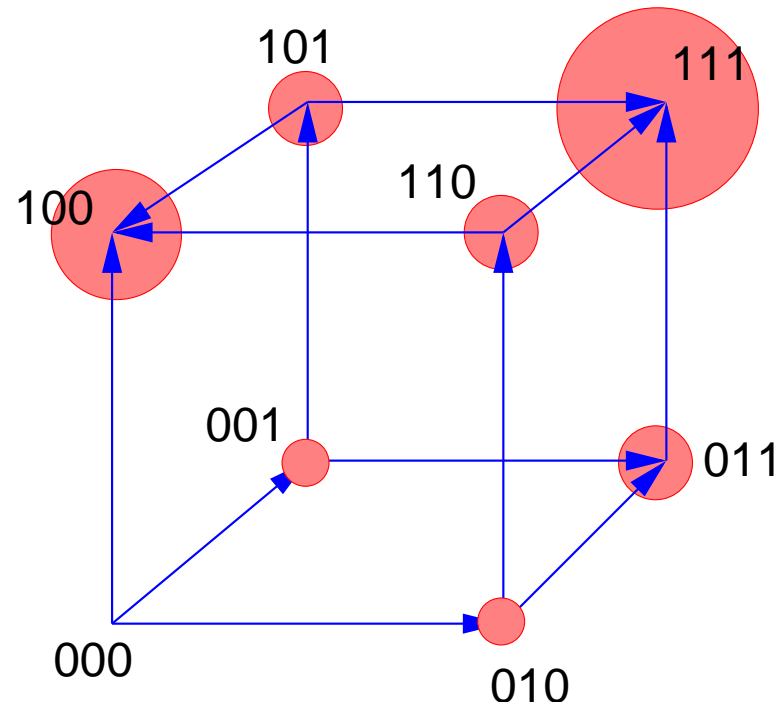
Poelwijk et al., JTB 2011



- Multi-peakedness is guaranteed if all instances of pairwise sign epistasis are reciprocal

Crona et al., JTB 2013

Mutational pathways



- $L = 3$ mutational steps from wildtype 000 to adapted type 111
- Mutations can occur in $3 \times 2 \times 1 = 3! = 6$ different orders corresponding to 6 possible pathways
- Only a subset of pathways are “uphill” (= increasing in fitness)

SSWM dynamics

- **SSWM** = Strong Selection/Weak Mutation Gillespie 1983, Orr 2002
- **Weak mutation**: Each new mutation goes to fixation or is lost before the next one arrives
- **Strong selection**: The fixation probability of a mutation of selective advantage s in a population of size N is Kimura 1963

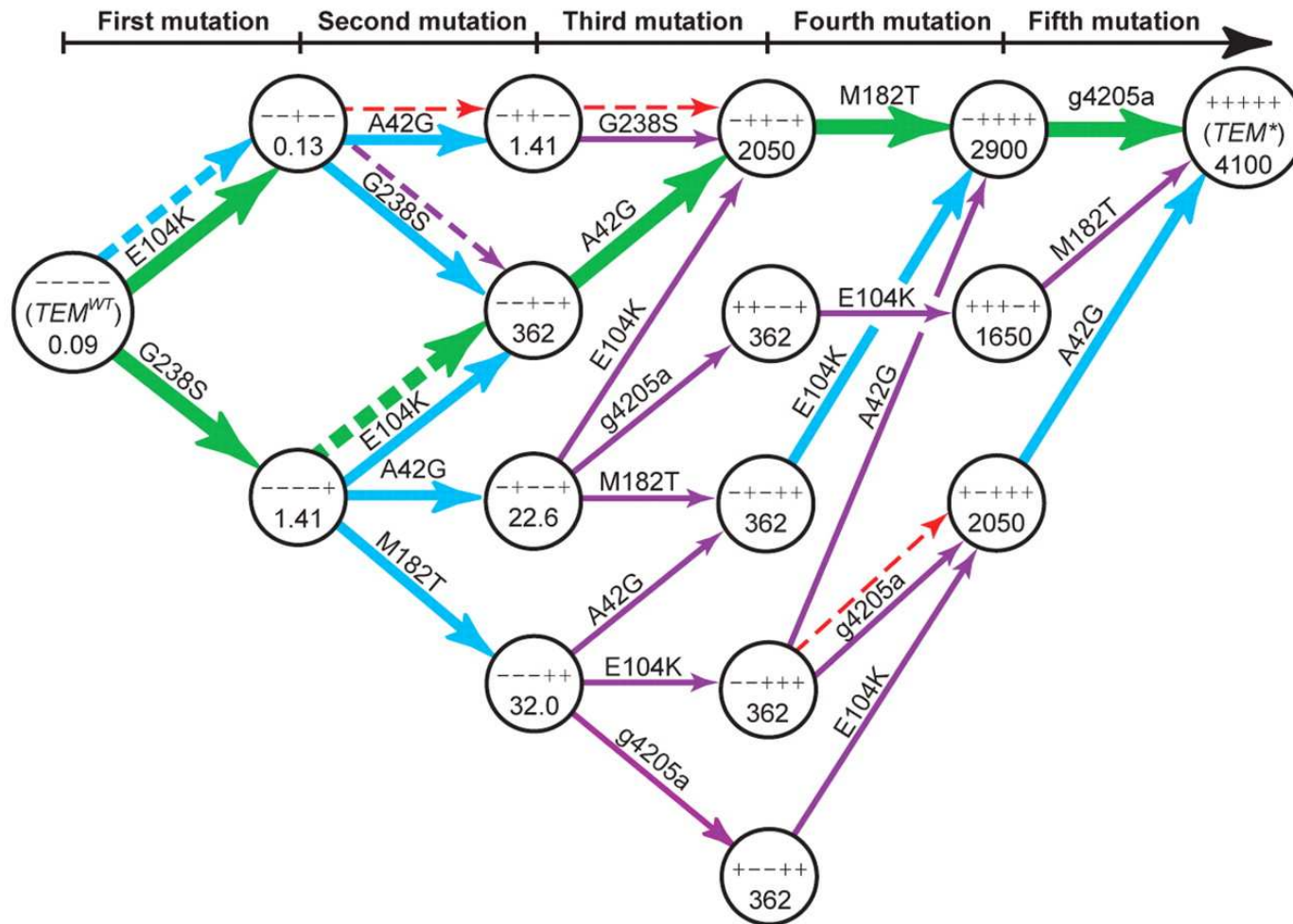
$$p_{\text{fix}}(s, N) \approx \frac{1 - \exp[-2s]}{1 - \exp[-2Ns]} \approx 2s$$

