

Quantum Creep and VRH in 1D Disordered Electron Systems

Transport in Luttinger liquids and 1D CDWs: strong pinning and global regime diagram

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Outline

- Systems and model
- Single impurity
- Gaussian impurities

- Weak and strong pinning
- Instantons for strong pinning
- T, E-dependence of (non-) linear conductivities
- Conclusions

Systems under consideration

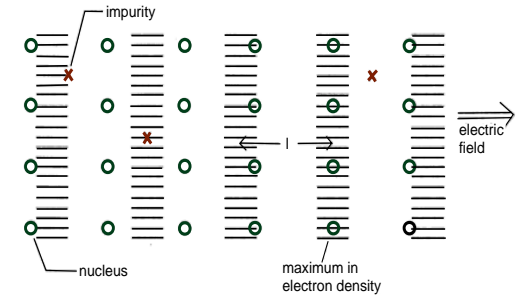
1D Quantum Fluids (Luttinger '63, Haldane '81)

$$\hat{\psi}^\dagger_B(x) = \sqrt{\hat{\rho}(x)} e^{i\hat{\theta}(x)}, \quad [\partial_x \hat{\theta}(x), \hat{\phi}(x')] = i\pi \delta(x - x')$$

1D Mass-, Charge-, Spin-, Flux-Density Waves

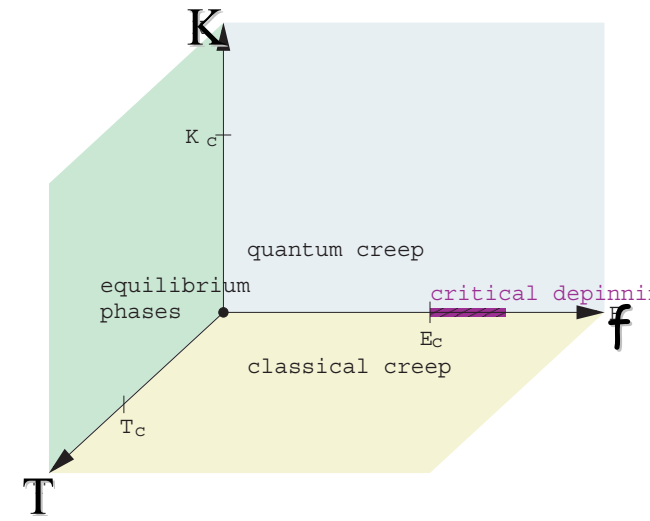
$$\rho(\mathbf{x}, \phi) \sim \nabla \phi + \rho_1 \cos(Q\mathbf{x} + 2\phi(\mathbf{x})) + \dots$$

charge density wave



Minimal model - energy densities

$$\rho(\mathbf{x}, \phi) \sim \nabla\phi + \rho_1 \cos(\mathbf{Q}\mathbf{x} + 2\phi(\mathbf{x})) + \dots$$



- elastic energy:

$$\underline{\kappa}^{-1} (\nabla \hat{\phi}(\mathbf{x}))^2$$

- kinetic energy:

$$\underline{K}^2 \cdot \hat{P}^2(\mathbf{x}), \quad K \sim \frac{\hbar}{\sqrt{m}}$$

$$[\hat{P}(\mathbf{x}), \hat{\phi}(\mathbf{x}')] = \frac{\pi}{i} \delta(\mathbf{x} - \mathbf{x}')$$

$K=1$: free electrons
 $K \sim 10^{-1} \dots 10^{-2}$ CDWs, SDWs

- random potential:

$$\underline{u} \sum_{i=1}^N \delta(\mathbf{x} - \mathbf{x}_i) \cdot \rho(\mathbf{x}, \hat{\phi})$$

- driving force:

