Weakly interacting Bose gas in disordered environment*

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arXiv: 0811.1269 (Phys. Rev. B)

Outline

Introduction

Interacting Bose gas in weak random potential

Strong disorder

Bosons in lower dimensions

Bosons in traps

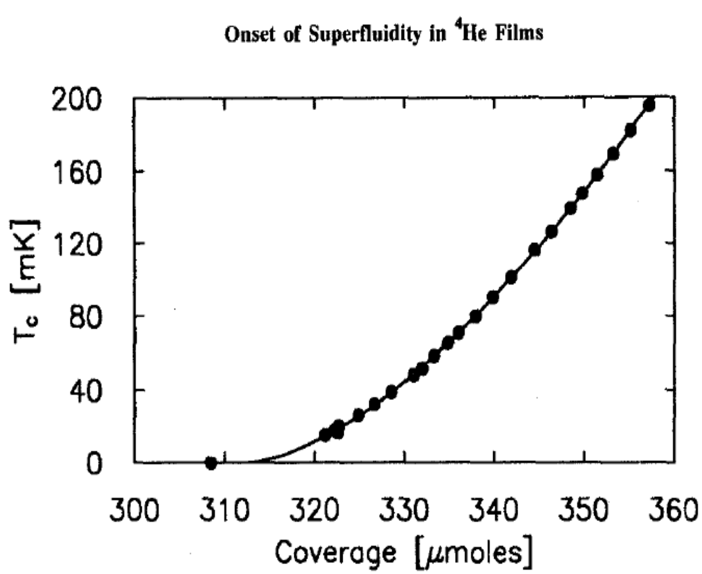
Introduction

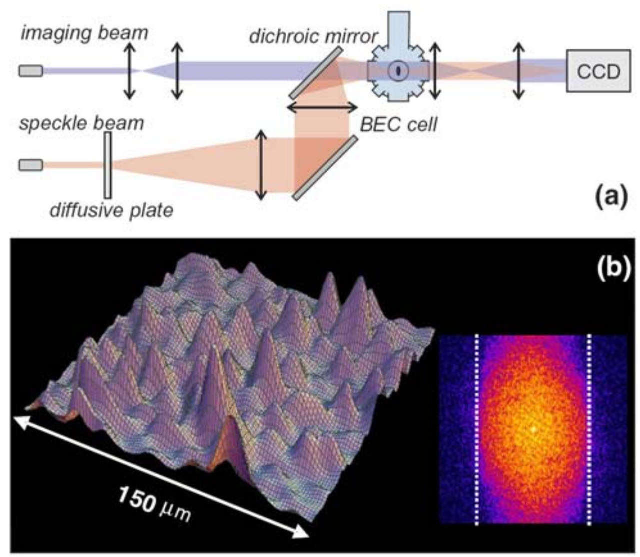
BEC : finite part of atoms in the state with minimal energy.

Examples: Superfluid ^{4}He, laser cooled atoms in a trap*

Disorder: Superfluid He in porous media (J.D. Reppy et al '92)
Cold atoms in speckle potential (R.G. Hulet et al. '08)

Breakdown of superfluidity at strong disorder





*Other examples: excitons in semiconductors, BEC of spin waves

Phase Coherence and Superfluid-Insulator Transition in a Disordered Bose-Einstein Condensate

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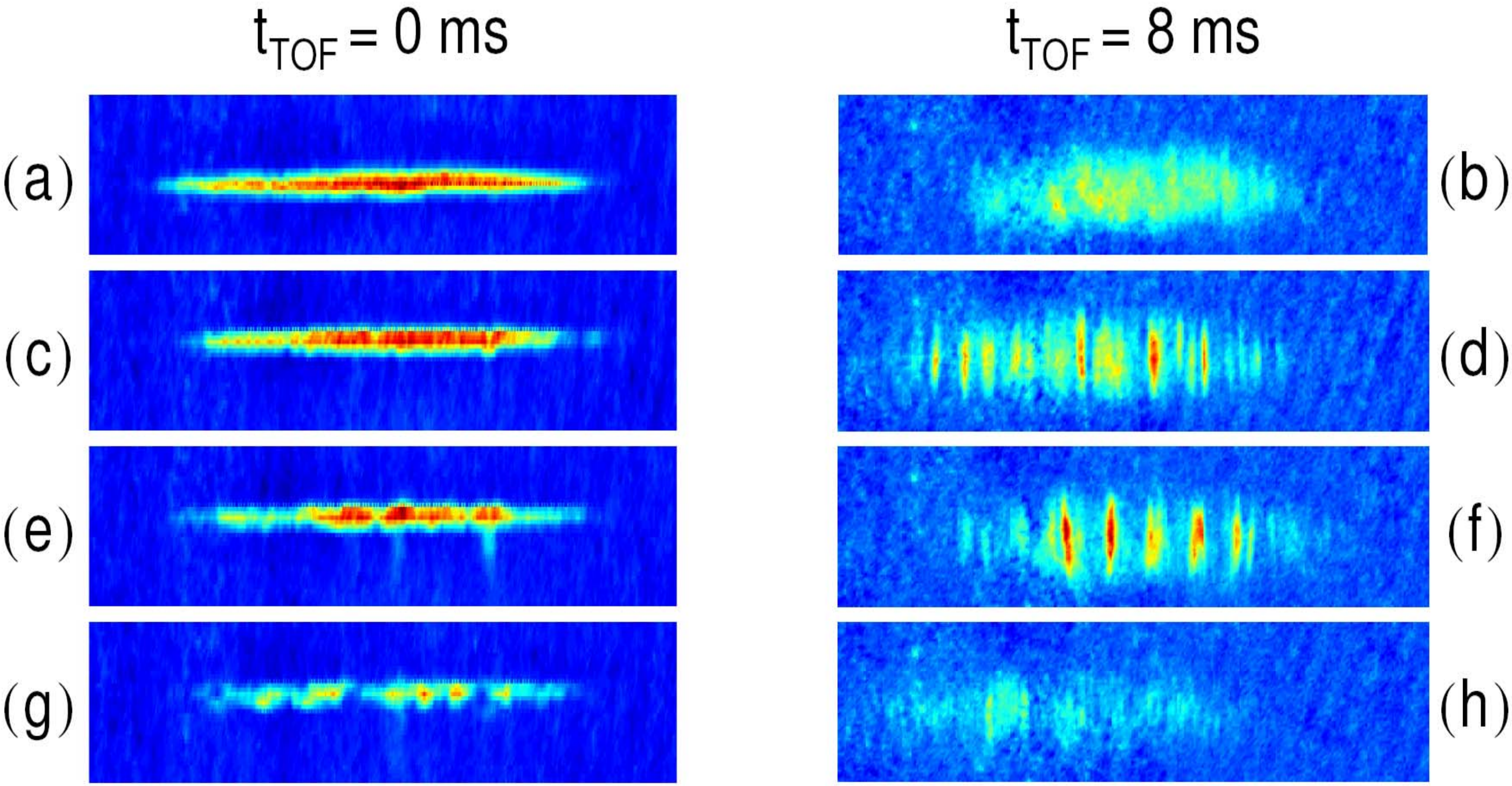
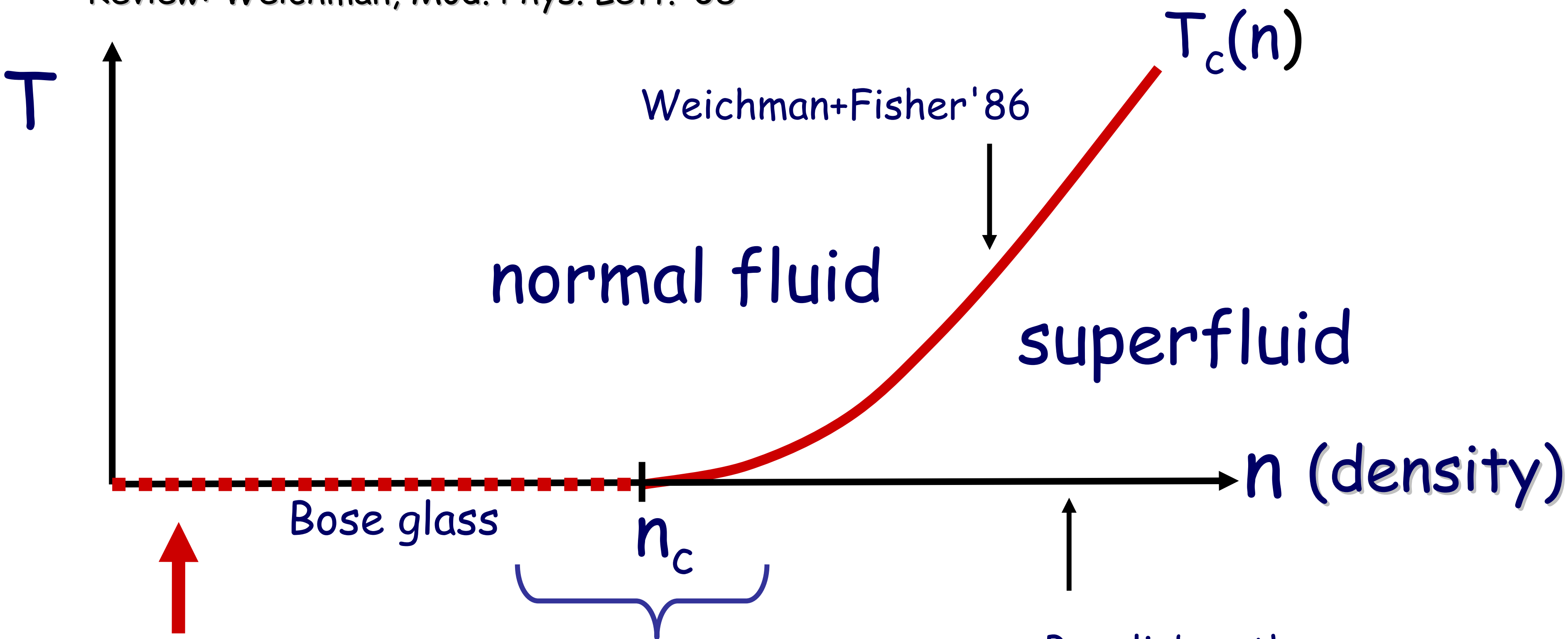
(Dated: October 30, 2007)

