# What is quantum mechanics trying to tell us?[1]

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I explore whether it is possible to make sense of the quantum mechanical description of physical reality by taking the proper subject of physics to be correlation and only correlation, and by separating the problem of understanding the nature of quantum mechanics from the hard problem of understanding the nature of objective probability in individual systems, and the even harder problem of understanding the nature of conscious awareness. The resulting perspective on quantum mechanics is supported by some elementary but insufficiently emphasized theorems. Whether or not it is adequate as a new *Weltanschauung*, this point of view toward quantum mechanics provides a different perspective from which to teach the subject or explain its peculiar character to people in other fields. © *1998 American Association of Physics Teachers.*

*[W]e cannot think of any object apart from the possibility of its connection with other things.* Wittgenstein, *Tractatus*, 2.0121.

*If everything that we call ''being'' and ''non-being'' consists in the existence and non-existence of connections between elements, it makes no sense to speak of an element's being (non-being)... .* Wittgenstein, *Philosophical Investigations*, 50.

*It happened to him as it always happens to those who turn to science... simply to get an answer to an everyday question of life. Science answered thousands of other very subtle and ingenious questions... but not the one he was trying to solve.* Tolstoy, *Resurrection*, Part 2, Chapter 30.

*[I]n our description of nature the purpose is not to disclose the real essence of the phenomena but only to track down, so far as it is possible, relations between the manifold aspects of our experience.* Bohr[2]

## I. WHAT QUANTUM MECHANICS IS TRYING TO TELL US

I would like to describe an attitude toward quantum mechanics which, whether or not it clarifies the interpretational problems that continue to plague the subject, at least sets them in a rather different perspective. This point of view alters somewhat the language used to address these issues—a glossary is provided in Appendix C—and it may offer a less perplexing basis for teaching quantum mechanics or explaining it to nonspecialists. It is based on one fundamental insight, perhaps best introduced by an analogy.

My complete answer to the late 19th century question ''what is electrodynamics trying to tell us'' would simply be this:

> *Fields in empty space have physical reality; the medium that supports them does not.*

Having thus removed the mystery from electrodynamics, let me immediately do the same for quantum mechanics:

> *Correlations have physical reality; that which they correlate does not.*

The first proposition probably sounded as bizarre to most late 19th century physicists as the second sounds to us today; I expect that the second will sound as boringly obvious to late 21st century physicists as the first sounds to us today.

And that's all there is to it. The rest is commentary.

## II. CORRELATIONS AND ONLY CORRELATIONS

Let me expand on my ten-word answer to what quantum mechanics is all about, which I have called elsewhere[3] the Ithaca interpretation of quantum mechanics (IIQM).

Note first that the term ''physical reality'' is not necessarily synonymous with unqualified ''reality.'' The distinction is of no interest in understanding what classical electrodynamics is trying to tell us, but it may be deeply relevant to why quantum mechanics has not been widely seen to be a theory of correlation without correlata. I shall set aside for now the tension between *reality* and *physical reality*, but as noted in Sec. IV below, it will come back to force itself upon us.[4]

According to the IIQM the only proper subjects for the physics of a system are its correlations. The physical reality of a system is entirely contained in (a) the correlations among its subsystems and (b) its correlations with other systems, viewed together with itself as subsystems of a larger system. I shall refer to these as the *internal* and *external* correlations of the system. A *completely isolated* system is one that has no external correlations or external dynamical interactions.

The wave function of a physical system (when it has one) or, more generally, its quantum state (pure or mixed) is nothing more than a concise encapsulation of its internal correlations. Insofar as the state or the wave function (when the state is pure) has physical reality, that reality does not extend beyond the reality of the internal correlations that the state encodes. In this respect the IIQM agrees with Bohr and Heisenberg, who viewed the wave function as nothing more than a computational tool. It disagrees with Schrödinger's early view of the wave function, or with the views of currently active deviant subcultures, such as the Bohm–de Broglie interpretation,[5] and its recent refinements, or efforts to modify quantum mechanics by making wave function ''collapse'' a dynamical physical process.[6]

The IIQM does not emerge from a general view of the world out of which quantum mechanics is extracted; the strategy is rather to take the formalism of quantum mechanics as given, and to try to infer from the theory itself what quantum mechanics is trying to tell us about physical reality.

Thus by systems and subsystems I simply mean the conventional representation of a complex system by products of subsystem state spaces. If the system, for example, is a Heisenberg model of a number of magnetic ions, the subsystems are the spin degrees of freedom of the individual ions. If the system is a hydrogen atom, the subsystems could be the electron and the proton, further resolved, if this is of interest, into their spin and orbital degrees of freedom. In an example that preoccupied the founders of the theory, the system is an experiment, and the subsystems are the microscopic object of study and the macroscopic apparatus used to study it.

The crucial formal property of a resolution into subsystems is that all observables associated with one subsystem must commute with all observables associated with any other distinct subsystem. So if the subsystems are interacting, then we are dealing with subsystem correlations at a given time. A further requirement is that the subsystem subspaces whose product makes up the state space for the entire system can be straightforwardly identified in the standard way with physically meaningful subsystems of a real (or model) physical system—i.e., that the resolution into subsystems is in some sense natural, as it is in the above examples.[7]

By correlations among subsystems I have in mind the mean values, at any given time, of all system observables (Hermitian operators) that consist of products over subsystems of individual subsystem observables. Among the observables of a subsystem are the projection operators onto its linear subspaces, so the set of all correlations among the subsystems contains the set of all joint probability distributions over subsystems. Since these distributions are in turn enough to determine the means of the products of all observables, it does not matter whether one interprets ''correlations'' to mean joint distributions, or means of products of observables. I shall use whichever interpretation is more appropriate to the case at hand, but I should emphasize that I use the term ''correlation'' in a sense in which the absence of correlation (arising when a joint distribution factors) is regarded as correlation of a degenerate (*trivial*) form.

It is a remarkable (but not often remarked upon) feature of the quantum mechanical formalism that all the joint distributions associated with any of the possible resolutions of a system into subsystems and any of the possible choices of observable within each subsystem, are mutually compatible: They all assign identical probabilities within any sets of subsystems to which they can all be applied.[8] The physical reality of subsystem correlations therefore need not be restricted to any particular resolution of a system into subsystems or to particular choices of observable within each subsystem, even though different observables for a given subsystem fail, in general, to commute. It is only when one tries to go beyond their intersubsystem *correlations* to actual *correlata*—particular *values* for the subsystem observables—that noncommuting observables are incapable of sharing simultaneous physical reality.[9]

The central conceptual difficulty for the IIQM is the puzzle of what it means to insist that correlations *and only* correlations have physical reality. The ''and only'' part is an inescapable consequence of many different ''no-hidden-variables'' theorems, as discussed in Sec. IX below. These theorems require that if all correlations have simultaneous physical reality, then all the correlated quantities themselves cannot. This problem—how to make sense of correlations without correlata—brings us up against two major puzzles:

(1) How is probability to be understood as an intrinsic objective feature of the physical world, rather than merely as a tactical device for coping with our ignorance? How is one to make sense of fundamental, irreducible correlation?

(2) Physics, at least as we understand it today, has nothing to say about the phenomenon of consciousness. Conscious reality has more content than physical reality.

I propose to set aside both of these puzzles. Many of the difficulties one encounters in interpreting quantum mechanics stem from our inadequate understanding of objective probability and of conscious awareness. It seems worth inquiring whether one can make sense of quantum mechanics *conditional* on eventually making sense of these two even more difficult problems. I shall therefore take the notion of *correlation* as one of the primitive building blocks from which an understanding of quantum mechanics is to be constructed. And I shall take the extraordinary ability of consciousness to go beyond its own correlations with certain other subsystems to a direct perception of its own underlying correlata as a deep puzzle about the nature of consciousness, that, however, ought not to be a stumbling block in constructing an understanding of the quantum mechanical description of the nonconscious world.

Before moving to the effort to make sense of quantum mechanics, let me expand on the two puzzles to be set aside.

## III. THE PUZZLE OF OBJECTIVE PROBABILITY

If correlations constitute the *full* content of physical reality, then the fundamental role probability plays in quantum mechanics has nothing to do with ignorance. The correlata—those properties we would be ignorant of—have no physical reality. There is nothing for us to be ignorant of.

A probability that deals only with correlation cannot be based on an ensemble of copies of a given system, with properties having definite values in each copy, for the physical absence of correlata applies separately to each copy. The only physical description it is possible to give each individual member of such an ensemble is in terms of its own internal correlations. There is thus no physical or conceptual role for such an ensemble to play. All its members are physically identical, each completely characterized by the identical set of internal probabilities. The appropriate context for a theory of correlations without correlata is one in which probabilistic notions have meaningful application to individual systems.

It is entirely appropriate for a physics that is both fundamental and probabilistic to apply directly to individual systems. The natural world, after all, consists of individual systems; ensembles are an artificial contrivance or, at best, a very special kind of composite individual system. One motivation behind the desire for an ensemble interpretation of quantum probabilities is a yearning (not always acknowledged) for hidden variables (of which values for correlata constitute the most important example). The view that probabilistic theories are about ensembles implicitly assumes that probability is about ignorance; the hidden variables include whatever it is we are ignorant of. But in a nondeterministic world, probability has nothing to do with incomplete knowledge. Quantum mechanics is the first example in human experience where probabilities play an essential role even when there is nothing to be ignorant about. The correlations quantum mechanics describes prevail among quantities whose in-

dividual values are not just unknown: they have no physical reality. We lack an adequate understanding of how probability or correlation is to be understood under such conditions, but ensemble interpretations fail to capture this central feature.

Another motivation for an ensemble interpretation of quantum probability is the intuition that because the *predictions* of quantum mechanics are fundamentally probabilistic rather than deterministic, quantum mechanics only can make sense as a theory of ensembles. Whether or not this is the only way to understand probabilistic predictive power, physics ought to be able to *describe* as well as *predict* the behavior of the natural world. The fact that physics cannot make a deterministic prediction about an individual system does not excuse us from pursuing the goal of being able to construct a description of an individual system at the present moment, and not just a fictitious ensemble of such systems.

I shall not explore further the notion of probability and correlation as objective properties of individual physical systems, though the validity of much of what I say depends on subsequent efforts to make this less problematic. My instincts are that this is the right order to proceed in: Objective probability arises *only* in quantum mechanics. We will understand it better only when we understand quantum mechanics better. My strategy is to try to understand quantum mechanics contingent on an understanding of objective probability, and only then to see what that understanding teaches us about objective probability.[10]

So throughout this essay I shall treat correlation and probability as primitive concepts, ''incapable of further reduction... a primary fundamental notion of physics.'' [11] The aim is to see whether all the mysteries of quantum mechanics can be reduced to this single puzzle. I believe that they can, provided one steers clear of another even greater mystery: the nature of one's own personal consciousness.