$$\underline{-f} \cdot \hat{\phi}(\mathbf{x}), \quad f \sim E, H, j, \dots$$

Length scales

l : mean impurity spacing



$\gg k_F^{-1}$

$K = \pi \hbar v \kappa$

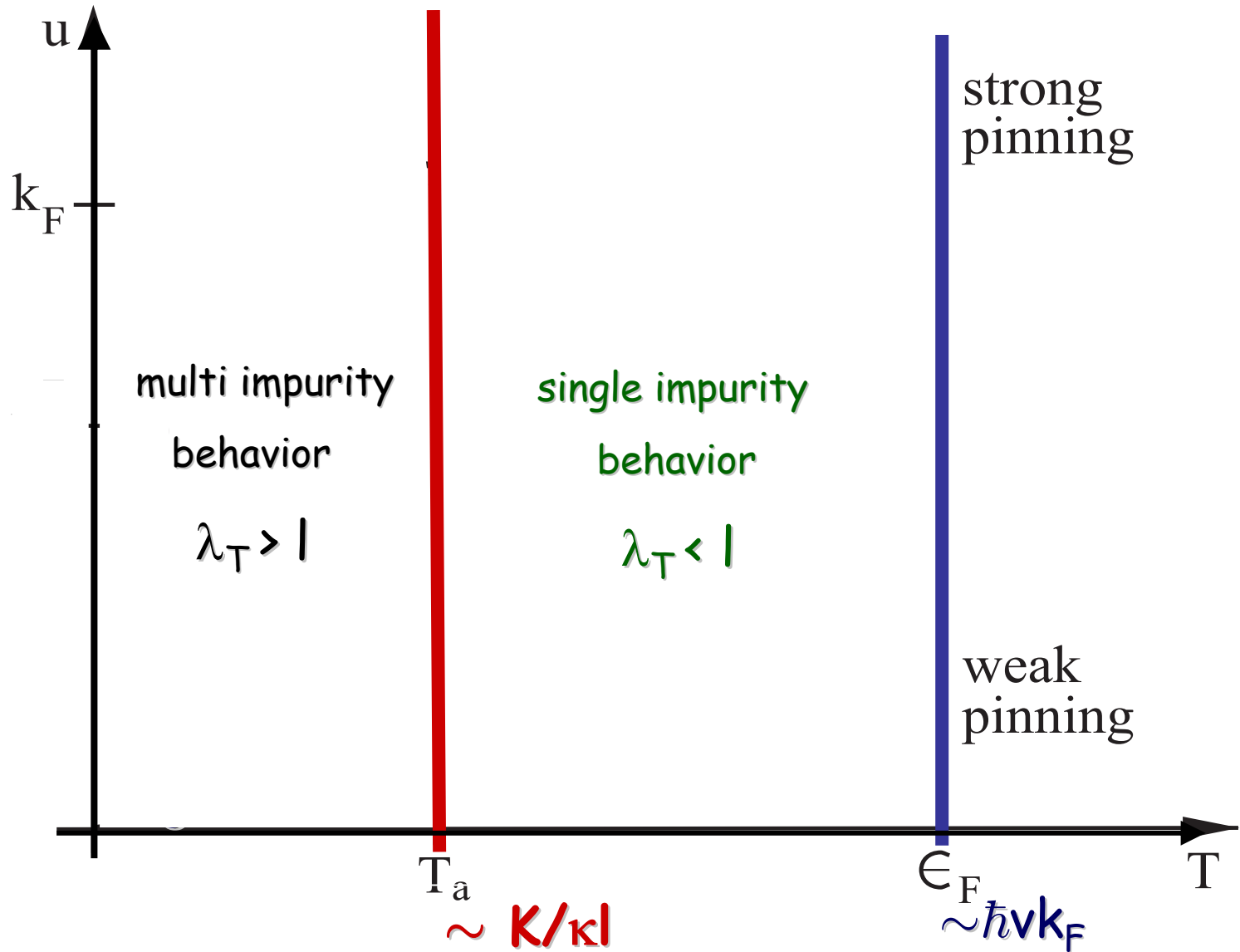


$\lambda_T \sim K / \kappa T$ thermal de Broglie wave length

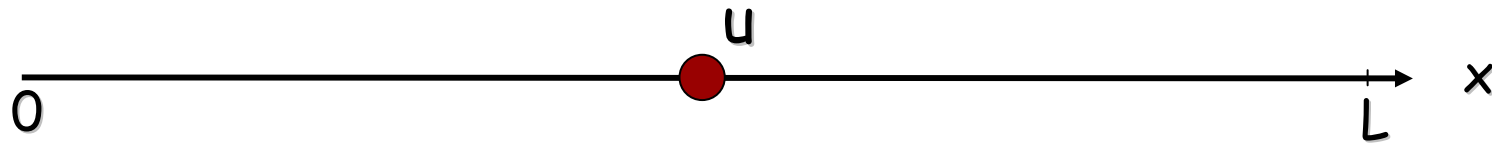


$\lambda_T \underset{<}{>} l \iff T \underset{>}{<} T_I \sim K / \kappa l$
many
single impurity

Temperature scales



Single impurity (weak or strong), $L \rightarrow \infty$



$K < 1$ (repulsive interaction)

→ impurity relevant

Kane & Fisher '92,
Furusaki & Nagaosa '93

$$G = \sigma/L \sim T^{2/K-2}$$

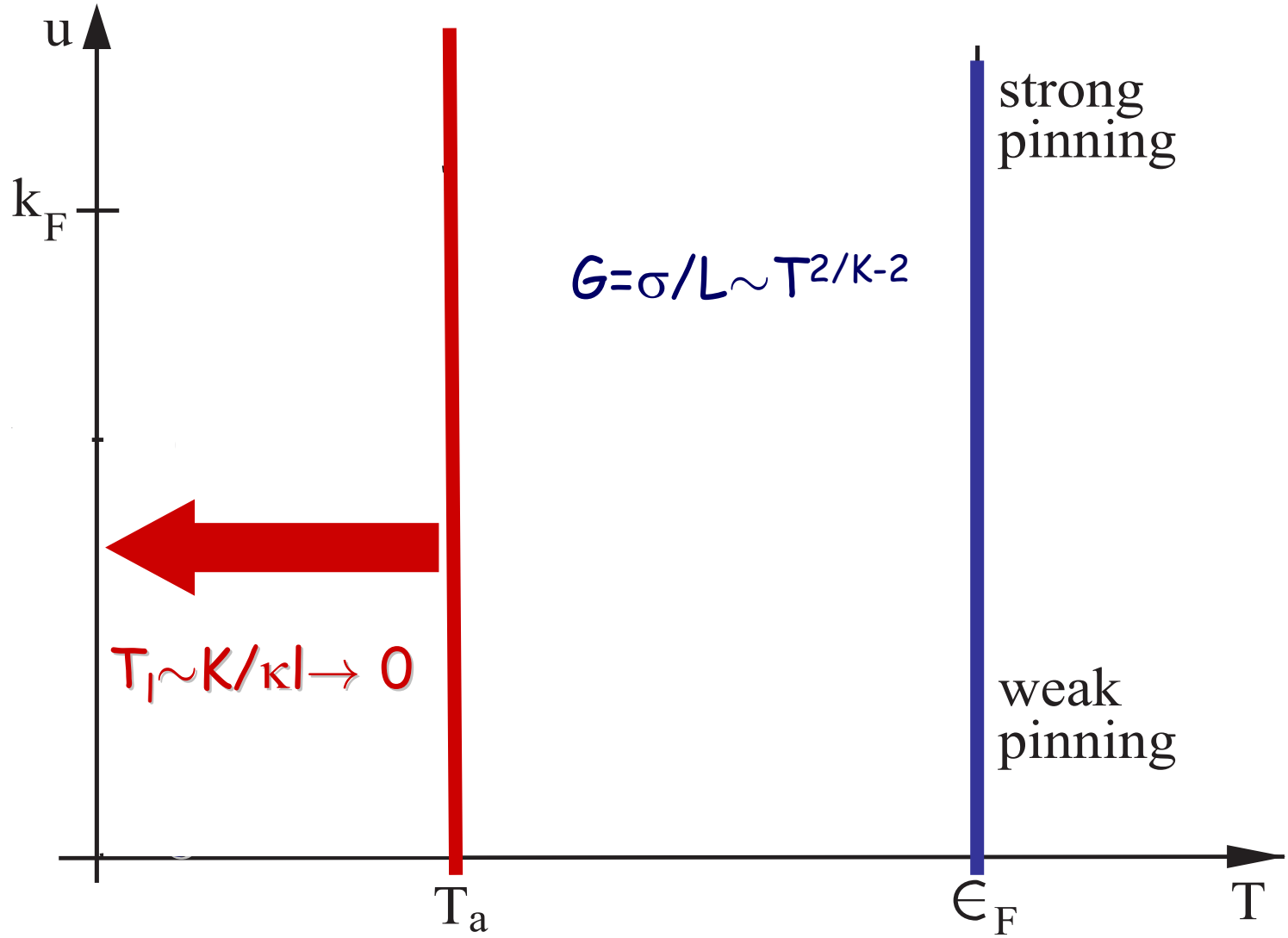
$K > 1$ (attractive interaction)

