Vortex walls in helical magnets

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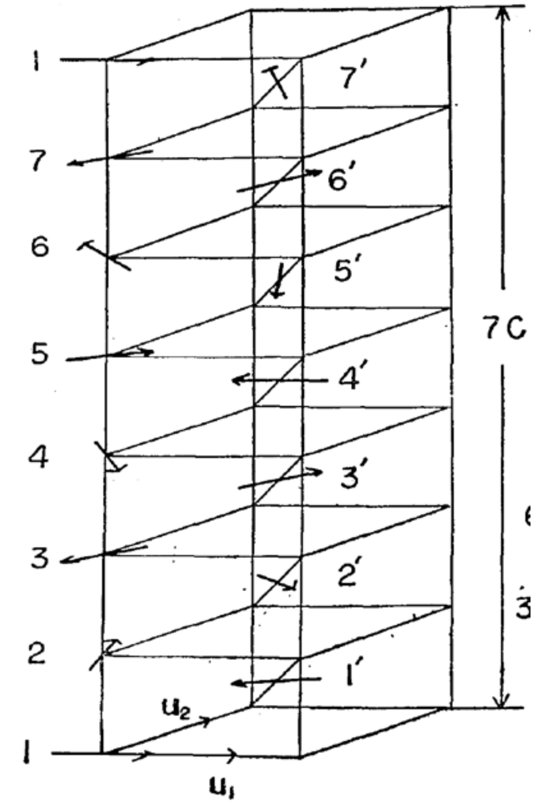
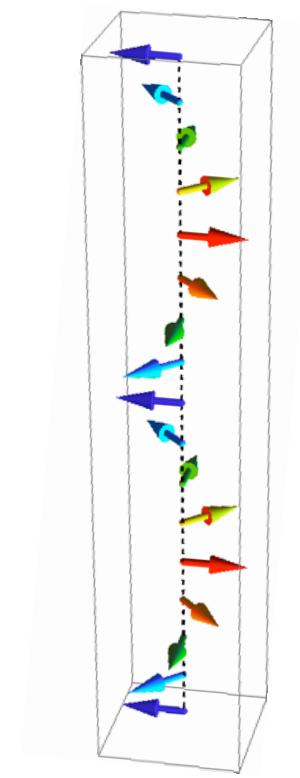
A New Type of Antiferromagnetic Structure in the Rutile Type Crystal

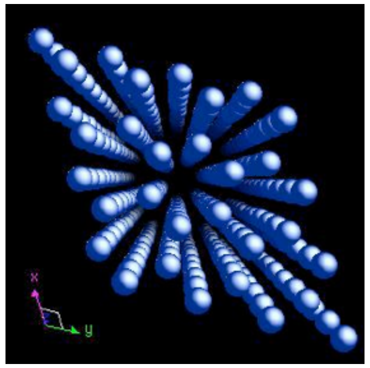
By Akio YOSHIMORI

Department of Physics, University of Osaka Prefecture, Mozu-Higashi, Sakai



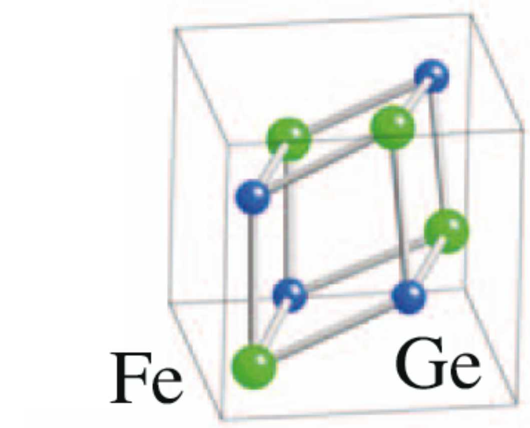


Chiral Magnets: time reversal + spatial inversion symmetry broken



simultaneously

separately

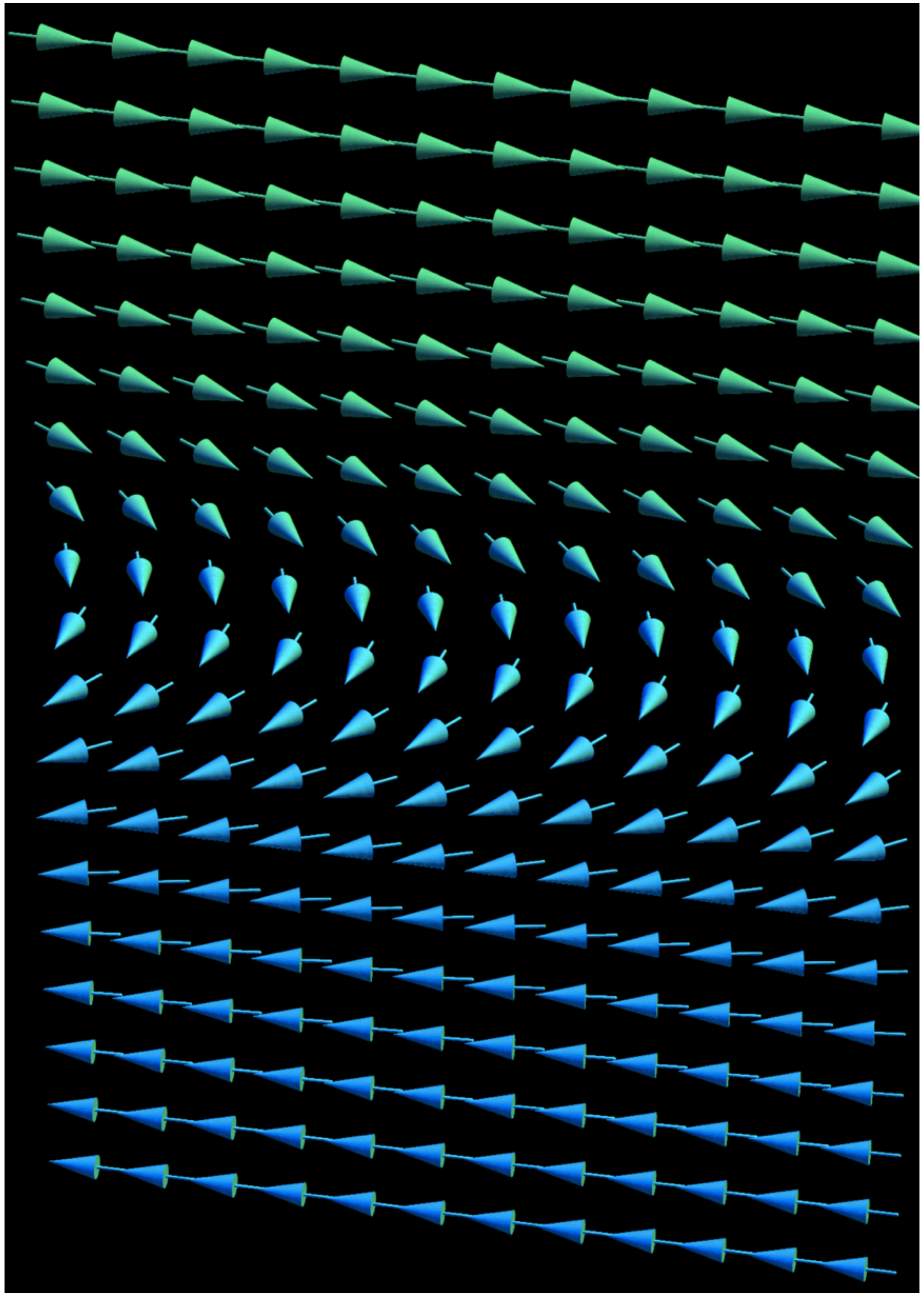


centrosymmetric crystals:
space group contains
inversion center

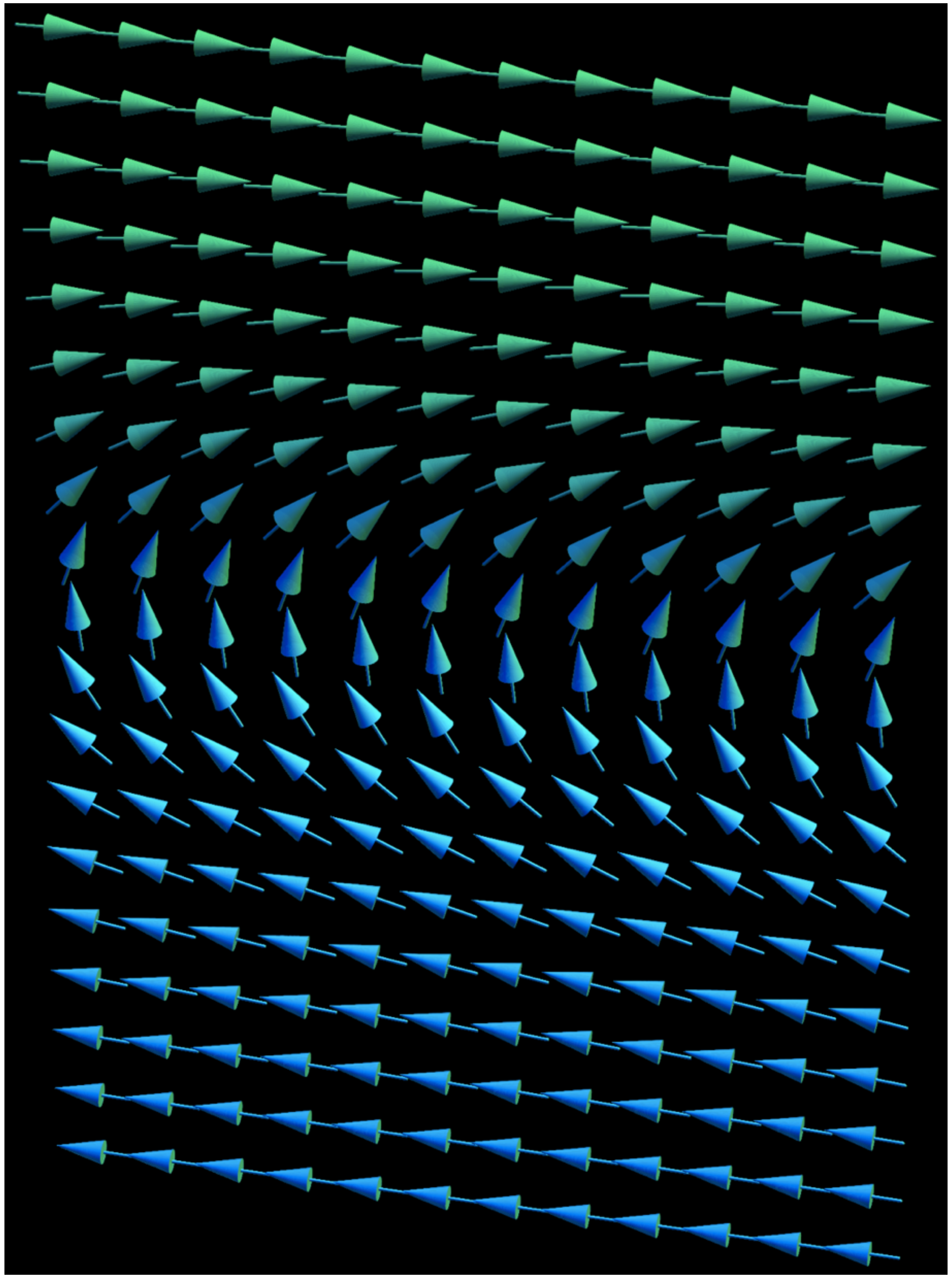
non-centrosymmetric
crystals:
space group contains no
inversion center

How do the domain walls look like?

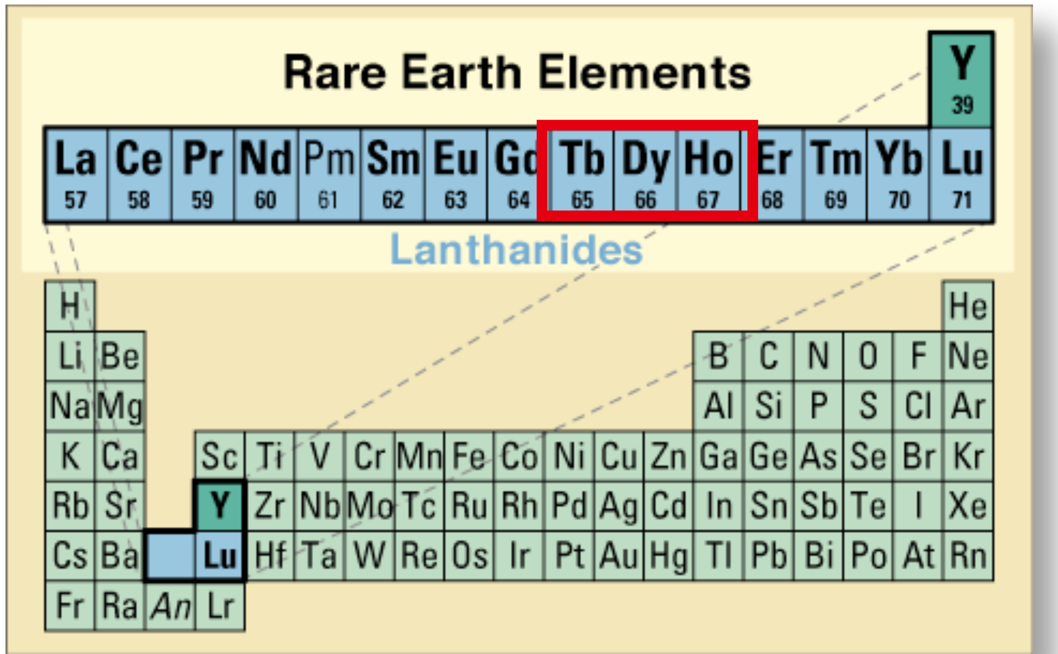
Bloch wall (Bloch 1932)