for $0 < s \ll 1$ and $p_{\text{fix}} = 0$ for $s \leq 0$, provided $N|s| \gg 1$

- Under these conditions the evolution is restricted to uphill (= fitness monotonic) mutational pathways, which are called **accessible**
- The **weight of an accessible path** in the SSWM regime is the product of the normalized fixation probabilities $\pi_i = \frac{s_i}{\sum_j s_j}$

“Darwinian evolution can follow only very few mutational paths to fitter proteins”

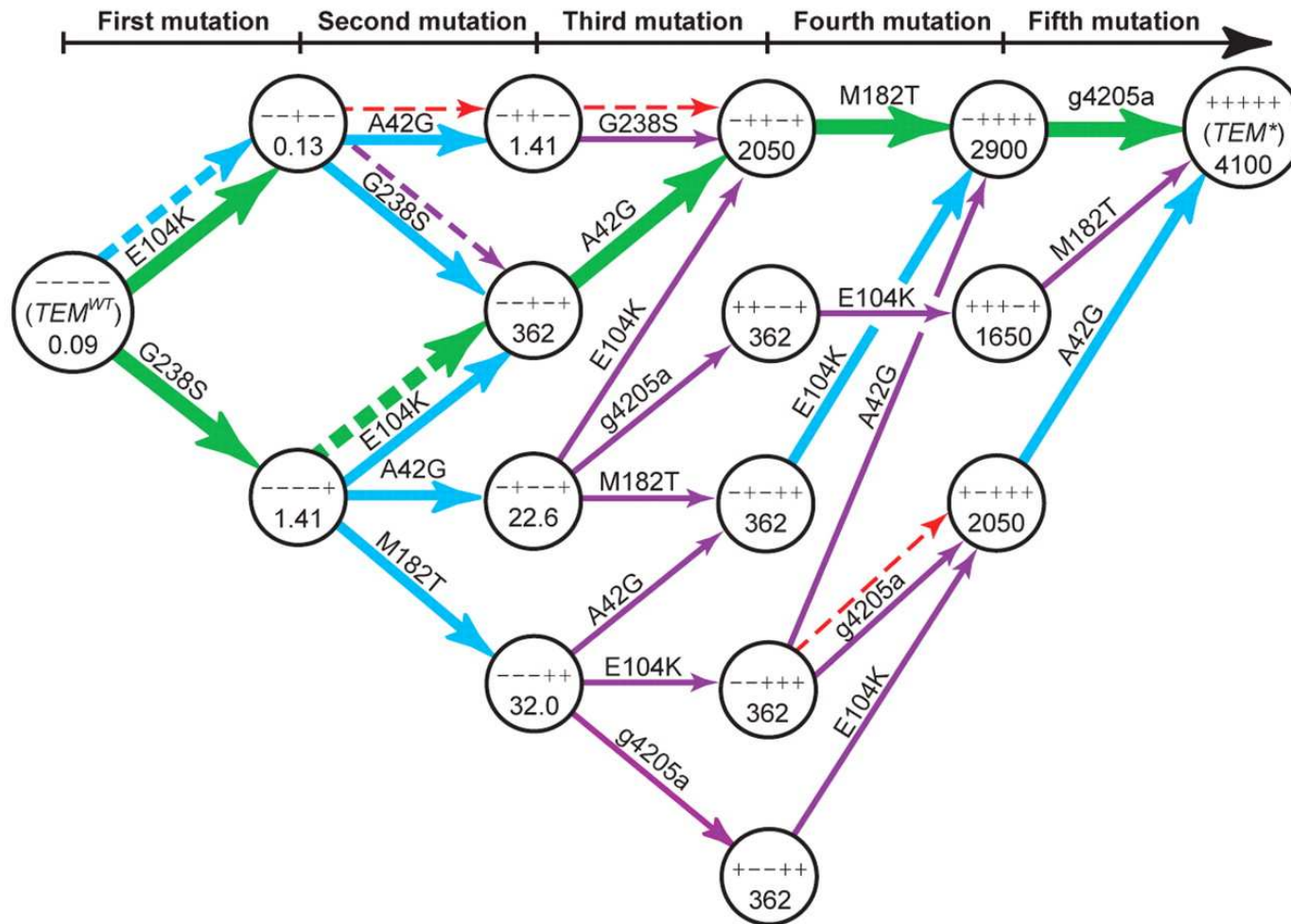
D.M. Weinreich et al., Science 2006



- 5 mutations increase resistance of TEM-1 β -lactamase by $\sim 10^5$

“Darwinian evolution can follow only very few mutational paths to fitter proteins”

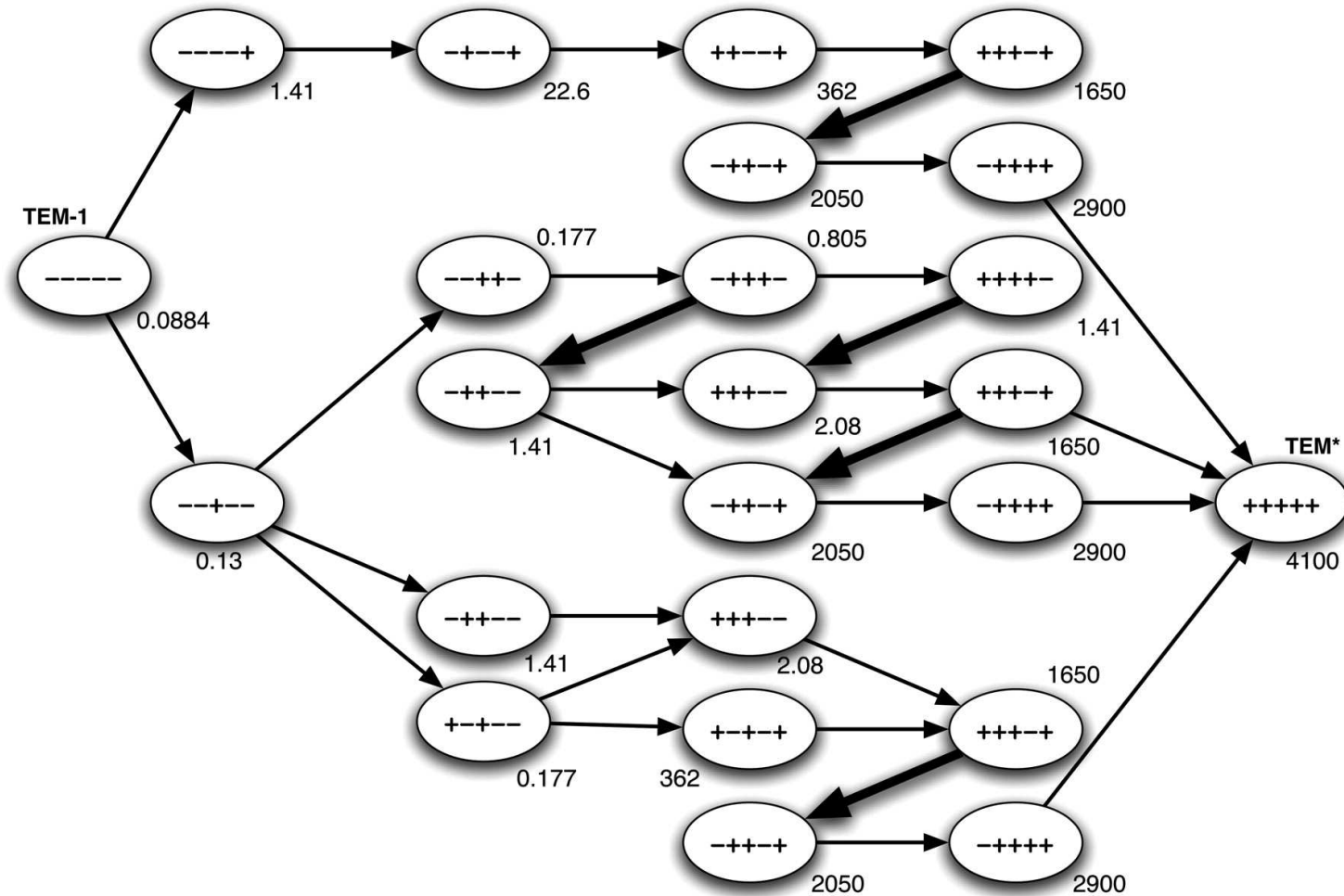
D.M. Weinreich et al., Science 2006



- 18 out of $5! = 120$ directed mutational pathways are increasing...