→ impurity irrelevant

$$G \sim (K) e^2/h$$

Pinning strength

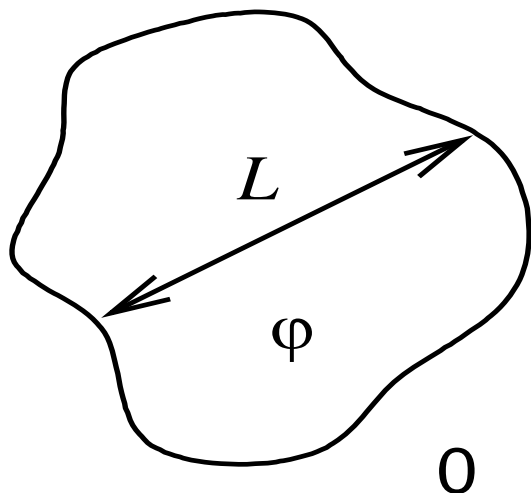
Single impurity



Disorder average

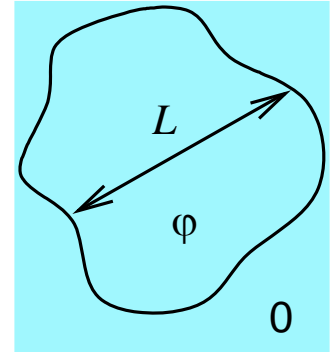
$$\langle v_R(\mathbf{x}) \rangle_R = 0, \quad \langle v_R(\mathbf{x}) v_R(\mathbf{x}') \rangle_R = v_R^2 \delta(\mathbf{x} - \mathbf{x}')$$

$$\left\langle \left(\int_0^L d^D x V_R(\mathbf{x}, \varphi) \right)^2 \right\rangle_R \approx \left\langle \left(\int_0^L d^D x V'_R(\mathbf{x}, 0) \varphi + \dots \right)^2 \right\rangle_R \approx v_R^2 L^D \varphi^2$$



Relevance of weak disorder?

Free energy of domain of size L :



$$F \approx cL^D(\varphi/L)^2 - v_R L^{D/2} \varphi - T - c \left(\frac{K}{\varphi} \right)^2 L^{-D} - f \varphi L^D$$

$$= cL^{D-2} \left[\varphi^2 - \frac{v_R}{c} L^{\frac{4-D}{2}} \varphi - \frac{T}{c} L^{2-D} - \left(\frac{K}{\varphi} \right)^2 L^{2(1-D)} - \frac{f}{c} L^2 \right]$$

$$f = 0 : \text{Min} \Rightarrow \varphi \approx (L/L_p)^\zeta, \quad \zeta = \frac{4-D}{2}$$

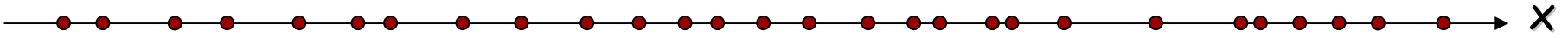
“roughness exponent”

self similar ground state

Larkin-length $L_p \approx \left(\frac{c}{v_R} \right)^{\frac{2}{4-D}} \gg 1,$

Larkin 1970, Imry & Ma 1975, Fukuyama & Lee 1978

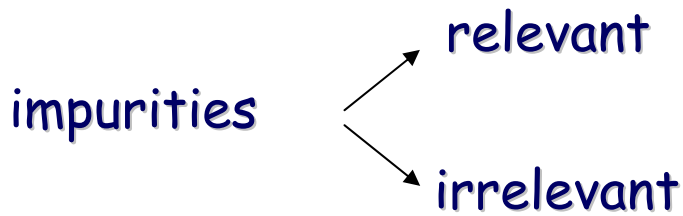
Many weak impurities (Gaussian)



$u, l \rightarrow 0, \quad (l/u^2)^{1/3} \sim \xi_0 \gg k_F^{-1}, \quad \text{fixed}$

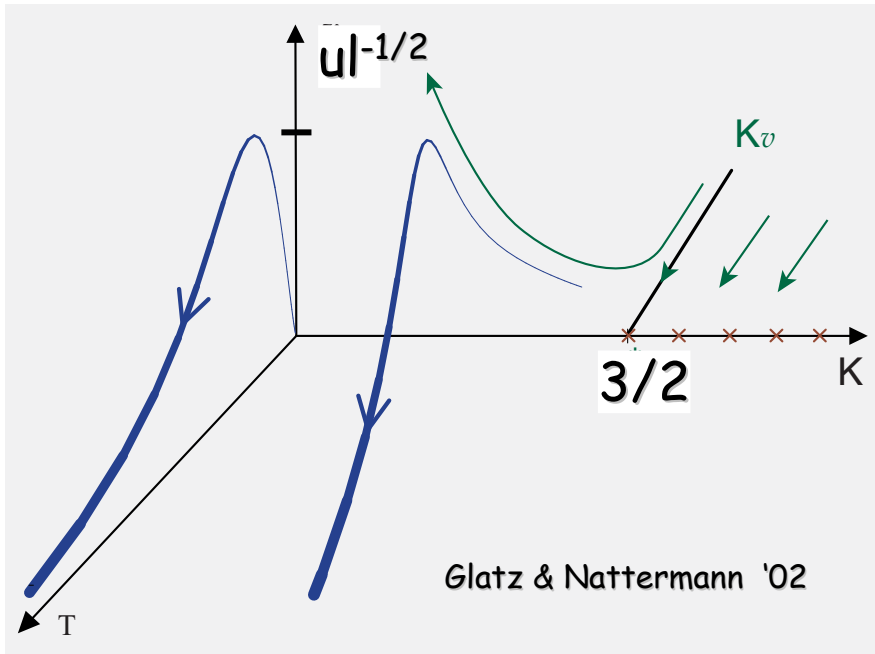
$K < 3/2$

$K > 3/2$



Suzumura & Fukuyama '83
Giamarchi & Schulz '88

$T_{loc} \sim K/\kappa\xi$



$\xi_0 \xrightarrow{\text{quantum fluctuations}} \xi \approx k_F^{-1} (\xi_0 k_F)^{3/(3-2K)}$