## IV. THE PUZZLE OF CONSCIOUSNESS

Consciousness enters the picture through the disquieting but indisputable fact that *I* know perfectly well that my individual particular *perceptions* of certain kinds of subsystems *do* have a reality that goes beyond the correlation my perceptions have acquired with the subsystem through my interaction with it. It has become traditional in this context to call such subsystems classical or macroscopic. I *know* that photomultiplier #1 fired and photomultiplier #2 did not. I directly perceive the particularity of my conscious representation of the photomultipliers from which I infer the particularity of the photomultiplier excitations themselves.

To the extent that ''I'' am describable by physics, which deals only with the correlations between me and the photomultipliers, physics can only (correctly) assert that photomultiplier #$n$ firing is perfectly correlated with my knowing that photomultiplier #$n$ fired for either value of $n$. The question that physics does not answer is how it can be that *I know* that it is #1 and is *not* #2. This is indeed a problem. It is part of the problem of consciousness.

The problem of consciousness is an even harder problem than the problem of interpreting quantum mechanics, and it is important not to confuse the two. As with the puzzle of objective probability, here too it seems sensible to attempt first to understand quantum mechanics in full awareness of the fact that we do not understand consciousness, taking the view that consciousness is beyond the scope of physical science, at least as we understand it today. This (and only this) is why I distinguish between *reality* and *physical reality*. Physical reality is narrower than what is real to the conscious mind. Quantum mechanics offers an insufficient basis for a theory of everything if everything is to include consciousness.

Before relegating the problem of consciousness to the filing cabinet of harder problems to be examined after satisfactorily interpreting quantum mechanics—we shall be forced on various occasions in the pages that follow to acknowledge the existence of that cabinet—let me note some manifestations even in classical physics of the ability of consciousness to apprehend what physics cannot.

The notion of *now*—the present moment—is immediately evident to consciousness as a special moment of time (or a brief interval—of order perhaps a few tenths of a second). It seems highly plausible to me that your *now* overlaps with my *now* or, if you are very far away from me, with a region space-like separated from my *now*. On the other hand, I can conceive of it not working this way—that your *now* is two weeks behind or 15 min ahead of my *now*. In that case when we have a conversation each of us is talking to a mindless hulk. I mention this not because I believe in mindless hulks but because you encounter them in discussions of the ''many worlds'' interpretation of quantum mechanics. I do not believe in many worlds any more than I believe in many *nows*, but I find it significant that the imagery evoked in thinking about a purely classical puzzle of consciousness is the same as that encountered in the many worlds attempt to extend quantum mechanics to account for our conscious perceptions.[12]

Physics has nothing to do with such notions. It knows nothing of *now* and deals only with correlations between one time and another. The point on my world line corresponding to *now*, obvious as it is to *me*, cannot be identified in any terms known to today's physics. This *particularity* of consciousness—its ability to go beyond time differences and position itself absolutely along the world line of the being that possesses it—has a similar flavor to its ability to go beyond its own correlations with a subsystem, to a direct awareness of its own particular correlatum and therefore, by inference, an awareness of a particular subsystem property.[13]

An even simpler example of an elementary constituent of consciousness which physics is silent on, is the quality of the sensation of *blueness*. Physics can speak of a certain class of spectral densities of the radiation field, it can speak of the stimulation of certain receptors within the eye, it can speak of nerve impulses from the eye to the visual cortex, but it is absolutely silent about what is completely obvious to me (and I assume to you)—the characteristic and absolutely unmistakable *blue* quality of the experience of blueness itself.[14]

Consciousness enters into the interpretation of quantum mechanics because it and it alone underlies our conviction that a purely relational physics—a physics of correlations without correlata—has insufficient descriptive power. Consciousness cannot easily be banished from such discussions, because the conviction arises in contexts where the underlying conscious perception may only be implicit.[15] One must therefore remain aware of its ramifications, as a mystery in its own right, so one can disentangle the characteristic puzzles of consciousness from efforts to come to terms with the lesser puzzle of understanding the quantum mechanical description of the nonconscious world.[16]

## V. A THEOREM ABOUT QUANTUM CORRELATIONS

There is a common-sense appeal to the idea of a physics that is mute on absolute subsystem properties, restricted in its scope to the correlations among such properties. Why should physics be able to produce more than a description of the world in the world's own terms, by relating some parts of the world to other parts? More substantially, it is pertinent to note that I am on firm ground in insisting that the entire content of the physics of a system consists of a specification of the correlations among its subsystems, because this happens to be true. It is the content of an insufficiently noted but quite elementary theorem, important enough to deserve a section of its own.

It is well known that if you are given the mean value of *all* the observables of a system, then this uniquely determines its quantum state (pure or mixed). Suppose, however, that the mean values you are supplied with are restricted to those of observables that are products of subsystem observables over some specific resolution of the system into subsystems—i.e., you are only supplied with the set of all correlations among a particular set of subsystems that combine to make up the entire system. How well is the state of the whole system pinned down when the set of specified mean values is restricted to such products over subsystems of subsystem observables, excluding observables that extend globally over the entire system?

The surprising (if you've never thought about it) answer is this: Completely! *Subsystem correlations (for any one resolution of the system into subsystems) are enough to determine the state of the entire system uniquely.* This theorem must have been noticed early on, but the oldest statements of it that I know of are improbably recent.[17] I shall refer to it as the Theorem on the Sufficiency of Subsystem Correlations or SSC Theorem. It follows immediately from three facts:

(1) As noted, the means of *all* observables for the entire system determine its state.
(2) The set of all products over subsystems of subsystem observables contains a basis for the algebra of *all* such system-wide observables.
(3) The algorithm that supplies observables with their mean values is linear on the algebra of observables.

As a result if you are given the mean values of all such product-over-subsystem observables, it is a matter of simple arithmetic to compute the mean values of whatever set of global system observables you need to pin down the state.

This is spelled out in detail in Appendix A. As a simple example, if a system consists of two spin-$\frac{1}{2}$ subsystems, then the projection operator on the singlet state—the state of zero *total* spin—is a global system observable. It has the well-known form

$$P_{\text{singlet}} = \tfrac{1}{4}(1 - \sigma_x^1 \otimes \sigma_x^2 - \sigma_y^1 \otimes \sigma_y^2 - \sigma_z^1 \otimes \sigma_z^2), \tag{1}$$

and therefore its mean value is entirely determined by the mean values of the products of the $x$, $y$, and $z$ components of the individual spins. Since the singlet state is that unique state in which $P_{\text{singlet}}$ has the mean value 1, the system will be in the singlet state provided these three quantities all have the value $-1$ that expresses perfect anticorrelation.

That like components of the individual spins are perfectly anticorrelated in the singlet state is a famously familiar fact; that perfect anticorrelations of three orthogonal components

is enough to ensure that the global state *is* the singlet state—a particularly simple playing out of the possibility guaranteed by the SSC Theorem—is not as familiar.

Though the proof of the SSC Theorem is elementary, its conceptual implications are profound. If the quantum theoretical description of the physical reality of a system is complete, then so is the description of the system entirely in terms of all the correlations that prevail among any specified set of its subsystems, because the information contained in either of those two descriptions is the same. Anything you can say in terms of quantum states—and some strange things can be stated in that language—can be translated into a statement about subsystem correlations—i.e., about joint probability distributions. At a minimum, whether or not the IIQM can be made into a coherent whole, this simple fact ought to be stressed in all introductory expositions of the quantum theory:

> *The quantum state of a complex system is nothing more than a concise encapsulation of the correlations among its subsystems.*

The quantum state is a remarkably powerful encoding of those correlations. It enables us to calculate them for any resolution of the system into subsystems and for any set whatever of subsystem observables. The fact that all the different sets of subsystem correlations can be encoded in a single quantum state provides an explicit demonstration of the mutual consistency of the correlations associated with all of the different ways of dividing a system into subsystems. While I am not convinced that this shift in point of view from quantum state to subsystem correlations eliminates all conceptual problems from the foundations of quantum mechanics, it does alter how you look at many of those problems and, I believe, offers a better way to tell people encountering the subject for the first time what it is all about. In Secs. VI–XI, I describe some of the shifts in perspective that take place when you start taking seriously the notion that the physics of a system is only about the correlations among its subsystems.

## VI. ELIMINATION OF MEASUREMENT FROM THE FOUNDATIONS

The notion of ''measurement'' plays a fundamental role in conventional formulations of quantum mechanics. Indeed, quantum mechanics is often presented as merely an algorithm that takes you from one measurement (''state preparation'' involves selecting a particular output channel from a measurement apparatus) to another. John Bell railed eloquently against this.[18] Why should the scope of physics be restricted to the artificial contrivances we are forced to resort to in our efforts to probe the world? Why should a fundamental theory have to take its meaning from a notion of ''measurement'' external to the theory itself? Should not the meaning of ''measurement'' emerge from the theory, rather than the other way around? Should not physics be able to make statements about the unmeasured, unprepared world?

> To restrict quantum mechanics to be exclusively about piddling laboratory operations is to betray the great enterprise. A serious formulation will not exclude the big world outside of the laboratory.

I argue here that the very much broader concept of correla-

tion ought to replace measurement in a serious formulation of what quantum mechanics is all about.[19]

The key to freeing quantum mechanics from the tyranny of measurement is to note that a measurement consists of the establishment of a particular kind of correlation between two particular kinds of subsystems, and to insist that everything that can be said about the physical reality of the correlations established in a measurement applies equally well to the correlations among any subsystems of a quantum system. If physics is about correlations among subsystems then it is *a fortiori* about measurement. But to insist that physics is exclusively about measurement is unnecessarily to relegate to an inferior ontological status the more general correlations among arbitrary subsystems.

Expanding on this, let me review in its simplest form the standard characterization of a measurement. In a measurement a particular interaction brings about a particular kind of correlation between two particular subsystems. One of the subsystems, the one one wishes to learn about, is arbitrary, but in many important applications it describes something on the atomic scale. Call this subsystem the *specimen*. The other subsystem has enormously many degrees of freedom, describing a piece of laboratory equipment that includes some sort of readily readable output (which could be in the form of a pointer, a digital display, or a printout). It is usually called the *apparatus*.