“Darwinian evolution can follow only very few mutational paths to fitter proteins”

D.M. Weinreich et al., Science 2006



- ...and 27 out of 18651552840 undirected pathways

De Pisto et al. 2007

Accessibility and predictability

- Pathways are accessible if fitness/resistance increases monotonically
- Existence of a **small but nonzero** fraction of accessible pathways implies high (retrospective) predictability

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Questions for mathematical theory

- How does accessibility depend on the genetic interactions and on the boundary conditions of the paths?
- How typical is it that a small but nonzero fraction of pathways are accessible?
- These questions can be addressed systematically using **probabilistic models** of fitness landscapes

Null model: House-of-Cards

J. Franke, A. Klözer, J.A.G.M. de Visser & JK, PLoS Comp. Biol. 2011

- In the **house-of-cards model** fitness is assigned randomly to genotypes, for example, from a uniform distribution [Kingman 1978, Kauffman & Levin 1987](#)
- Then the probability that a given path is accessible is $1/L!$ and hence the expected number of accessible paths is $L! \times \frac{1}{L!} = 1$ which suggests very high predictability.

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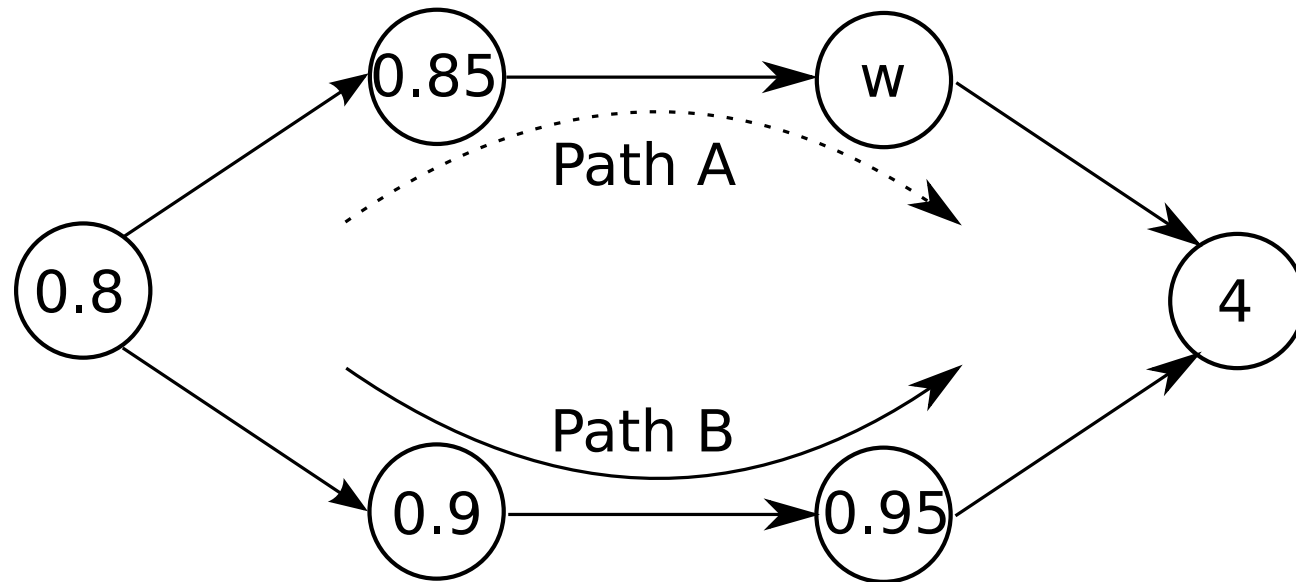
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- Then the probability that a given path is accessible is $1/L!$ and hence the expected number of accessible paths is $L! \times \frac{1}{L!} = 1$ which suggests very high predictability.
- This is however misleading, because most landscape realizations do not possess a single accessible path.
- Accessibility is determined primarily by initial fitness and transitions sharply from high to low at a threshold fitness $\sim \log L/L$ Hegarty & Martinsson 2014
- As a consequence, **conditioned on accessibility** (or low initial fitness) the typical number of paths is of order $L \ll L!$