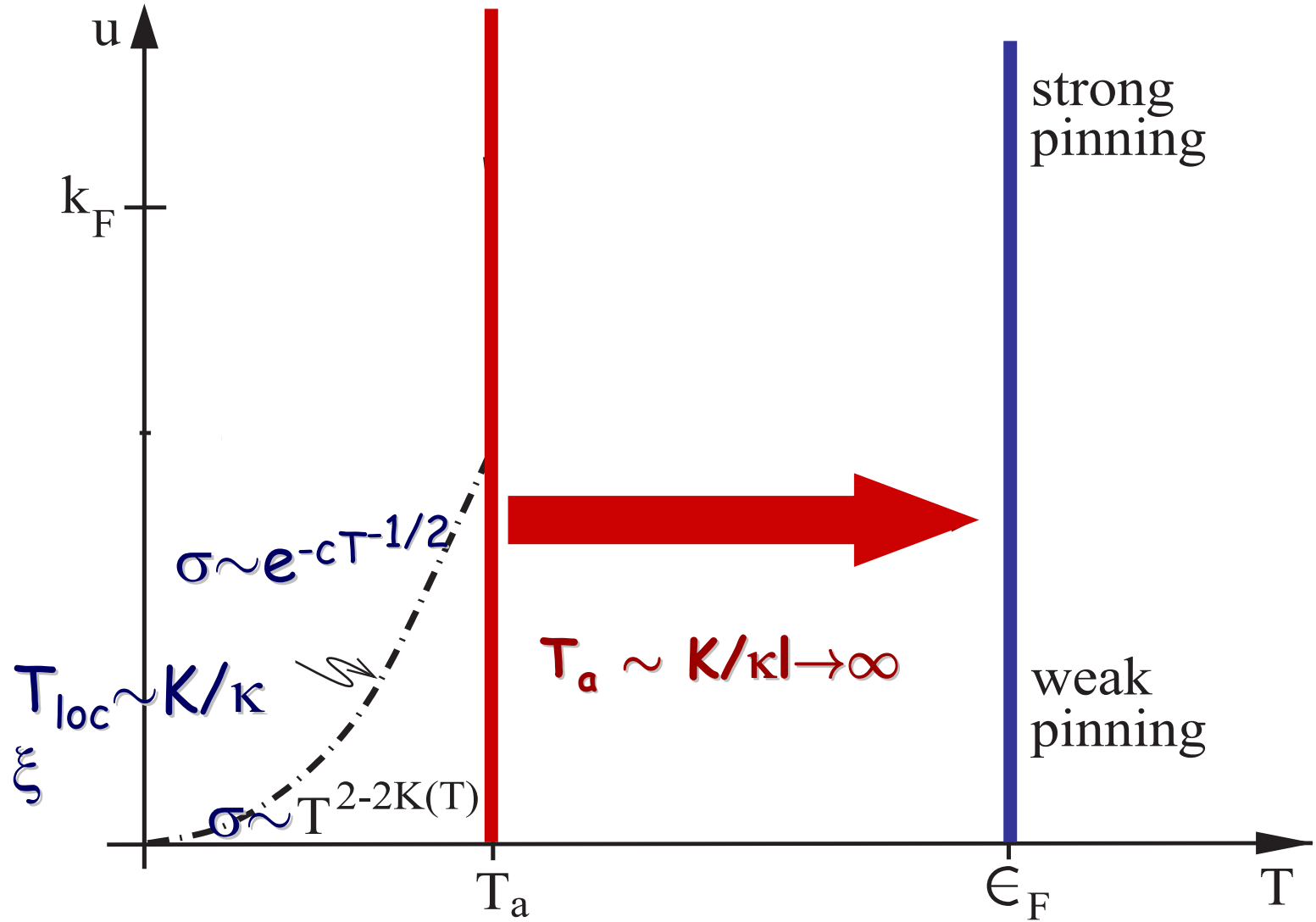
$\sigma \sim$

- $T \gg T_{loc} \sim \hbar v/\xi$
Giamarchi & Schulz
- $T \ll T_{loc} \sim \hbar v/\xi$

$e^{-cT^{-1/2}}$

Pinning strength

Gaussian impurities



Here: Many strong Poissonian impurities

1. What is a strong impurity? $T < T_a$

no interaction ($K=1$): $u > u_c = k_F$

interaction ($K < 1$):

→ integrate out fluctuations with wave vector $\Lambda < |k| < k_F$

$$u \rightarrow u_{\text{eff}} \approx u (k_F / \Lambda)^{-K} \quad \text{Glazman et al. '92}$$

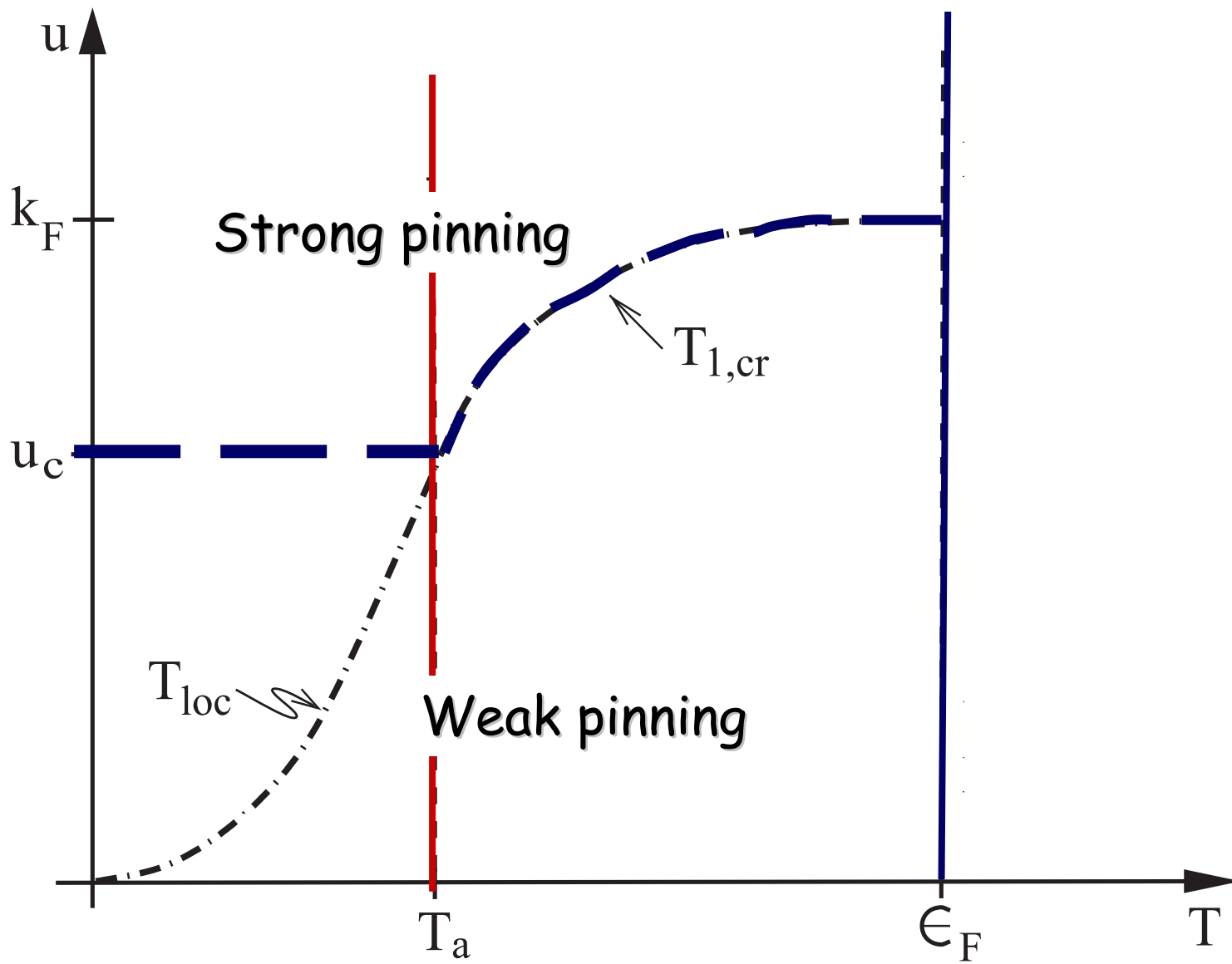
strong pinning : $|u_{\text{eff}}| \gg 1$ Fukuyama & Lee '78