Neel wall



Centrosymmetric Crystals: Systems

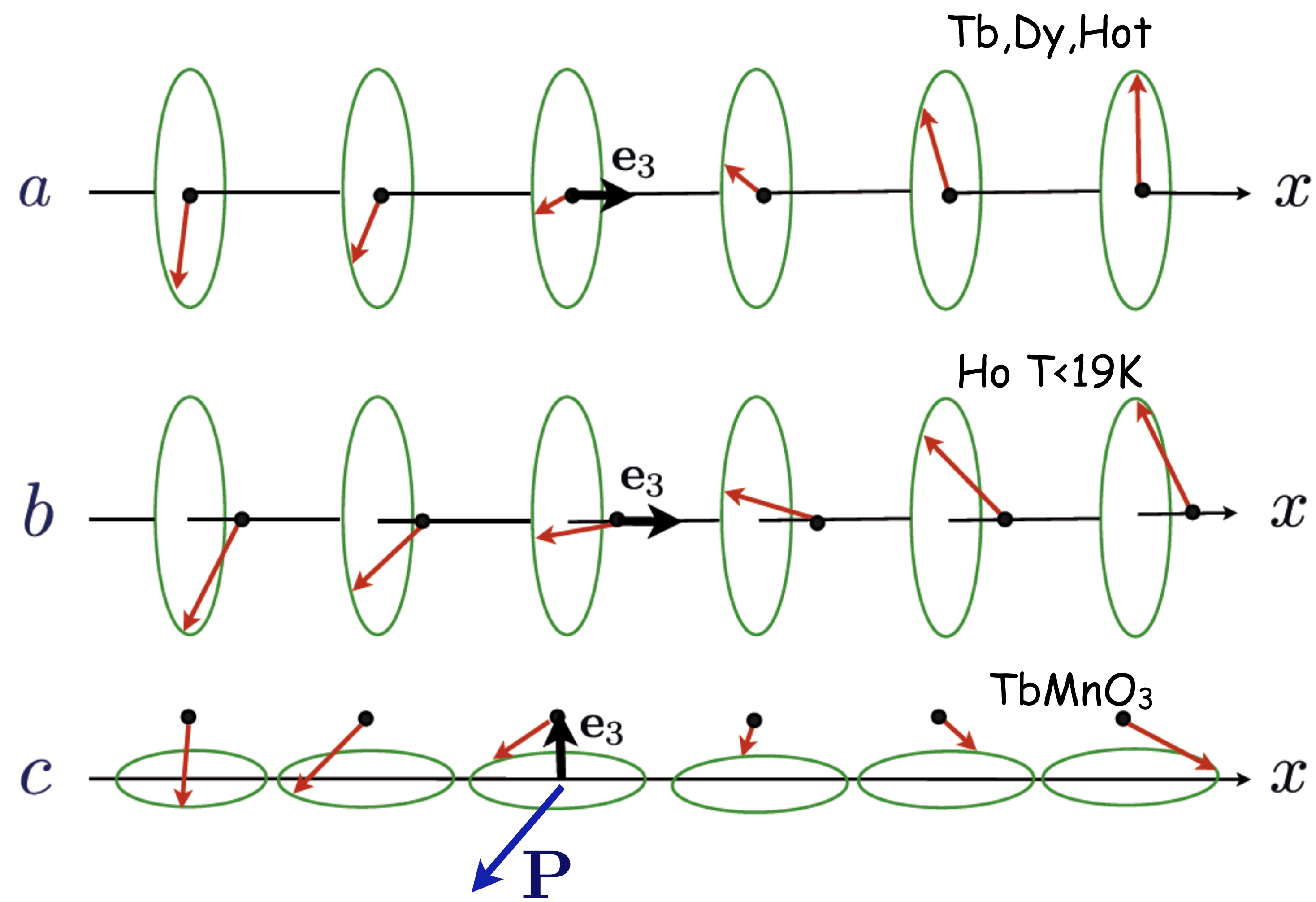


Tb, Dy, Ho :
RKKY interaction $\rightarrow$
frustration $\rightarrow$ helical structure

$RMnO_3$ $\quad R \in \{Y, Tb, Dy\}$

$R_2Mn_2O_5$, $R \in \{Tb, Bi\}$

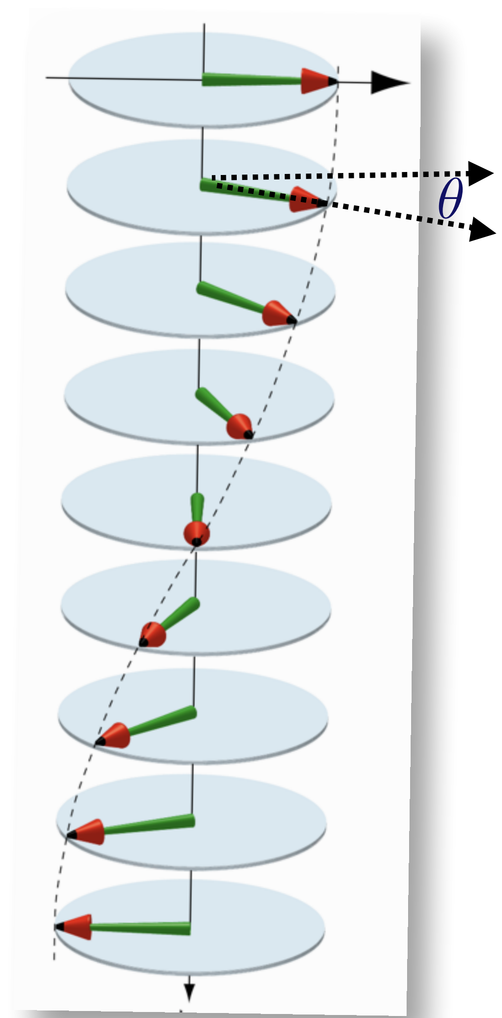
$Ni_3V_2O_8$ and $LiCu_2O_2$



Centrosymmetric Crystals: Model (i)

$$\mathcal{H} = \frac{J}{a}\int_r \Big\{ -\frac{\theta^2}{2}(\partial_x \mathbf{m}_\perp)^2 + \frac{a^2}{4}(\partial_x^2 \mathbf{m}_\perp)^2 + (\nabla_\perp \mathbf{m})^2 + (\partial_x m_3)^2 + V(m_3)\Big\}.$$

$$\mathbf{r} \to -\mathbf{r} \quad t \to -t$$

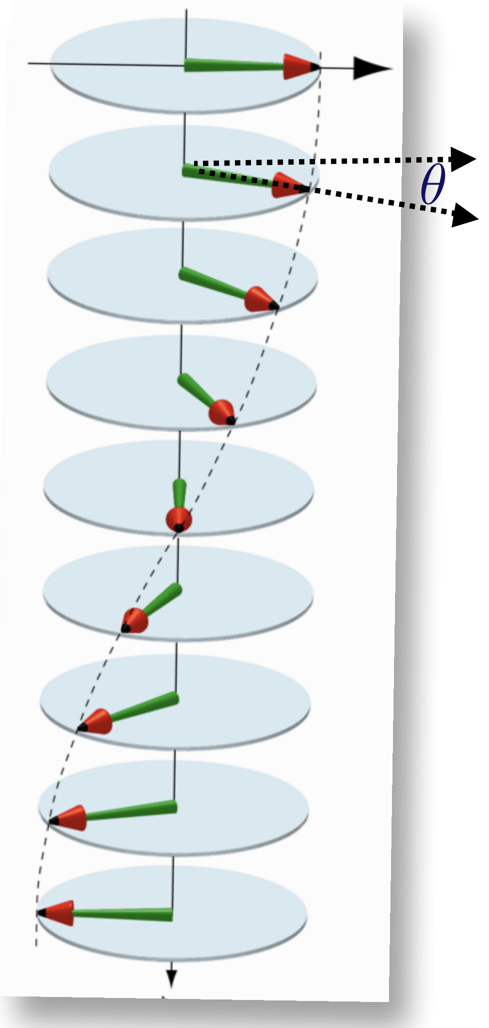


$$\mathbf{m}^2 = 1, \to \mathbf{m} = \mathbf{m}(\vartheta, \varphi), \qquad \theta = qa \ll 1$$

$$\boxed{\mathbf{m} = |m_\perp|\,(\mathbf{e}_1 \cos qx + \chi \mathbf{e}_2 \sin qx) + \zeta |m_3| \mathbf{e}_3}$$

$\zeta = \pm 1$ conicity, $\chi = \pm 1$ chirality

Centrosymmetric Crystals: Model (ii)



$$\mathbf{m} = \mathbf{m}(\varphi, \vartheta)$$

$$\mathcal{H} = \frac{J}{a} \int_r \left\{ (\nabla_\perp \varphi)^2 + \frac{a^2}{4} \left[\left((\partial_x \varphi)^2 - q^2 \right)^2 + (\partial_x^2 \varphi)^2 \right] \right\}.$$

Topological defects

$$\left\{ 4\nabla^2 + a^2 \left[6(\partial_x \varphi)^2 - 2q^2 - \partial_x^2 \right] \partial_x^2 \right\} \varphi = 0.$$

Centrosymmetric Crystals: Domain Walls (i)

JOURNAL OF APPLIED PHYSICS VOLUME 95, NUMBER 11 1 JUNE 2004

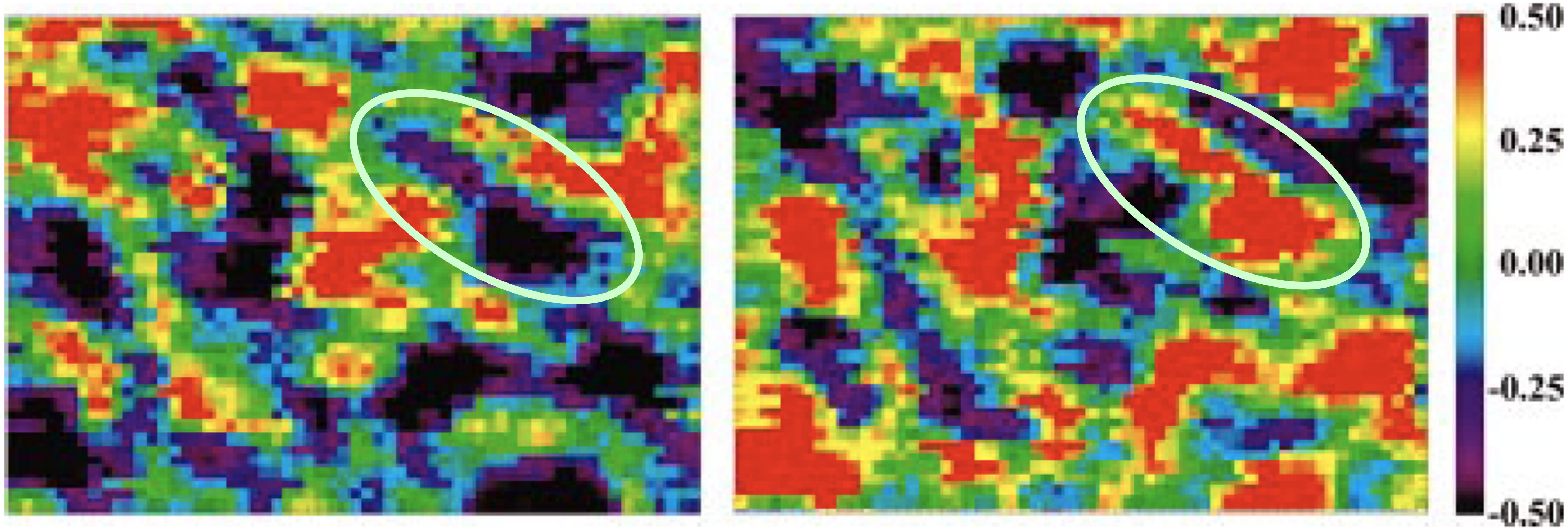
Imaging spiral magnetic domains in Ho metal using circularly polarized Bragg diffraction

J. C. Lang, D. R. Lee, D. Haskel, and G. Srajer
Advanced Photon Source, Argonne National Laboratory, Argonne, Illinois 60439

(Presented on 6 January 2004)

We have used circularly polarized x rays to image the spiral magnetic domain structure in a single crystal of Ho metal. In these structures, the magnetization direction rotates between successive atomic layers forming a helix. At magnetic Bragg diffraction peaks, circularly polarized x rays are sensitive to the handedness of such a helix (i.e., either right or left handed). By reversing the helicity of the incident beam with phase-retarding optics and measuring the difference in the Bragg scattering, contrast between domains of opposing handedness can be obtained. © *2004 American Institute of Physics*. [DOI: 10.1063/1.1688252]

film plane perpendicular to helical axis



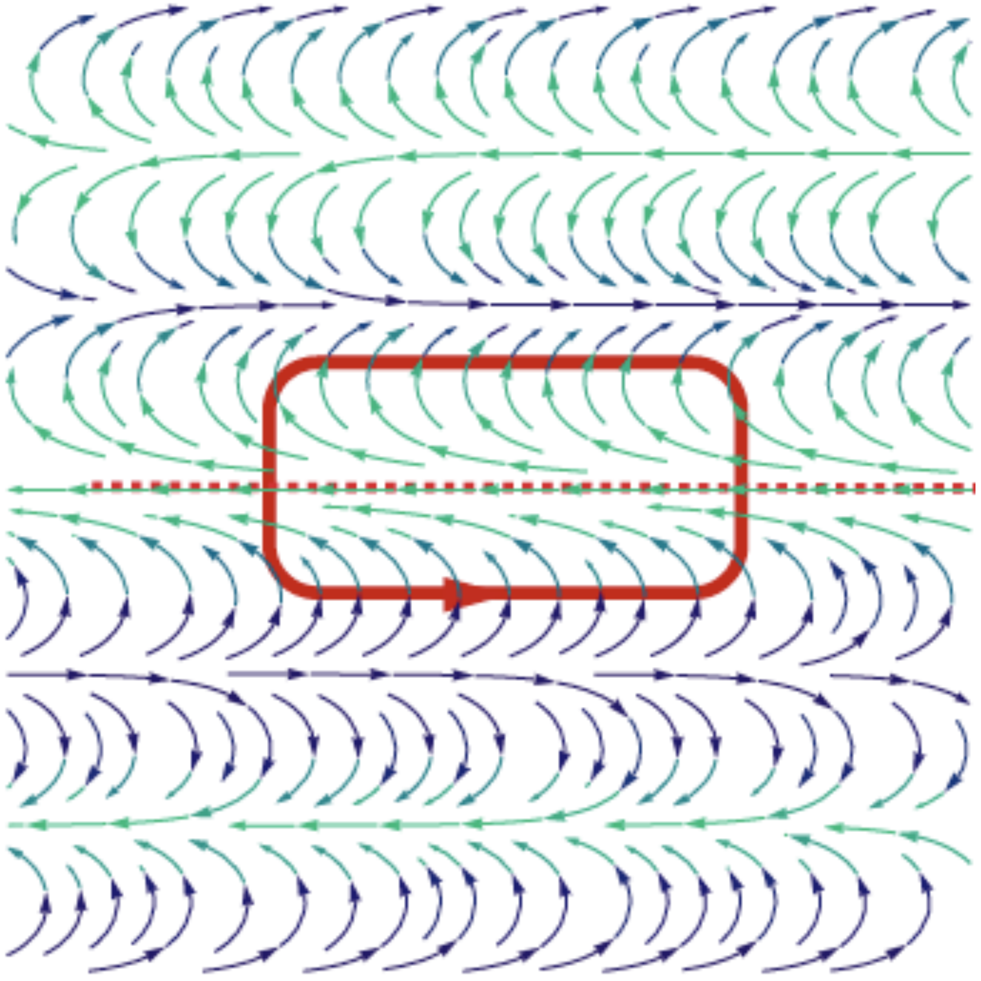
Centrosymmetric Crystals: Domain Walls (ii)