Paths to evolutionary rescue

Jan Schmidt

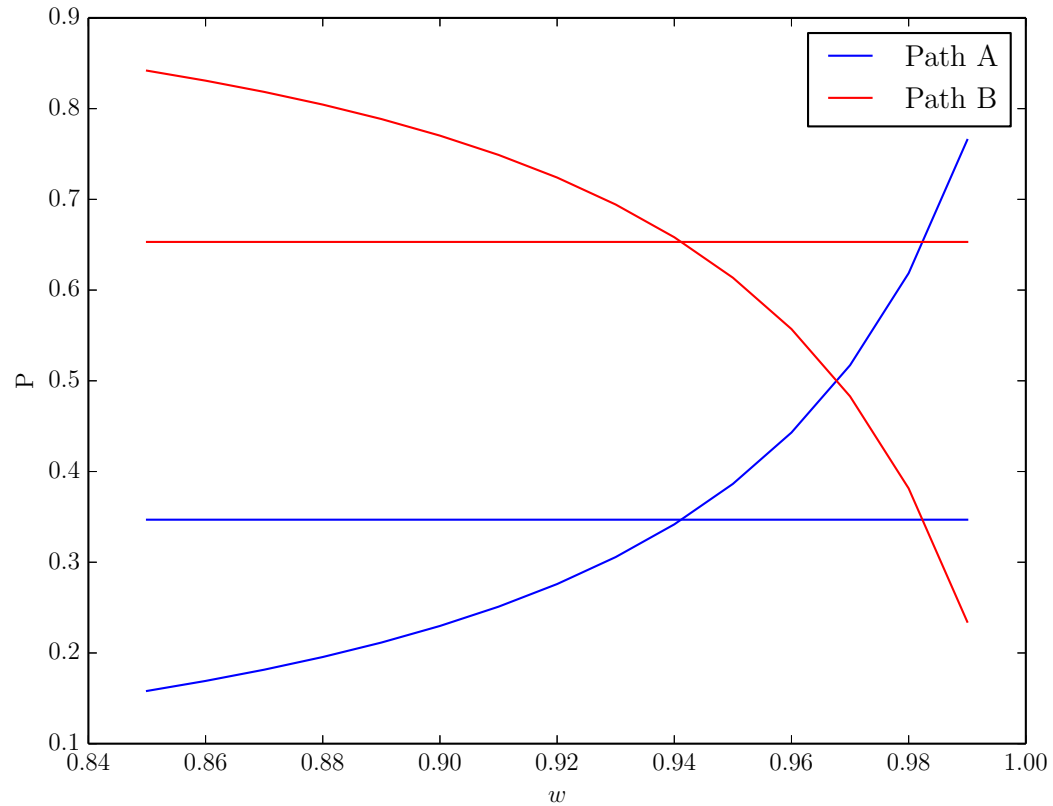
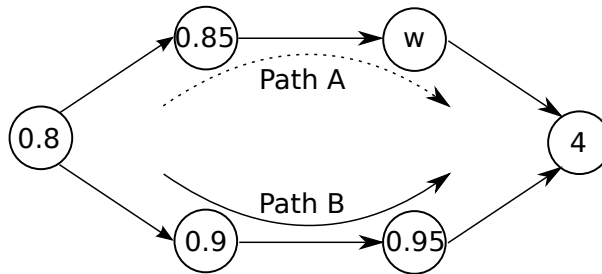
- A population with negative absolute fitness that is destined for extinction can be rescued by *de novo* beneficial mutations
- If this requires multiple mutational steps there can be different **rescue paths**
- Evolutionary rescue is the process underlying the evolution of drug resistance
- Path weights for evolutionary rescue can be determined using path-resolved branching process theory [Iwasa et al. 2003](#); [Bauer & Gokhale 2015](#)
- SSWM weights and the weights of rescue paths can differ substantially
- In particular, evolutionary rescue does not require fitness to be monotonically increasing

Example: Two competing accessible paths



- Initial and all intermediate types have multiplicative fitness/reproductive ratio < 1
- SSWM weights are determined by the first step and do not depend on the intermediate fitness value w
- By contrast, rescue occurs preferentially along path A as $w \rightarrow 1$

Example: Two competing accessible paths



Landscapes of antibiotic resistance

Patterns of epistasis

- Comparative studies of empirical fitness landscapes reveals generic features but also characteristic differences