Initially, at the start of a measurement, the specimen and the apparatus are uncorrelated: The state of the specimen–apparatus system is a product state

$$|I\rangle = |s\rangle \otimes |a\rangle. \tag{2}$$

To measure a specimen observable $S$ with eigenstates $|s_i\rangle$ one must establish an interaction between specimen and apparatus that takes an initial state $|s_i\rangle \otimes |a\rangle$ of the combined system into the final state $|s_i\rangle \otimes |a_i\rangle$, where the $|a_i\rangle$ are a set of orthogonal apparatus states associated with macroscopically distinguishable scale readings:

$$|s_i\rangle \otimes |a\rangle \rightarrow |s_i\rangle \otimes |a_i\rangle. \tag{3}$$

Because the transformation (3) takes orthogonal states into orthogonal states it can indeed be realized by a unitary transformation—i.e., as a time development under a suitable choice of Hamiltonian. Because unitary transformations are linear, if the initial state of the specimen has an expansion

$$|s\rangle = \sum \alpha_i |s_i\rangle, \tag{4}$$

then when the measurement interaction has completed its action, the state of the system will be

$$|F\rangle = \sum \alpha_i |s_i\rangle \otimes |a_i\rangle. \tag{5}$$

A correlation has therefore been established between specimen and apparatus characterized by the joint probability distribution

$$p(s_i, a_j) = \langle F | P_{s_i} P_{a_j} | F \rangle = |\alpha_i|^2 \delta_{ij} \tag{6}$$

(where the $P$'s are the appropriate projection operators: $P_{s_i} = |s_i\rangle\langle s_i|$, $P_{a_i} = |a_i\rangle\langle a_i|$). This joint distribution describes a perfect correlation between apparatus and specimen states: The probability of the $j$th apparatus state being associated with the $i$th specimen state is zero unless $i = j$. And the over-

all probability of the $j$th apparatus state is $\Sigma_i p(s_i, a_j) = |\alpha_j|^2$, which is just the probability the Born rule assigns to "the result of a measurement of $S$ on a specimen in the state $|s\rangle$ yielding the value $s_j$."

So a measurement of a specimen observable $S$ is an interaction between the specimen and the apparatus designed to extend the Born probabilities from the specimen states $|s_i\rangle$ to corresponding apparatus states $|a_i\rangle$. This is a useful thing to do because although we humans are incapable of directly perceiving the condition of a microscopic specimen, we are able to perceive the condition of a macroscopic apparatus. Both this ability of ours and its limitation presumably arise from our having evolved under the selective pressure of having to deal with macroscopic things like tigers and oranges, but not (at least at the stage of development when consciousness first arose) with microscopic things like atoms and molecules. As noted above, how we manage this conscious perception is deeply mysterious, but it should be viewed as a mystery about *us* and should not be confused with the problem of understanding quantum mechanics.

The great emphasis even today on the particular kinds of correlation established in a measurement finds its origins in the early history of the subject. In the beginning, when people were groping for an understanding of microscopic specimens, it was natural to express everything in terms of the more familiar macroscopic apparatuses with which they were able to correlate the microscopic specimens, through measurement interactions. Measurements produced the only correlations people felt comfortable with. Today, three quarters of a century later, having accumulated a vast body of experience dealing with microscopic specimens, we have developed enough intuition about them to contemplate usefully a much broader class of correlations in which no subsystems are required to be of the macroscopic or "classical" kind directly accessible to our perception, and in which the correlations are neither necessarily of the one-to-one type established in a measurement nor necessarily restricted to just a pair of subsystems.

The emphasis on measurement in conventional formulations of quantum mechanics, and the accompanying emphasis on a classical domain of phenomena, ought to be viewed as historic relics. The classical domain plays a central role only if one restricts the correlations one is willing to call physically real, to those between specimens and apparatuses, where an apparatus is a subsystem large enough that we can perceive it directly—i.e., a "classical" subsystem. We ought by now to have outgrown this point of view. The bipartite specimen–apparatus correlations produced by a measurement are not the only kinds of subsystem correlations worthy of being granted physical reality. The quantum theory allows us to contemplate together *all* the correlations among arbitrary subsystems, and it is simply a bad habit not to grant micro–micro–⋯–micro correlations as much objective reality as the traditional emphasis on measurements has granted to micro–macro correlations.

This reluctance to shift the emphasis from measurement to correlation lies behind statements one often encounters to the effect that interactions with its environment are in some not very well specified way continually *measuring* a specimen. This is to characterize a very general state of affairs by a very special and rather atypical case. Interactions with its environment have the precise effect of correlating a specimen with that environment. Interactions with a measurement apparatus correlate a specimen with that apparatus. In both

cases interaction produces correlation. In measurements the interactions are designed so that the correlations that develop have the particular form (6) of special interest to us. It is only the reluctance to acknowledge that all correlations are real and objective—not just those produced by a measurement—that leads one to view the more general specimen–environment correlations in terms of the more special specimen–apparatus correlations produced in a measurement.

## VII. ELIMINATION OF KNOWLEDGE FROM THE FOUNDATIONS

There has always been talk to the effect that quantum mechanics describes not the physical world but our knowledge of the physical world. This intrusion of human knowledge into physics is distastefully anthropocentric. In the IIQM such talk is replaced with talk about objective correlations between subsystems. Human knowledge has intruded for two reasons:

(1) The restriction of attention to the correlations established in measurements has led to an excessively narrow focus on the correlations between a specimen and what we *know* about it (or what our mechanical surrogate—the apparatus—records about it).
(2) There is a confusion between the strange and unprecedented role of probability in the quantum theory as an objective feature of the physical world, and the older better understood uses of probability as a practical device for coping with human ignorance. Because we understand probability reasonably well in the latter sense, and have only a glimmering of an understanding of probability in the more fundamental former sense, it is tempting to interpret probabilistic assertions incorrectly as statements about human ignorance or knowledge.

As an important illustration, consider how people distinguish between pure and mixed states. It is often said that a system is in a pure state if *we* have maximum *knowledge* of the system, while it is in a mixed state if *our knowledge* of the system is incomplete. But from the point of view of the IIQM, *we* are simply a particular subsystem, and a highly problematic one at that, to the extent that our consciousness comes into play. This characterization of the difference between pure and mixed states can be translated into a statement about objective correlation between subsystems that makes no reference to us or our knowledge.

By definition, a system $\mathscr{S}_1$ is in a pure state if all the correlations among any of its own subsystems can be characterized in terms of a density matrix that is a projection operator onto a one-dimensional subspace. This in turn can be shown (Appendix B) to be possible if and only if any conceivable larger system $\mathscr{S} = \mathscr{S}_1 + \mathscr{S}_2$ that contains $\mathscr{S}_1$ as a subsystem has only trivial correlations (i.e., only factorizable joint distributions) between its subsystems $\mathscr{S}_1$ and $\mathscr{S}_2$. Thus *a system is in a pure state if and only if its internal correlations are incompatible with the existence of any nontrivial external correlations.*

The absence or presence of nontrivial external correlations is the objective fact. The anthropocentrisms simply express the consequences of this fact for us, should we be told all the internal correlations of $\mathscr{S}_1$. It is another remarkable feature of quantum mechanics (not shared with classical physics, where external correlations are always possible) that the to-

tality of all possible internal correlations is enough to determine whether or not any nontrivial external correlations are possible. For Appendix A shows that the internal correlations of a subsystem are enough to determine its density matrix; and Appendix B shows that nontrivial external correlations are possible if and only if that density matrix is not a one-dimensional projection operator. To characterize the situation in which the internal correlations are of the kind that prohibit any external correlations as a situation in which ''we have maximum knowledge'' is to let *ourselves* intrude on a formulation that has no need of *us*.

This intrusion of ''knowledge'' into the distinction between pure and mixed states can lead to another kind of confusion. It is a common error always to view a mixed state as describing a system that is actually in one of a number of different possible pure states, with specified probabilities. While this ''ignorance interpretation'' of the mixed state can indeed be a useful practical way to describe an ensemble of completely isolated systems, it entirely misses the deep and fundamental character of mixed states: If a system has any external correlations whatever, then its quantum state cannot be pure. Pure states are a rarity, enjoyed only by completely isolated systems. The states of externally correlated individual systems are fundamentally and irreducibly mixed. This has nothing to do with ''our ignorance.'' It is a consequence of the existence of objective external correlation.

## VIII. THE MEASUREMENT PROBLEM

According to a conventional view, if a specimen is in a state

$$|s\rangle = \sum \alpha_i |s_i\rangle, \tag{7}$$

then after a measurement of an observable whose eigenstates are the $|s_i\rangle$, the state of the system discontinuously ''collapses'' to the state $|s_i\rangle$ with probability $|\alpha_i|^2$. At that point all information contained in the phases of the amplitudes $\alpha_i$ is irredeemably lost. The ''measurement problem'' is the problem of how to reconcile this with the continuous evolution of the specimen–apparatus system into the final state (5), which is clearly still capable of revealing interference effects in the form of probabilities that do depend on the phases of the $\alpha_i$.

According to the IIQM the state of a specimen is just a compact specification of all its internal subsystem correlations. To understand collapse, we should restate it not in terms of the *state* of the specimen, but in terms of the specimen's *internal correlations*. The physical content of the claim that after the measurement the system ''is in'' the state $|s_i\rangle$ with probability $|\alpha_i|^2$, is that after the measurement the specimen has the internal correlations appropriate to the state $|s_i\rangle$ with probability $|\alpha_i|^2$.

When it is put this way any discontinuity vanishes. For as noted above, during the course of the measurement interaction the combined specimen–apparatus system evolves continuously from its uncorrelated initial state (2) to the highly correlated final state (5). As soon as any nontrivial correlation develops, the state of the specimen ceases to be pure, and at the end of the interaction when the whole system is in the state (5), the state of the specimen has continuously evolved into the mixed state

$$\sum |\alpha_i|^2 |s_i\rangle\langle s_i|. \qquad (8)$$

In this mixed state the internal correlations of the specimen are identical to what they would be if it were in the pure state $|s_i\rangle$ with probability $|\alpha_i|^2$—i.e., the internal correlations are identical to those given by the collapse story.

This is another familiar tale. The IIQM shifts the way it is sometimes told, by emphasizing that the state of a nontrivially correlated subsystem is never pure: The state of the specimen evolves continuously from a pure state through a sequence of mixed states into the ''post-measurement'' mixed state (8) at the moment the measurement interaction completes its task. If at that stage one wishes to regard the state of the specimen as undergoing an abrupt change, it is at worst a collapse from a mixed state viewed in this fundamental way, to the same mixed state viewed under the ''ignorance interpretation.'' Since the internal correlations of the specimen are exactly the same regardless of which view you take, the collapse, if one chooses so to regard it, is rather ethereal.

There is thus no quantum measurement problem for the *internal* correlations of the specimen or the apparatus. After the measurement interaction is complete their states are *exactly*—not just FAPP[20]—the conventional post-measurement mixed states, which reveal no interference effects whatever in any probability distributions associated entirely with the specimen or entirely with the apparatus. These mixed states have evolved from the pre-measurement pure states in an entirely continuous fashion.[21]

The measurement problem survives only in the *specimen–apparatus correlations* that hold between specimen and apparatus observables, both of which differ from those characterized by the joint distribution (6) that the measurement interaction was designed to produce. Consider, for example, the specimen observable

$$S_{12} = |s_1\rangle\langle s_2| + |s_2\rangle\langle s_1| \qquad (9)$$

and the apparatus observable

$$A_{12} = |a_1\rangle\langle a_2| + |a_2\rangle\langle a_1|. \qquad (10)$$

In the final state (5) of the specimen–apparatus system these have nontrivial correlations

$$\langle F|S_{12}A_{12}|F\rangle = 2 \operatorname{Re} \alpha_1^* \alpha_2 \qquad (11)$$

that depend on the relative phases of the $\alpha_i$, even though those phases can affect no *internal* specimen or apparatus correlations in the state $|F\rangle$.