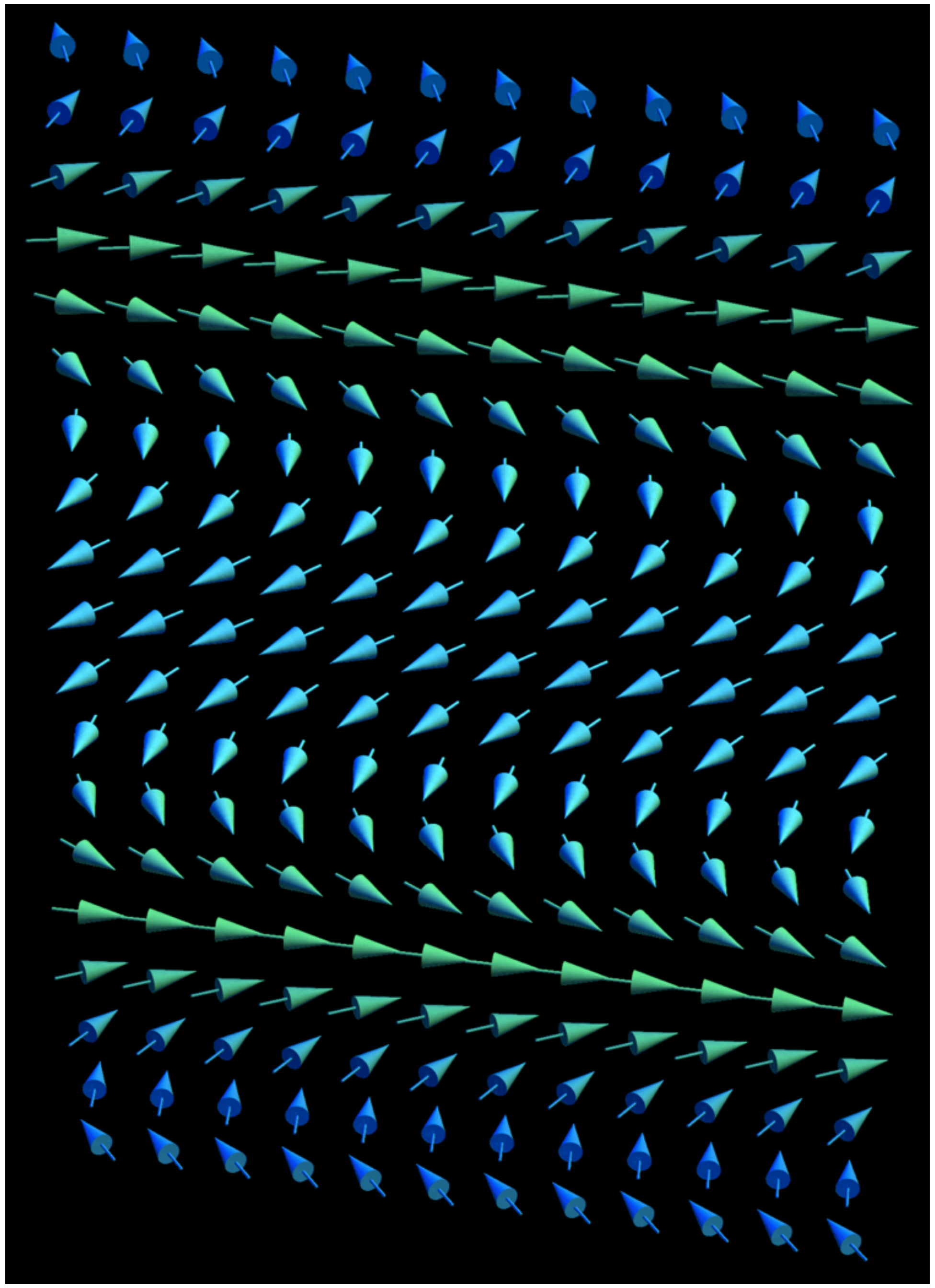
(a) Hubert walls (1975): domain walls perpendicular to helical axis

$$\varphi(x) = \varphi(x_0) + \ln\cosh(q(x - x_0))$$

$$\sigma_H \sim J\theta^3/a^2$$

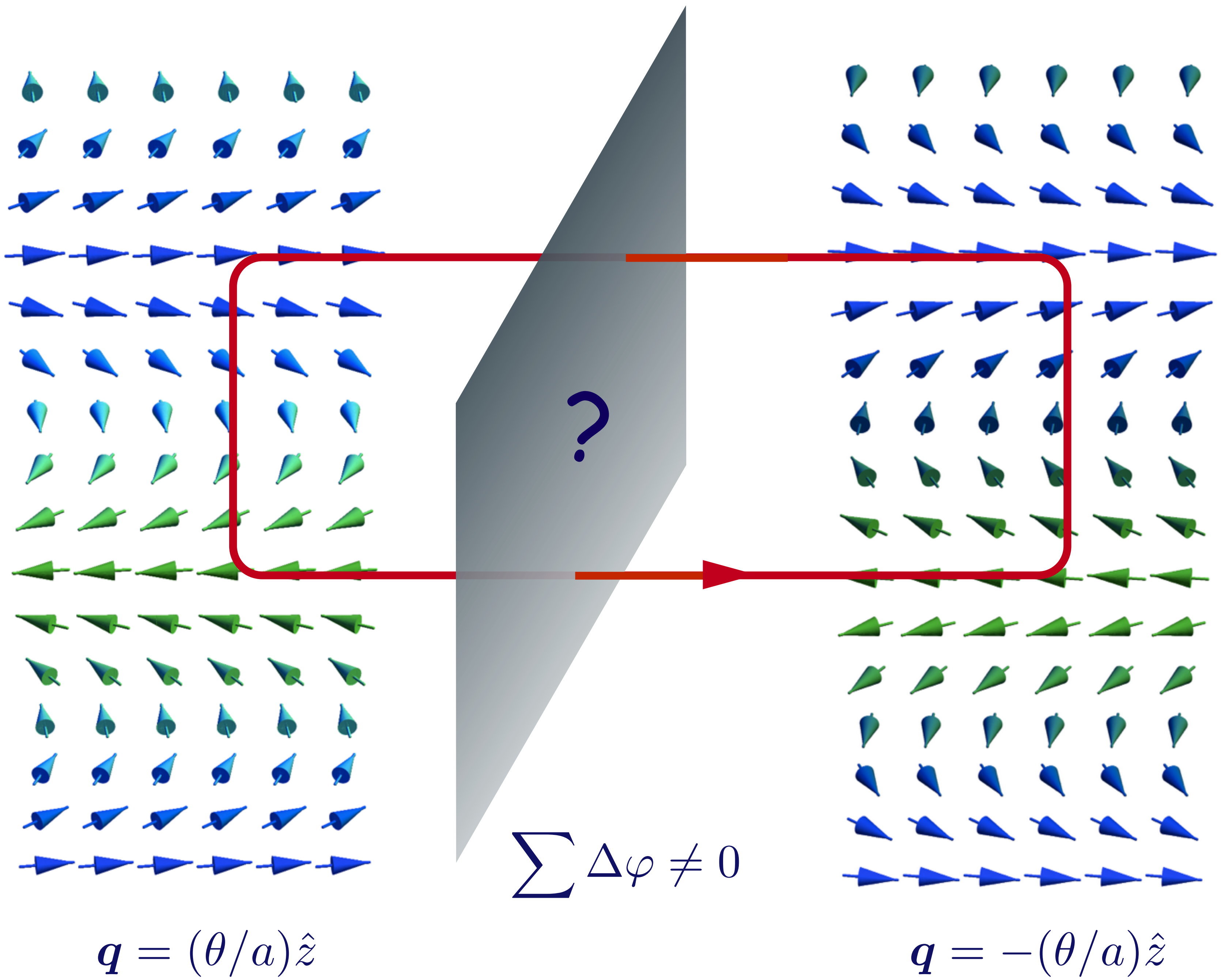
x





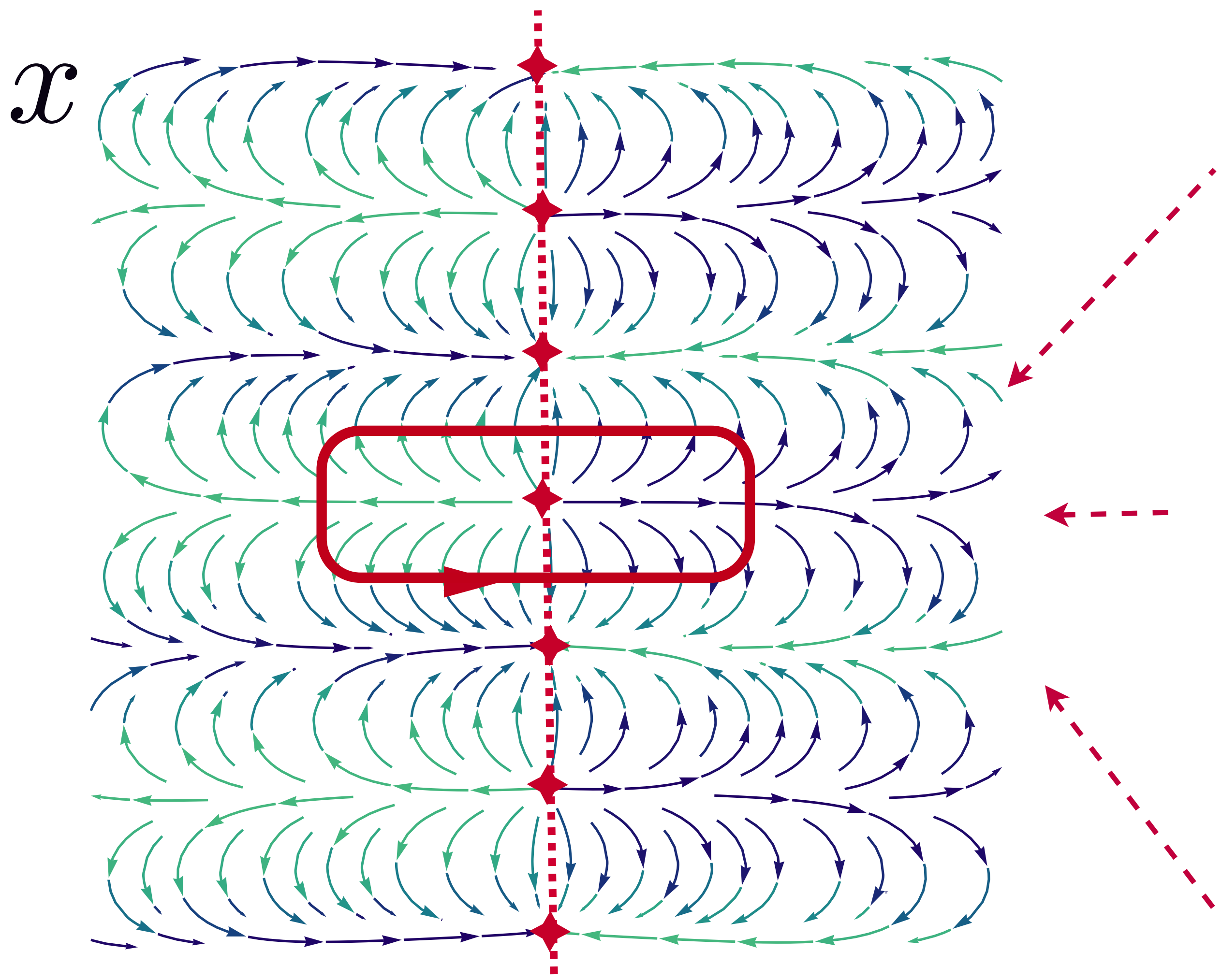
Centrosymmetric Crystals: Domain Walls (iii)

(b) Vortex walls :domain walls not perpendicular to helical axis , no exact solution exists

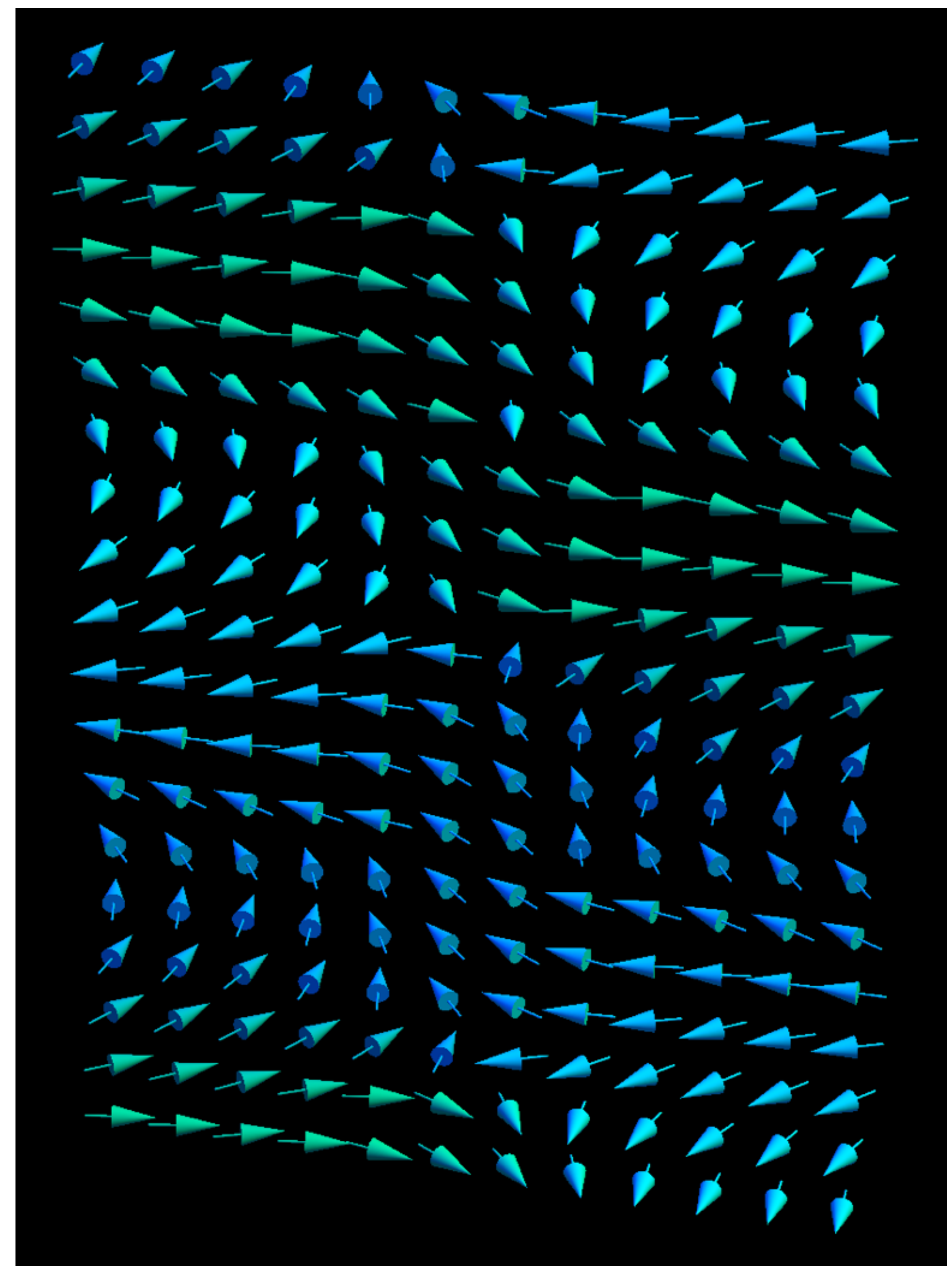


Centrosymmetric Crystals: Domain Walls (iv)