$$\rightarrow u > u_c = k_F (|k_F|)^{K-1}$$

2. What is a strong impurity? $T > T_a$

strong pinning: $\lambda_T u_{\text{eff}} \gg 1$ → $u > u_1(T) = k_F (T/\varepsilon_F)^{1-K}$

$$\rightarrow T < T_{1,\text{cr}} = \varepsilon_F (u/k_F)^{1/(1-K)}$$

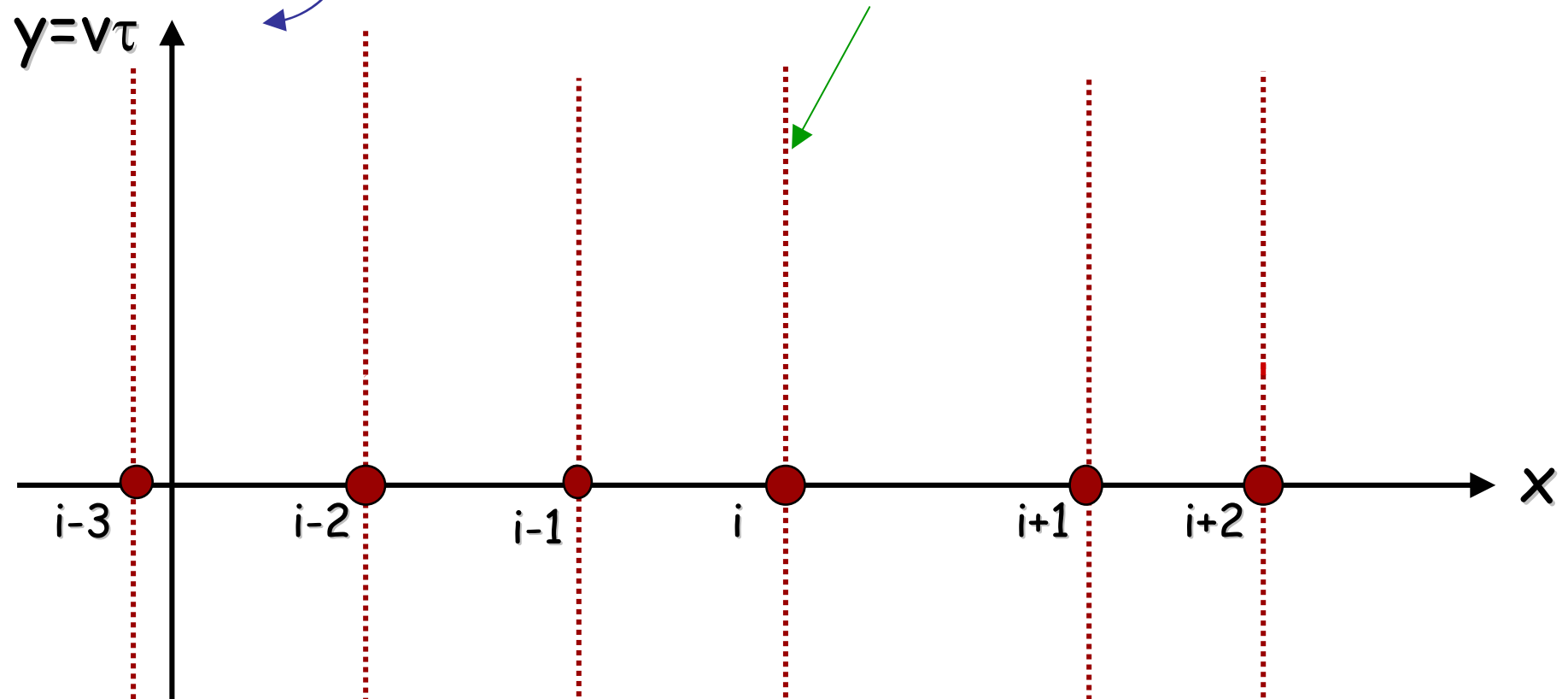


Many strong Poissonian impurities:

$$\frac{S}{\hbar} = \frac{1}{2\pi K} \int_0^L dx \int_0^{\lambda_T} d\tau \left\{ (\partial_x \phi + fx)^2 + (\partial_\tau \phi)^2 - u \sum_{i=1}^N \cos(2\phi - 2k_F x_i) \right\}$$