There need be nothing peculiar about the specimen observable $S_{12}$. If, for example, the specimen is a two-state system viewed as a spin-$\frac{1}{2}$ and $|s_1\rangle$ and $|s_2\rangle$ are the eigenstates of the $z$ component of spin, then $S_{12}$ is just the $x$ component. On the other hand the apparatus observable $A_{12}$ is quite bizarre, since its values $\pm 1$ discriminate between the apparatus being in either of the two superpositions $|a_1\rangle \pm |a_2\rangle$ of states with macroscopically distinguishable scale readings. ''Macroscopically'' is, of course, crucial. Were the ''apparatus'' merely another microscopic spin-$\frac{1}{2}$, then (11) would give just the correlation in the two $x$ components. Under those conditions there would be little trouble introducing a further straightforward coupling between specimen and apparatus that undid the measurement interaction, transforming the perfectly correlated system state with both sub-

systems in mixed states back into the entirely uncorrelated system state with both subsystems back in their initial pure states. For the same reasons that classical macroscopic systems are hard to run backwards, the measurement interaction cannot so readily be undone when the apparatus is macroscopic. The apparatus observables whose correlation with the specimen depend on the critical phases necessary for the reconstruction of the original state are correspondingly difficult to realize.

But *in principle* it could be done. This is the measurement problem. What makes it so much more vexing than the old classical problem of irreversibility at the macroscopic level is only what happens when *I* get into the story. When *I* look at the scale of the apparatus *I know* what it reads. Those absurdly delicate, hopelessly inaccessible, global system correlations *obviously* vanish completely when they connect up with *me*. Whether this is because consciousness is beyond the range of phenomena that quantum mechanics is capable of dealing with, or because it has infinitely many degrees of freedom or special super-selection rules of its own, I would not presume to guess. But this is a puzzle about consciousness which should not get mixed up with efforts to understand quantum mechanics as a theory of subsystem correlations in the nonconscious world.

It is here that the IIQM comes closest to the many worlds extravaganza.[22] Many worlds (or many minds) enter the story only when the formalism is taken to apply to consciousness itself. In that case, even though *I know* that photomultiplier #1 fired, this correlation between me and the photomultipliers is associated with merely one component of a superposition of states of the me–photomultipliers system. There is another component in which *I know* that photomultiplier #2 fired. If quantum mechanics applies to my conscious awareness (and if there is no objective physical process of ''wave-function collapse'') then there is no evading this, and away we go to Fairyland. But since there are so many other aspects of conscious awareness that physics has nothing to say about, I find it naive to assume that it can sensibly be extended to account for the characteristic particularity of conscious experience that takes it beyond the correlations between me and the objects of my knowledge.

If we leave conscious beings out of the picture and insist that physics is only about correlation, then there is no measurement problem in quantum mechanics. This is not to say that there is no problem. But it is not a problem for the science of quantum mechanics. It is an everyday question of life: the puzzle of conscious awareness.

## IX. ABSENCE OF CORRELATA

In maintaining that subsystem correlations *and only* correlations have physical reality, I have not been very precise about what ''and only'' is meant to exclude. One thing that it does *not* exclude is the existence of global probability distributions for an individual subsystem, since these are special cases of its external correlations with the observables for all the external subsystems taken to be identically unity. Indeed, as remarked upon in Sec. II, it is a conceptually remarkable (though analytically trivial) feature of the quantum mechanical formalism that every one of the many different joint distributions in which a given subsystem $\mathscr{S}_1$ appears gives exactly the same set of marginal distributions for that given subsystem. It does not matter which other subsystems $\mathscr{S}_2, \ldots, \mathscr{S}_n$ appear in the resolution $\mathscr{S} = \mathscr{S}_1 + \mathscr{S}_2 + \cdots + \mathscr{S}_n$

of the full system $\mathscr{S}$ into subsystems, and it does not matter which observable one chooses for each of the other subsystems.

This is conceptually remarkable because if one takes the orthodox view that joint distributions apply only to the results of measurement, then different joint distributions leading to the same marginal distribution for $\mathscr{S}_1$ characterize mutually exclusive experimental arrangements, and it is hard to understand why the marginal distributions for $\mathscr{S}_1$ should be invariant under such changes. I have remarked on this elsewhere.[23] It is not remarkable—on the contrary, it is essential for the consistency of the whole point of view—if the joint distributions are regarded as characterizing coexisting aspects (all possible subsystem correlations) of physical reality. The price one pays for this broader vision of the nature of joint distributions is the need to deny physical reality to a complete collection of correlata underlying all these correlations.

The correlata cannot all have physical reality because in spite of the existence of all subsystem joint distributions and of unique marginal distributions for individual subsystems, it is impossible to construct, in the standard way, a full and mutually consistent set of *conditional* distributions from the joint and individual subsystem distributions. Let me illustrate this extraordinary feature of quantum probabilities with what is probably the simplest example of it, discovered by Lucien Hardy in a rather different context.[24]

Take a system consisting of two subsystems, each describable by a two-dimensional state space. Consider just two noncommuting observables for each subsystem, named 1 and 2 for one subsystem, and $1'$ and $2'$ for the other. Label the two eigenstates of each observable by the name of the observable and one of the two letters $R$ (for ''red'') or $G$ (for ''green''), and consider the subsystem correlations in the system state

$$|\Psi\rangle \propto |2R,2'R\rangle - |1R,1'R\rangle\langle 1R,1'R|2R,2'R\rangle \qquad (12)$$

(where $|X,Y\rangle$ means $|X\rangle \otimes |Y\rangle$). According to the IIQM the $11'$, $22'$, $12'$, and $21'$ subsystem correlations all have simultaneous physical reality and indeed, we can compute from (12) the four joint distributions $p(iX,j'Y)$ where each of $i$ and $j$ can be 1 or 2, and each of $X$ and $Y$ can be $R$ or $G$.

Furthermore, the marginal distributions, characterizing one of the two systems,

$$p(iX) = p(iX,j'R) + p(iX,j'G) \qquad (13)$$

and

$$p(j'Y) = p(iR,j'Y) + p(iG,j'Y) \qquad (14)$$

are indeed independent of whether the observable for the other (summed over) system is its #1 or #2 observable. There is therefore no formal obstacle to defining in the conventional way conditional distributions satisfying

$$p(iX|j'Y)p(j'Y) = p(iX,j'Y) \qquad (15)$$

and

$$p(j'Y|iX)p(iX) = p(iX,j'Y). \qquad (16)$$

Yet these conditional distributions are mutually inconsistent.

The proof of this for the Hardy state (12) is simple. Inspection of (12) reveals that $|\Psi\rangle$ is constructed to be orthogonal to the state $|1R,1'R\rangle$, and it is also orthogonal to the states $|1G,2'G\rangle$, and $|2G,1'G\rangle$, since the $R$ and $G$ eigenstates of any one subsystem observable are orthogonal. But

$|\Psi\rangle$ is not orthogonal to $|2G,2'G\rangle$, since for either subsystem the eigenstates of the #2 observable are neither orthogonal to nor identical to those of the #1 observable. Consequently the probabilities $p(1R,1'R)$, $p(1G,2'G)$, and $p(2G,1'G)$ are zero, but $p(2G,2'G)$ is not:

$$p(1G,2'G) = p(1R,1'R) = p(2G,1'G) = 0, \qquad (17)$$
$$p(2G,2'G) \neq 0.$$

The vanishing of $p(1G,2'G)$ requires that

$$p(1R|2'G) = 1, \qquad (18)$$

the vanishing of $p(1R,1'R)$ requires that

$$p(1'G|1R) = 1, \qquad (19)$$

and the vanishing of $p(2G,1'G)$ requires that

$$p(2R|1'G) = 1. \qquad (20)$$

Combining these, if observable $2'$ has the value $G$, then (18) requires 1 to have the value $R$, in which case (19) requires $1'$ to have the value $G$, in which case (20) requires 2 to have the value $R$. So if $2'$ has the value $G$ then 2 must have the value $R$:

$$p(2R|2'G) = 1. \qquad (21)$$

But this is inconsistent with the nonzero value of $p(2G,2'G)$. The statistics (17) are incompatible with these straightforwardly constructed conditional distributions.

The conventional interpretation of quantum mechanics finds the above line of reasoning unacceptable. According to the conventional view, probabilities like $p(iX,j'Y)$ are not measures of some preexisting set of objective correlations between all four pairs of subsystem observables. These probabilities apply only to the results of actual measurements. The probability $p(1R,2'G)$ is the probability that a joint *measurement* of observables 1 and $2'$ yields the values $R$ and $G$. The three conditional distributions (18)–(20) do not characterize coexisting states of being, but the results of mutually exclusive experiments. Since at most one of the experiments can actually be performed, at most one of the distributions is meaningful, and it makes no sense to combine them as I have done.

But the IIQM takes a broader view of joint distributions. All correlations among all possible subsystem observables have simultaneous physical reality. In particular all four pair distributions have physical reality, whether or not one chooses to extend the correlations between a particular one of these pairs to a pair of apparatuses by means of an appropriately chosen measurement interaction. What the preceding argument demonstrates is that if all the subsystem joint distributions do share a common physical reality, then the *conditional* distributions constructed from them cannot, even though all the joint distributions yield unique mutually consistent marginal distributions for the subsystems. But if it makes no physical sense to talk about the probability of 1 being $R$, *given* that $2'$ is $G$, this can only be because absolute subsystem properties are not ''given.'' If physical reality consists of all the correlations among subsystems then physical reality cannot extend to the values for the full set of correlata underlying those correlations.

The way we conventionally speak of probability makes it hard to express this state of affairs. One tends, for example, to speak of $p(1R,2'G)$ as the probability that 1 is $R$ and $2'$

is $G$. But if it makes sense to speak of 1 *being* $R$ and $2'$ *being* $G$, why should it not make equal sense to speak of the probability of 1 being $R$, *given* that $2'$ is $G$? The answer has to be that $p(1R,2'G)$ cannot be viewed as the probability that 1 *is* $R$ and $2'$ *is* $G$. This would make sense were probability a device for coping with ignorance, but the objective probabilities of quantum mechanics exist even though there is nothing to be ignorant of. They express correlations in the absence of correlata. To avoid such linguistic traps it would be better to speak not of ''probabilities'' but of ''propensities'' or ''dispositions,'' or to eschew all talk of probability in favor of talk about correlation.