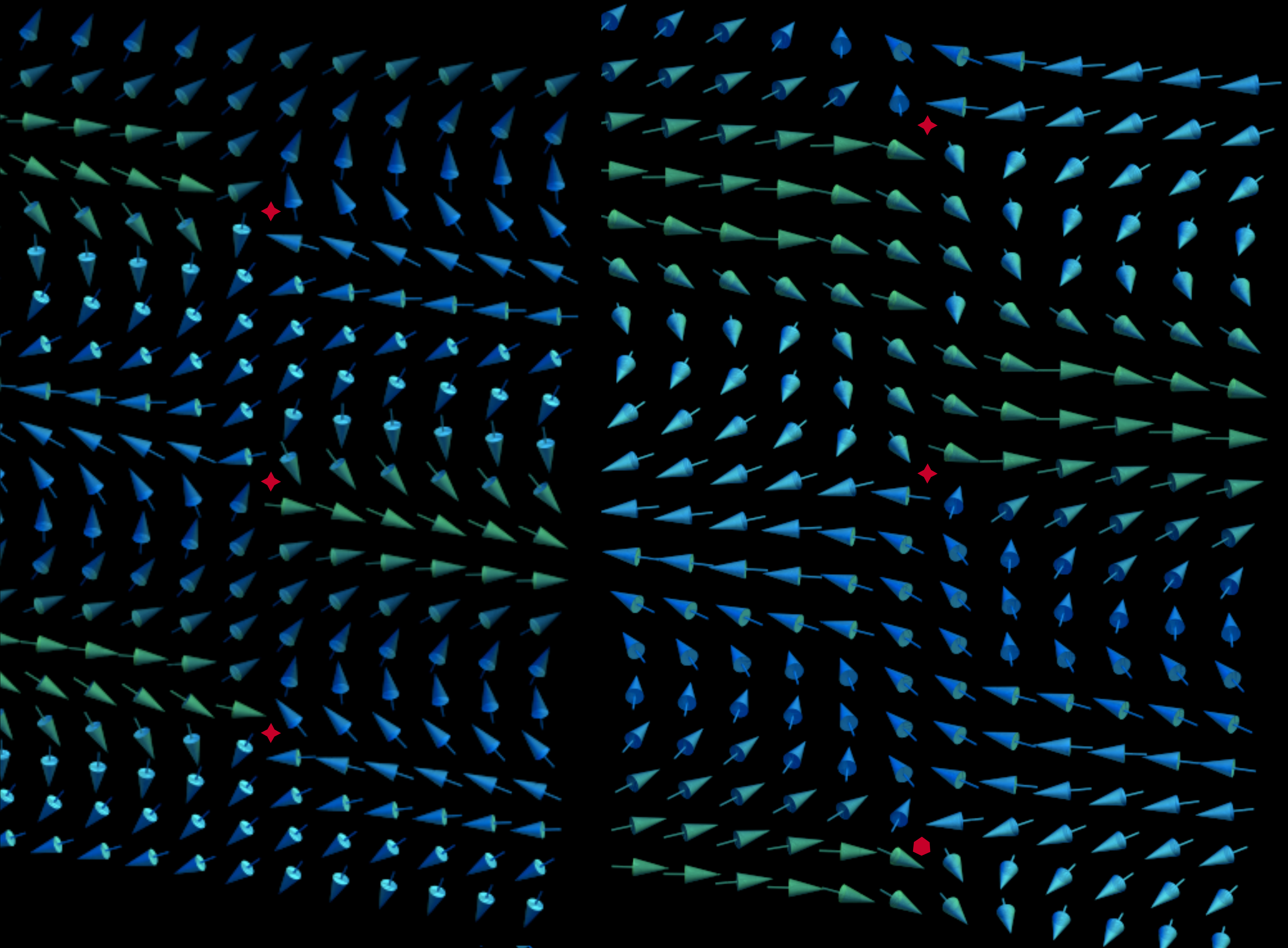
<u>Defects:</u> Vortex domain wall parallel to helix



$$n_w = \oint_C d\varphi/2\pi = 1$$

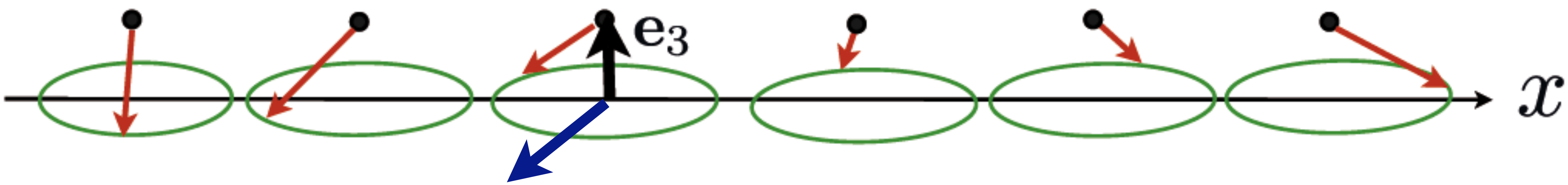


$$\sigma_v \sim J(\theta/a^2)\,\ln^{1/2}\left(\frac{\pi}{\theta}\right)$$



Centrosymmetric Crystals: Multiferroics

Mostovoy 2006: Coupling to electric polarization



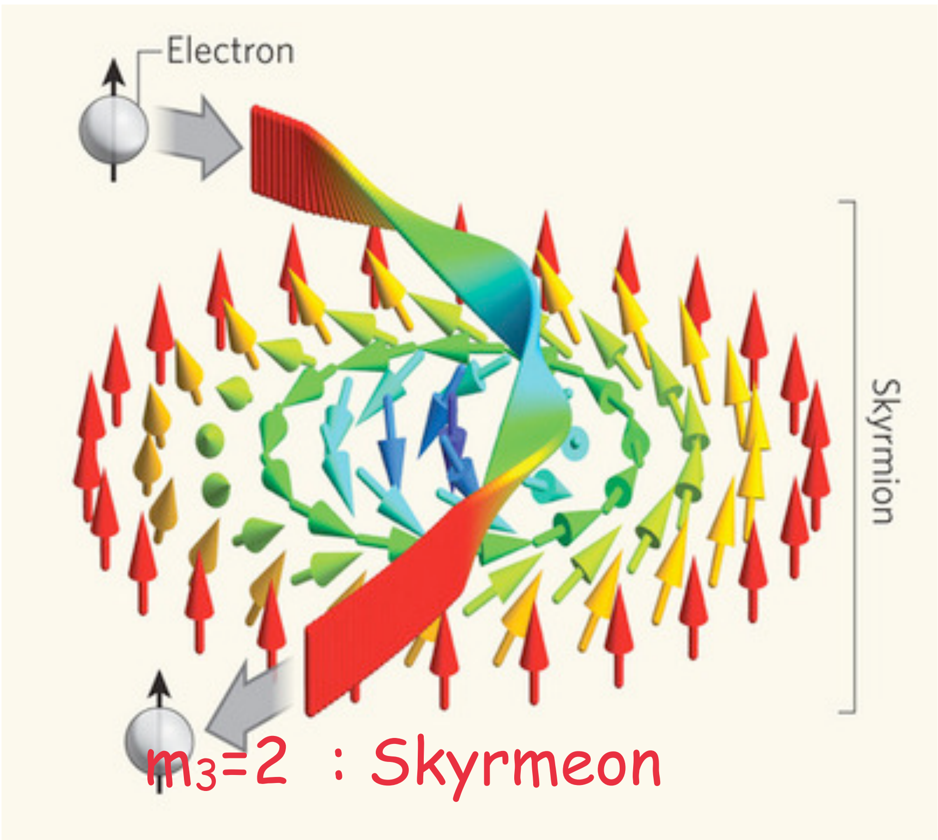
$$\mathbf{P} = \kappa\left[\mathbf{m}(\nabla\cdot\mathbf{m}) - (\mathbf{m}\cdot\nabla)\mathbf{m}\right],$$

vortex lines are charged

$$\rho^{(1)} = 2\pi n_w \kappa\left[\mathbf{e}_3 \times \hat{x}\right]\hat{\mathbf{n}}$$

Centrosymmetric Crystals: Topological Hall effect*

*Ye, Kim, Millis, Shraiman, Majumdar, and Tesanovic



Electrons moving adiabatically in exchange field of magnetization experience Berry's magnetic field

$$\boxed{\begin{aligned} B_\alpha &= \frac{\phi_0}{8\pi}\epsilon_{\alpha\beta\gamma}\mathbf{m}\cdot(\partial_\beta\mathbf{m}\times\partial_\gamma\mathbf{m}) \\ \phi_B &= \frac{\phi_0}{4\pi}\int d\cos\vartheta d\varphi = \frac{\phi_0}{2}m_3. \end{aligned}}$$

force on vortex line

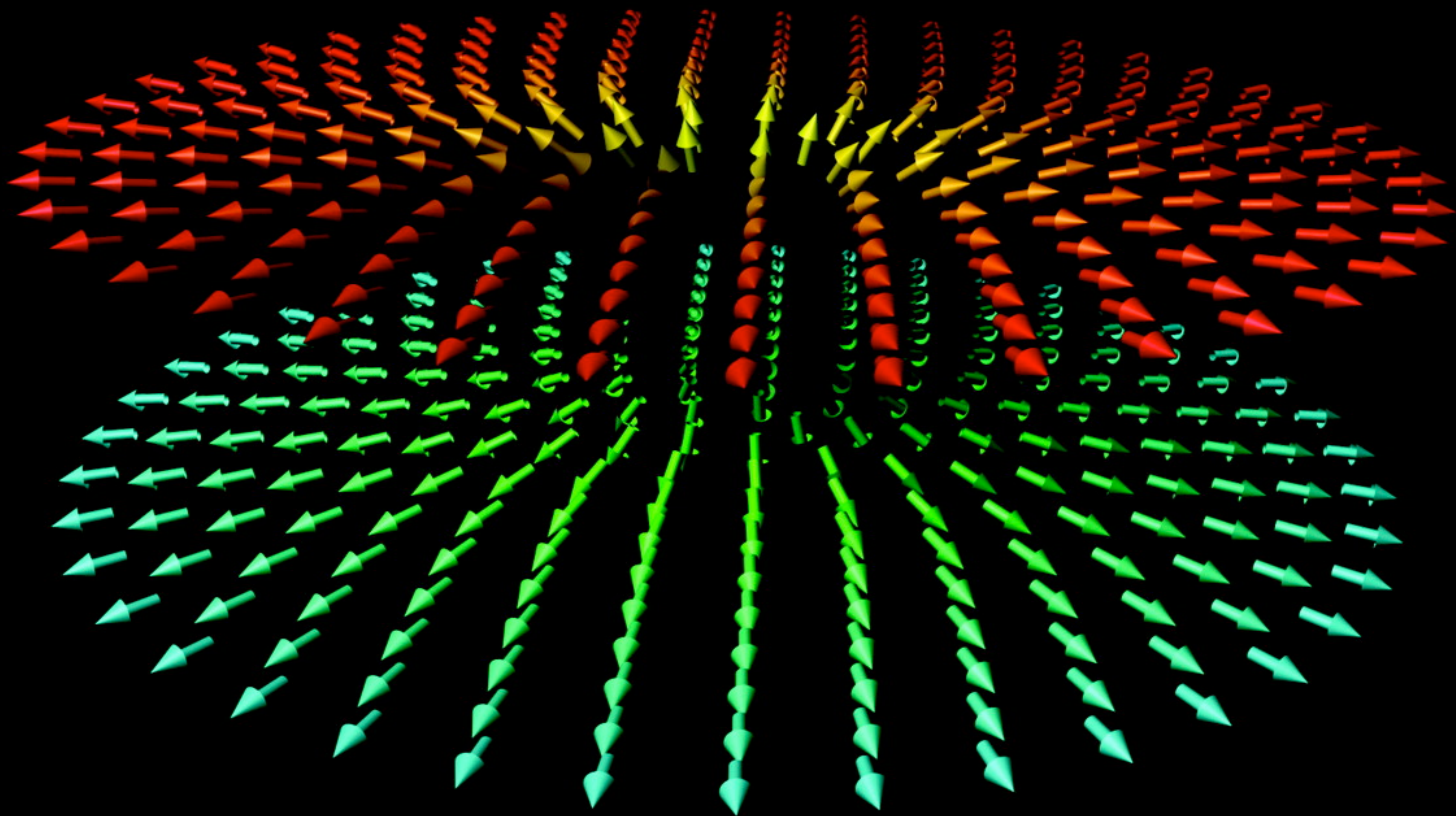
$$\mathbf{f} = \frac{1}{c}\mathbf{j}\times\mathbf{e}_3\phi_B \ .$$

$$p = m_3\theta\frac{j}{10^5\,Am^{-2}}Nm^{-2}$$

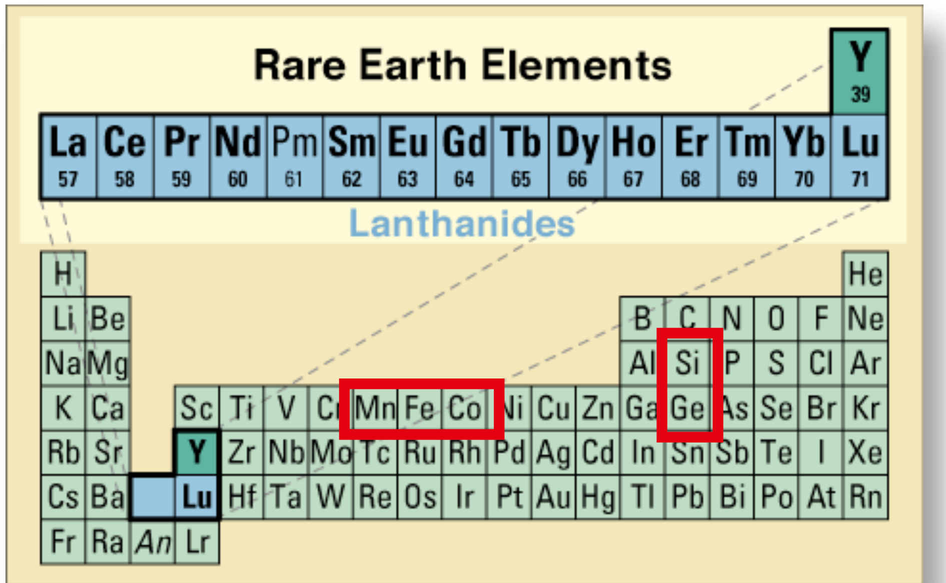
$$p_c = J\theta n_i a/6 \approx \theta\frac{T_c}{20K}\frac{n_i}{10^{17}cm^{-3}}Nm^{-2}$$

