UNIVERSITÄT ZU KÖLN

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# Probability theory and stochastic processes for physicists

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**Problem 19: A multiplicative Langevin equation**

The stochastic process $Y(t)$, $-1 \leq Y(t) \leq 1$, is defined through the multiplicative Langevin equation

$$\frac{dY}{dt} = \alpha Y + \sqrt{1 - Y^2}\xi(t) \tag{1}$$

with $\langle \xi(t) \rangle = 0$, $\langle \xi(t)\xi(t') \rangle = \delta(t - t')$.

a.) Write down the corresponding Fokker-Planck equation in the Itŏ and Stratonovich interpretations.

b.) Compute the stationary distribution of the process. For which values of $\alpha$ is the stationary distribution normalizable?

**Problem 20: Fokker-Planck and Schrödinger operators**

We look for a stochastic process $Y(t)$ with the stationary distribution

$$P_s(y) = \frac{K}{2} \exp[-K|y|] \tag{2}$$

with $K > 0$.

a.) Write down an additive Langevin equation whose stationary distribution is (2).

b.) Write down the corresponding Fokker-Planck operator and map it to a Schrödinger operator. Using a standard result of elementary quantum mechanics, this allows you to determine the spectrum of the Fokker-Planck operator. What is the time scale governing the relaxation to the stationary distribution (2)?