I am not suggesting that banishing ''probability'' from our vocabulary will remove all puzzles from quantum mechanics; only that it can help avoid misuses of that term. As noted in Sec. III, the problem of what objective probability or objective correlation or propensity might mean—of what it means to have correlation when values cannot be assigned to the correlata—is one I propose to set aside to explore whether one can make better sense of quantum mechanics, contingent on acquiring a better understanding of this admittedly peculiar notion. What Hardy's state (12) tells us is that if all correlations between subsystems *do* have joint physical reality, then distributions conditional on particular subsystem properties *cannot* in general exist, and therefore such correlations *must* be without correlata.[25]

## X. NONLOCALITY?

Hardy did not come up with the state (12) to demonstrate that the joint existence of pair distributions is incompatible with the joint existence of conditional distributions. He produced it as a succinct and powerful contribution to the tradition of ''nonlocality'' arguments stemming from Bell's theorem.[26]

Under the IIQM, such arguments do not work as demonstrations of nonlocality. If two subsystems are spatially separated then the *local* properties of each are limited to their *internal* correlations. These are completely determined by the density matrix of each. The density matrix of either subsystem is unaffected by any dynamical process acting only on the other subsystem, even when the dynamical process consists of letting the other subsystem undergo a measurement interaction with a third subsystem that functions as an apparatus. The choice and performance of a measurement on one subsystem cannot alter the local properties of the other, far away subsystem. Otherwise one could use ''quantum nonlocality'' to send instantaneous signals. The impossibility of doing this should be called *physical locality*.

Quantum mechanics obeys physical locality. ''Quantum nonlocality'' (a violation, so to speak, of *metaphysical locality*) arises when one tries to reconcile the *actual* results of specific experiments to the *hypothetical* results of other experiments that might have been performed but were not. In talking of ''actual results'' one is going beyond the subsystem correlations with which physics can deal, to our mysterious ability to perceive—i.e., become consciously aware of—a particular one of the correlated possibilities, when we ourselves are among the subsystems. While it is surely unreasonable to insist that we have no right to try to make sense of our own direct perceptions, this kind of reasoning goes beyond what can be expressed in proper *physical* terms.

Nevertheless, the following line of thought has a powerful appeal. Consider a series of experiments in which two particles interact in such a way as to leave them in the Hardy state, and then fly apart to separate measurement apparatuses in a manner that preserves the Hardy state correlations of all the #1 and #2 observables. This is possible if those observables are, for example, polarizations along different nonorthogonal directions.

Consider a series of measurements in which the choice of which observable to measure is decided by tossing a coin at the site of the measurement. Consider a run in which the coin tosses result in observables 2 and $2'$ being measured, and in which the result of each measurement is perceived to be $G$. [The nonvanishing of $p(2G,2'G)$ guarantees that such runs are possible.] Suppose the measurement interactions take place in space-like separated space-time regions, so there is a frame of reference (Alice's—let her be in the vicinity of the unprimed measurement as it takes place) in which the perception of $G$ at the unprimed system occurs before the toss of the coin at the primed system, and another frame (Bob's—let him be in the space-time neighborhood of the primed measurement) in which $G$ is perceived at the primed system before the toss at the unprimed system.

Once Alice perceives $G$ for the 2 measurement, she is surely entitled to conclude that if the (yet to be performed in her frame) coin toss results in a $1'$ measurement on the primed system, the result will be perceived to be $R$, since $p(2G,1'G)=0$. By the same token once Bob perceives $G$ for the $2'$ measurement he can correctly conclude that if the (still unperformed in his frame) toss at the unprimed system results in a 1 measurement the perceived result must be $R$.

How can these two valid conclusions be reconciled with the fact that $1R$ and $1'R$ are never jointly perceived? There are two options. The first is to abandon the implicit assumption that the perceived result of a later measurement is unaffected by the choice and/or outcome of an earlier one. This is a route taken by those who embrace quantum nonlocality. It has the disconcerting feature that which measurement process affects which depends on whether you are using Alice's frame of reference or Bob's, but since the influence is of one of two space-like separated events on another, this is unavoidable. The most determined efforts to extract nonlocality from this kind of reasoning are those of Henry Stapp.[27]

The second option (which I prefer) is to deny that the combined predictions of Alice and Bob have any relevance to *what would have been perceived* if both measurements had actually been of type 1. Indeed, it is hard to give ''what would have been perceived'' any meaning in this case, since both predictions are based on actual perceptions of type-2 measurements. Alice, for example, having perceived $G$ in her type-2 measurement is perfectly correct in concluding that if the toss of Bob's coin results in a $1'$ measurement then Bob will necessarily perceive $R$. Bob's experience is similar. But to extract from this a contradiction with the impossibility of joint $1R$ and $1'R$ perceptions, it is necessary to slide from statements about actual perceived results of actual experiments to possible perceived results of experiments that were not actually performed. This is to extend the peculiar but undeniable ability of consciousness to experience the particularity of a correlation in an actual individual case to a hypothetical ability to experience the fictional particularity of a correlation in a fictional case, and to impose a consistency on the actually and fictionally perceived particularities.

It is hard to see how to make this compelling, unless what consciousness is directly perceiving are actual correlata un-

derlying all the correlations. If these had physical reality in an individual case, locality would indeed require the value of a correlatum in one subsystem to be the same, regardless of what local operations were performed on the other subsystem. But since quantum mechanics is about correlations that exist without correlata, such an argument does not work as a demonstration of nonlocality.[28]

There is another tradition of nonlocality arguments, starting with the very first version of Bell's theorem, which tests whether all the correlations between currently noninteracting and far-apart subsystems can be explained in terms of information commonly available to the subsystems at the time of their last interaction. This ''common-cause'' explanation for correlation assumes that it makes sense to condition all joint subsystem distributions on the detailed features of such hypothetical common information. One then imposes some reasonable locality conditions on these hypothetical conditional distributions and shows that the resulting forms imply certain inequalities that are inconsistent with the joint distributions given by quantum mechanics.

From the perspective of the IIQM, if the pair of systems is completely isolated from the rest of the world, such a conditioning on common information is highly problematic, independent of the subsequent imposing of locality conditions on such conditional distributions. Refining the subsystem joint distributions according to ''conditions'' at the source makes little sense from the perspective of the SSC Theorem, which assures us that the correlations contain in themselves complete information about the physical reality (encoded in the state) of the two-subsystem system. Such a refinement would grant physical reality to further features of the correlations going beyond what is contained in their joint (pure) state. The only thing such arguments show to be nonlocal is any such supplementation of the quantum mechanical description. Indeed, that was how Bell put it in his first paper, and for some time thereafter the theorem was viewed not as a proof that the physical world is nonlocal, but only as a nonlocality proof for any hidden variables theory underlying the correlations.

It is, to be sure, a remarkable fact that the common-cause explanation for correlation between noninteracting subsystems fails when applied to quantum correlations, but this ought to be understood in terms of the broader (equally remarkable) fact that correlation and only correlation constitutes the full content of physical reality.

## XI. COMMENTS ON OTHER APPROACHES

I first encountered the view that correlations are fundamental and irreducible when I heard it advocated as the proper way to think about Einstein–Podolsky–Rosen (EPR) correlations, in talks by Paul Teller[29] and Arthur Fine.[30] It did not then occur to me that this might be the proper way to think about much more general correlations, but it should have, since this is an important part of Bohr's reply[31] to EPR.[32] Nor did it occur to me that objective reality might consist *only* of correlations until I heard Lee Smolin[33] sketch an approach to quantum mechanics that treated *symmetrically* a physical system and the world external to that physical system. Shortly thereafter I received a beautiful paper from Carlo Rovelli[34] arguing from a very different point of view that quantum states were expressions of relations between subsystems. Recently Gyula Bene[35] has written interestingly along these lines.

This general attitude toward quantum states—that the information they contain is necessarily *relational*—goes at least back to Everett's original ''relative-state'' formulation of quantum mechanics.[36] What is special to the IIQM is (a) its insistence, justified by the SSC Theorem, on replacing all talk about quantum *states* with talk about *subsystem correlations*, (b) its insistence that *all* correlations among subsystem observables for *all* resolutions into subsystems have *joint* validity—simultaneous physical reality, if you will, and (c) its insistence that the *correlata* that underlie those correlations lie beyond the descriptive powers of physical science or, equivalently, that although all subsystem *joint* distributions are meaningful the corresponding *conditional* distributions are not.

The IIQM evokes the Everett interpretation in stressing that a measurement is nothing more than a particular kind of interaction between two particular types of subsystems, designed to yield a particular kind of correlation, and in stressing the fact that a system $S_1$ that has nontrivial external correlations with a system $S_2$, has no pure state of its own, even when the joint system $S = S_1 + S_2$ is in a pure state $|\Psi\rangle$. The IIQM assigns a fundamental status to the reduced density matrix of $S_1$ as the complete embodiment of all its internal correlations. Everett, on the other hand, characterizes $S_1$ by a multitude of pure states, each conditional on the assignment of an (almost arbitrary) pure state to $S_2$. Specifically, if

$$P = |\chi\rangle\langle\chi| \tag{22}$$

is a projection operator on any pure state $|\chi\rangle$ of $S_2$ and

$$\langle\Psi|P|\Psi\rangle \neq 0, \tag{23}$$

then one easily establishes that there is a unique pure state $|\phi\rangle$ of $S_1$ for which the mean value of any observable $A$ of $S_1$ is given by

$$\langle\phi|A|\phi\rangle = \langle\Psi|AP|\Psi\rangle / \langle\Psi|P|\Psi\rangle. \tag{24}$$

Everett calls $|\phi\rangle$ the state of $S_1$ *relative to* $|\chi\rangle$ being the state of $S_2$.

According to the IIQM Everett's relative states have no physical significance, because the internal correlations of the subsystem $S_1$ in the relative state $|\phi\rangle$ are given by a distribution that is *conditioned* on the other subsystem $S_2$ being in the state $|\chi\rangle$. While the *correlations* between arbitrary observables of $S_1$ and the observable $P = |\chi\rangle\langle\chi|$ of $S_2$, or the corresponding *joint* distributions, do have physical reality, the *conditional* distribution for $S_1$ obtained by conditioning on $P$ *having the value* 1 in $S_2$ does not. As discussed in Sec. IX, one cannot condition on the values of correlata, because such values have no physical reality. Thus Everett's relative states of a subsystem give rise to internal correlations for that subsystem that are specified by conditional distributions that have no physical meaning in the IIQM. It is the insistence on the simultaneous reality of all these *conditional* distributions that sends one off into the cloud-cuckoo-land of many worlds.

Christopher Fuchs has suggested[37] that the distinction between the many worlds interpretation and the ''correlations without correlata'' of the IIQM, is most succinctly expressed by characterizing many worlds as *correlata without correlations*. In the many worlds interpretation particular individual values of physical properties exist in (over) abundance; but the problem of relating probabilities to the branching of the

worlds of different correlata has not been satisfactorily resolved, in spite of many efforts going all the way back to Everett's original paper.