$$j_c \approx 6\cdot 10^7 Am^{-2}$$

$m_3=1$: Meron



Non-Centrosymmetric Crystals: systems and model



FeGe , MnSi, $Fe_{1-x}Co_xSi$

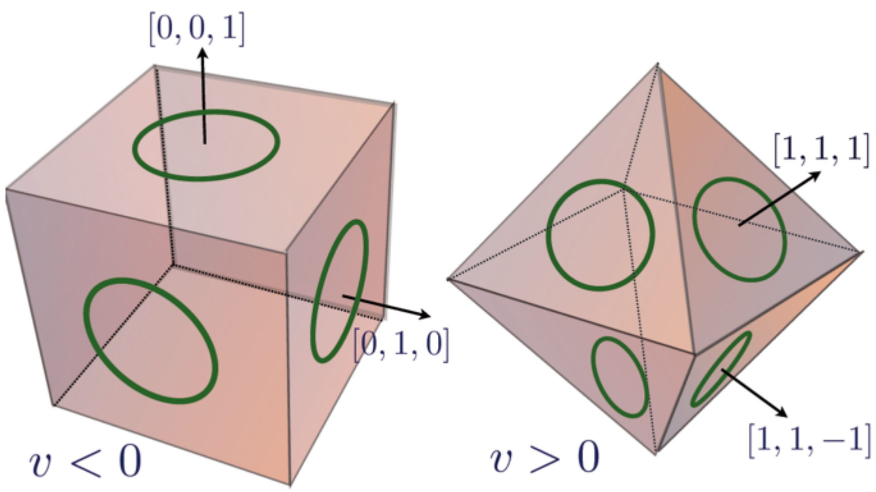
Heisenberg-spins + weak cubic anisotropy

Dzyaloshinskii-Moriya interaction $g\mathbf{M}_i \times \mathbf{M}_j$

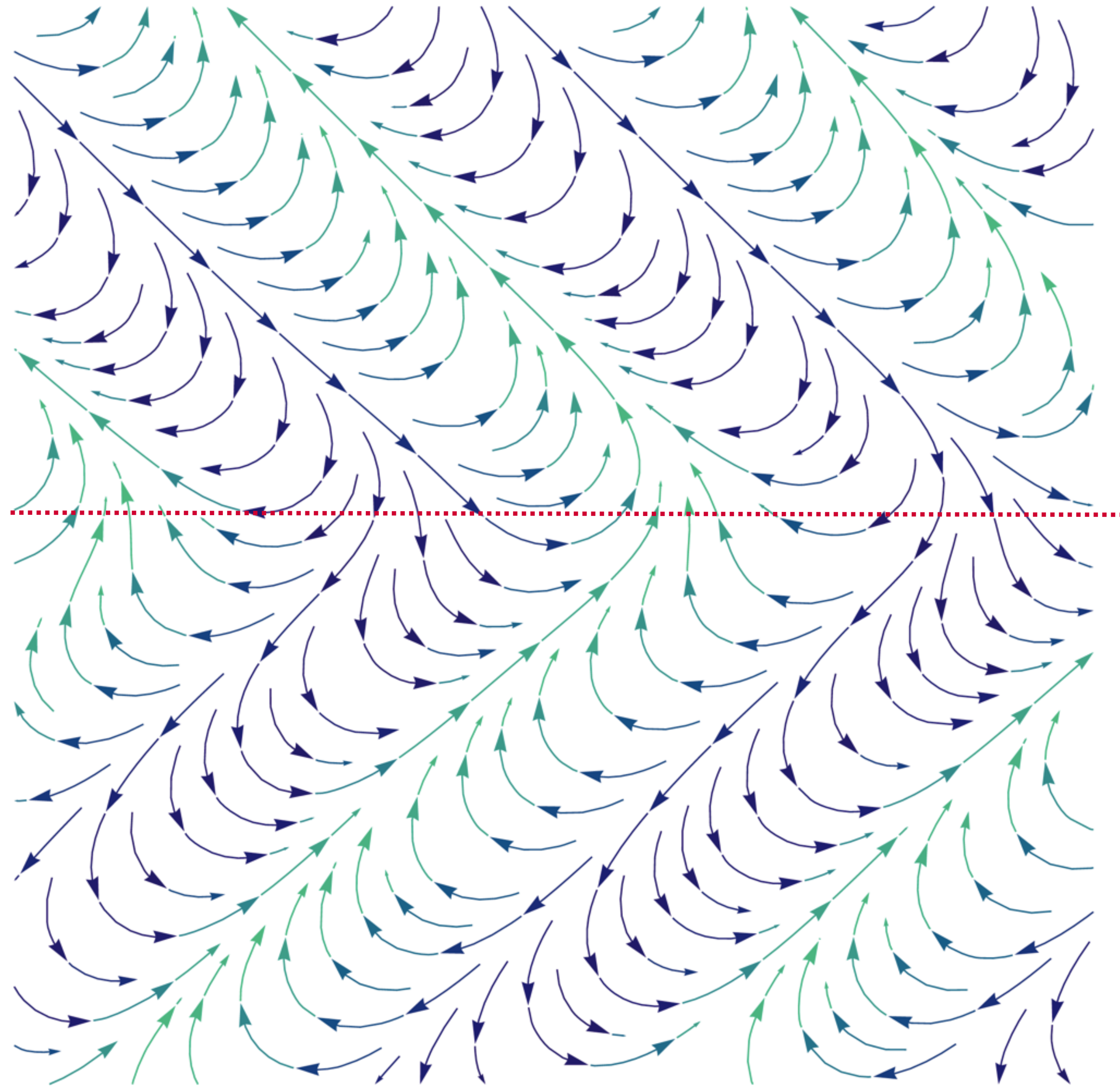
$$\mathcal{H}_{ncs} = J/a \int d^3r \left\{ (\boldsymbol{\nabla} \boldsymbol{m})^2 + 2q\, \boldsymbol{m} \cdot [\boldsymbol{\nabla} \times \boldsymbol{m}] + v \cancel{\sum_\alpha m_\alpha^4} \right\}$$

$$v = O(q^4) \qquad \boldsymbol{m}(\boldsymbol{r}) = \mathbf{e}_1 \cos\varphi(\boldsymbol{r}) + q^{-1} \boldsymbol{\nabla}\varphi \times \mathbf{e}_1 \sin\varphi(\boldsymbol{r}),$$

Non-Centrosymmetric Crystals: Hubert walls

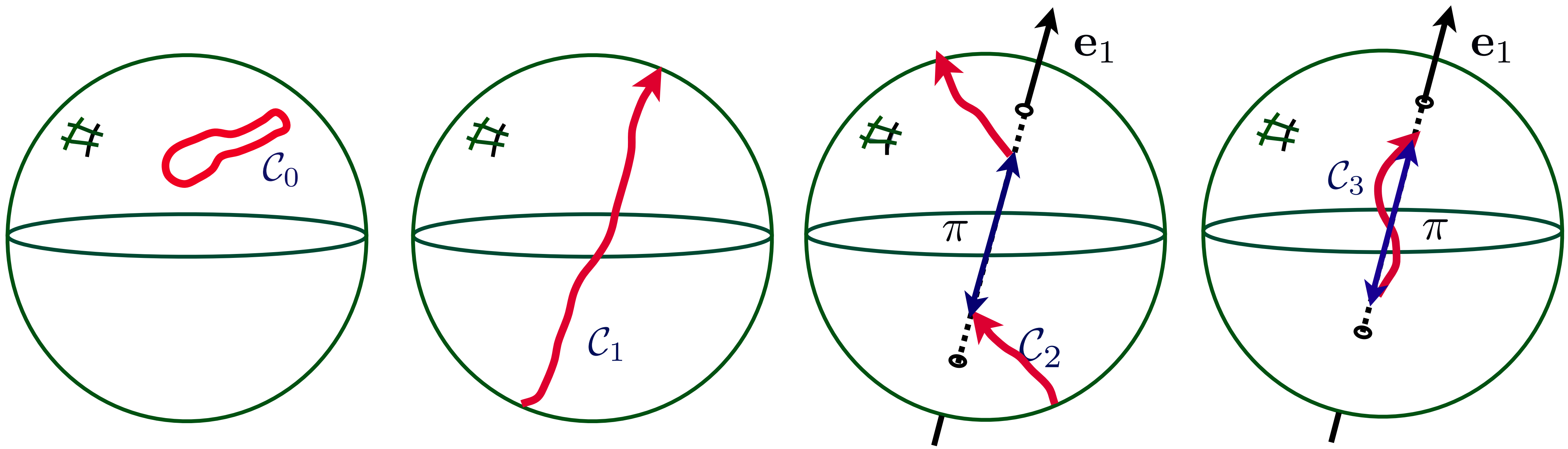


Domain wall bisector to
wave vectors of adjacent domains

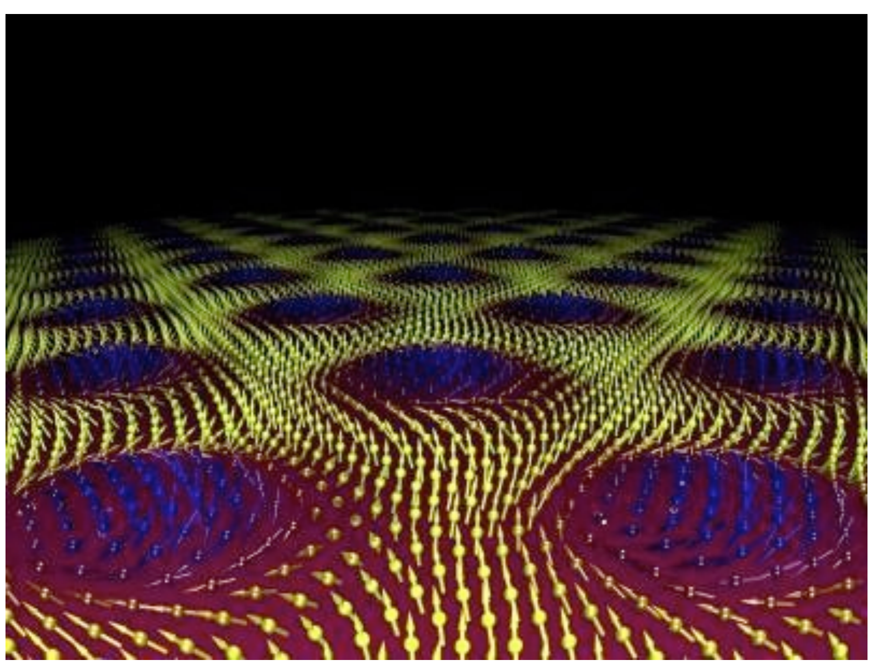


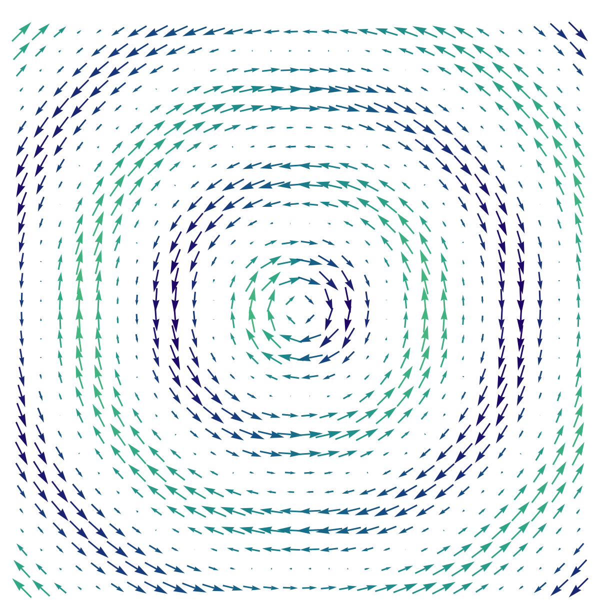
$$\mathcal{R} = SO(3)/Z_2$$

$$\pi_3(\mathcal{R}) = Z \text{ (non-trivial texture)}$$
$$\pi_2(\mathcal{R}) = 0 \text{ (no point defects)}$$
$$\pi_1(\mathcal{R}) = Z_4 \text{ (stable line defects)}$$

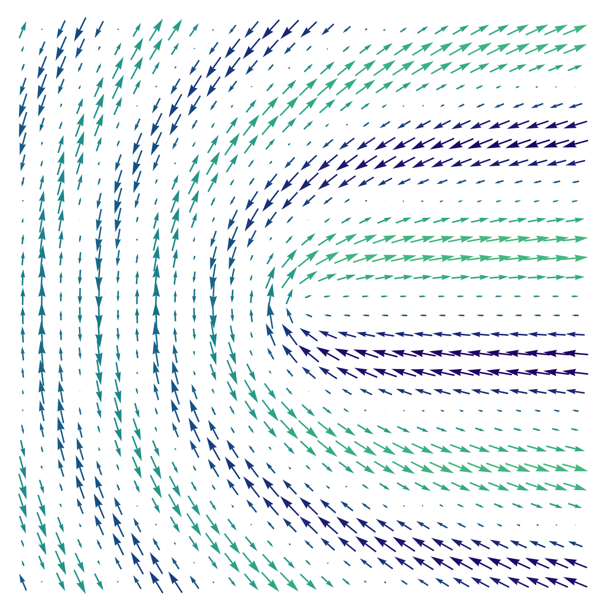


G.E. Volovik, V.P. Mineev, 1977, "Investigation of singularities in superuid 3He and liquid crystals by homotopic topology methods," Sov.Phys. JETP 45 1186 - 1196