Hulet et al



Bosons in disordered environment:

Review: Weichman, Mod. Phys. Lett. '08



this work

Shklovskii '08
Babichenko² '08
Shklovskii+Müller '08

Halperin, Lee+ Ma'86

Giamarchi+Schulz '87 (d=1)

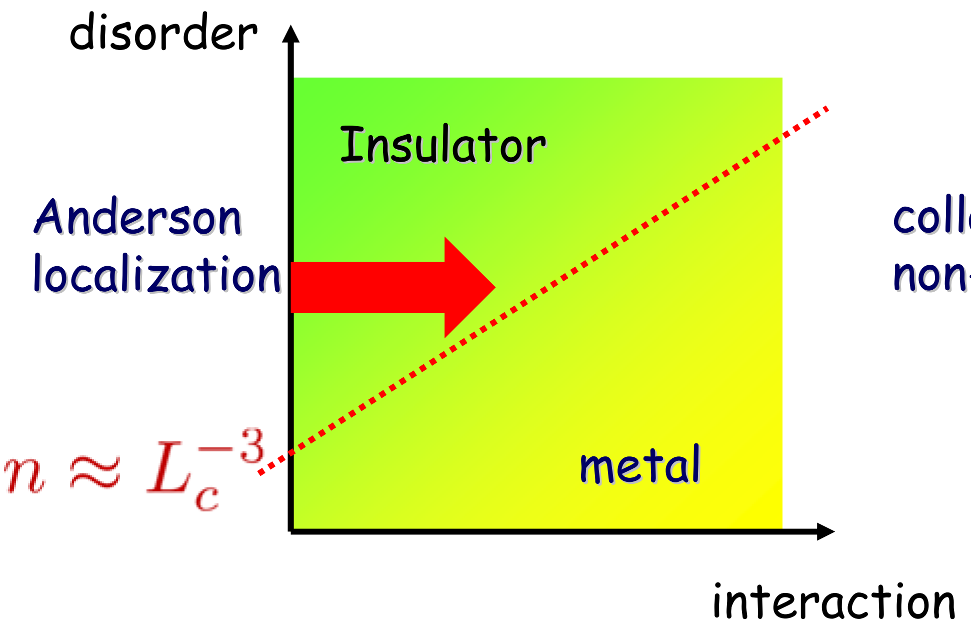
Fisher, Weichman, Grinstein+Fisher '89
$\rightarrow$ z=d

Bogoliubov theory
Huang+Meng '92

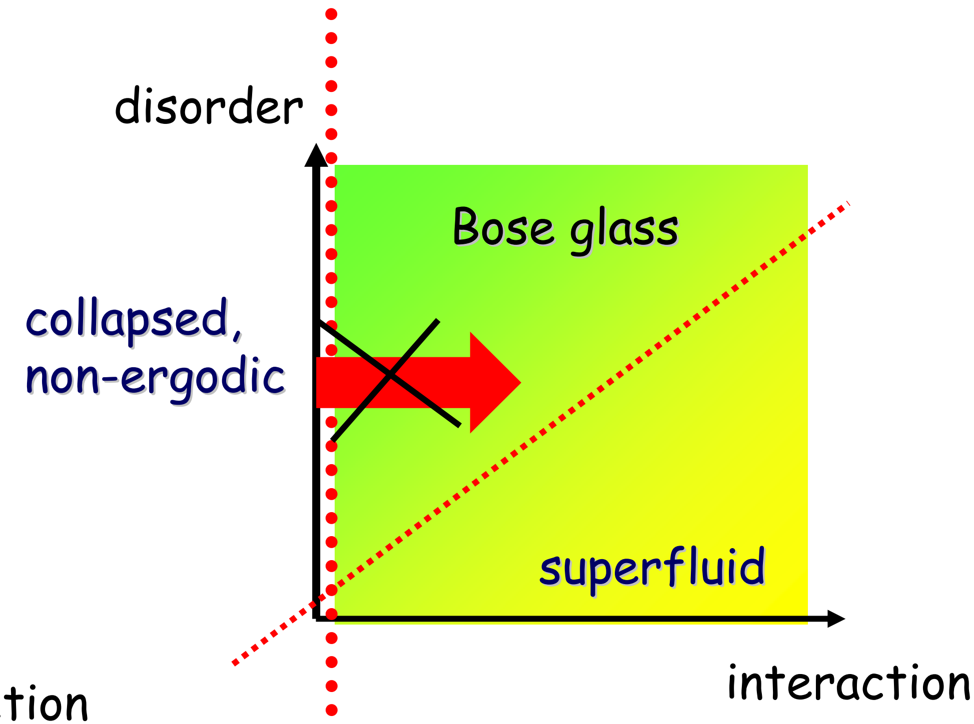
$$\rho_s \approx \rho_0[1 - c_1\sqrt{n_c/n}]$$

Cross-Pitaevskii equation

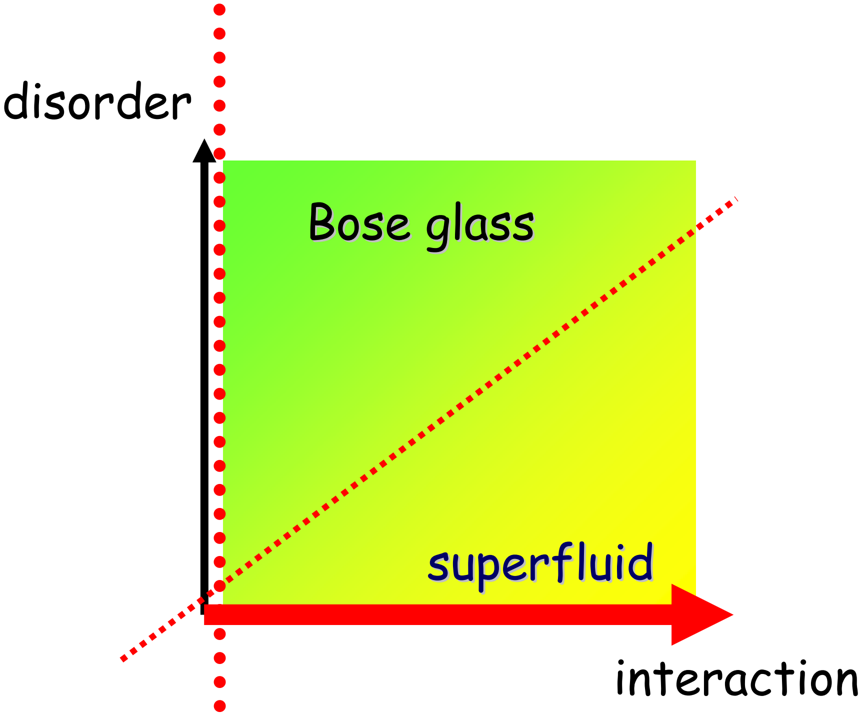
Fermions



Bosons



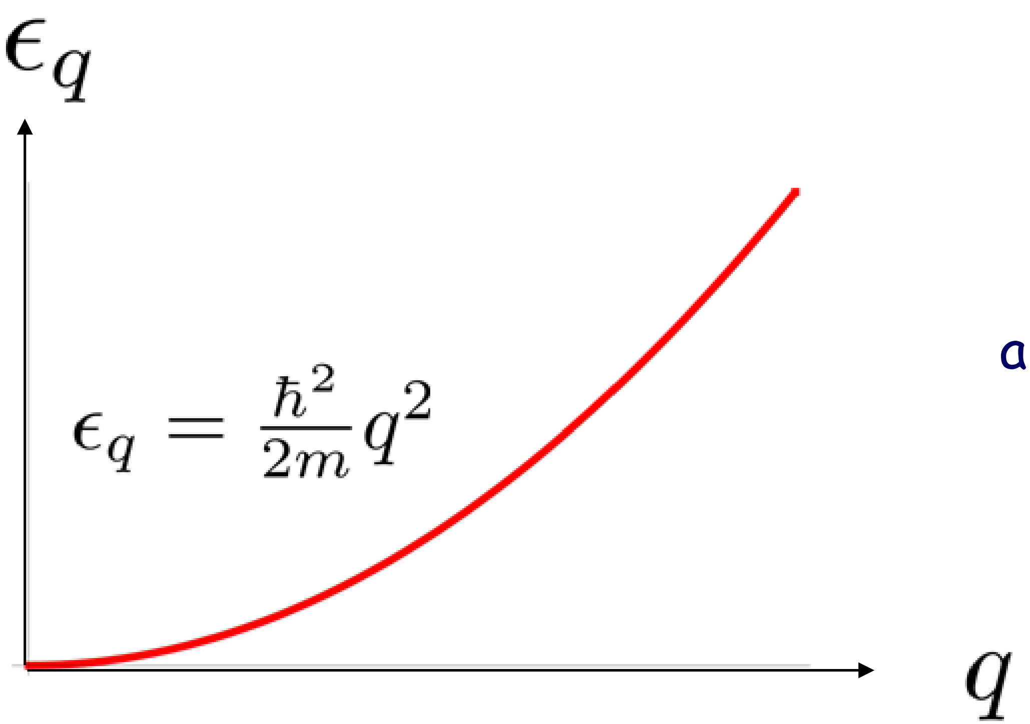
No Disorder



(a) BEC: Ideal bosons T=0

$$\mathcal{H} = -\frac{\hbar^2}{2m}\int d^d x \hat{\Psi}^\dagger(x)\nabla^2\hat{\Psi}(x), \quad \hat{\Psi} = \sum_q e^{iqx}\hat{a}_q$$