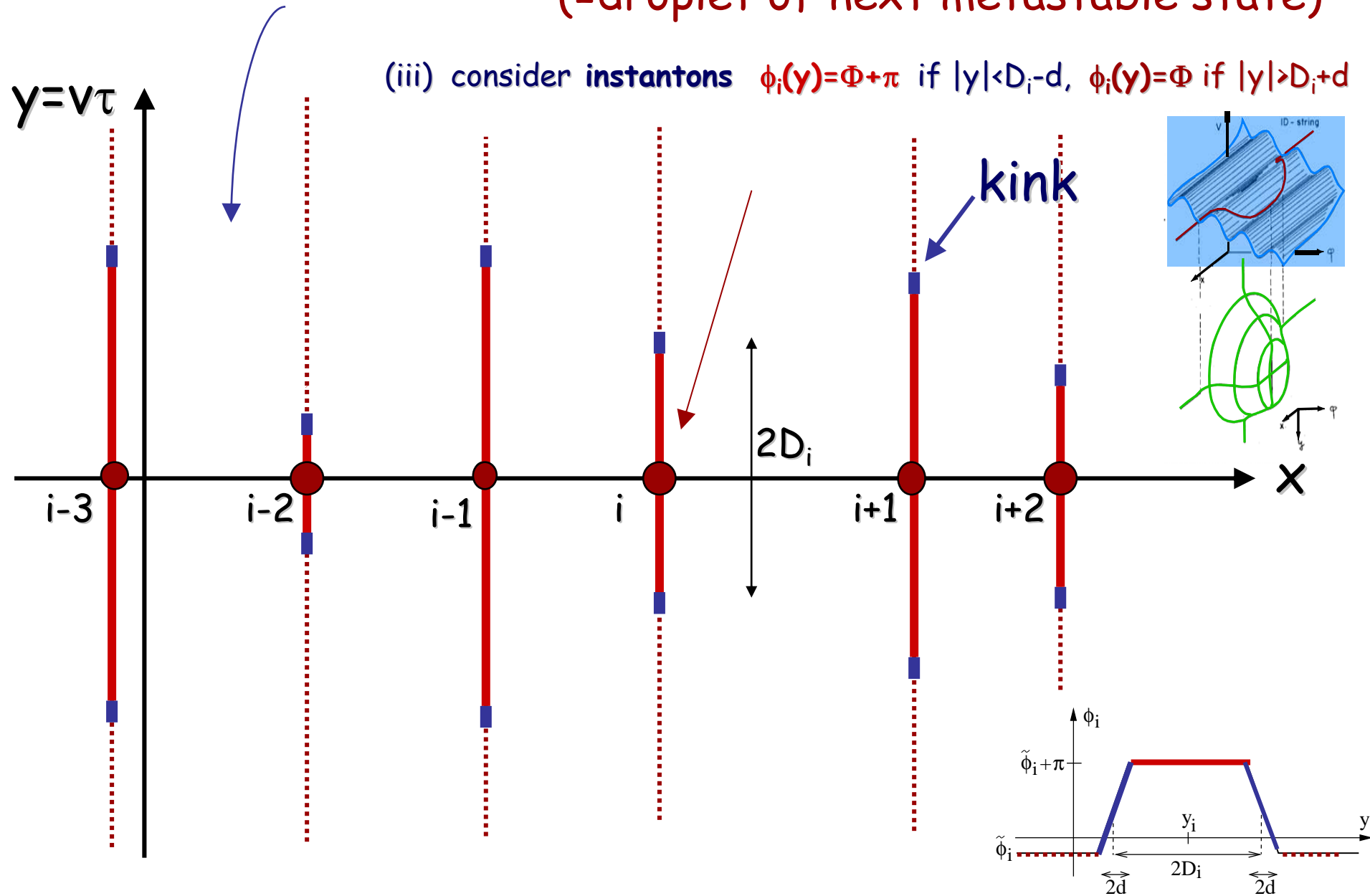
metastable states

- (i) integrate out $\phi(x,y)$ between impurities $\rightarrow \phi(x_i, y) \equiv \phi_i(y)$
- (ii) metastable state: $\Phi_i = \pi(n_i - \alpha_i)$, ground state $n_i = \sum_{j \geq i} [k_F(x_j - x_{j-1})]_G$,



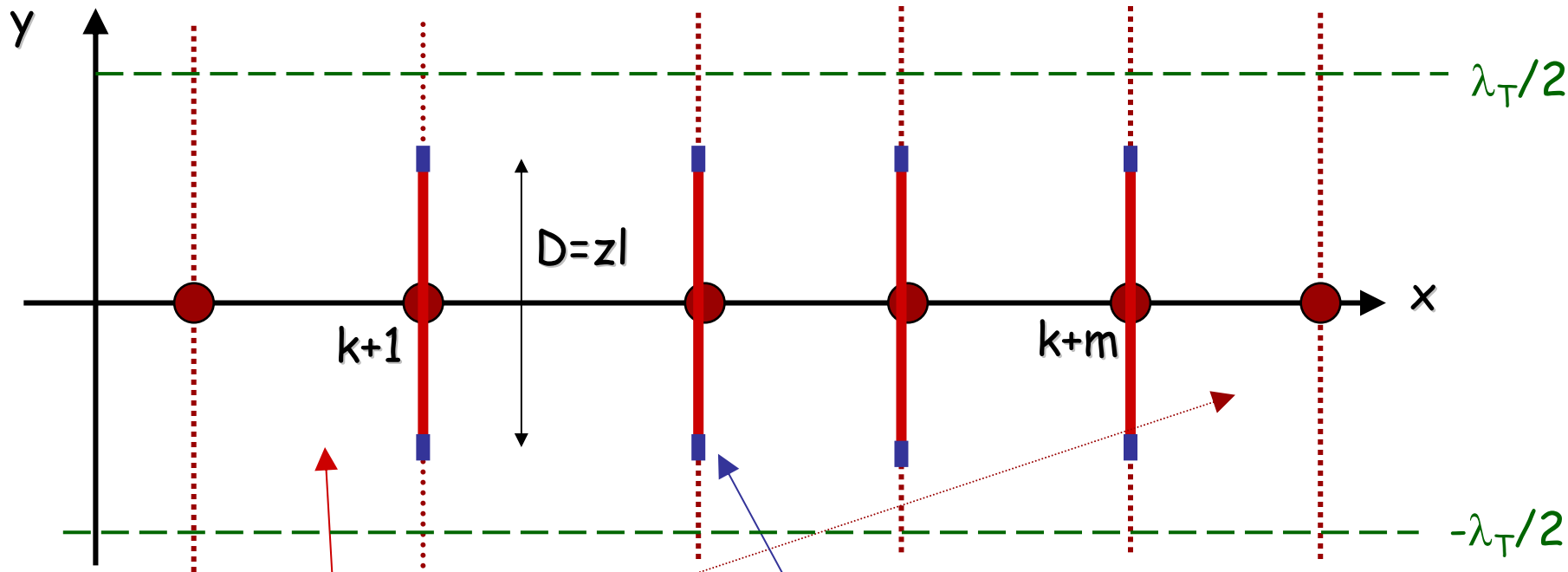
Tunneling under applied field: instantons (=droplet of next metastable state)

(iii) consider instantons $\phi_i(y) = \Phi + \pi$ if $|y| < D_i - d$, $\phi_i(y) = \Phi$ if $|y| > D_i + d$