There has also been a venerable tradition of talk about consciousness and quantum physics, almost from the beginning. My own talk is closest to that which gives consciousness the power of ultimately ''reducing the wave packet.'' The difference is that the IIQM does not speak of wave packet reduction at all, because if physical reality consists only of correlations, nothing physically real ever changes discontinuously. To be sure a vestige of this point of view is retained in my warnings to separate the problem of our mysterious ability directly to perceive the particularity of our own correlation with another macroscopic system from the problem of understanding quantum mechanics. But as noted in Sec. IV, the IIQM takes the view that this ability poses a very hard problem about the nature of our consciousness which ought not to be confused with the merely hard problem of understanding the nature of quantum mechanics as applied to a world devoid of consciousness.[38]

This point of view toward consciousness is in sharp contrast to a more recent tradition, which tries to find an explanation for consciousness based on quantum physics.[39] The IIQM takes quite the opposite position, that consciousness experience goes beyond anything physics is currently (and perhaps ever) capable of coming to grips with.

Two other interpretive schemes—the modal interpretations[40] and the consistent histories approach[41]—also dethrone measurement. Both can be distinguished from the approach described here in terms of how they treat correlations and correlata. Modal interpretations grant reality to more than just relational quantities, at the price of restricting this stronger reality to very special circumstances. Subsystem correlations *and* the associated correlata can be real provided there are just two subsystems, and provided the correlations have the strong form (6). This is made interesting by the Schmidt (polar) decomposition theorem,[42] which guarantees that the state of any two-subsystem system leads to such correlations for some choice of the two subsystem observables. But it leaves the status of other observables up in the air, is embarrassed when the Schmidt decomposition of the two subsystem state is not unique, and has nothing to say about three or more subsystems.

The consistent histories interpretation of quantum mechanics applies to time-dependent as well as equal-time correlations. In contrast to the IIQM, consistent historians are not at all shy about dealing with the correlata that underlie a given set of correlations. They gain this interpretive flexibility by insisting that any talk about either correlations or correlata must be restricted to sets of observables singled out by certain quite stringent consistency conditions. Thus in the example of Sec. VII consistent historians may speak of the correlations *and* the correlata for the observables 1 and 1′ or those for 1 and 2′ or those for 2 and 1′ or those for 2 and 2′. But they are forbidden to combine features of all these cases into a single description. These various incompatible descriptions constitute mutually exclusive ''frameworks'' for describing a single physical system.

I view the consistent histories interpretation as a formalization and extension of Bohr's doctrine of complementarity.[43] The consistent historians liberate complementarity from the context of mutually exclusive experimental arrangements, by stating the restrictions in terms of the quantum mechanical formalism itself, without any reference to measurement. This enables one within a given framework to contemplate what *is* whether or not anything has actually been *measured*—indeed measurements in the consistent histories interpretation (as in the IIQM and the Everett interpretation) are simply a special case in which some of the subsystems function as apparatuses.

The price one pays for this liberation is that the paradoxical quality of complementarity is stripped of the protective covering furnished by Bohr's talk of mutually exclusive experimental arrangements, and laid bare as a vision of a single reality about which one can reason in a variety of mutually exclusive ways, provided one takes care not to mix them up. Reality is, as it were, replaced by a set of complementary representations, each including a subset of the correlations and their accompanying correlata. In the consistent histories interpretation it is rather as if the representations have physical reality but the representata do not.

The IIQM, in contrast, allows one to contemplate together *all* subsystem correlations, associated with *all* complementary sets of subsystem observables. In justification of treating all such correlations as simultaneously real, one notes that quantum mechanics allows one, given the state of the global system, to calculate together the values of all such correlations; that the joint (but not the conditional) distributions arrived at in this way are all mutually consistent; and that quantum mechanics ensures that the catalog of all such joint subsystem distributions completely pins down the global state. The IIQM achieves this capability by denying to physics the possibility of dealing with the individual correlata at all.

Whether this is a fatal defect of the IIQM, whether it is a manifestation of the primitive state of our thinking about objective probability, or whether it is a consequence of the inability of physics to encompass conscious awareness, remains to be explored.

## XII. A FEW FINAL REMARKS

At the risk of losing the interest of those who (like myself) read only the first and last sections before deciding whether the rest is worth perusing, I conclude with some brief comments about loose ends.

As noted at the beginning, what I have been describing is more an attitude toward quantum mechanics than a systematic interpretation. The only proper subject of physics is how some parts of the world relate to other parts. Correlations constitute its entire content. The actual specific values of the correlated quantities in the actual specific world we know, are beyond the powers of physics to articulate. The answer to the question ''What has physical reality?'' depends on the nature of ''what.'' The answer is ''Everything!'' if one is asking about correlations among subsystems, but ''Nothing!'' if one is asking about particular values for the subsystem correlata.

This alters the terms of the traditional debates. Traditionally people have been asking what *correlata* have physical reality. The many different schools of thought differ by answering with many different versions of ''Some'' while the IIQM answers ''None!'' The question of what *correlations* have physical reality, which the IIQM answers with ''All!'' has not, to my knowledge, been asked in this context. While I maintain that abandoning the ability of physics to speak of correlata is a small price to pay for the recognition that it can speak simultaneously and consistently of all possible corre-

lations, there remains the question of how to tie this wonderful structure of relationships down to anything particular, if physics admits of nothing particular.

At this stage I am not prepared to offer an answer, beyond noting that this formulates the conceptual problem posed by quantum mechanics in a somewhat different way, and suggesting that there may be something to be learned by thinking about it along these lines. I suspect our unfathomable conscious perceptions will have to enter the picture, as a way of updating the correlations. To acknowledge this is not to acknowledge that ''consciousness collapses the wave-packet.'' But it is to admit that quantum mechanics does not describe a world of eternally developing correlation (described by ''the wave-function of the universe''), but a phenomenology for investigating what kinds of correlations can coexist with each other, and for updating current correlations and extrapolating them into the future. This phenomenology applies to any system that can be well approximated as completely isolated.

A skeptic might object that the problem of how to update correlations is nothing more than the measurement problem, under a new name. Perhaps it is, but at least the problem is posed in a new context: How are we to understand the interplay between correlation as the only objective feature of physical reality and the absolute particularity of conscious reality? Is something missing from a description of nature whose purpose is not to disclose the real essence of the phenomena but only to track down relations between the manifold aspects of our experience? Is this a shortcoming of our description of nature or is it a deep problem about the nature of our experience?

Besides ''measurement'' John Bell[44] also disapproved of the word ''system''—a word I have used uncritically more than a hundred times (not counting ''subsystem,'' which occurs even more often). If the purpose of physics is to track down relations between the manifold aspects of our experience, then there is nothing wrong in leaving the specifications of the systems to ourselves, however we manage to do it—sometimes by direct conscious perception, sometimes by deductions from what we have learned from the correlations we have managed to induce between the systems we can perceive and the ones we cannot. Admitting ''system'' to the proper vocabulary of physics is not the same as admitting ''correlata''—the (physically inaccessible) particular values of the quantifiable properties of an individual system.

By acknowledging that in our description of nature the purpose is not to disclose the real essence of the phenomena, we free ourselves to construct from the manifold aspects of our experience formal representations of the systems we want to talk about. We have learned how to express their possible correlations by an appropriate state space, and the evolution of those correlations by an appropriate Hamiltonian. By setting aside ''the real essence of the phenomena'' we also acquire the ability to replace the befuddling spectre of an endlessly branching state of the universe—as disturbing in the self-styled down-to-earth Bohmian interpretation as it is in the wildest extravagances of the many worlds interpretation—with a quantum mechanics that simply tells us how we can expect some of the manifold aspects of our experience to be correlated with others. While this may sound anthropocentric, it is my expectation that *anthropos* can be kept out of everything but the initial and final conditions, and often (but not always) even out of those.

But this remains to be explored.

## APPENDIX A: THE SSC THEOREM—SUBSYSTEM CORRELATIONS DETERMINE THE STATE

Given a system $\mathscr{S} = \mathscr{S}_1 + \mathscr{S}_2$ with density matrix $W$, then $W$ is completely determined by the values of Tr $W(A \otimes B)$ for an appropriate set of observable pairs $A$, $B$, where $A = A \otimes 1$ is an observable of subsystem $\mathscr{S}_1$ and $B = 1 \otimes B$ is an observable of subsystem $\mathscr{S}_2$. The proof is straightforward.

Give the state spaces for $\mathscr{S}_1$ and $\mathscr{S}_2$ orthonormal bases of states $|\psi_\mu\rangle$ and $|\phi_\alpha\rangle$, respectively. Let the $A$'s consist of the Hermitian operators on $\mathscr{S}_1$,

$$A_r^{(\mu\nu)} = \tfrac{1}{2}(|\psi_\mu\rangle\langle\psi_\nu| + |\psi_\nu\rangle\langle\psi_\mu|) \tag{25}$$

and

$$A_i^{(\mu\nu)} = \frac{1}{2i}(|\psi_\mu\rangle\langle\psi_\nu| - |\psi_\nu\rangle\langle\psi_\mu|), \tag{26}$$

and let the $B$'s consist of the Hermitian operators on $\mathscr{S}_2$,

$$B_r^{(\alpha\beta)} = \tfrac{1}{2}(|\phi_\alpha\rangle\langle\phi_\beta| + |\phi_\beta\rangle\langle\phi_\alpha|) \tag{27}$$

and

$$B_i^{(\alpha\beta)} = \frac{1}{2i}(|\phi_\alpha\rangle\langle\phi_\beta| - |\phi_\beta\rangle\langle\phi_\alpha|). \tag{28}$$

The states $|\psi_\mu, \phi_\alpha\rangle = |\psi_\mu\rangle \otimes |\phi_\alpha\rangle$ are a complete orthonormal set of states for the composite system $\mathscr{S}$, and the density matrix $W$ for the entire system $\mathscr{S}$ is determined by its matrix elements

$$\langle\psi_\nu, \phi_\beta|W|\psi_\mu, \phi_\alpha\rangle = \text{Tr } W(|\psi_\mu, \phi_\alpha\rangle\langle\psi_\nu, \phi_\beta|). \tag{29}$$

But this can be expressed entirely in terms of quantities of the form Tr $W(A \otimes B)$—i.e., in terms of subsystem correlations:

$$\begin{aligned}
\langle\psi_\nu, \phi_\beta|W|\psi_\mu, \phi_\alpha\rangle &= \text{Tr } W(|\psi_\mu, \phi_\alpha\rangle\langle\psi_\nu, \phi_\beta|) \\
&= \text{Tr } W((A_r^{(\mu\nu)} + iA_i^{(\mu\nu)}) \\
&\quad \otimes (B_r^{(\alpha\beta)} + iB_i^{(\alpha\beta)})) \\
&= \text{Tr } W(A_r^{(\mu\nu)} \otimes B_r^{(\alpha\beta)}) - \text{Tr } W(A_i^{(\mu\nu)} \\
&\quad \otimes B_i^{(\alpha\beta)}) + i \text{ Tr } W(A_r^{(\mu\nu)} \otimes B_i^{(\alpha\beta)}) \\
&\quad + i \text{ Tr } W(A_i^{(\mu\nu)} \otimes B_r^{(\alpha\beta)}). \tag{30}
\end{aligned}$$

Thus the values of the subsystem correlations between all the $A$'s and $B$'s are enough to determine all the matrix elements of $W$ in a complete set of states for the total system $\mathscr{S}$, and

hence they are enough to determine the density matrix $W$ for the total system.