$$N = nV = \int d^d x \langle \hat{\Psi}^\dagger(x)\hat{\Psi}(x)\rangle$$



$\boxed{\mu=0}$

all particles in ground state

BEC : Interacting bosons

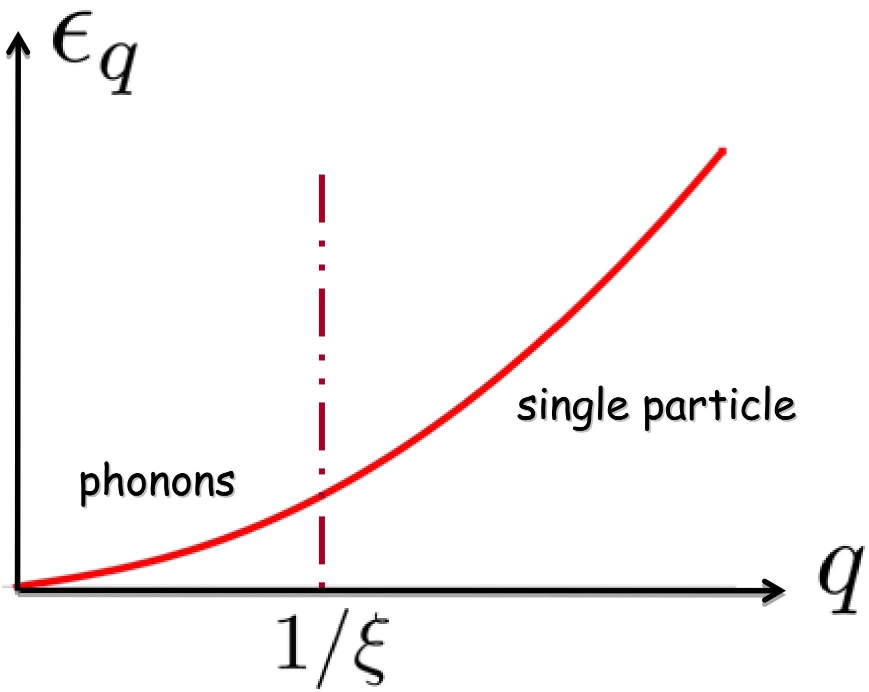
$$\mathcal{H} = \frac{\hbar^2}{2m} \int d^3x \hat{\Psi}^\dagger [-\nabla^2 + 4\pi a \, \hat{\Psi}^\dagger \hat{\Psi}] \, \hat{\Psi}$$

$$\hat{\Psi}^\dagger \hat{\Psi} = \hat{n}$$

Scattering length

$$\hat{\Psi}^{(\dagger)} = \Psi_0 + \delta \hat{\Psi}^{(\dagger)}$$

Bogoliubov transform $\Rightarrow$ $$\epsilon_q^2 = \frac{\hbar^2 q^2}{m} \left(\frac{\hbar^2}{m\xi^2} + \frac{\hbar^2 q^2}{4m} \right)$$

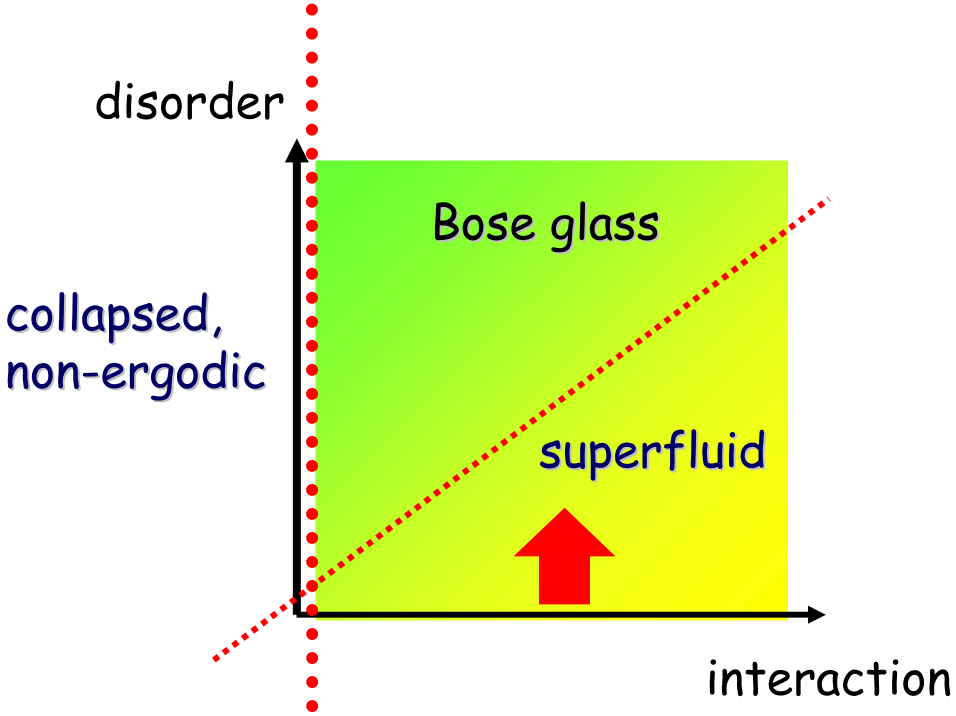


$$\mu = \frac{\hbar^2}{m\xi^2} \left(1 + \frac{c}{n^{1/3}\xi} \right)$$

Lee & Yang '57

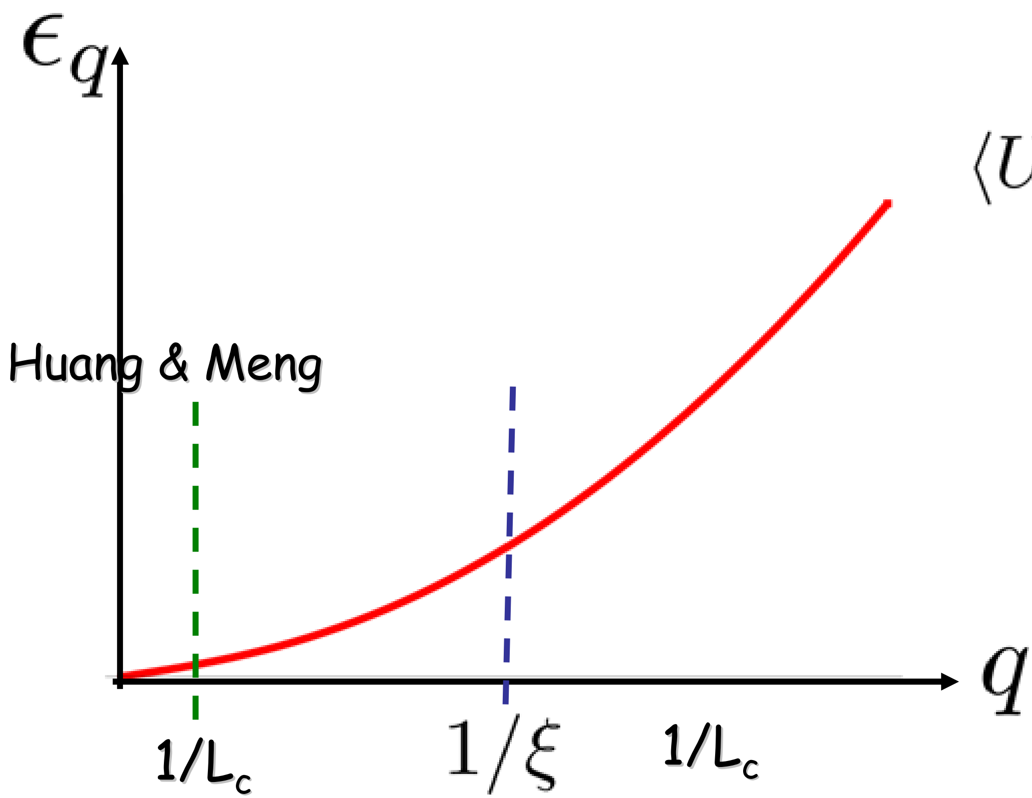
healing length $$\xi = \frac{1}{\sqrt{4\pi a n}}$$

