Transport in disordered Luttinger liquids: strong pinning and global phase diagram

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Zusammenfassung We calculate the tunnel current of a Luttinger liquid with a finite density of strong impurities using using combined RG and instanton approach. For very low temperatures T (or electric fields E) the (nonlinear) conductivity is of variable range hopping type as for weak pinning. For higher temperatures or fields the conductivity shows power law behavior corresponding to a crossover from multi- to single-impurity tunneling. For even higher T and not too strong pinning there is a second crossover to weak pinning. The determination of the position of the various crossover lines both for strong and weak pinning allows the construction of the global phase diagram.

1 Introduction

One-dimensional electron systems exhibit a number of peculiarities which destroy the familiar Fermi-liquid behavior known from higher dimensions. Main reason is the geometrical restriction of the motion in one dimension where electrons cannot avoid each other. As a consequence, excitation are density waves (plasmons) similar to sound waves in solids. The corresponding phase is called a Luttinger liquid [1,2]. Renewed interest in Luttinger liquids arises from progress in manufacturing narrow quantum wires with a few or a single conducting channel. Examples are carbon nanotubes [3], polydiacetylen [4], quantum Hall edges [5] and semiconductor cleave edge quantum wires [6]. From a theoretical point of view 1D quantum wires allow the investigation of the interplay of interaction and disorder effects since short range interaction can be treated already within a harmonic bosonic theory [7]. Central quantity is the interaction parameter K which plays (under certain conditions) the role of a dimensionless conductance of a clean Luttinger liquids [8,1].

The effect of disorder on transport in Luttinger liquids has been so far considered in two limiting cases: (i) The effect of a *single* impurity was considered in [8–11]. Here the conductance depends crucially on the strength K of the interaction. Impurities are irrelevant for attractive ($K > 1$) and strongly relevant for repulsive interaction ($K < 1$), respectively. For finite voltage V and $K < 1$, the conductance is $\sim V^{2/K-2}$ [8] These considerations can be extended to two impurities. Depending on the applied gate voltage, Coulomb blockade effects may give rise to resonant tunneling [8,10].

(ii) In the opposite case of a *finite density* of *weak* impurities, (Gaussian) disorder is a relevant perturbation for $K < 3/2$ leading to the strong localization of electrons. The shift of the critical values of K to $K_c = 3/2$ can be traced back to an additional factor $1/\sqrt{a}$ arising in the effective disorder strength from the impurity concentration a^{-1} [2]. For weak external electric field E the conductivity is highly nonlinear: $\sigma(E) \sim e^{-c/\sqrt{E}}$ [12–15]. At low but finite temperatures T this result goes over into the variable range hopping expression for the linear conductivity $\sigma \sim e^{-c'/\sqrt{T}}$ [13–17]. The derivation of the result requires a weak coupling to a dissipative bath to allow energy relaxation [14,18]. At higher temperatures there is a crossover to $\sigma \sim T^{2-2K}$ [19].

On the contrary, much less is known in the case of a *finite density* of *strong* pinning centers [20,2], which is the topic we will address in the present paper. In particular we determine both the temperature and electric field dependence of the (nonlinear) conductivity for this case in a broad temperature and electric field region. The main results of the paper are the conductivities (10), (11), (13) and (14) as well as the crossover behavior summarized in Fig. 1 and Fig. 2, respectively.

The T-dependence of the linear conductivity shows three distinct regions. For $T \ll T_a = \hbar v/a$ the thermal de Broglie wave length $\lambda_T = \hbar v/T$ of the plasmons is larger than a and hence the hopping processes includes many impurities. v denotes the plasmon velocity. The conductivity is of variable range hopping typ $\sigma \sim e^{-c'/\sqrt{T}}$ similar to the case of weak impurities [13].