Tunneling under applied field: instantons

(iv) For simplicity: $D_i \equiv D$, $k < i \leq k+m$, D_i elsewhere

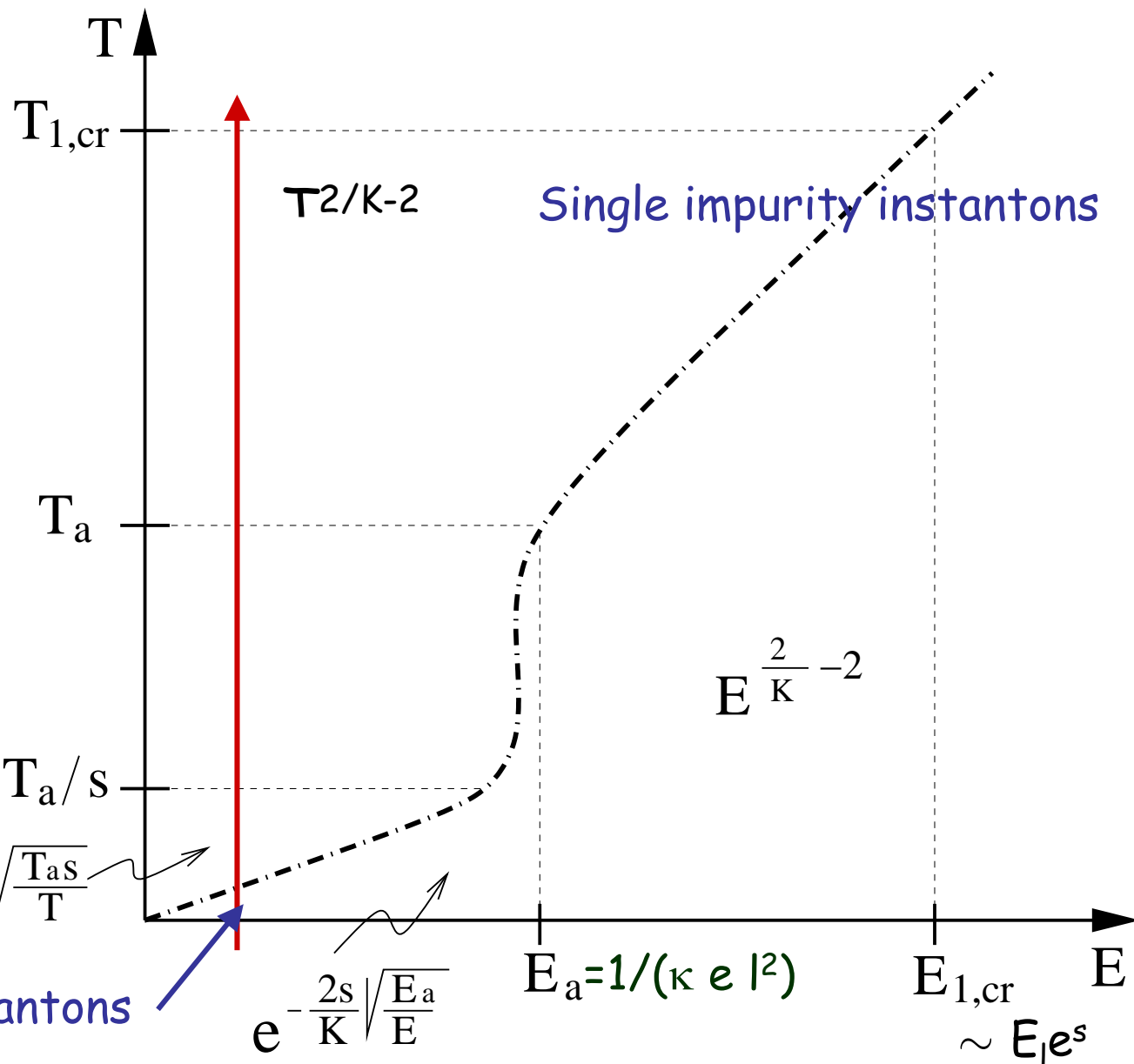
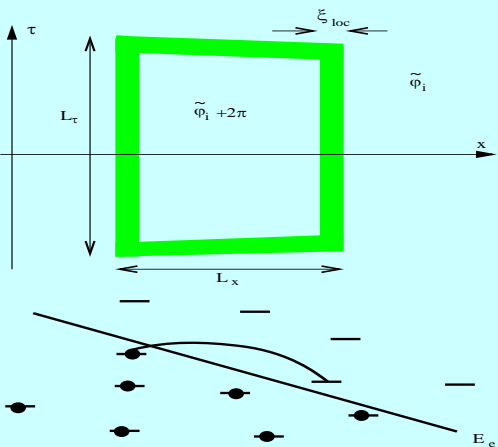
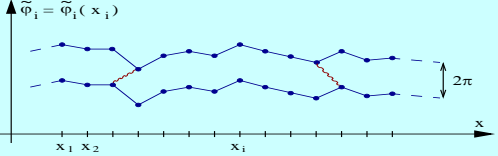


$$S/\hbar = 2/K \left\{ z\sigma_m(k) + \ln(1 + e^{-2z}) + m(s + \ln \tanh(z/2) - zfa^2) \right\}$$