Weak Disorder



(d) Weakly repulsive bosons in a weak random potential

$$\mathcal{H} = \int d^3x \Psi^\dagger \left(-\frac{\hbar^2}{2m}\nabla^2 + U(\mathbf{x}) + \underbrace{\frac{2\pi\hbar^2 a}{m}}_{g} \Psi^\dagger \Psi \right) \Psi$$



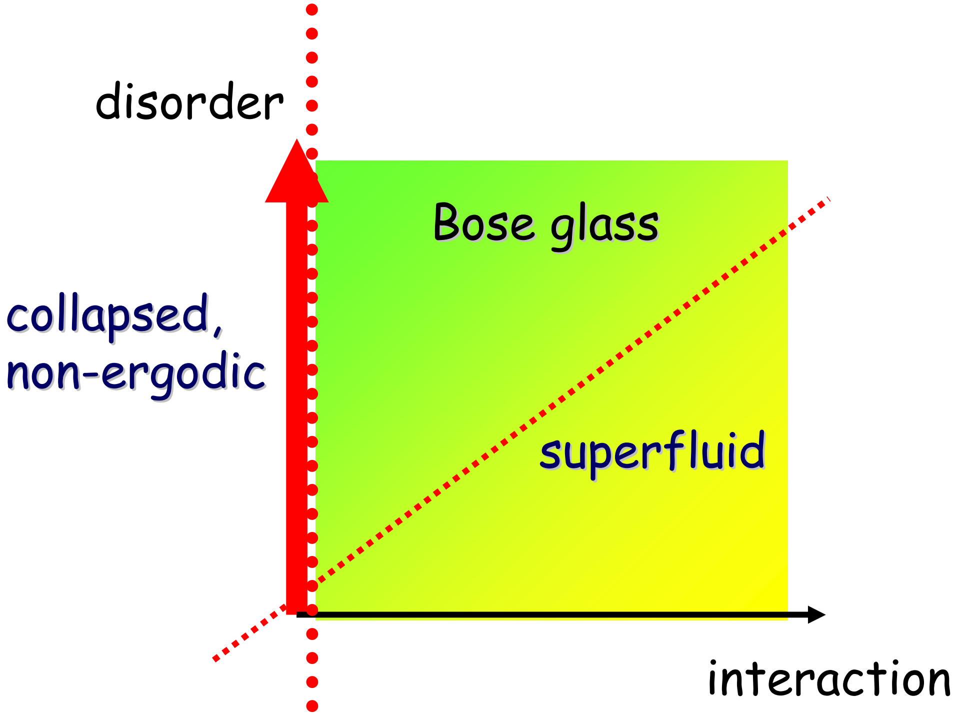
$$\langle U(\mathbf{x}) U(\mathbf{x}') \rangle = \kappa^2 \delta(\mathbf{x} - \mathbf{x}')$$

mean free path

$$L_c = \frac{\hbar^4}{m^2 \kappa^2}$$

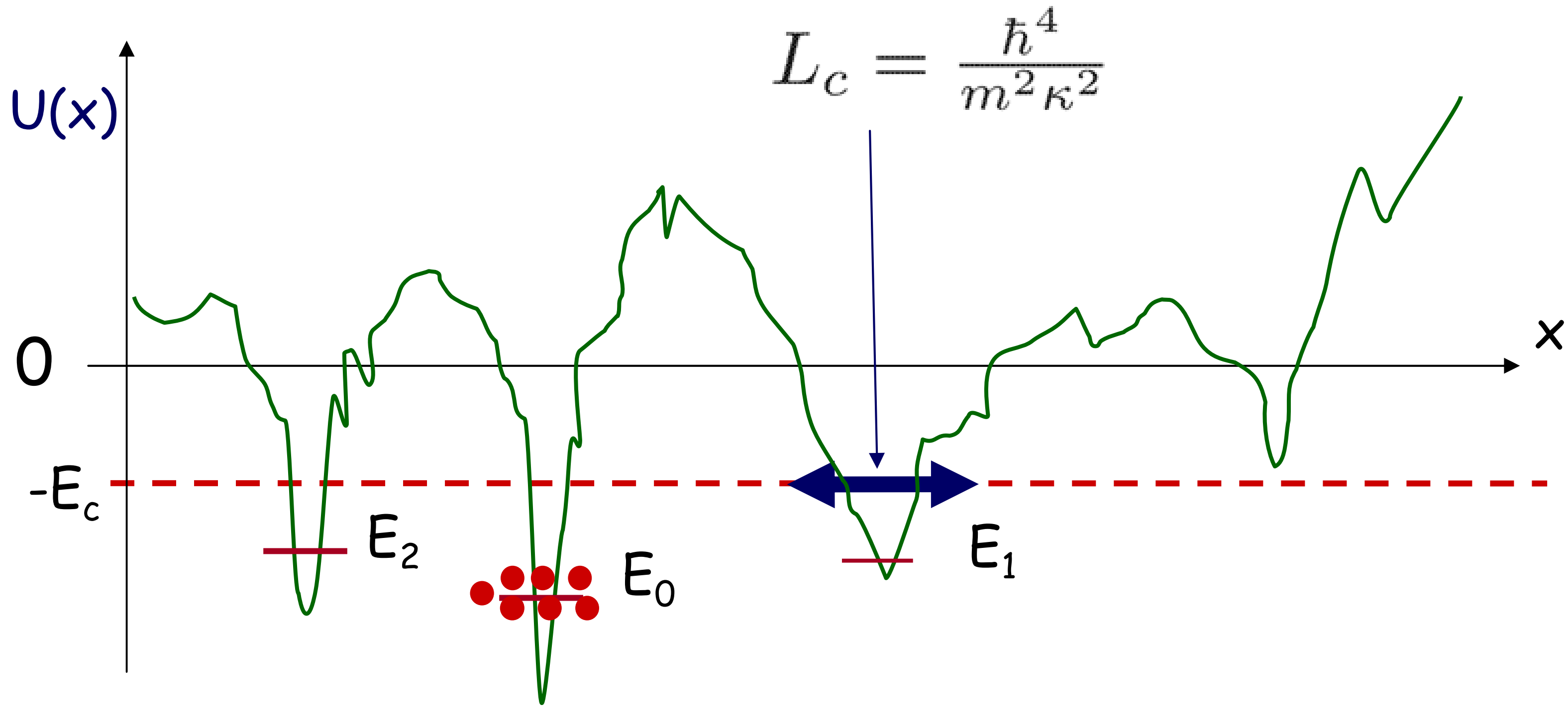
$$\Delta\mu = \Delta\epsilon \approx \int_0^{\xi^{-1}} \frac{d^d q}{h^3} \frac{\kappa^2}{c_s q} \sim \frac{\hbar^2}{m\xi^2} \left(\frac{\xi}{L_c} \right)^{4-d}, \quad d > 1$$

Strong Disorder, no Interaction



Ideal 3d Bose gas in random potential

$$\frac{\hbar^2}{2m}\nabla^2\psi + (E - U(\mathbf{x}))\psi = \mathbf{0} \quad \langle U(\mathbf{x})U(\mathbf{x}')\rangle = \kappa^2\delta(\mathbf{x}-\mathbf{x}')$$

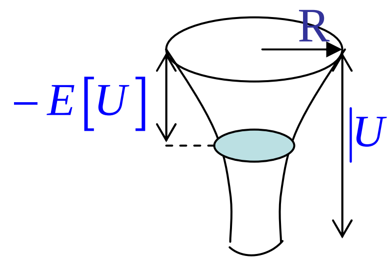


$$E_c = \frac{\hbar^2}{2m{L_c}^2}$$ energy scale of localized states

T=0: All particles in ground state $E_0 \approx -E_c \ln^2(L_0/L_c)$

Single particle density of states DOS $E \to -\infty$

$$\nu(E,V) = \frac{1}{V}\int \delta\left(E - E\left[U(\mathbf{x})\right]\right) dW\left[U(\mathbf{x}), V\right]$$