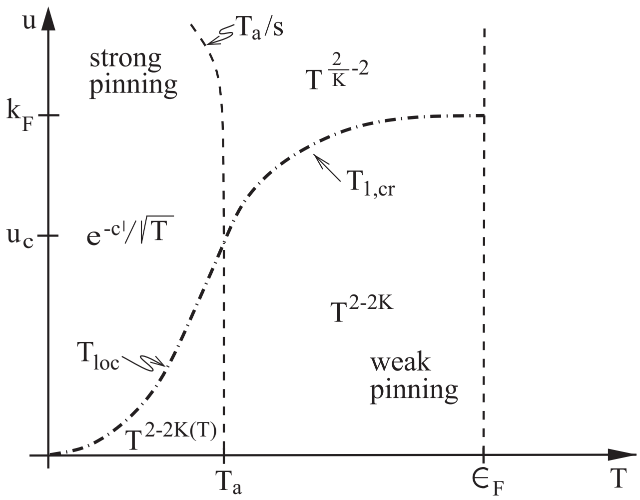

Abbildung1. $u - T$ phase diagram of the linear conductivity of disordered Luttinger liquids. For strong pinning, $u > u_c \sim k_F(k_F a)^{K-1}$ and $T < T_{1,\mathrm{cr}} \sim T_a(u/u_c)^{1/(1-K)}$, T_a/s separates the variable range hopping from the single impurity hopping regime. For $T > T_{1,\mathrm{cr}}$ impurities become weak. For weak pinning, $u < u_c$, $T_{\mathrm{loc}} \sim T_a(u/u_c)^{2/(3-2K)}$ separates variable range hopping from renormalized power law behavior. For $T > T_a$ the power law is unrenormalized.

In an intermediate temperature region $T_a/s < T < T_{1,\mathrm{cr}} \approx T_a(u/u_c)^{1/(1-K)}$, hopping occurs through single impurities which act now independently. Here $u_c \sim k_F(k_F a)^{K-1}$ describes the borderline between weak and strong pinning (at low T), and s is a large number explained below. The conductivity behaves as $\sigma \sim T^{2/K-2}$ as found before for isolated weak links [8,10,11] and discussed for Luttinger liquids with many impurities in [20]. At even higher temperature $T > T_{1,\mathrm{cr}}$ individual impurities become weak and conductivity follows $\sigma \sim T^{2-2K}$ [20]. A related crossover occurs for the nonlinear conductivity as a function of the electric field strength E and will be discussed below.

2 Model and Instantons

Starting point of our calculation is the action of interacting electrons subject to an external uniform electric field and strong pinning centers. In bosonized form the action takes the form

$$S = \frac{\hbar}{2\pi K}\int_0^L\int_0^{\lambda_T} dxdy \Big\{(\partial_y\varphi)^2 + (\partial_x\varphi + fx)^2 - \sum_{i=1}^{N} u\delta(x-x_i)\cos(2\varphi + 2k_F x_i)\Big\} \qquad (1)$$

The phase $\varphi(x)$ is related to the electron density $\rho(x)$ by $\pi\rho(x) = (k_F + \partial_x\varphi)(1 + 2\cos(2\varphi + 2k_F x))$. k_F is the Fermi wave vector, $\tau = y/v$ and $f = eEK/v\hbar$. The action (1) depends (besides on L) on five dimensionless parameters K, u/k_F, $ak_F(\gg 1)$, $\lambda_T k_F = \epsilon_F/T(\gg 1)$ and $f/k_F^2(\ll 1)$, where $\epsilon_F \sim \hbar v k_F$ denotes the energy cutoff (bandwidth). For $u > k_F$ the effective cut-off is k_F. For $u < k_F$ it is convenient to integrate out the small scale fluctuations in the momentum shell $\Lambda < |k| < k_F$ [9] which gives:

$$u \to u_{\mathrm{eff}} \approx u(k_F/\Lambda)^{-K}, \qquad \Lambda = \max(u_{\mathrm{eff}}, a^{-1}, \lambda_T^{-1}). \qquad (2)$$

In particular, $u_{\mathrm{eff}} \approx k_F(u/k_F)^{1/(1-K)}$ if $u_{\mathrm{eff}} > a^{-1}, \lambda_T^{-1}$. For strong pinning, $au_{\mathrm{eff}} > 1$ [21] (as long as $\lambda_T > a$), this condition results in $u > u_c \approx k_F(ak_F)^{K-1}$. Only in the absence of interaction, $K = 1$, u_c becomes independent of a. For $\lambda_T < a$ this condition has to be replaced by $\lambda_T u_{\mathrm{eff}} > 1$ since impurities are not longer coupled. This results in $u > u_1(T) \approx k_F(T/\epsilon_F)^{1-K}$. For $u < u_1(T)$, i.e. $T > T_1 \approx \epsilon_F(u/k_F)^{1/(1-K)}$, isolated impurities become weak [20]. After this renormalization the effective cut-off is u_{eff} and the number of dimensionless parameters is reduced by one.