This proof straightforwardly generalizes to a system $\mathscr{S} = \mathscr{S}_1 + \cdots + \mathscr{S}_n$ composed of more than two subsystems: Given any resolution of $\mathscr{S}$ into $n$ subsystems, the density matrix of $\mathscr{S}$ is entirely determined by the correlations among appropriate observables belonging to those subsystems. In such cases the structure of quantum mechanics guarantees the important fact that it doesn't matter whether we pin down the density matrix, for example, of $\mathscr{S} = \mathscr{S}_1 + \mathscr{S}_2 + \mathscr{S}_3$ from correlations between observables of $\mathscr{S}_1$ with observables that act globally on $\mathscr{S}_2 + \mathscr{S}_3$, or from correlations between observables of $\mathscr{S}_3$ with observables acting globally on $\mathscr{S}_1 + \mathscr{S}_2$, or from tripartite correlations among observables acting only on the three subsystems.

Thus the density matrix of a composite system determines all the correlations among the subsystems that make it up and, conversely, *the correlations among all the subsystems completely determine the density matrix for the composite system they make up*. The mathematical structure of quantum mechanics imposes constraints, of course, on what those correlations can be—namely they are restricted to those that can arise from some global density matrix. The particular form of that density matrix is then completely pinned down by the correlations themselves.

That the correlations cannot be more general than that is the content of Gleason's Theorem.[45] It would be interesting to explore the extent to which the underlying structure of probabilities assigned to subspaces of a Hilbert space on which Gleason's Theorem rests is itself pinned down by the requirement of consistency among the different possible resolutions of a system into subsystems.

## APPENDIX B: THE EXTERNAL CORRELATIONS OF A SYSTEM ARE NECESSARILY TRIVIAL IF AND ONLY IF ITS STATE IS PURE

We first show that if the state of a subsystem $\mathscr{S}_1$ is pure, i.e., if its density matrix $W_1$ is a one-dimensional projection operator,

$$W_1 = P_\phi = |\phi\rangle\langle\phi|, \tag{31}$$

then the density matrix $W$ of any larger system system $\mathscr{S} = \mathscr{S}_1 + \mathscr{S}_2$ containing $\mathscr{S}_1$ as a subsystem must be of the form

$$W = P_\phi \otimes W_2, \tag{32}$$

and therefore all external correlations of $\mathscr{S}_1$ are trivial.

This is easily established in the representation in which $W$ is diagonal:

$$W = \sum_i w_i |\Psi^i\rangle\langle\Psi^i|, \tag{33}$$

where the weights $w_i$ are non-negative. If the reduced density matrix $W_1$ for $\mathscr{S}_1$ has the form (31), then its diagonal elements $\langle\phi'|W_1|\phi'\rangle$ must vanish for any state $|\phi'\rangle$ in the state space of $\mathscr{S}_1$ orthogonal to $|\phi\rangle$; i.e., if the $|\chi_n\rangle$ are any orthonormal basis for the state space of $\mathscr{S}_2$, then

$$0 = \sum_n \langle\phi',\chi_n|W|\phi',\chi_n\rangle = \sum_{i,n} w_i |\langle\phi',\chi_n|\Psi^i\rangle|^2. \tag{34}$$

Since the $w_i$ are non-negative, for every nonzero $w_i$ we have

$$0 = \langle\phi',\chi_n|\Psi^i\rangle. \tag{35}$$

Now if the $|\phi_j\rangle$ are an orthonormal basis for the state space of $\mathscr{S}_1$ with $|\phi_1\rangle = |\phi\rangle$, then each $|\Psi^i\rangle$ appearing in the expansion (33) of $W$ is of the form

$$|\Psi^i\rangle = \sum_{j,n} |\phi_j,\chi_n\rangle\langle\phi_j,\chi_n|\Psi^i\rangle. \tag{36}$$

It follows from (35) that

$$|\Psi^i\rangle = \sum_n |\phi,\chi_n\rangle\langle\phi,\chi_n|\Psi^i\rangle$$

$$= |\phi\rangle \otimes \sum_n |\chi_n\rangle\langle\phi,\chi_n|\Psi^i\rangle. \tag{37}$$

This [with the form (33) of $W$] shows that $W$ is indeed of the form (32).

Conversely, if the state of $\mathscr{S}_1$ is mixed, then its density matrix has the form

$$W_1 = \sum p_i |\phi_i\rangle\langle\phi_i|, \tag{38}$$

where the states $|\phi_i\rangle$ are orthonormal and at least two of the $p_i$ (which we can take to be $p_1$ and $p_2$) are nonzero. This density matrix can arise if $\mathscr{S}_1$ is a subsystem of a larger system $\mathscr{S} = \mathscr{S}_1 + \mathscr{S}_2$ with pure-state density matrix

$$W = |\Psi\rangle\langle\Psi|, \tag{39}$$

where the state $|\Psi\rangle$ is given by

$$|\Psi\rangle = \sum_i \sqrt{p_i} |\phi_i\rangle \otimes |\chi_i\rangle, \tag{40}$$

and the $|\chi_i\rangle$ are an orthonormal set of states for $\mathscr{S}_2$. If observables $A_1$ and $A_2$ are defined for each subsystem by

$$A_1 = |\phi_1\rangle\langle\phi_2| + |\phi_2\rangle\langle\phi_1| \tag{41}$$

and

$$A_2 = |\chi_1\rangle\langle\chi_2| + |\chi_2\rangle\langle\chi_1|, \tag{42}$$

then $A_1$ and $A_2$ are nontrivially correlated, since

$$\text{Tr } W A_1 \otimes A_2 = 2\sqrt{p_1 p_2} \tag{43}$$

but

$$\text{Tr } W A_1 \otimes 1 = 0, \quad \text{Tr } W 1 \otimes A_2 = 0. \tag{44}$$

## APPENDIX C: GLOSSARY OF TERMS

*Internal correlations.* The internal correlations of a system are the correlations prevailing among any of its subsystems.

*External correlations.* The external correlations of a system are those it has with other systems which together constitute the subsystems of a larger system.

*Trivial correlations.* Subsystem correlations arising from joint probabilities that are products of subsystem probabilities.

*Nontrivial correlations.* Correlations that are not trivial—i.e., in which the mean of some products differs from the product of the means.

*State.* The state of a system is the complete set of all its internal correlations. These are concisely encoded in its density matrix.

*SSC Theorem.* The theorem on the sufficiency of subsystem correlations for a complete determination of the quantum state of a composite system. It is stated and proved in Appendix A; see also Ref. 17.

*Pure state.* The state of a system whose density matrix is a one-dimensional projection operator. Or, equivalently, the state of a system that has no nontrivial external correlations.

*Mixed state.* The state of a system whose density matrix is not a one-dimensional projection operator. Or, equivalently, the state of a system that can have nontrivial external correlations.

*Dynamically isolated system.* A system that has no external interactions.

*Completely isolated system.* A system that has no external interactions or correlations.

*Specimen.* A subsystem (usually microscopic) that we wish to learn something about.

*Apparatus.* A macroscopic subsystem we dynamically correlate with a specimen.

*Measurement.* The dynamical process by which the correlations between a specimen and an apparatus are brought into the particular canonical form (6).

*Physical locality.* The fact that the internal correlations of a dynamically isolated system do not depend on any interactions experienced by other systems external to it.

*Metaphysical locality.* The requirement (often violated) that the external correlations of a dynamically isolated system should make sense in terms of internal correlata.

*Correlatum.* The particular value of a property of an individual system (represented in the formalism by a particular eigenvalue of the corresponding hermitian operator). According to the IIQM, correlations among the correlata of different subsystems have physical reality but the correlata themselves do not.

*Physical reality.* That whereof physics can speak. For example the physical reality of *blue* includes a certain class of Fourier decompositions of the radiation field, and the excitations in the retina produced by fields with such Fourier decompositions, and the signals transmitted by such excitations to the visual cortex.

*Reality.* Physical reality plus that on which physics is silent, its conscious perception. For example for me the reality of *blue* consists of its physical reality augmented by my accompanying sensation of *blueness*.

*IIQM.* ''The Ithaca Interpretation of Quantum Mechanics''—the constellation of ideas put forth above, more accurately characterized as ''An Ithaca Interpretation of Quantum Mechanics.''

[1] Notes for a lecture given at the Symposium in Honor of Edward M. Purcell, Harvard University, October 18, 1997.

[2] Introductory Survey to *Atomic Theory and the Description of Nature* (Cambridge U.P., Cambridge, 1934), p. 18. Reprinted in *Niels Bohr, Collected Works* (North-Holland, Amsterdam, 1985), Vol. 6, p. 296.

[3] N. David Mermin, ''The Ithaca Interpretation of Quantum Mechanics,'' Pramana (to be published); a version can be found in quant-ph 9609013, Los Alamos e-Print archive at xxx.lanl.gov (1996). The nomenclature was intended to indicate a resemblance to the body of interpretational lore named after a grander city in northern Europe, and also, by its geographic modesty and lack of descriptive content, to suggest that what was being promulgated was not so much a logical foundation for the subject as a philosophical perspective or pedagogical approach. It does not imply that others in Ithaca share these views, or that others outside of Ithaca have not expressed similar thoughts. None of the ingredients of the IIQM are novel, but I have cooked them together into a somewhat different stew.

[4] These are examples of the kinds of terms or distinctions that have a special character in the IIQM, and are collected together in Appendix C.

[5] D. Bohm and D. J. Hiley, *The Undivided Universe* (Routledge, New York, 1993).

[6] For a recent review and a set of references, see for example Philip Pearle, ''Wavefunction Collapse Models With Nonwhite Noise,'' in *Perspectives on Quantum Reality*, edited by Rob Clifton (Kluwer, Dordrecht, 1996), pp. 93–109.

[7] A technical point: In taking the state space of the system to be a product of subsystem state spaces I am restricting the discussion to cases where the significant manifestations of quantum mechanical indistinguishability of particles—the symmetry or antisymmetry of many-particle wave functions—are limited to the constituents of the individual subsystems. This is conventional (though rarely noted) in discussions of the foundations of quantum mechanics. Thus in discussing the measurement of the spin of an atom, one does not antisymmetrize the combined wave function for the atom–apparatus system over, for example, the electronic variables occurring in both subsystems. The overlap between atomic and apparatus electronic wave functions is taken to be zero. Since the major conceptual problems posed by quantum mechanics—nonlocality and the measurement problem—are present even in the quantum mechanics of distinguishable particles, this simplification does not appear to evade essential features. A more rigorous approach will probably require a field-theoretic formulation. There seems no point in trying to cross that bridge unless one can first cross the simpler one attempted here. In a similar vein, I also consider here only nonrelativistic quantum mechanics, because the conceptual problems are already present in the nonrelativistic theory. A treatment of relativistic quantum mechanics will also require a field theoretic reformulation.