Consider potential fluctuation of depth U and width R

probability $W[U] \sim \exp\left[-U^2 R^d / 2\kappa^2\right]$

$\to$ localized state of energy $E \sim \hbar^2/(2mR^2) + U$

Contribution of DOS at energy $E \sim \max W[E - \hbar^2/(2mR^2)]$

Maximize W with respect to R $\Rightarrow$ $R = L_c(E_c/|E|)^{1/2}$

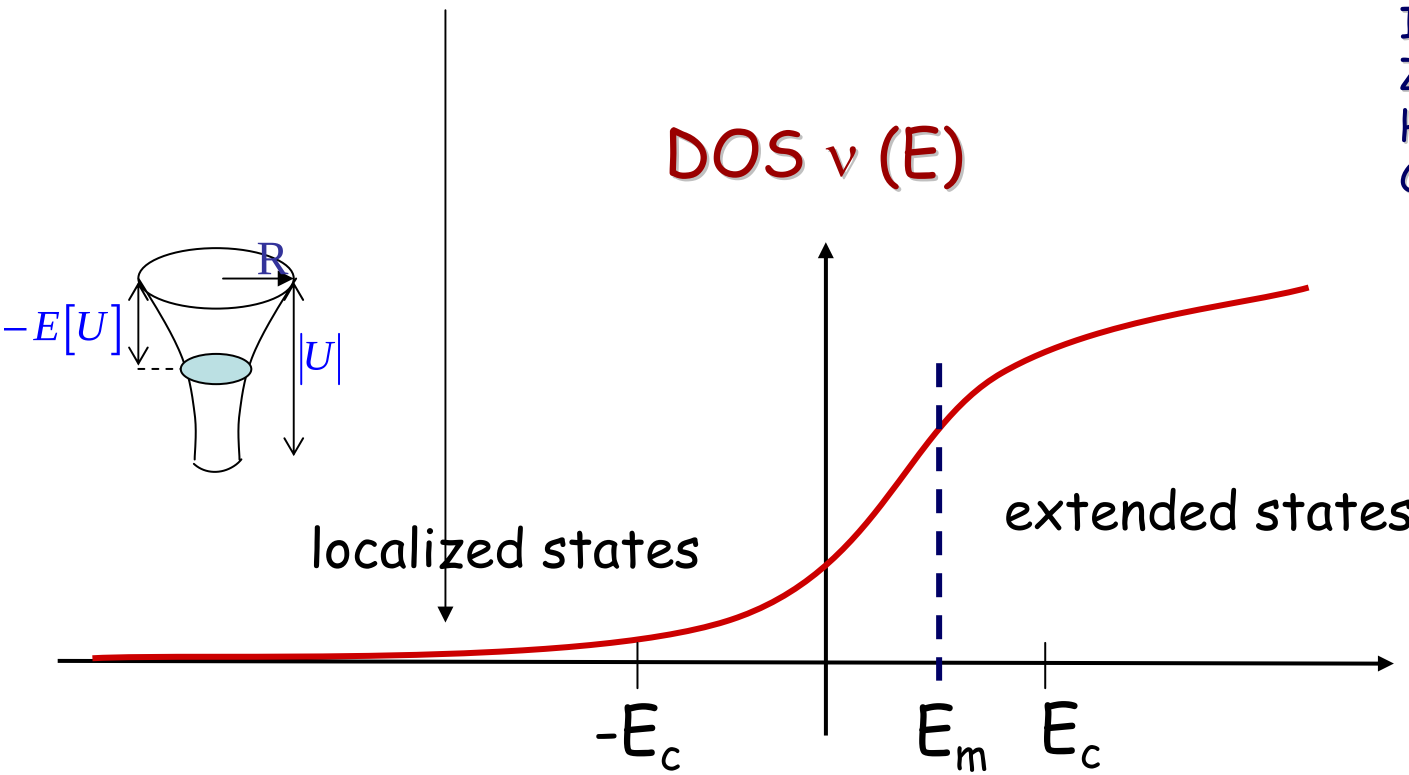
$\to$ $\boxed{\nu \sim \exp\{-(|E|/E_c)^{(4-d)/2}\}}$

Ideal Bose gas in random potential

DOS for E « - E_c dominated by wells of width $R \sim \hbar/\sqrt{m|E|} \ll L_c$

$$\nu(E) = \frac{1}{V}\langle\delta(E - E[U(\mathbf{x})])\rangle \sim |E|^{3/2} e^{-\sqrt{|E|/E_c}}$$

I.M. Lifshitz '66,
Zittartz and Langer '66,
Halperin and Lax,'66
Cardy '78

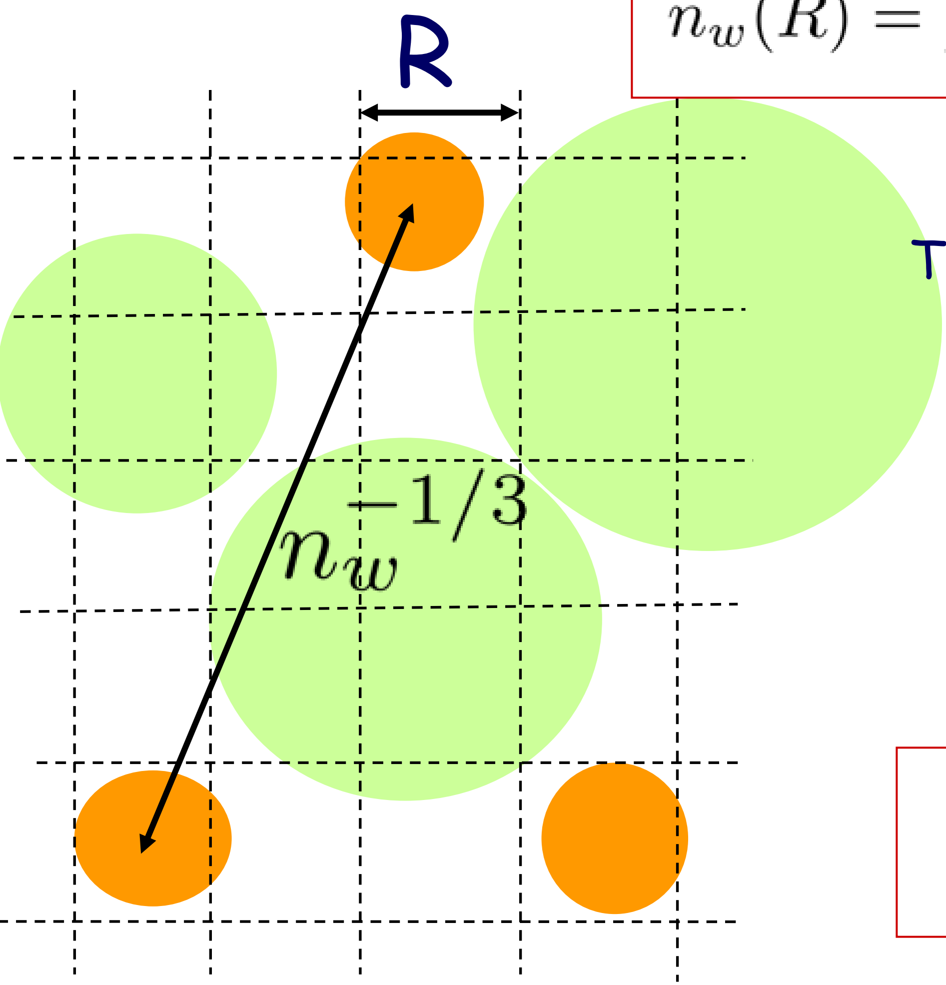


$E \sim -\hbar^2/mR^2$

Ideal Bose gas in random potential

Spatial density $n_W(R)$ of wells with radius < R<< L_c (E <- $h^2/(2mR^2)$<<E_c)

$$n_w(R) = \int_{-\infty}^{-\frac{\hbar^2}{2mR^2}} dE\ \nu(E) \sim \frac{L_c}{R^4} e^{-L_c/R}$$



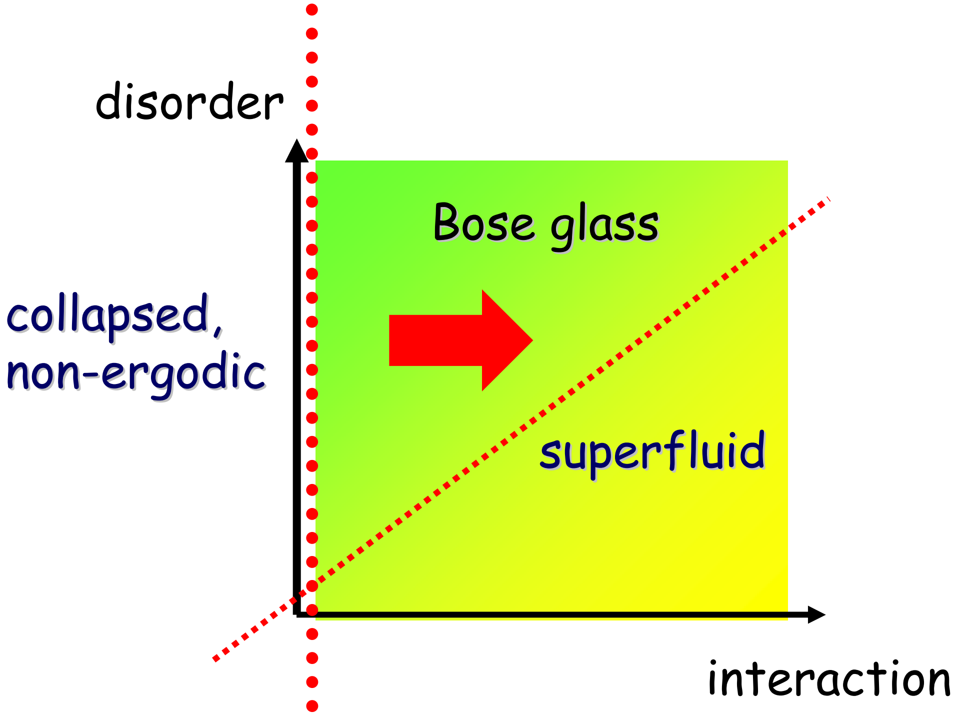
Tunneling amplitude t(R) between wells with radius < R :