The phase field between the impurities can now be easily integrated out leaving only its values $\varphi(x_i, y) \equiv \phi_i(y)$ *at* the impurity sites x_i which are assumed to be randomly distributed. The procedure is equivalent to solving the saddle point equation between impurities with the appropriate boundary conditions $\partial_x^2\varphi + \partial_y^2\varphi = -f, \quad \varphi(x, y + \lambda_T) = \varphi(x, y)$. Since the solution is periodic in

y-direction, it can be expressed in terms of Fourier components with Matsubara frequencies $\phi_i(y) = {\lambda_T}^{-1} \sum_{\omega_n} \phi_{i,\omega_n} e^{-i\omega_n y}$, $\omega_n = 2\pi n/\lambda_T$. The action can now be rewritten in terms of the ϕ_{i,ω_n}

$$S = \frac{\hbar}{2\pi K} \sum_{i=0}^{N} \left\{ \sum_{\omega_n} \frac{\omega_n}{\lambda_T} \left(\frac{|\phi_{i+1,\omega_n} - \phi_{i,\omega_n}|^2}{\sinh \omega_n a_i} + \left(|\phi_{i,\omega_n}|^2 + |\phi_{i+1,\omega_n}|^2\right) \tanh \frac{\omega_n a_i}{2} \right) \right.$$
$$\left. - f(a_{i-1} + a_i)\phi_{i,0} + u_{\text{eff}} \int dy \left[1 - \cos\left(2\phi_i(y) + 2\pi\alpha_i\right) \right] \right\} \qquad (3)$$

where $a_i = x_{i+1} - x_i$ and $\alpha_i = k_F x_i/\pi$. Since $k_F a_i \gg 1$ below we will assume the α_i to be random phases but keep the impurity distance approximately constant $a_i \approx a$ (we come back to this point later).

In the following we consider the current to result from tunneling processes between metastable states within an instanton approach, assuming strong pinning and weak quantum fluctuations, i.e. $K \ll 1$. It is then assumed that the tunneling process starts from a classical *metastable* configuration $\tilde{\phi}_i$ which minimizes the impurity potential for all values of y. Hence $\tilde{\phi}_i = \pi(n_i - \alpha_i)$ where n_i is integer. Among the many metastable states there is one (modulo π) zero field *ground* state $\tilde{\phi}_i^0$ where $n_i = n_i^0 = \sum_{j \le i} [\Delta\alpha_{j-1}]_G$ [13]. $[\alpha]_G$ denotes the closest integer to α and here and henceforth $\Delta Q_i = Q_{i+1} - Q_i$ for an any quantity Q_i depending on the impurity index i. An new metastable state follows from the ground state by adding integers $q_i = \pm 1$ to the n_i^0. This change corresponds to a redistribution of electrons in the wire. The energy is correspondingly increased by $\left(\nu_k(q) + \nu_{k+m}(-q)\right)/\pi\kappa a$ where $\nu_k(q) = q^2 - 2q\left(\Delta\alpha_k - [\Delta\alpha_k]_G\right)$ is proportional to the surface energy of the such a region. Apart from rare bifurcation sites where $\nu_k(\pm 1) = 0$ (which correspond to resonance conditions) this energy is always positive.

Next we consider an *instanton* configuration which connects the original state $\tilde{\phi}_i$ with the new state $\tilde{\phi}_i + \pi$, n_i depends in general on y. Since the instanton connects two neighboring meta-stable states, we assume a double kink configuration for the instanton at each impurity site:

$$\begin{aligned} \phi_i(y) &= \tilde{\phi}_i + \pi\,, \qquad && |y - y_i| < D_i - d, \\ \phi_i(y) &= \tilde{\phi}_i\,, && |y - y_i| > D_i + d \end{aligned} \qquad (4)$$

with a linear interpolation between the two values at the kink walls in the regions $\big||y - y_i| - D_i\big| < d$. $y_i \pm D_i$ is the kink/anti-kink position, $d \sim 1/u$ is the approximate width of the kinks and $2D_i$ their distance. This form of the instanton has the Fourier components

$$\phi_{i,\omega_n} = \lambda_T \tilde{\phi}_i \delta_{\omega_n,0} + 2\pi e^{i\omega y_i} \frac{\sin \omega_n D_i}{\omega_n} \frac{\sin \omega_n d}{\omega_n d}\,. \qquad (5)$$