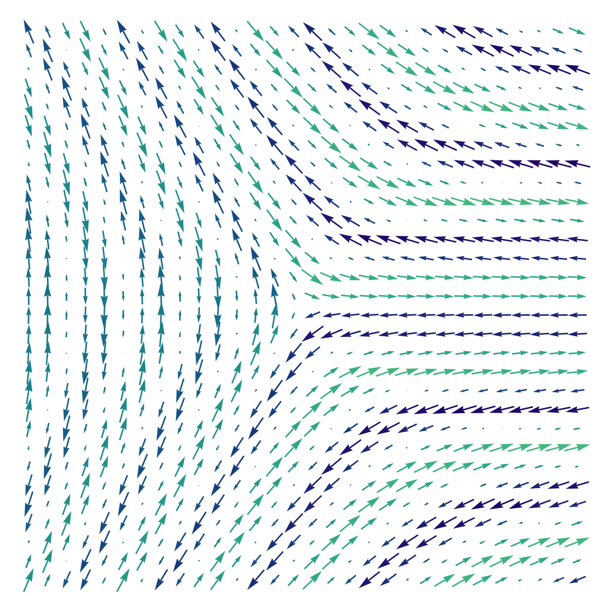




2π-disclination

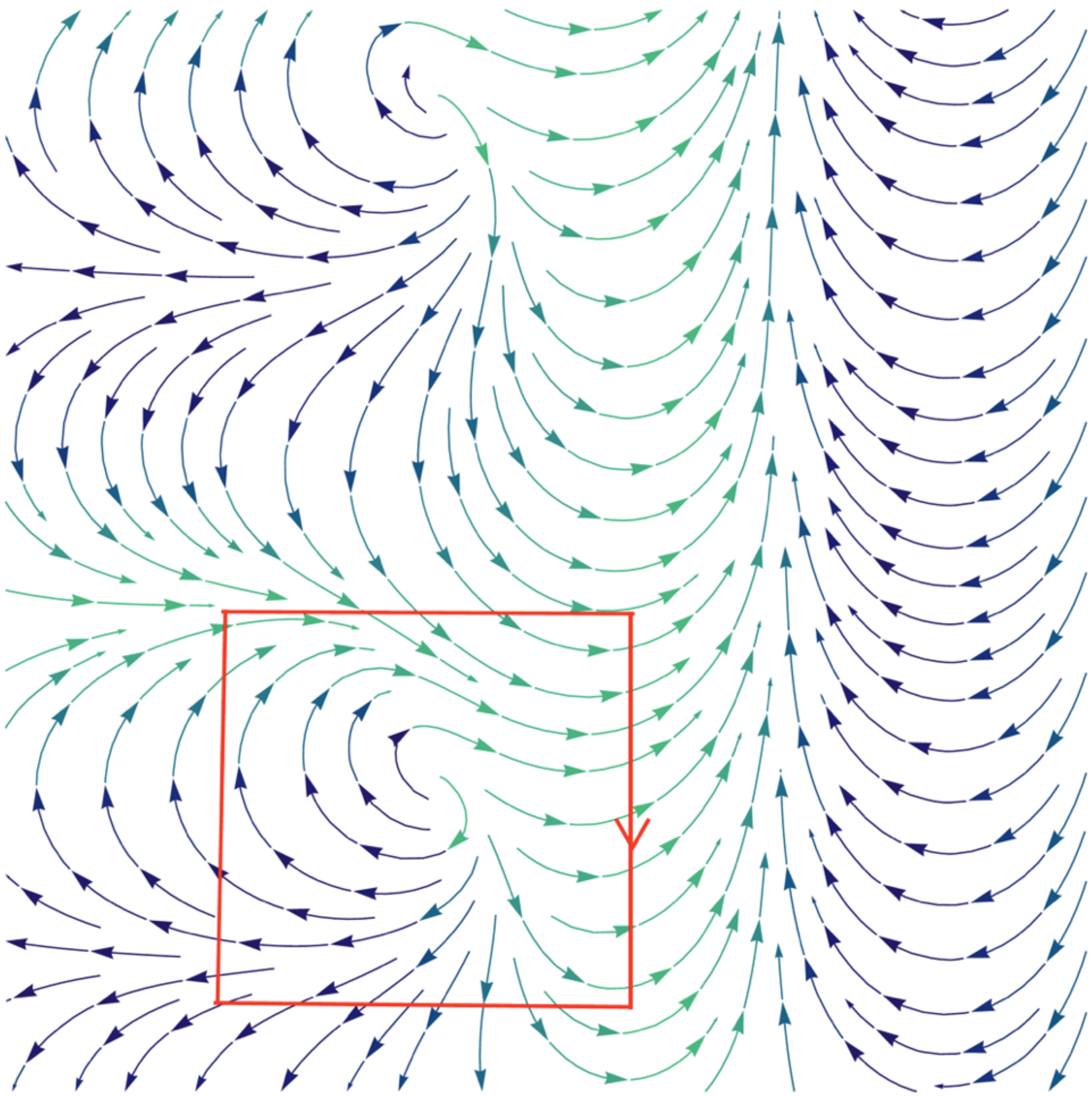


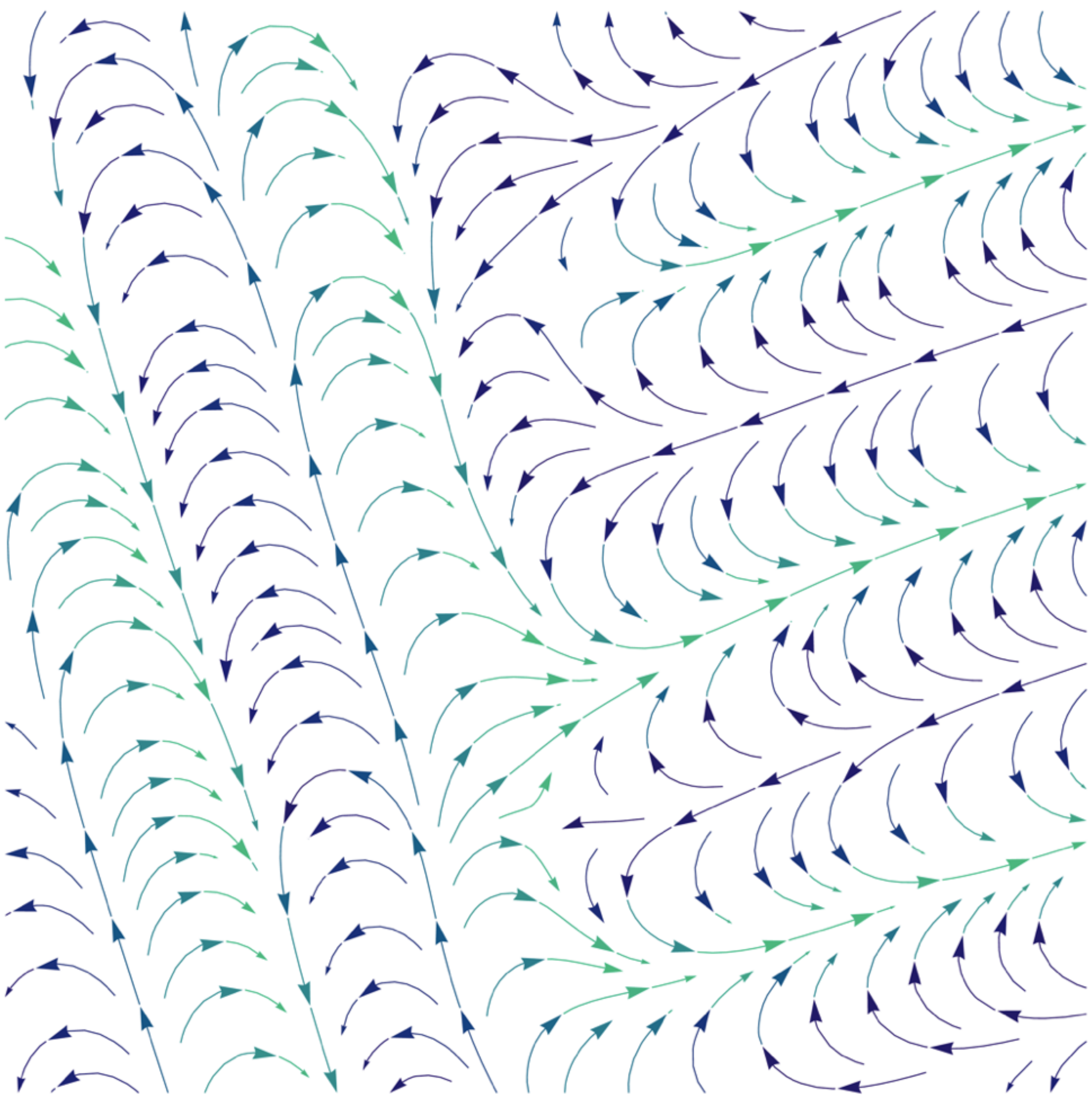
π-disclination



-π-disclination

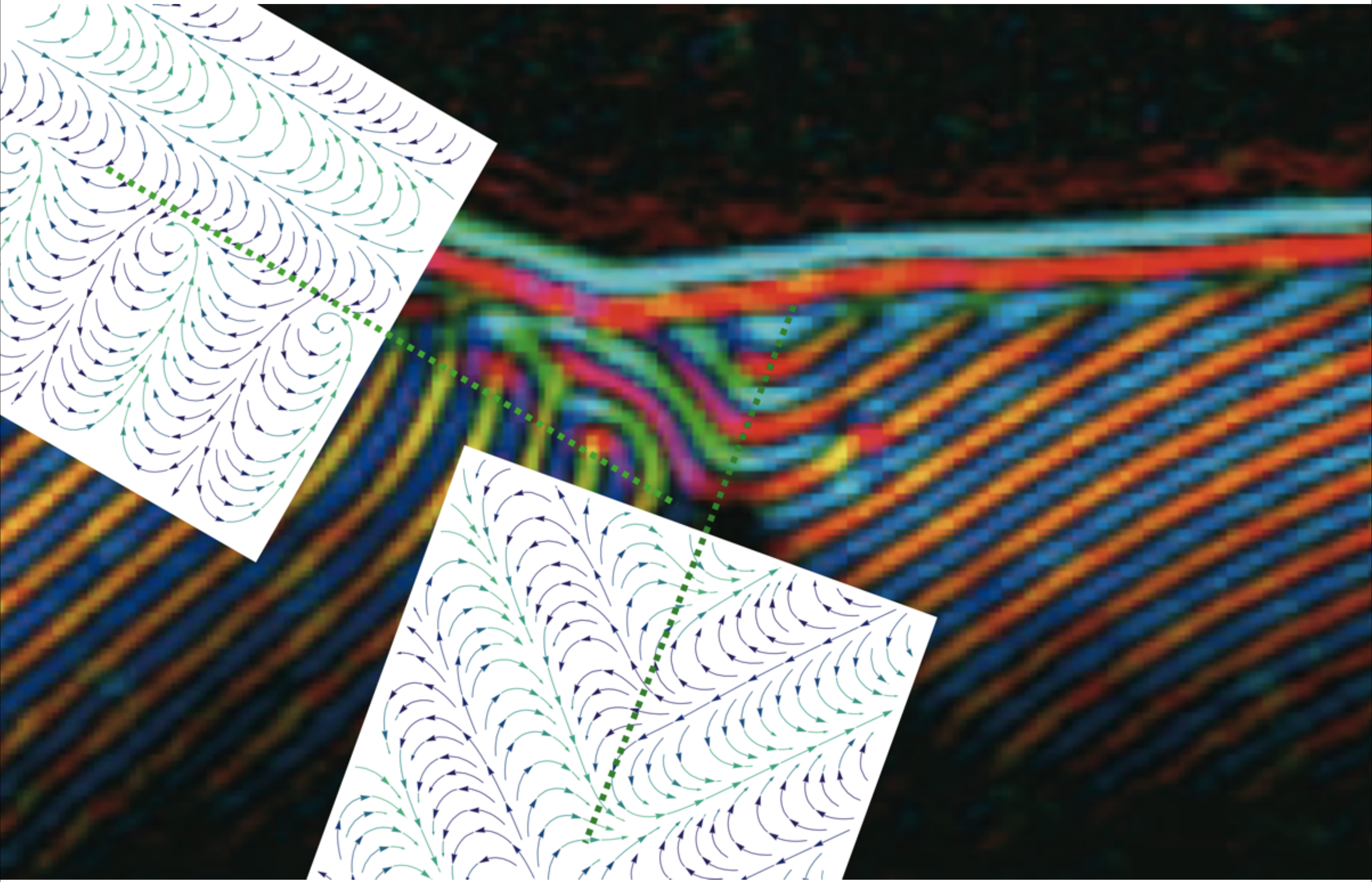
Domain wall is not bisector to the wave vectors of the adjacent domains





Numerically calculated vortex domain wall

Vortex domain walls much heavier than Hubert walls,
may decay in zig-zag structure of vortex-free domain walls



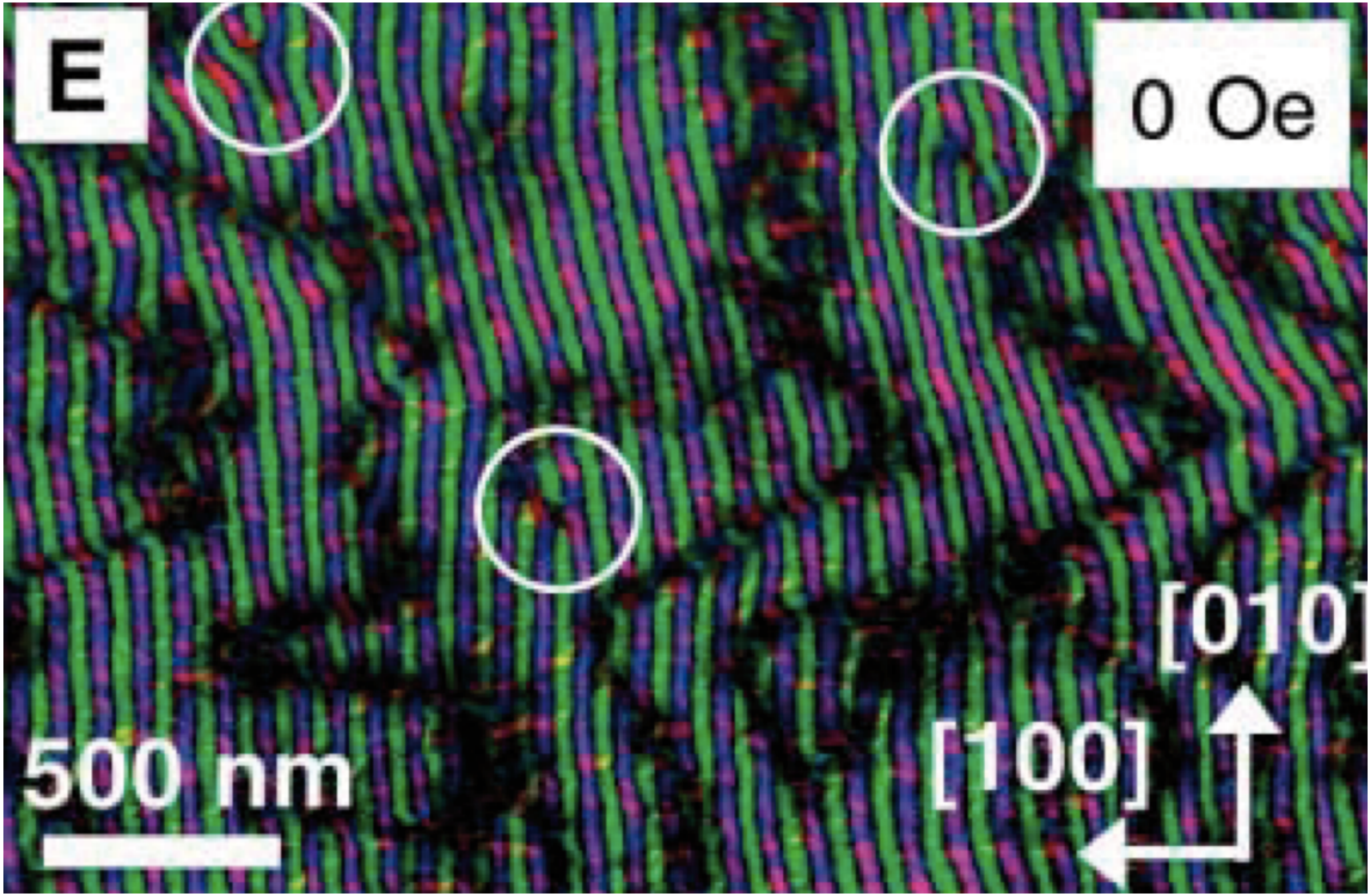
FeGe, Uchida et al. 2008

Conclusions

Domain walls in helical magnets are always two dimensional textures (vortex lines), in contrast to Bloch or Neel walls, with the exception of special orientations (Hubert walls).