[8] This is spelled out more explicitly at the beginning of Sec. IX.

[9] For example, the joint distribution for electron position and proton position in a hydrogen atom exists simultaneously with the joint distribution for electron momentum and proton position, even though the position and momentum of the electron do not have joint physical reality or a meaningful joint distribution of their own. And both the position–position and momentum–position distributions return the same distribution for the proton position, when the electronic variables are integrated out.

[10] The essential role of objective probability in the quantum mechanical description of an individual system was stressed by Popper, who used the term ''propensity.'' See Karl Popper, *Quantum Theory and the Schism in Physics* (Rowman and Littlefield, Totowa, NJ, 1982). Heisenberg may have had something similar in mind with his term ''potentia.'' While I agree with Popper that quantum mechanics requires us to adopt a view of probability as a fundamental feature of an individual system, I do not believe that he gives anything like an adequate account of how this clears up what he called the ''quantum mysteries and horrors.'' See N. David Mermin, ''The Great Quantum Muddle,'' Philos. Sci. **50**, 651–656 (1983); reprinted in *Boojums All the Way Through* (Cambridge U.P., Cambridge, 1990), pp. 190–197.

[11] Wolfgang Pauli, ''Probability and physics,'' in *Writings on Physics and Philosophy* (Springer-Verlag, New York, 1994), pp. 43–48.

[12] I comment further on the Everett interpretation, which was subsequently transformed into the many worlds interpretation, in Sec. XI.

[13] Einstein was apparently resigned to the inaccessibility of *now* to physics. According to Carnap [''Intellectual Autobiography,'' in *The Philosophy of Rudolf Carnap*, edited by P. A. Schilpp (Open Court, LaSalle, IL, 1963), pp. 37–38] in a conversation in the early 1950's ''Einstein said that the problem of the Now worried him seriously. He explained that the experience of the Now means something special for man, something essentially different from the past and the future, but that this important difference does not and cannot occur within physics. That this experience cannot be grasped by science seemed to him a matter of painful but inevitable resignation.'' This is particularly interesting in view of Einstein's notorious unwillingness to extend his resignation over the inability of physics to deal with the special character of *now*, to its inability to deal with the special character of correlata underlying the quantum correlations.

[14] To my surprise, this point—a banality among philosophers, who speak of *qualia*—is extremely hard, if not impossible, to put across to some physicists. I have sometimes managed to do it by citing a theory I had as a child to account for the fact that different people have different favorite colors. My idea—a kind of chromo-aesthetic absolutism—was that there was, in fact, only one most pleasurable color sensation, but the reason *your* favorite color was blue while *mine* was red was that the sensation you experienced looking at blue objects was identical to the sensation I experienced looking at red ones. I recently found precisely this example (complete to the choice of colors—only ''you'' and ''me'' are interchanged) on a list of possibly meaningless questions in P. W. Bridgman, *The Logic of Modern Physics* (Macmillan, New York, 1927), p. 30.

[15]For an example see the discussion of quantum nonlocality in Sec. X.

[16]For an engaging discussion of these issues and many references, see Euan Squires, *Conscious Mind in the Physical World* (Hilger, Bristol and New York, 1990).

[17]S. Bergia, F. Cannata, A. Cornia, and R. Livi, ''On the actual measurability of the density matrix of a decaying system by means of measurements on the decay products,'' Found. Phys. **10**, 723–730 (1980). See also W. K. Wootters, in *Complexity, Entropy and the Physics of Information*, edited by W. H. Zurek (Addison–Wesley, Redwood City, CA, 1990), pp. 39–46.

[18]J. S. Bell, ''Against measurement,'' Phys. World 33–40 (August, 1990). This critique elicited interesting rejoinders from Rudolf Peierls [''In defense of measurement,'' Phys. World 19–20 (January, 1991)] and Kurt Gottfried [''Does quantum mechanics carry the seeds of its own destruction?'' 31–40 (October, 1991)]. Bell's death deprived us of his response.

[19]I defer to Sec. VIII any discussion of ''the measurement problem''—the constellation of issues arising in the context of ''wave-packet collapse.''

[20]FAPP=For all practical purposes. See J. S. Bell, in Ref. 18.

[21]The exact absence of interference effects for any observables associated entirely with either the system or the apparatus (i.e., of the form $S \otimes 1$ or $1 \otimes A$ for arbitrary system and apparatus observables $S$ and $A$), is, of course, also directly evident from the form (5) of the post-measurement (pure) state of the total specimen–apparatus system, in which the phases of the $\alpha_i$ still appear.

[22]Hugh Everett III, ''Relative-State Formulation of Quantum Mechanics,'' Rev. Mod. Phys. **29**, 454–462 (1957). Everett says virtually nothing about many worlds except, perhaps, in a note added in proof. I discuss the relation of the IIQM to Everett's relative-state formulation in Sec. XI.

[23]See N. David Mermin, ''Hidden Variables and the Two Theorems of John Bell,'' Rev. Mod. Phys. **65**, 803–815 (1993), especially Sec. VII.

[24]Lucien Hardy, ''Quantum mechanics, local realistic theories, and Lorentz-invariant realistic theories,'' Phys. Rev. Lett. **68**, 2981–2984 (1992). The version of Hardy's argument given here uses the notation in N. David Mermin, ''Quantum mysteries refined,'' Am. J. Phys. **62**, 880–897 (1994).

[25]Nor is the absence of subsystem correlata a peculiarity of a small class of specially contrived states. Hardy has shown (in the context of a ''nonlocality'' argument, but the theorems apply equally well in the present context) that this state of affairs is generic, holding for appropriate subsystem observables whenever a system $\mathcal{S} = \mathcal{S}_1 + \mathcal{S}_2$ has any nontrivial correlations between its subsystems $\mathcal{S}_1$ and $\mathcal{S}_2$ (unless the individual subsystem probabilities are completely random—i.e., unless the individual subsystem density matrices are proportional to the unit matrix). See Lucien Hardy, ''Nonlocality for two particles without inequalities for almost all entangled states,'' Phys. Rev. Lett. **71**, 1665–1668 (1993).

[26]John S. Bell, ''On the Einstein–Podolsky–Rosen paradox,'' Physics **1**, 195–200 (1964).

[27]Most recently in Henry P. Stapp, ''Nonlocal character of quantum theory,'' Am. J. Phys. **65**, 300–304 (1997).

[28]I have given a detailed analysis of how Stapp's carefully constructed derivation of nonlocality from the Hardy state can be used to illuminate Bohr's reply to Einstein, Podolsky, and Rosen. See N. David Mermin, ''Nonlocal character of quantum theory?'' Am. J. Phys. (to be published). Stapp responds in ''Quantum nonlocality,'' *ibid.* (to be published).

[29]Paul Teller, in *Philosophical Consequences of Quantum Theory*, edited by James T. Cushing and Ernan McMullin (Notre Dame U.P., Notre Dame, IN, 1989), pp. 208–223.

[30]Arthur Fine, in Ref. 29, pp. 175–194.

[31]Niels Bohr, ''Can Quantum Mechanical Description of Physical Reality Be Considered Complete?'' Phys. Rev. **48**, 696–702 (1935). One of Bohr's points is that there is nothing new or unusual about EPR correlations: Precisely the same kinds of correlations are set up in the measurement process, and therefore there is no cause for alarm because he has already straightened out that problem.

[32]Albert Einstein, Boris Podolsky, and Nathan Rosen, ''Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?'' Phys. Rev. **47**, 777–780 (1935).

[33]This point of view is expressed in Lee Smolin, *The Life of the Cosmos* (Oxford U.P., New York, 1997).

[34]Carlo Rovelli, ''Relational Quantum Mechanics,'' Int. J. Theoret. Phys. **35**, 1637–78 (1996). See also quant-ph 9609002, Los Alamos e-Print archive at xxx.lanl.gov (1996).

[35]Gyula Bene, ''Quantum Reference systems: a new framework for quantum mechanics,'' quant-ph/9703021, Los Alamos e-Print archive at xxx.lanl.gov (1996); ''Quantum phenomena do not violate the principle of locality—a new interpretation with physical consequences,'' Physica A (to be published); quant-ph/9706043, Los Alamos e-Print archive at xxx.lanl.gov (1996), Am. J. Phys. (submitted).

[36]H. Everett, III, in Ref. 22. It was later swept off into the many worlds interpretation.

[37]Christopher Fuchs (private communication).

[38]A similar attitude has been expressed by Rudolf Peierls, *Surprises in Theoretical Physics* (Princeton U.P., Princeton, NJ, 1979), p. 33: ''We are confident today that, if we could solve the Schrödinger equation for all the electrons in a large molecule, it would give us all the knowledge that chemists are able to discover about it... . Many people take it for granted that the same must be true of the science of life. The difficulty about how to formulate the acquisition of information, which we have met, is a strong reason for doubting this assumption.'' Even closer is the view of Robert Geroch, ''The Everett interpretation,'' Nous **18**, 617–633 (1984), p. 629: ''[W]hat must be accounted for... is, not the specific classical outcomes deemed to have occurred for a specific experiment, but rather the general human impression that classical outcomes do occur. This problem may well be soluble, but is probably beyond our present abilities; and, in any case, is basically not a problem in quantum mechanics.''

[39]See, for example, Roger Penrose, *The Emperor's New Mind* (Oxford U.P., New York, 1989), and *Shadows of the Mind* (Oxford U.P., New York, 1994); Henry Stapp, *Mind, Matter, and Quantum Mechanics* (Springer-Verlag, New York, 1993).

[40]See, for example, Jeffrey Bub, *Interpreting the Quantum World* (Cambridge U.P., Cambridge, 1997) and Bas C. Van Fraassen, *Quantum Mechanics: An Empiricist View* (Clarendon, Oxford, 1991), and references cited therein.

[41]For a recent formulation and references see Robert B. Griffiths, ''Consistent Histories and Quantum Reasoning,'' Phys. Rev. A **54**, 2759–2774 (1996).

[42]See, for example, Asher Peres, *Quantum Theory: Concepts and Methods* (Kluwer, Dodrecht, 1993), pp. 123–126.

[43]Indeed, I believe its character would be clarified if consistent historians were to characterize mutually exclusive families of correlations and correlata not as *inconsistent* or *incompatible* but as *complementary*. This terminology is suggested on p. 162 of Roland Omnès, *The Interpretation of Quantum Mechanics* (Princeton U.P., Princeton, 1994).

[44]J. S. Bell, Physics World, in Ref. 18.

[45]A. M. Gleason, ''Measures on the Closed Subspaces of a Hilbert Space,'' J. Math. Mech. **6**, 885–893 (1957).

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## THERE IS NO TEACHING IN PHYSICS

As I have grown older, I have come more and more to the conclusion that there is no teaching in physics, there is only inspiration to learn. ... The teacher may stimulate the mind of the student... but the journey to that goal must be made by the student himself.

W. F. G. Swann, ''The Teaching of Physics,'' Am. J. Phys. **19**(3), 182–187 (1951).

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