It is plausible that in the saddle point configuration all y_i will be the same, an approximation we will use in the following. It is convenient to introduce the dimensionless quantity $z_i = \pi D_i/a$. The instanton action can then be rewritten as

$$S_{\rm inst} \approx \frac{2\hbar}{K}\sum_i{}' \{ \frac{1}{\pi}(z_{i+1}-z_i)\Delta\tilde{\phi}_i - fa^2 z_i + s + \ln\left[\frac{\cosh((z_{i+1}-z_i)/2)}{\cosh((z_{i+1}+z_i)/2)}\tanh\frac{z_i}{2}\cosh z_i\right]\} \qquad (6)$$

where the sum goes only over impurities with $z_i > 0$. We consider s as a constant resulting from the scales smaller than a that includes the core action of a kink and an anti-kink: $s = \ln(Cau_{\rm eff}) \gg 1$, where $\ln C/K \gg 1$.

For a given initial metastable state $\{\tilde{\phi}_i\}$, $S_{\rm inst}$ is a function of the variational parameters $\{D_i\,;\ i = 1, ..., N\}$. The total action can be understood as the action corresponding to the first term of the partition sum expansion in tunneling amplitudes [22]. The nucleation rate Γ and hence the current I is given by

$$I \propto \Gamma \propto \prod_{i=0}^{N}\int_0^{i\infty} dD_k \exp(-S/\hbar). \qquad (7)$$

The calculation of (7) for the general case is possible only if the $\{\tilde{\phi}_i\}$ are given, the latter depend on the particular realization of the disorder. Instead we employ an approximate treatment in which we assume $D_i \equiv D = az/\pi$ for $k < i \leq k+m$ and $D_i = 0$ elsewhere, i.e. tunneling is assumed to occur simultaneously through m neighboring impurities. The instanton is then a rectangular object with extension ma and $2D$ in x and y direction, respectively. The instanton action can then be written as

$$S_{\rm inst} = \frac{2\hbar}{K}\left\{z\sigma_m(k) + \ln(1+e^{-2z}) + m(s + \ln\tanh\frac{z}{2} - z\tilde{f})\right\}. \qquad (8)$$

Here we introduced the dimensionless field strength $fa^2/\pi \equiv \tilde{f} = E/E_a$ where $E_a = 1/(e\kappa a^2)$, $\kappa = K/\pi\hbar v$ denotes the compressibility. $\sigma_m(k) = \big(\nu_k(1) + \nu_{k+m}(-1)\big)/2$ plays the role of a surface tension of the vertical boundaries of the instanton. In the ground state $\sigma_m(k)$ is equally distributed in the interval $0 \leq \sigma_m(k) < 2$ [14]. The second and the third contribution result from the horizontal boundaries of the instanton and include their surface tension s/a and their attractive interaction. The last term describes the volume contribution resulting from the external field.

In addition, we have to include a small dissipative term $S_{\rm bath} = \frac{2\hbar}{K} m\eta \ln z$, $\eta \ll 1$, in the action in order to allow for energy dissipation [14]. However, we will omit η-dependent terms in all results where they give only small corrections (apart from possible pre-exponential factors which we do not consider).

A necessary condition for tunneling is $\partial S_{\rm inst}/\partial z < 0$ for $z \to \infty$, i.e. $\sigma_m(k) < m\tilde{f}$. The tunneling probability follows from the saddle point value of the instanton action where z fulfils the condition

$$\sigma_m(k) - m\tilde{f} + \tanh z - 1 + \frac{m\,\eta}{z} + \frac{m}{\sinh z} = 0\,. \qquad (9)$$

3 Results and Conclusions

We discuss now several special cases:

(i) For sufficiently large fields $E \gg E_a$ the saddle point is $z_s \approx 1/\tilde{f} \ll 1$ which gives a tunneling probability $\Gamma \propto \tilde{f}^{2m/K-1}e^{-2ms/K}$. The exponent -1 results from the integration around the saddle point. Because of small K and correspondingly large kink core action, tunneling through single impurities ($m = 1$) is preferred and hence the nonlinear conductivity is given by

$$\sigma(E) \sim (E/E_a)^{\frac{2}{K}-2}\, e^{-2s/K}, \qquad E_a \ll E \ll E_{1,\mathrm{cr}}, \tag{10}$$

in agreement with previous results for tunneling through a single weak link [8] if we identify $e^{-s/K} \sim t$ with the hopping amplitude t through the link. The upper field strength for the validity of this result can be estimated from $D_s u_{\mathrm{eff}} \equiv z_s a u_{\mathrm{eff}} < 1$ since the instantons loose then their meaning. Using $u_{\mathrm{eff}} \approx k_F(u/k_F)^{1/(1-K)}$ we find $E_{1,\mathrm{cr}} \sim (k_F/e\kappa a)(u/k_F)^{1/(1-K)}$, which can be also read off directly from (10) as $E_{1,\mathrm{cr}} \sim E_a e^s$. Classically ($K = 0$), $E_{1,\mathrm{cr}}$ corresponds to the case when the field energy eEa the electron gains by moving to the next impurity is smaller than the pinning energy u/κ.

At finite temperatures there is a crossover to a temperature dependent conductivity

$$\sigma(T) \propto (T/T_a)^{\frac{2}{K}-2}\, e^{-2s/K}\,, \qquad EKea \ll T \ll T_{1,\mathrm{cr}} \tag{11}$$

when the instanton extension $2D_s$ reaches λ_T, i.e. for $E < E_a T/T_a$. For temperatures higher than $T_{1,\mathrm{cr}}$ isolated impurities are weak. Following the arguments of [20] one expects in this region $\sigma \sim T^{2-2K}$. Note that $E_{1,\mathrm{cr}} \sim E_a T_{1,\mathrm{cr}}/T_a$ as it should be.

(ii) In the opposite case of weak fields, $E \ll E_a$, tunneling happens simultaneously through many impurities and the saddle point $z_s \gg 1$. In this case we can estimate the typical surface tension as $\sigma_k(m) \approx 1/m$ for a chosen pair of sites k and $k + m$, respectively [14]. Then

$$S_{\mathrm{inst}} \approx \frac{2\hbar}{K}\left\{\frac{z}{m} + m(s + \eta \ln z) - zm\tilde{f}\right\}. \tag{12}$$

For very large values of m we can treat m as continuous and the saddle point condition gives $m_s \approx \tilde{f}^{-1/2} \gg 1$ and $z_s \approx s/2\tilde{f}$. The tunneling probability and hence the current is proportional to

$$I \sim \sigma(E) \sim e^{-\frac{2s}{K}\sqrt{E_a/E}},\ E \ll E_a. \tag{13}$$

If we write the result in the variable range hopping form [16] $I \sim e^{-2ma/\xi_{\mathrm{loc}}}$ we can identify the localization length $\xi_{\mathrm{loc}} \approx aK/s$ of the tunneling charges. (This localization length differs by a factor K from that used in [10,20]). There is a crossover to a temperature dependent conductivity if $\lambda_T \ll 2D_s$, i.e. for $E \ll sTE_a/T_a < E_a$ where

$$\sigma(T) \sim e^{-\frac{2}{K}\sqrt{csT_a/T)}}\,, \qquad EKea/s \ll T < T_a/s \tag{14}$$