$$t(R) = \exp\left(-\frac{1}{\hbar}\int |p| dl\right)$$

$$\frac{1}{\hbar}\int |p| dl \approx n_w^{-1/3}/R \sim e^{L_c/3R}$$

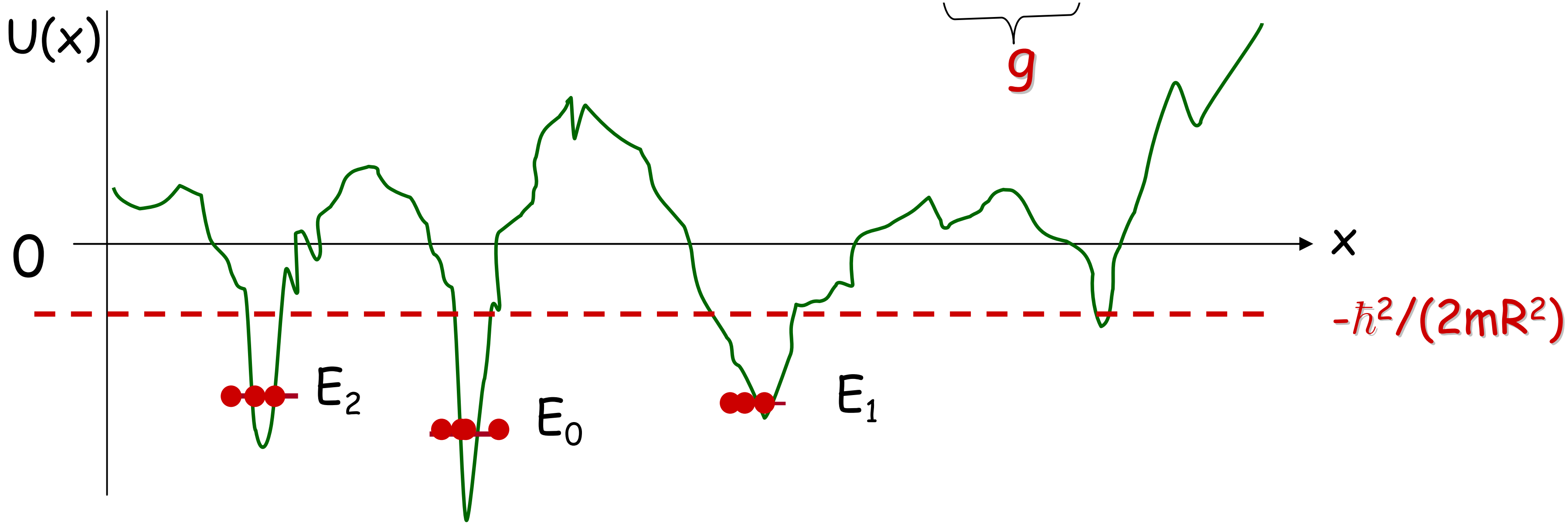
$$t(R) \sim e^{-\left(\frac{R}{L_c} e^{L_c/R}\right)^{1/3}}$$

Strong Disorder + Interaction



Weakly repulsive bosons in a random potential

$$\mathcal{H} = \int d^3x \Psi^\dagger \left(-\frac{\hbar^2}{2m}\nabla^2 + U(\mathbf{x}) + \underbrace{\frac{2\pi\hbar^2 a}{m}}_{g} \Psi^\dagger\Psi \right)\Psi$$



Assume that all potential wells with radii up to R are filled:

$\Rightarrow$ number of particles per well of size R : $N_w(R) = n/n_w(R) \gg 1$

$\Rightarrow$ repulsion energy per particle: $E_g(R) \approx g\, N_w/R^3 \sim g\, n\, e^{L_c/R}$

$\Rightarrow$ total energy per particle: $\mu(R) = -\hbar^2/(2mR^2) + E_g(R)$

Weakly repulsive bosons in a random potential

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Mininization over R: $\Rightarrow$ $R(n)=L_c/\ln(n_c/n)$,

$$\boxed{n \ll n_c \approx 1/(3L_c^2 a)}$$

(non-interacting Fermions: Ioffe-Regel $a \to L_c$)

$$\boxed{\mu(n) = -\frac{\hbar^2}{2mR^2(n)} = -\frac{1}{2}E_c(\ln\frac{n_c}{n})^2}$$

$$\frac{n_c}{n} = \frac{\xi^2}{{L_c}^2}$$

Variable hopping conductivity:

Absence of interaction: probability that two localized states have the same energy is zero.

Switch on interaction: energy levels split by amount gn_p. If $n \ll n_c$ wave function is still localized .

$\rightarrow$ T=0 conductivity (response to external force) in Bose-glass is still zero.

Tunneling probability between wells of distance L is $\sim \exp\{-2L/R\}$

$\rightarrow$ hopping probability $P(T) \sim \exp\{-2L/R-\Delta E/T\}$

$\Delta E\, \nu(E)\, L^3 \approx 1$, use relation R(n) and maximize P(T) with respect to hopping distance L $\Rightarrow$

$$\boxed{\sigma(T) \sim e^{-C[E_c n_c/(Tn)]^{1/4}}}$$

Preliminary conclusions

$\Rightarrow$ At n << n_c Bose gas decays into fragments,
particle density in fragments each of density $n_c \sim 1/(aL_c^2)$

$\Rightarrow$ tunneling exponentially suppressed: $t(n) \sim e^{-c(n_c/n)^{1/3}}$

$\Rightarrow$ particle number in fragments $N_w = L_c / \left[3a(\ln \frac{n_c}{n})^3\right]$ well defined

$\Rightarrow$ phase uncertain, no phase coherence $\Rightarrow$ no superfluidity

$\Rightarrow$ finite compressibility $\frac{n}{E_c} \ln\left(\frac{n_c}{n}\right)$ „Bose glass"

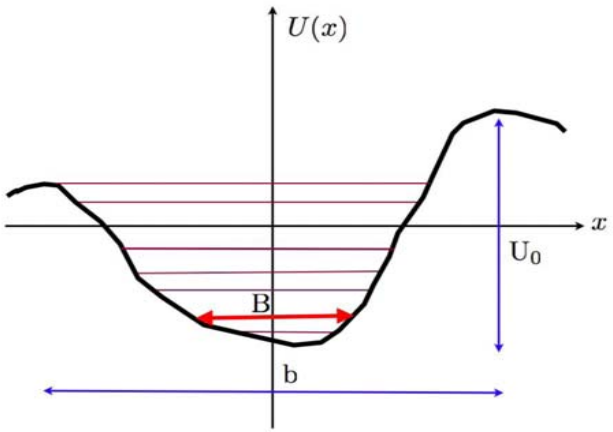
$\Rightarrow$ $\hat{H}_{\text{eff}} = \sum_j C_j(\hat{N}_j - \langle N_j \rangle)^2 - \sum_{i,j} t_{ij} \cos(\hat{\phi}_i - \hat{\phi}_j)$

$\Rightarrow$ charged bosons VRH $\sigma(T) \sim e^{-C[E_c n_c/(Tn)]^{1/4}}$

For $n \approx n_c$ i.e. fragments merge $\rightarrow$ transition to superfluid

Correlated disorder

$$\langle U(\mathbf{x})U(\mathbf{x}')\rangle = \frac{U_0^2}{b^3} e^{-|\mathbf{x}-\mathbf{x}'|/b}$$



$\Rightarrow$ 2 length scales b , $B=(\hbar^2/(mU_0))^{1/2}$

b << B $\Rightarrow$ uncorrelated disorder

b >> B $\Rightarrow$ new results

$$\nu(E) \sim |E|^3 \exp(-E^2/2U_0^2)$$

Keldysh & Proshko '63
Kane '63
Shklovskii and Efros '70
John & Stephen '84

$$\mu(b,n) \approx -U_0\sqrt{2\ln(\frac{n_c}{n})}$$

$n \ll n_c \sim 1/(B^2 a)$

$n_w(E)=b^{-3}\exp\{E^2/2U_0^2\}$

Generalization to d<3 dimensions

What is different?