Szendro et al., JSTAT 2013; de Visser & Krug, Nat. Rev. Genet. 2014

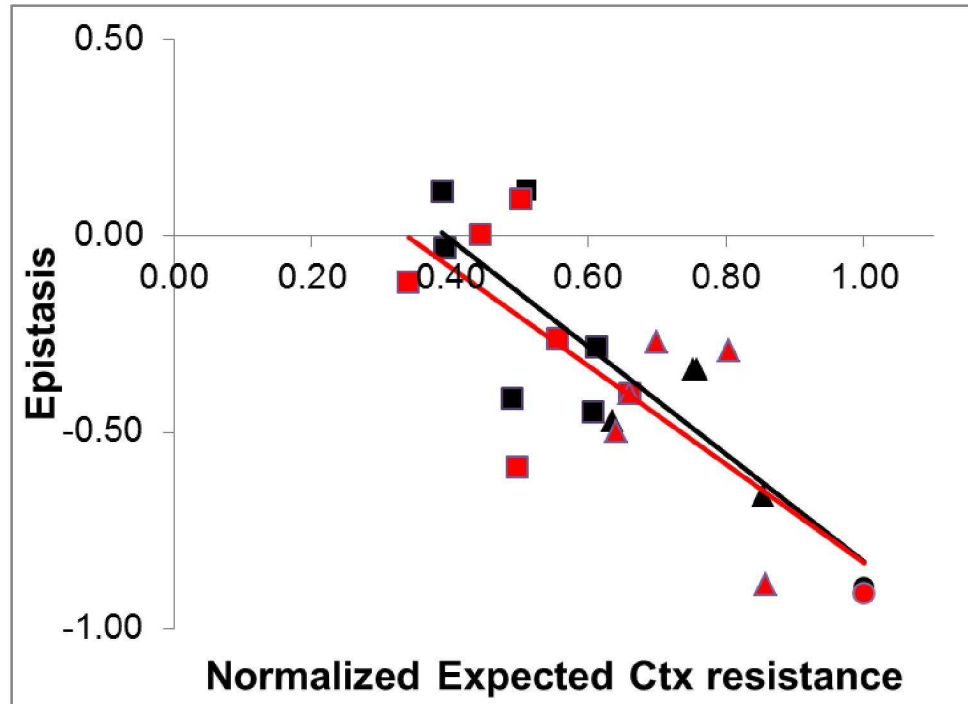
- In particular, the choice of the subset of mutations used to construct a landscape biases the patterns of epistasis:
 - singly beneficial vs. singly deleterious mutations
 - mutations chosen for individual or collective effects
 - mutations in the same gene or different genes
 - mutations occurring along an adaptive trajectory

Example:

M.F. Schenk et al., Mol. Biol. Evol. 2013

- Comparative analysis of two subsets of 4 mutations each chosen from a pool of 48 individually beneficial mutations in TEM-1 β -lactamase
- Mutations chosen according to effect on resistance (weakly vs. strongly beneficial)