Vortex walls generate Berry's field -> topological Hall effect -> domain walls can be manipulated by electric current.

In Multiferroics vortex lines are charged.



Real-Space Observation of Helical Spin Order

Masaya Uchida,[1*] Yoshinori Onose,[1†] Yoshio Matsui,[2] Yoshinori Tokura[1,3,4]

Conclusions and prospects

Helical magnets appear in centro- and noncentro-symmetric crystals.

Topological defects in a helical magnets are twisted vortices and domain walls.

In centrosymmetric materials as Ho,Er,Tb the wave vector of the helix can have either sign.

Domain walls are Hubert and vortex walls and their superposition.

Domain walls in helical magnets are caused by a weak anisotropy. Nevertheless neither their width nor energy depend on anisotropy.

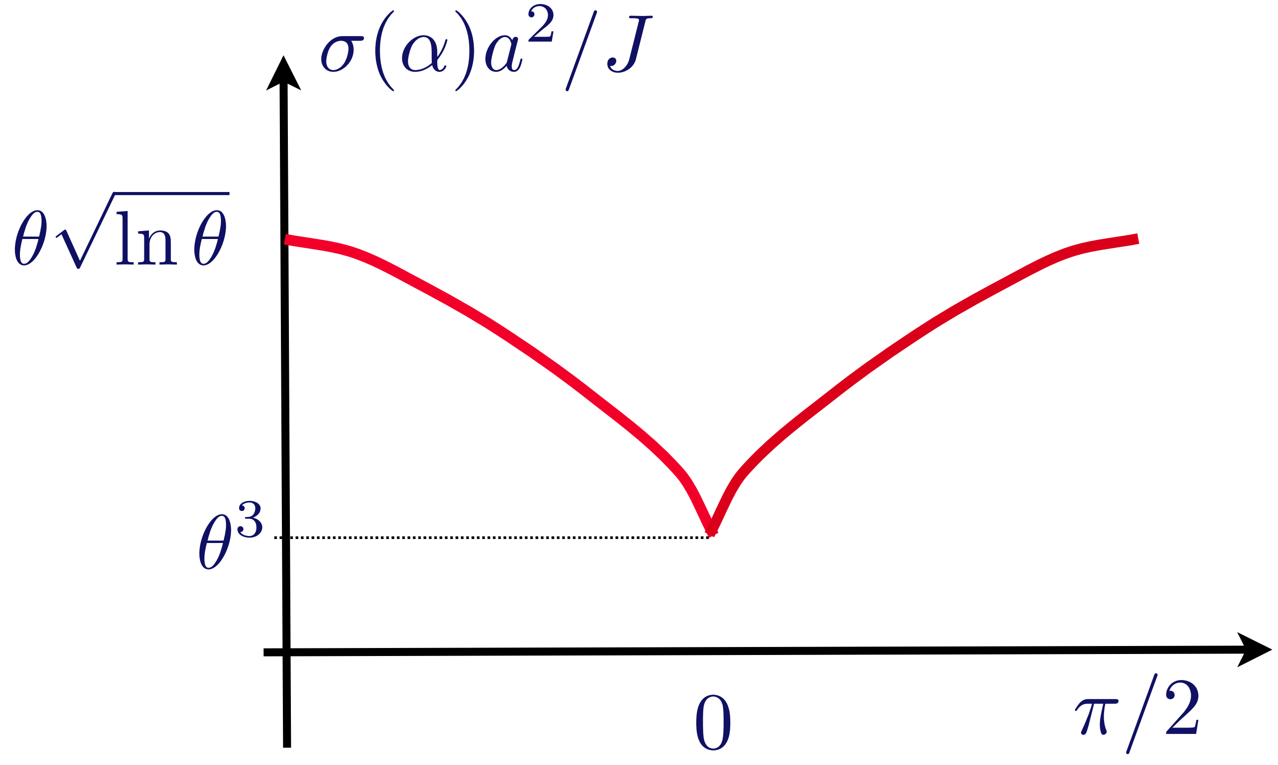
At almost any orientation of the plane domain wall it contains a chain of linear magnetic vortices whose energy is logarithmically large.

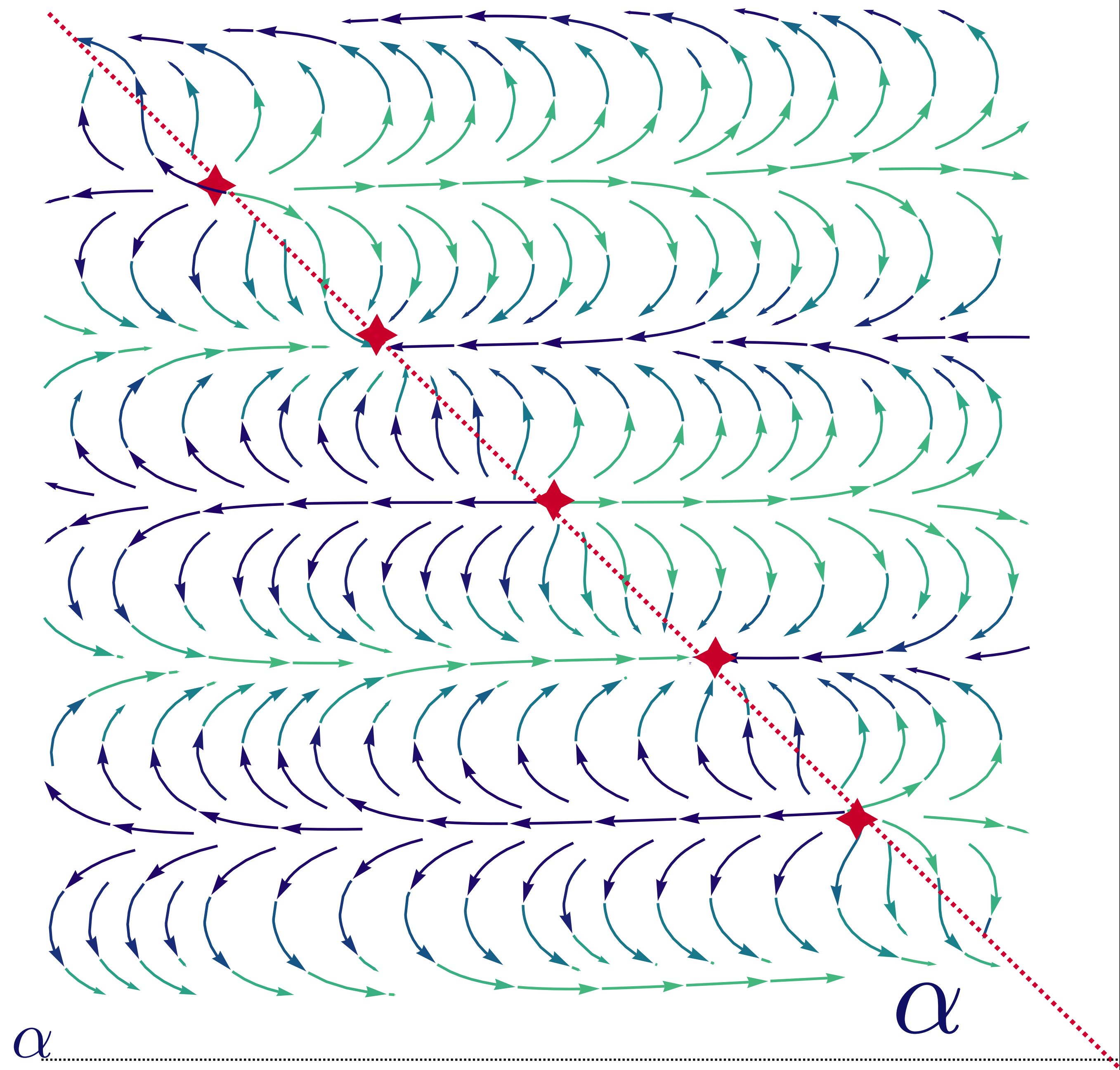
There are several exceptional orientations of domain walls – bisectors of two wave vectors determining the helixes far from domain wall – that correspond to “easy” vortex-free domain walls. Domain walls of other orientations can be alternatively formed as conjugated pieces of these “easy” walls (zig-zag structures).

Domain walls in helical magnets are always two dimensional textures. Domain walls generate Berry’s field interacting with electrons and bend their trajectories.

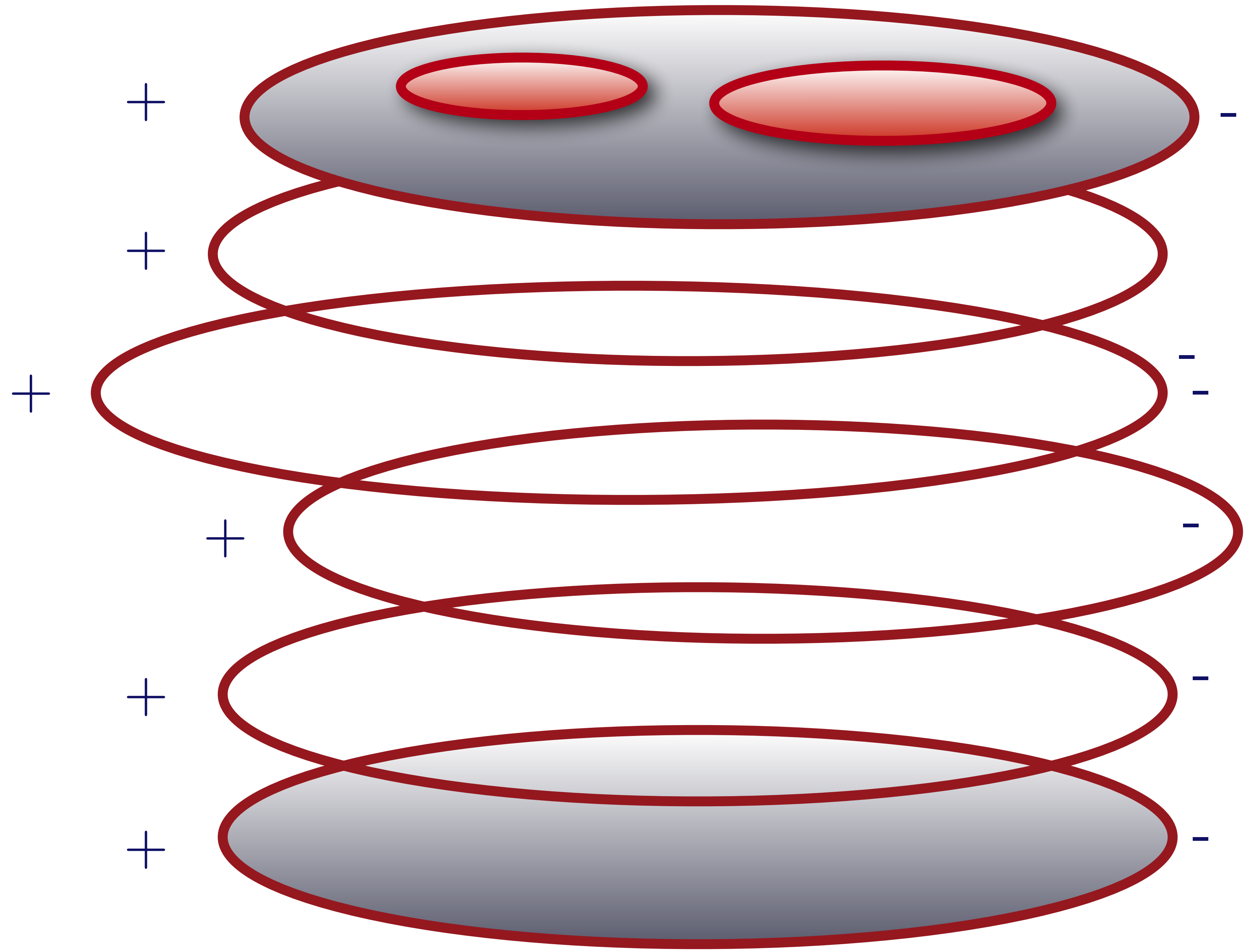
Centrosymmetric Crystals: Domain Walls (v)

Angular dependence of surface tension





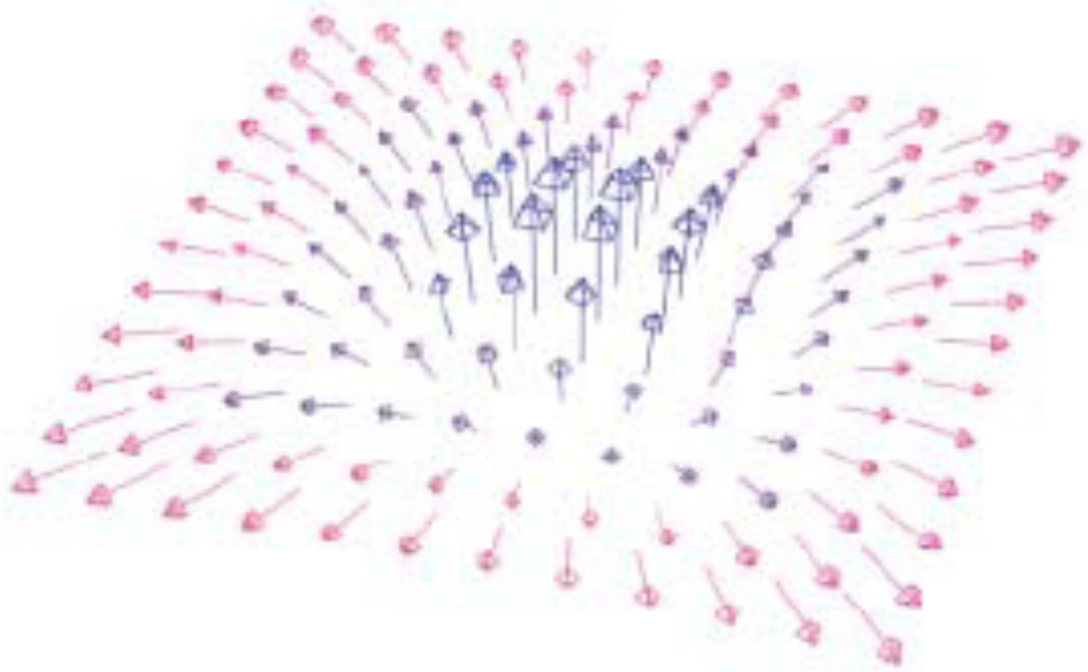
Centrosymmetric Crystals: Domain



$$F \approx 2\pi J L \theta \ln(Lq) - 2TL \ln 3 > 0$$

Hubert wall is always flat!