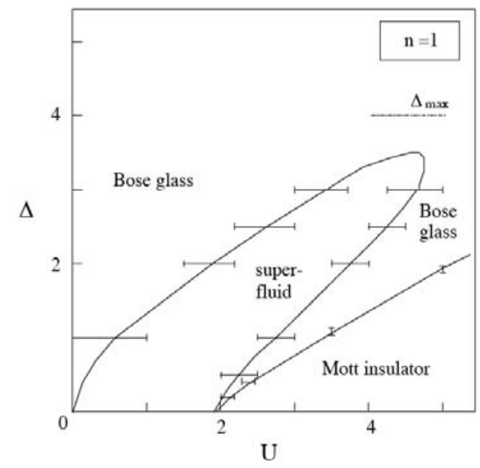
DOS,

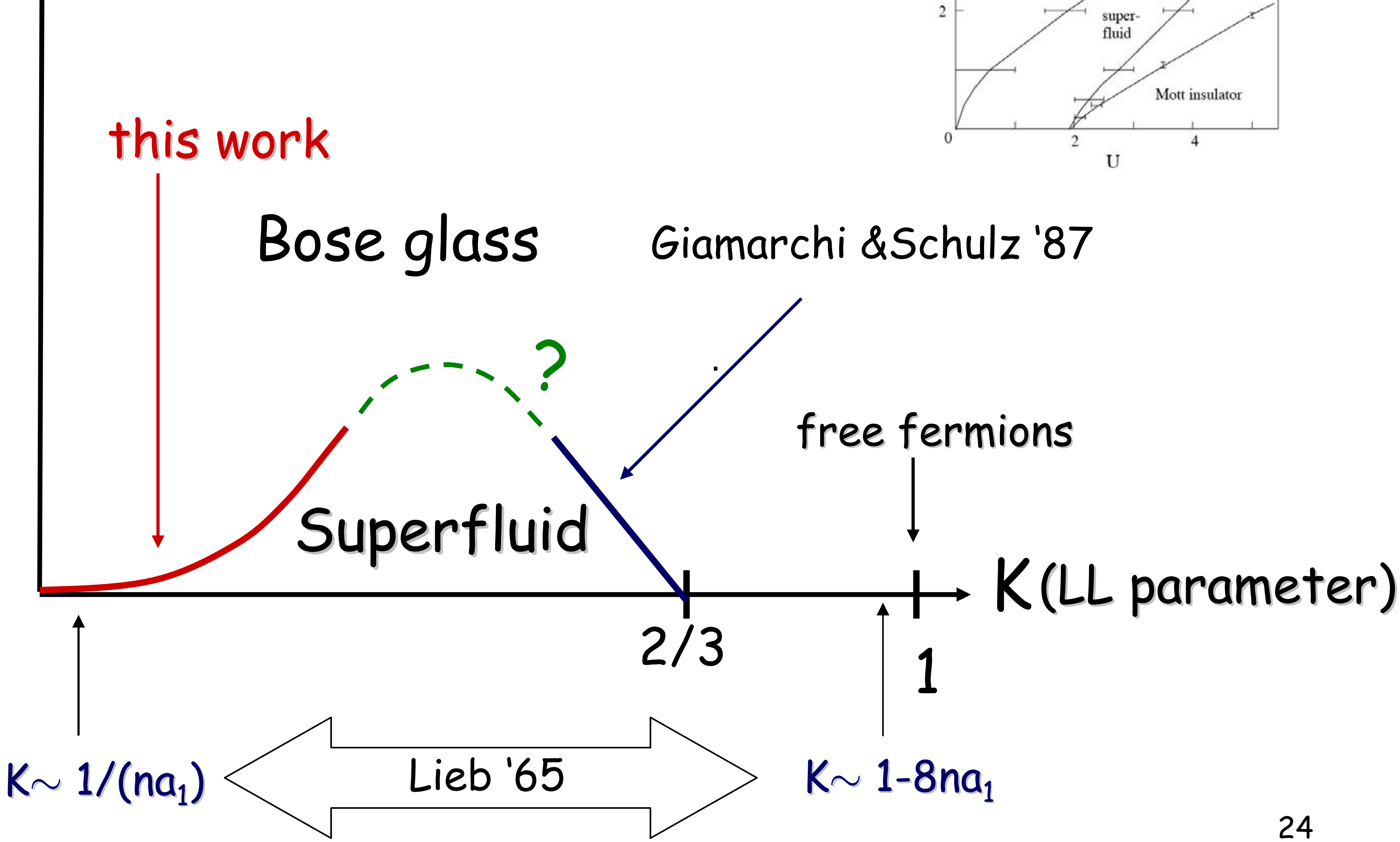
$a \to a_d^{d-2} = a\, r_\perp^{d-3}$

ξ, L_c, E_c

$n/n_c \sim n/L_c^2 a_d^{d-2}$

Bose gas in one dimensions





Bosons in traps

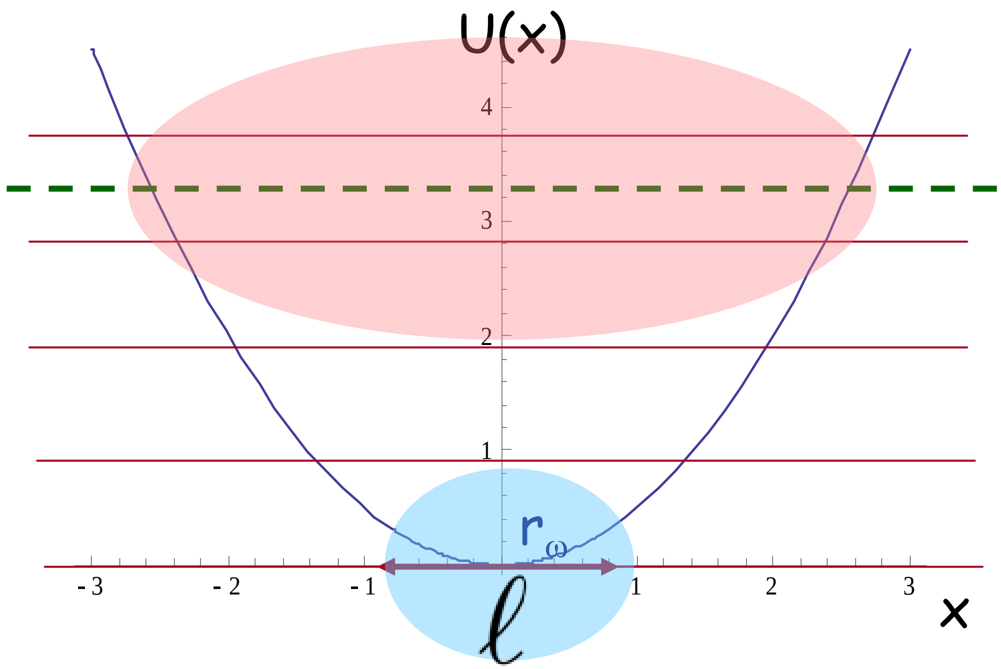
Ideal quantum gas in a harmonic trap

- oscillator length $\ell = (\hbar/m\omega)^{1/2}$ ($\approx$ 1000nm), $\hbar\omega \approx$ nK

- Bosons: T=0: all particles in ground state of size ℓ

- T_c : $\lambda_T^3 n \sim \lambda_T^3 N/R^3 \approx 1$ $\Rightarrow$ $\boxed{T_{c0} \sim \hbar\omega\, N^{1/3}}$

$\lambda_T = (h^2/Tm)^{1/2}$



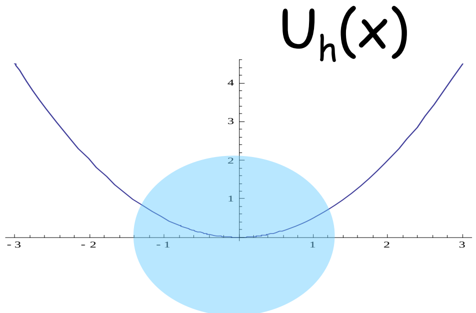
$m\omega^2 R^2 \approx T$ $N \approx 10^3 \ldots 10^8$

Fermions: $\varepsilon_F \sim T_{c0}$

$R_F \approx r_\omega N^{1/6}$

Bosons in traps (uncorrelated disorder)

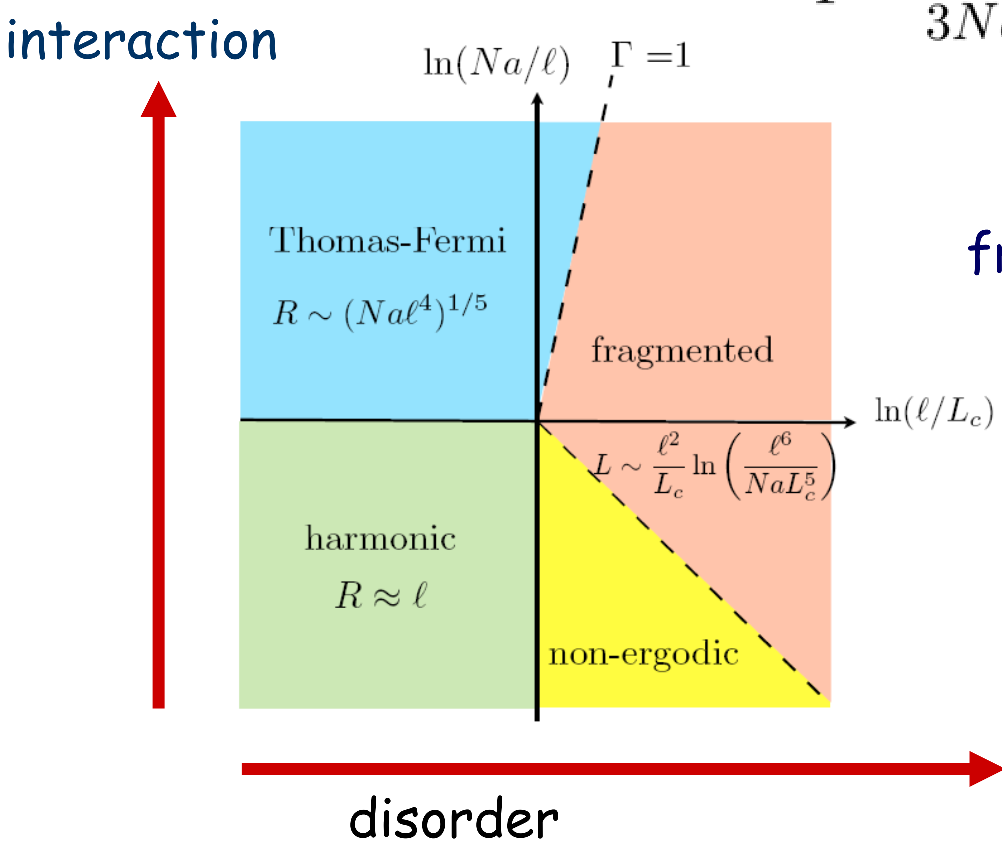
oscillator length $\ell = (\hbar/m\omega)^{1/2}$ ($\approx$ 1000nm), $\hbar\omega \approx$ nK