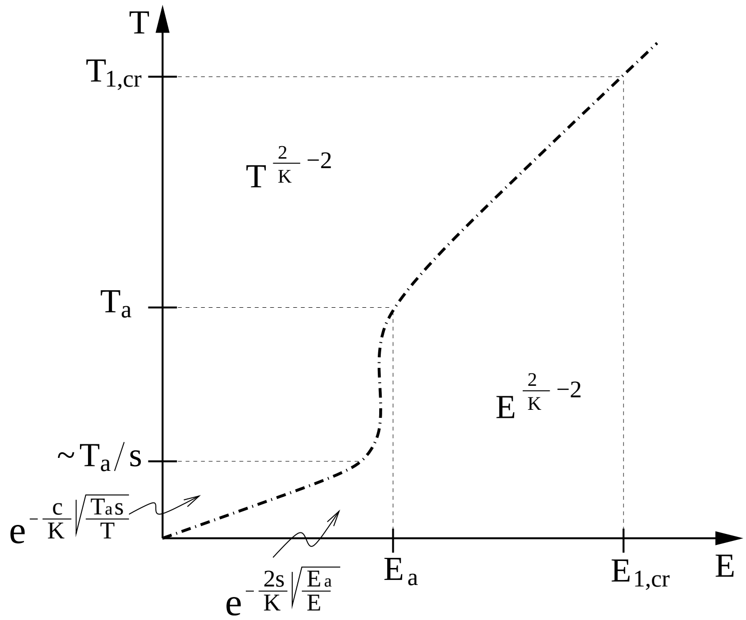

Abbildung2. Field and temperature dependence of the conductivity in the various regions of the $T-E$ plane. $T_a, T_{1,\mathrm{cr}}, s, E_a$ and $E_{1,\mathrm{cr}}$ are explained in the text.

Results (13) and (14) are in agreement with those obtained for weak pinning [13,14]. The results are summarized in Fig. 2.

(iii) If m is not too large (e.g. for large a) we have to take into account the discreteness of m. An instanton solution exists only for $m > \sqrt{E_a/E}$. Since $S_{\mathrm{inst}}(z(m), m)$ has always a negative derivative with respect to m at $m \to \sqrt{E_a/E}+0$, but for reasonably large values of s the interval of m with negative derivative is much shorter than 1 and hence the optimal hopping length $m_s(E)$ is the smallest integer exceeding $\sqrt{E_a/E}$, which we denote as $\left[\sqrt{E_a/E}\,\right]_{G+}$. To be more realistic we have to take into account the randomness of the impurity distances a_i such that decreasing the field (or the temperature), the current jumps by a factor $\sim e^{-2a_m/\xi_{\mathrm{loc}}}$. Clearly, for long wires these jumps will average out.

Finally, we briefly compare the present case of Poissonian strong disorder, $u_{\mathrm{eff}}a \gg 1$ with the Gaussian weak disorder, $u_{\mathrm{eff}}a \ll 1$ considered in [10,13,14,19]. In the latter case u and a are sent simultaneously to zero but the quantity $u^2/a \sim \xi_0^{-3} \ll k_F^3$ is assumed to be finite, ξ_0 denotes the bare correlation length. Fluctuations on scales smaller than ξ_0 renormalize $\xi_0 \to \xi \sim k_F^{-1}(\xi_0 k_F)^{3/(3-2K)}$. At low T the conductivity is of variable range hopping type (14) up to a temperature $T_{\mathrm{loc}} = \hbar v/\xi = T_a(u/u_c)^{2/(3-2K)}$. For higher T there is a direct crossover to $\sigma \sim T^{2-2K(T)}$ where K is now renormalized by disorder fluctuations [19,20]. This renormalization disappears only at much higher $T_a \sim \hbar v/a$. Both weak and strong pinning theories should roughly coincide for $u \to u_c$ where

$T_a \approx T_{1,\mathrm{cr}} \approx T_{\mathrm{loc}}$ which is indeed the case since $\xi \approx a$. In the strong pinning region ξ continues as $\xi \sim a/s$.

To summarize we have calculated the linear and nonlinear conductivity of Luttinger liquidss in the strong pinning regime using an instanton approach. For very low fields E and temperature T the conductivity is of variable range hopping type, eqs. (13) and (14), as for weak pinning. For higher and E and T there is a crossover to power law behavior (10) and (11). Quantum fluctuations reduce the regime of strong pinning to $u > u_c \sim k_F(k_F a)^{K-1}$ for $T < T_a$ and to $u > u_1 \sim k_F(T/\epsilon_F)^{1/(1-k)}$ for $T > T_a$. The determination of the position of the various crossover temperatures allows the construction of the global phase diagram Fig. 1 for the linear conductivity as well as the crossover between the linear and nonlinear conductivity Fig. 2 in the case of strong impurities.

Experimentally, a linear variable range hopping conductivity has been seen in carbon-nanotubes [3] and polydiacetylen [4].

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