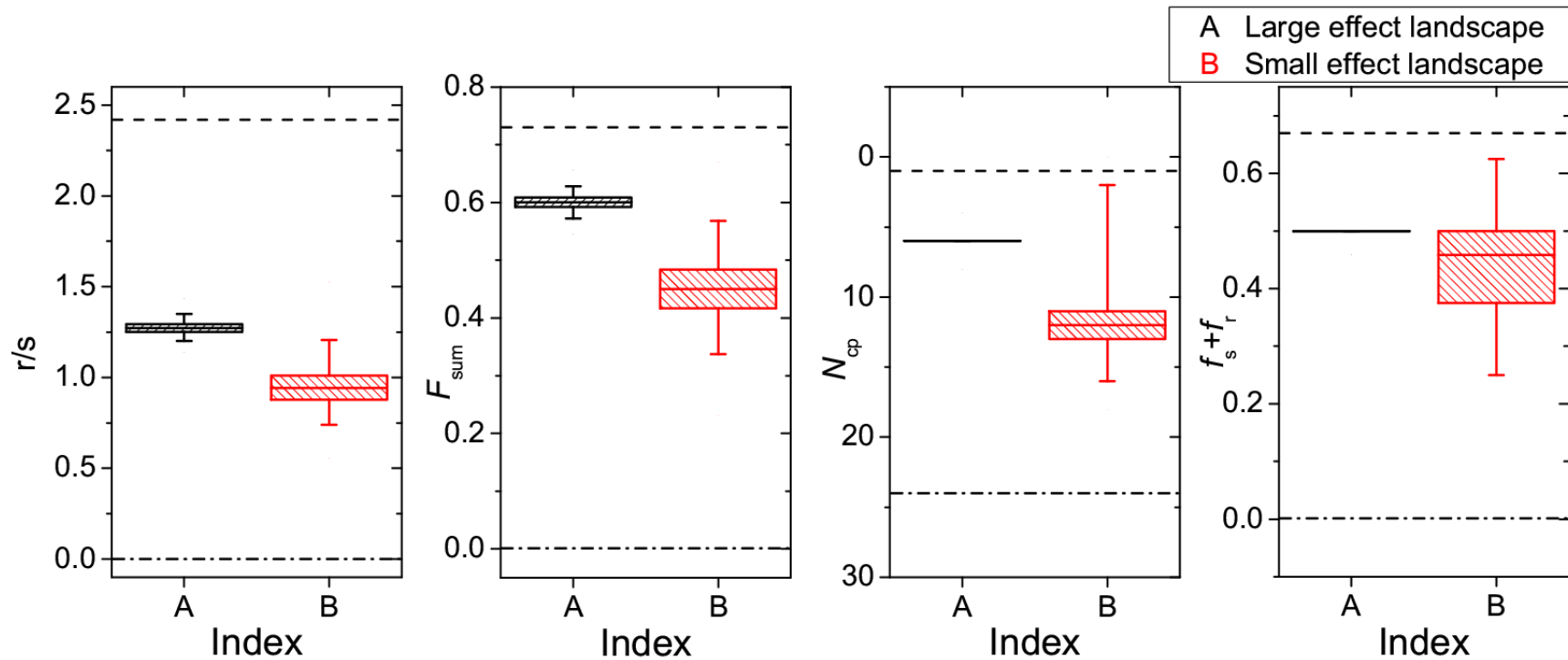
Diminishing returns epistasis



- Resistance of multiple mutants is lower than expected assuming multiplicative effects, and the deviation increases with effect strength
- Generic pattern that appears also in multicellular organisms