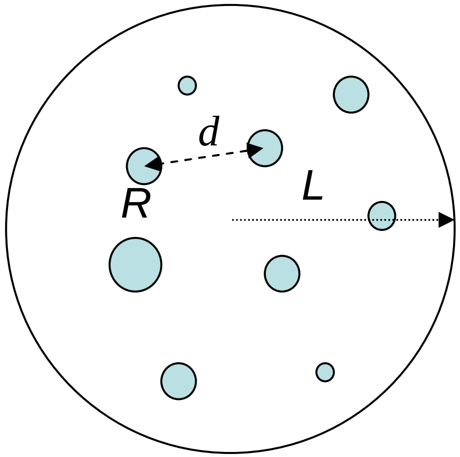
$$\mu(R) = -\frac{\hbar^2}{2mR^2} + E_{int}(R) + \frac{\hbar^2}{2m}\frac{R^2}{\ell^4}$$

$$\Gamma = \frac{\ell^6}{3NaL_c}$$

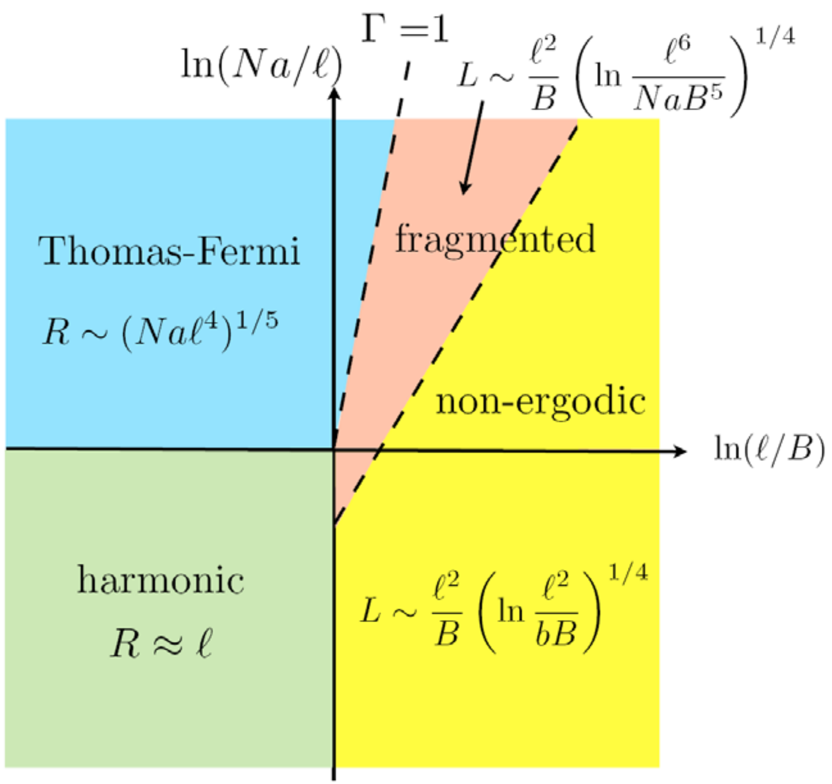
$$n(r) = n_c \left(\frac{n}{n_c}\right)^{\sqrt{1+r^2/r_F^2}},$$



fragmented state

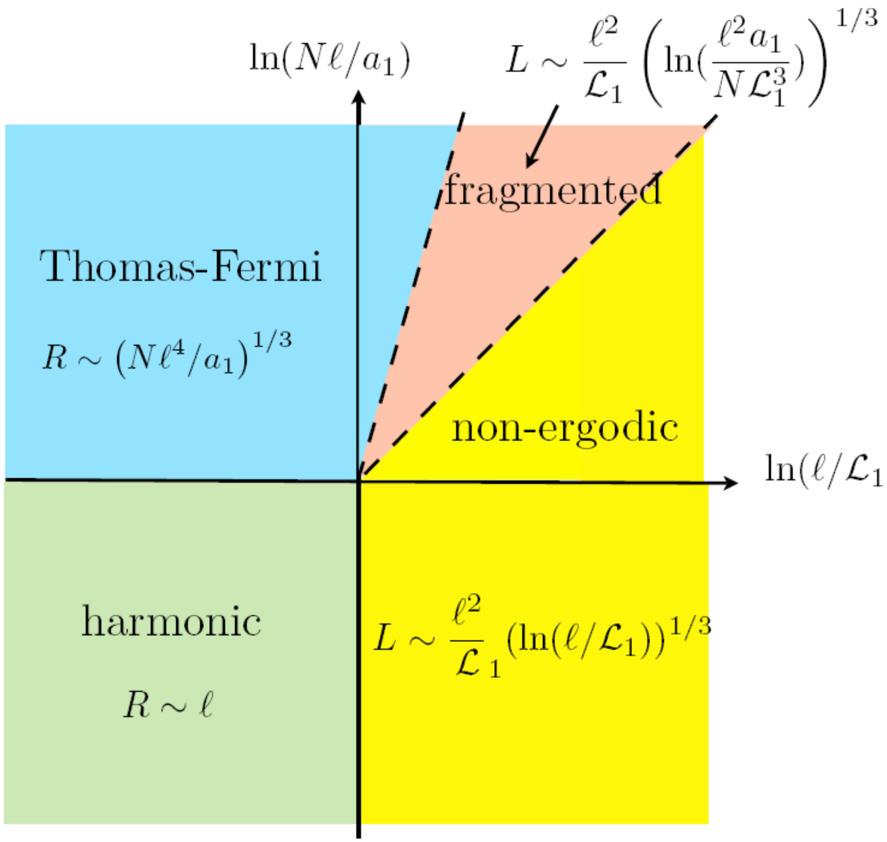


Bosons in traps (correlated disorder, d=3)

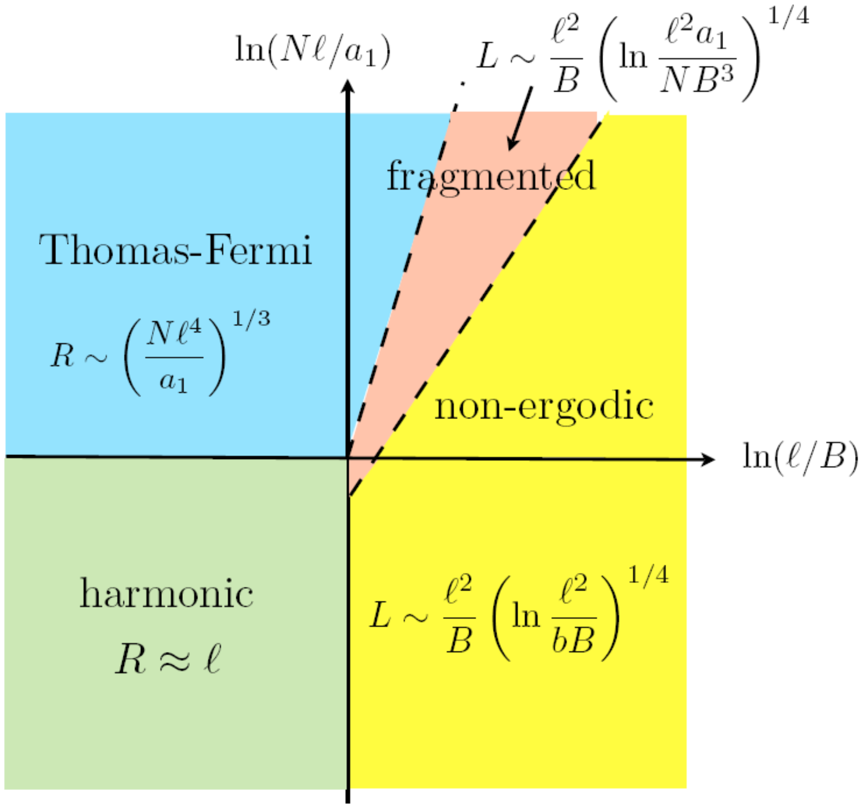


$$\Gamma = \frac{\ell^6}{NaB^5}$$

Bose gas in 1 dimensions: parabolic trap



Uncorrelated disorder



Correlated disorder

Prediction which could be tested

1. Cloud size as function of these parameters in fragmented state?

2. Cross-over from non-ergodic to ergodic state at critical N $N_c = L_c/3a$, $N_c = b^3/(3aB^2)$, number of particles in fragments?

3. Time of flight spectroscopy, $\Delta p \sim \hbar/R$, fragmented state should be visible.

4. Ground state reachable? According to our estimates ($L_c \approx 1\ \mu$ m) relaxation time $\approx$ 0.06s. Easier in lower dimension.

Changeable parameter: N, ω, a, U_0, b

Conclusions

- Semi-quantitative analysis of the phase states of a weakly interacting strongly diluted Bose gas in a random Gaussian potential.
- The system is charcterized by the mean free path L_c and the scattering length a (or a, U_0 and B for correlated disorder)
- At particle density $n \ll n_c \approx 1/(aL_c^2)$ the Bose particles occupy deep potential wells and exponentially weakly tunnel to other wells. The number of particles in each well is defined, but phases are uncertain.
- At average particle density $n \approx n_c$ the transition to the superfluid proceeds.
- In a trap the oscillator length l appears as a new length scale. Four different regimes are found, depending on the mutual strength of L_c, aN and l , respectively.
- All results can be extended to lower dimensions and to correlated disorder.