$$\begin{aligned} f &\sim E \\ s &\sim \ln(C\mu_{\text{eff}}) \end{aligned}$$

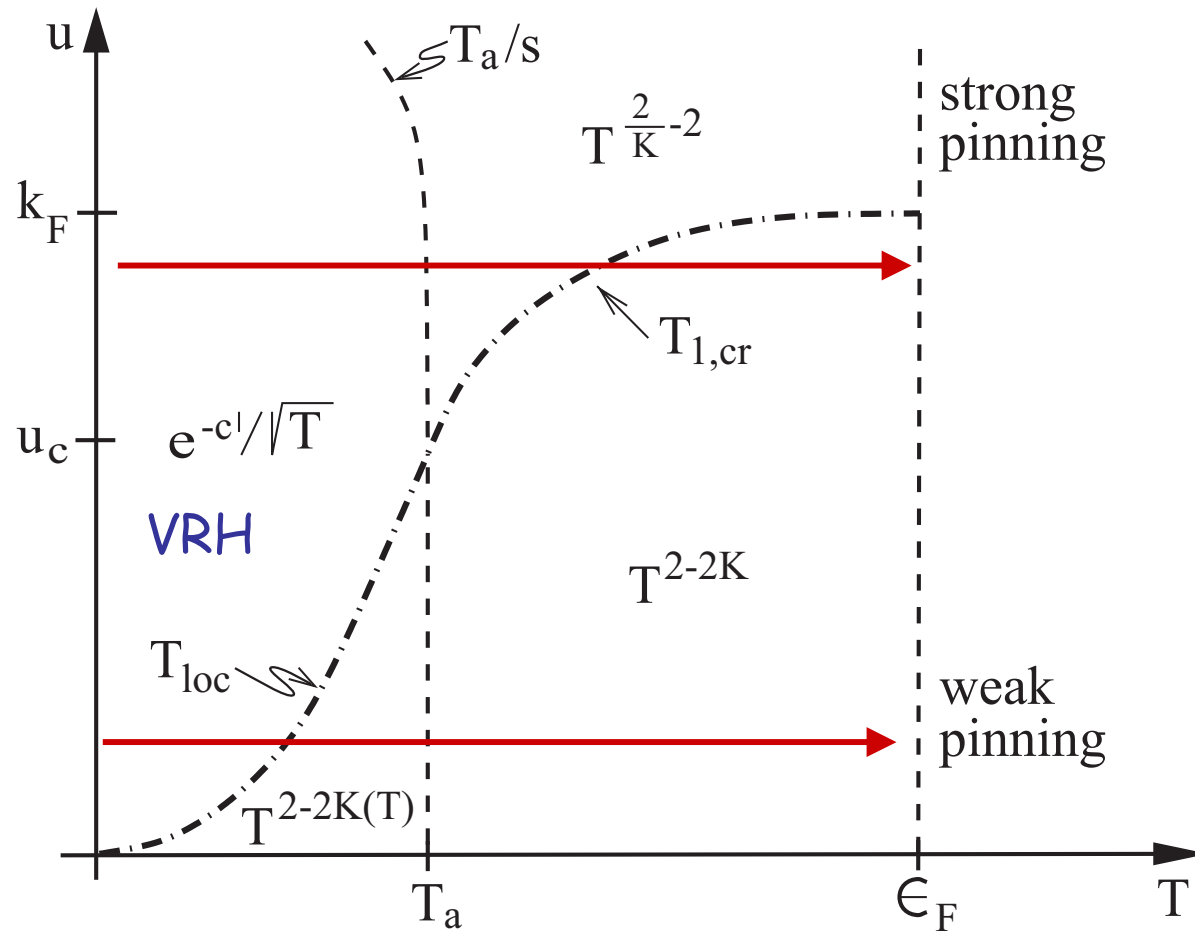
Current \sim tunneling probability $\sim \exp\{-S_{\text{saddle point}}(E)/\hbar\}$

If instanton hits $\pm \lambda_T/2 \rightarrow$ Cross-over to linear response



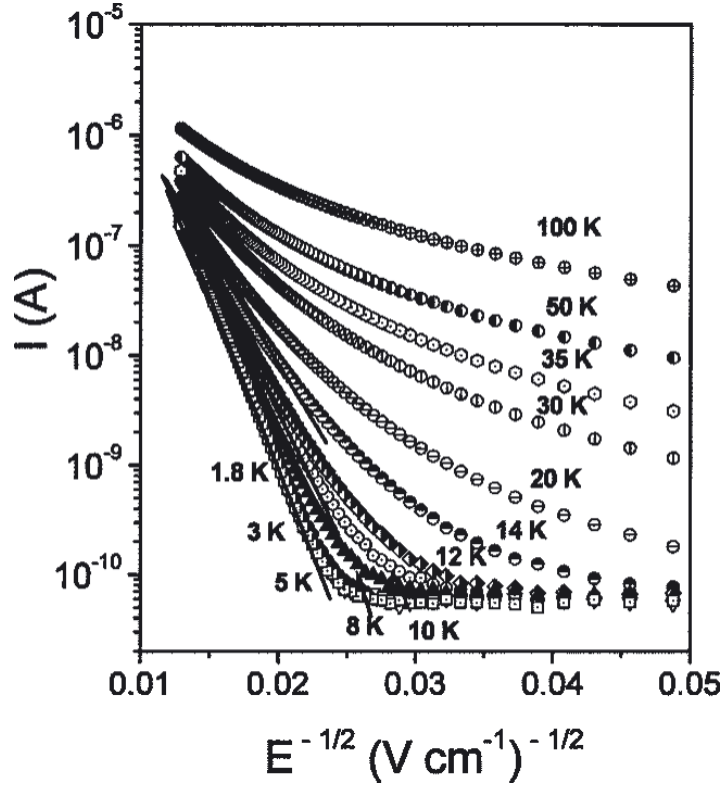
Multi impurity instantons

Different regimes of conductivity



Polydiacetylen

Aleshin et al. 2004



Carbon-nanotubes: Tang et al. 2000

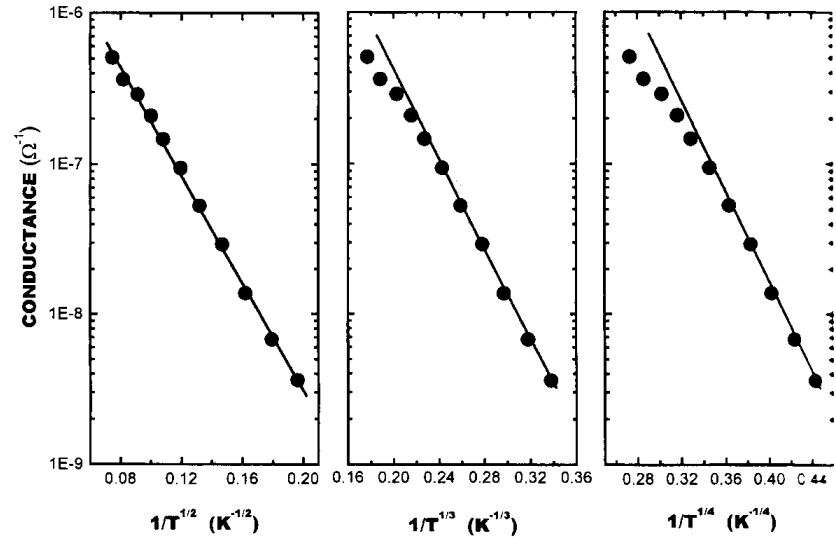


Fig. 4. The temperature dependence of the conductance measured at zero bias voltage.

Conclusions:

Considered 1D driven quantum model with periodic disorder:
CDWs and Luttinger liquids

Limit of strong disorder: instanton calculation

- linear and non-linear conductivity
- field and temperature cross-over between single and many impurity tunneling
- small field and temperature: Mott-Shklovskii-VRH
- larger E, T : Kane-Fisher-..... power law behavior
- global weak/strong pinning regime diagram