Schoustra et al., Proc. Roy. Soc. B 2016

Large effect landscape is consistently more rugged



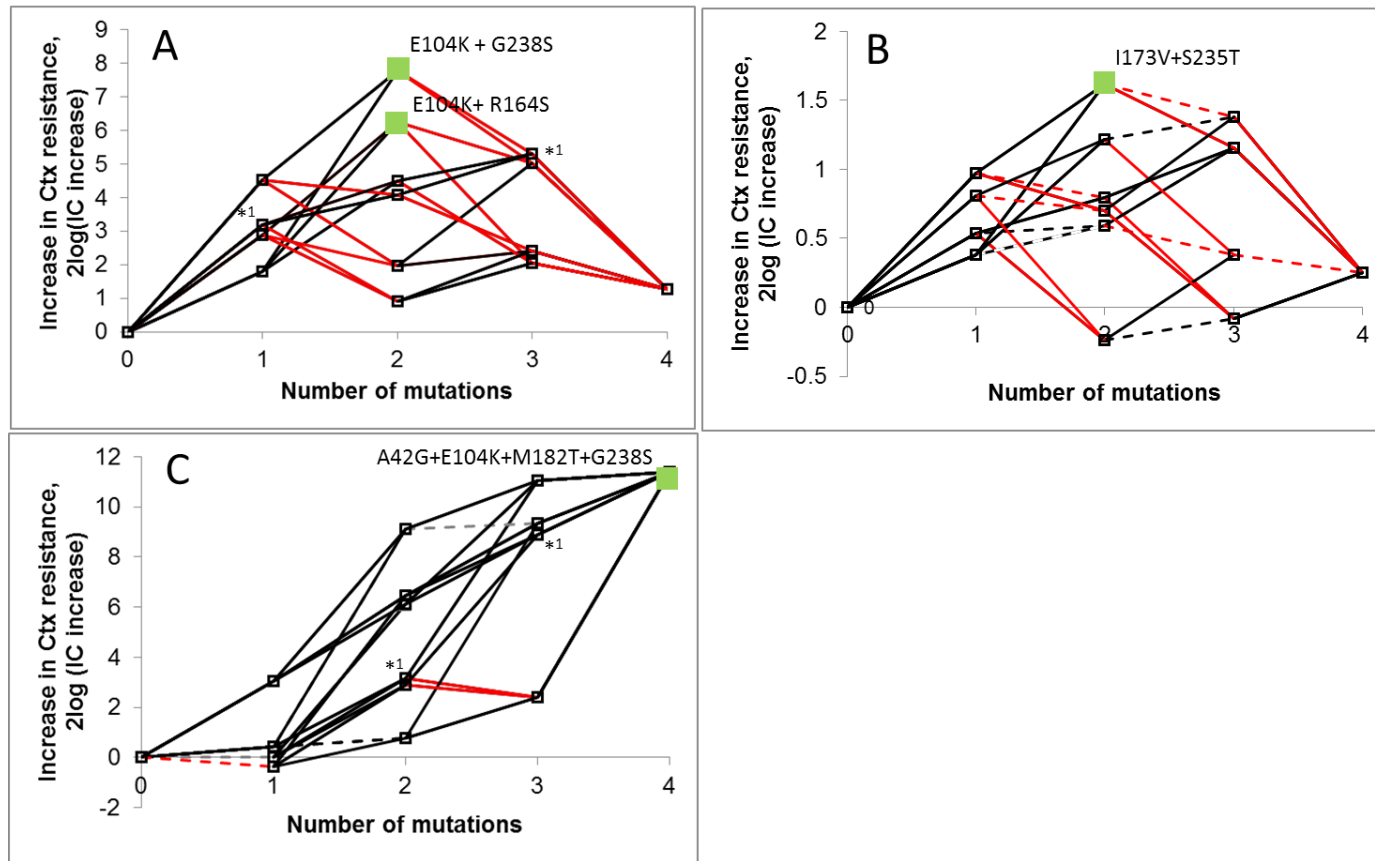
r/s : Roughness-to-slope ratio

F_{sum} : Relative weight of interactions

N_{cp} : Number of accessible paths

$f_s + f_r$: Fraction of sign-epistatic pairs

Mutations chosen for individual vs. collective effect



A: Large effect

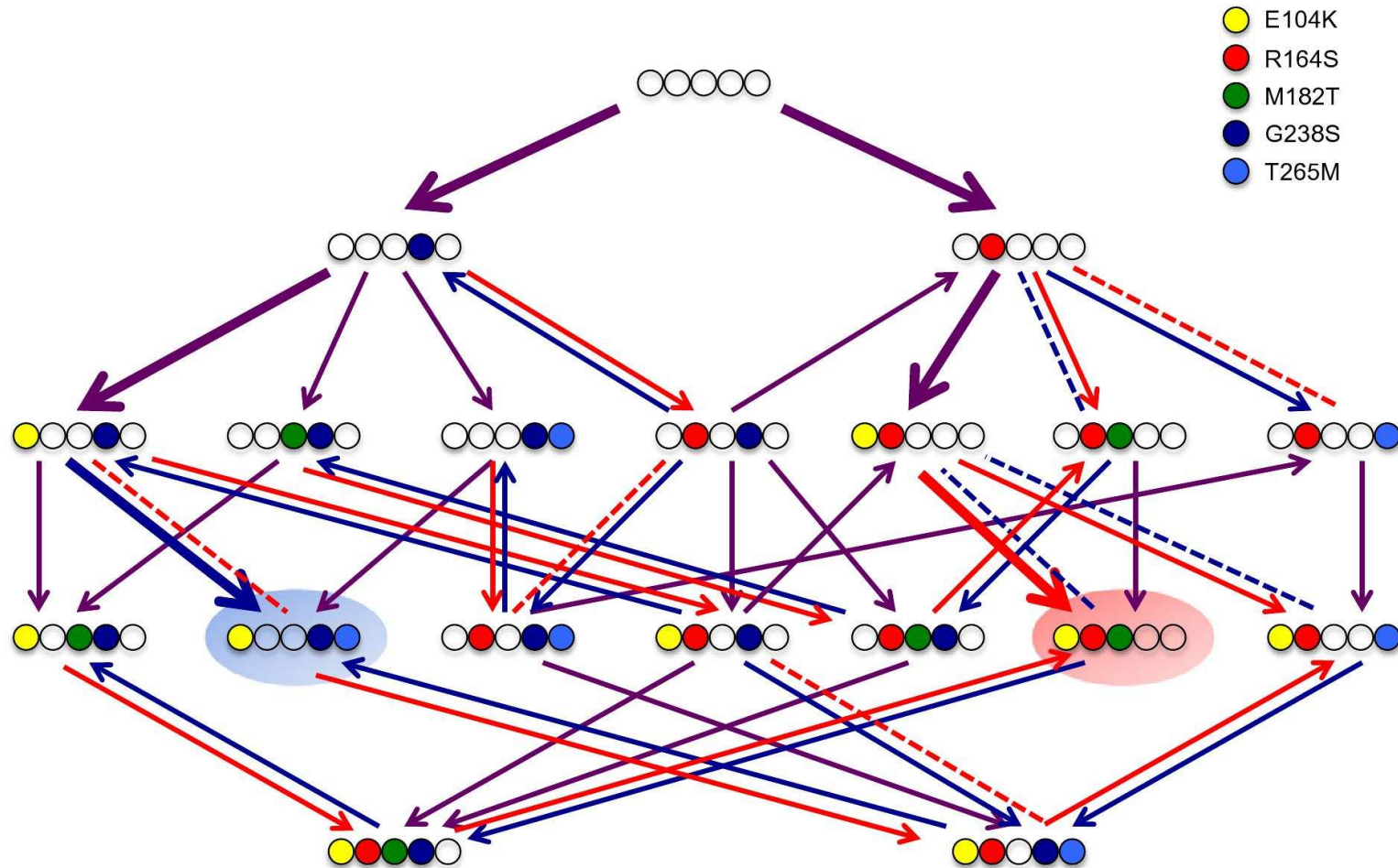
B: Small effect

C: Weinreich 2006

- Mutations chosen for individual effect interact more strongly and negatively than mutations chosen “with hindsight” because of their collective effect

Resistance landscapes for two different drugs

M.F. Schenk et al., *Evol. Appl.* (2015)



Arrows point to increasing resistance against **cefotaxime**, **ceftazidime** or **both**

Summary

- Empirical fitness landscapes are beginning to provide insights into the genetic constraints underlying evolutionary processes
- The weight and predictability of mutational pathways is determined by an interplay of landscape structure and population dynamics
- (Sign) epistatic interactions appear to be common across many different systems, but their mechanistic/phenotypic basis remains to be elucidated

Summary

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Thanks to

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