Centrosymmetric Crystals: Topological Hall effect*



m3=1 : Meron

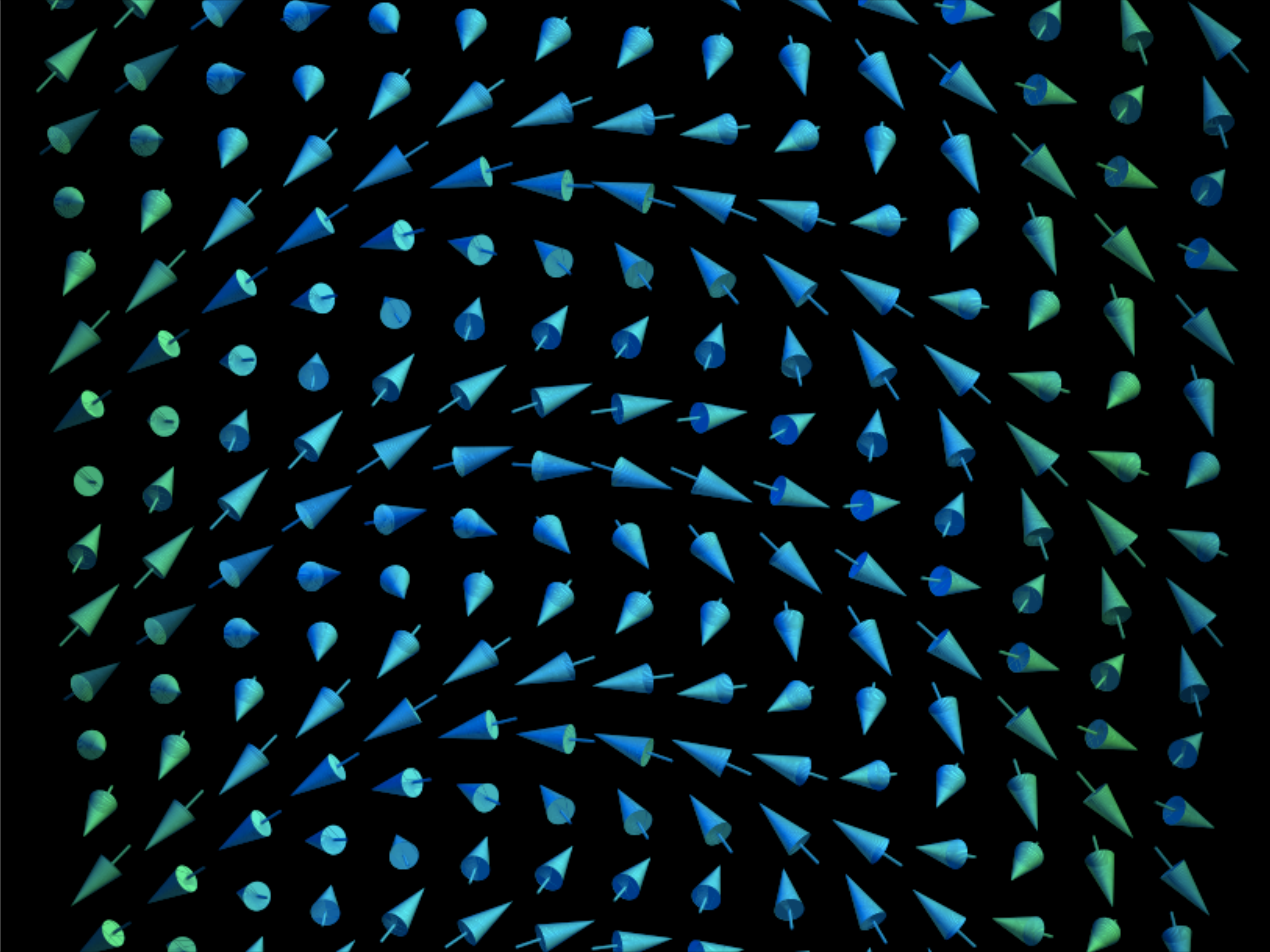
force on vortex line

$$\mathbf{f} = \frac{1}{c}\mathbf{j} \times \mathbf{e}_3 \phi_B \ .$$

$$p = m_3 \theta \frac{j}{10^5 Am^{-2}} Nm^{-2}$$

$$p_c = J\theta n_i a/6 \approx \theta \frac{T_c}{20K} \frac{n_i}{10^{17} cm^{-3}} Nm^{-2}$$

$$j_c \approx 6 \cdot 10^7 Am^{-2}$$



Detour:

Defect structures = deviation from perfect order

Classification according to homotopy groups : Toulouse & Kleman (1976)
Volovik & Mineev (1977)
Mermin (1979)

Degeneracy space $\mathcal{R}$

Consider mapping of a subspace $\mathcal{V}_d$ of $\mathbb{R}^3$ on $\mathcal{R}$

Ensemble of equivalent mappings: homotopy group $\pi_d(\mathcal{R})$

Continuum Hamiltonian

$$\mathcal{H}_{cs} = J/a \int d^3r \left\{ (\boldsymbol{\nabla}_\perp \boldsymbol{m})^2 - \frac{\theta^2}{2} (\partial_z \boldsymbol{m})^2 + \frac{a^2}{4} (\partial_z^2 \boldsymbol{m})^2 \right\}.$$

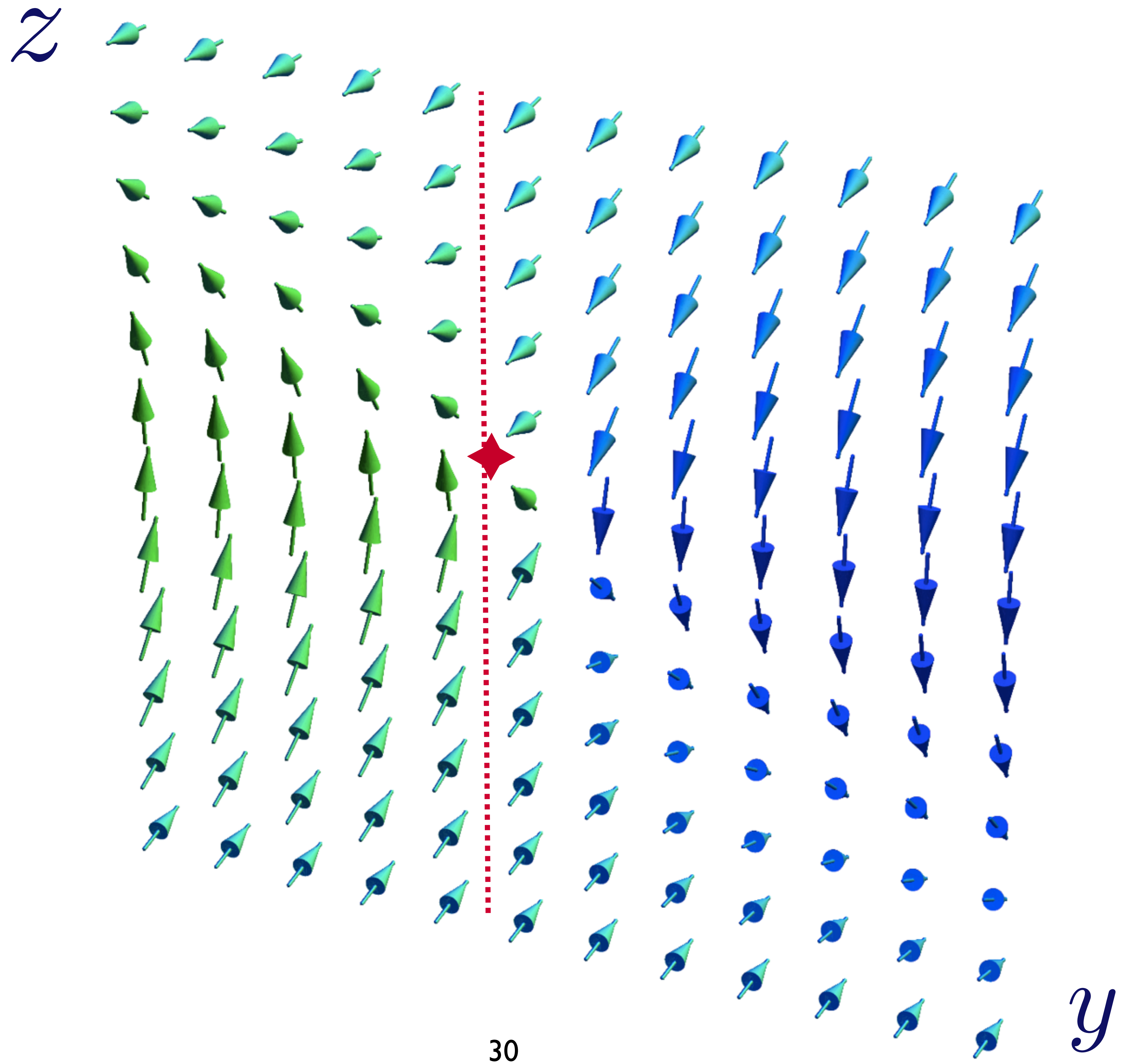
$$\boldsymbol{\nabla}_\perp = \hat{\mathbf{x}}\partial_x + \hat{\mathbf{y}}\partial_y \qquad \boldsymbol{m}^2 = 1 \qquad m_x + i m_y = e^{i\varphi}.$$

$$\text{symmetry } \mathbf{r} \to -\mathbf{r}$$

=> Two solutions of opposite chirality

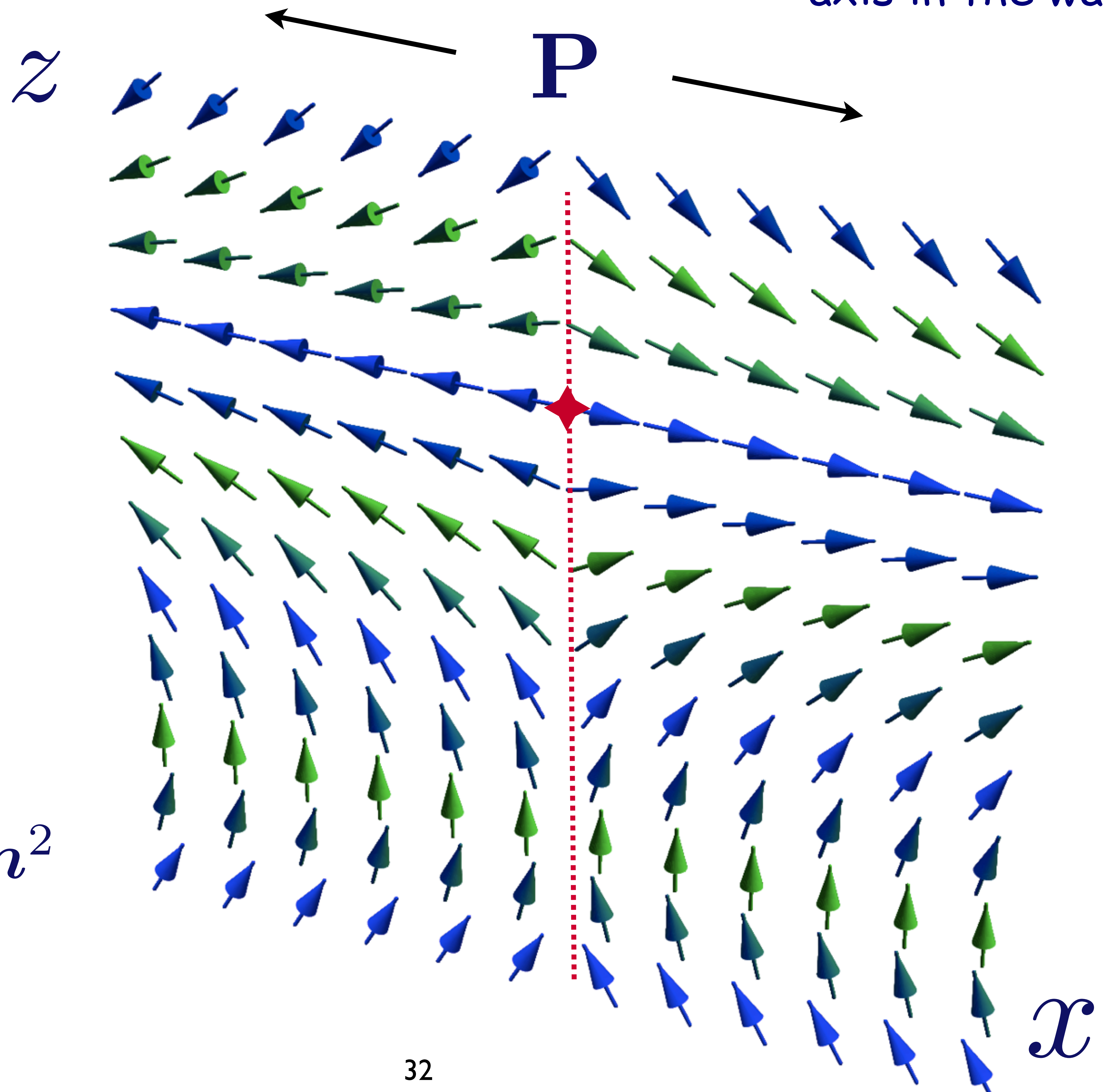
$$\varphi = \boldsymbol{q}\mathbf{z}, \qquad \boldsymbol{q} = \pm(\theta/a)\hat{z}, \qquad |q|a \ll 1.$$

(ii) wall in the xz-plane similar to Bloch wall: magmetization rotates around normal to the wall

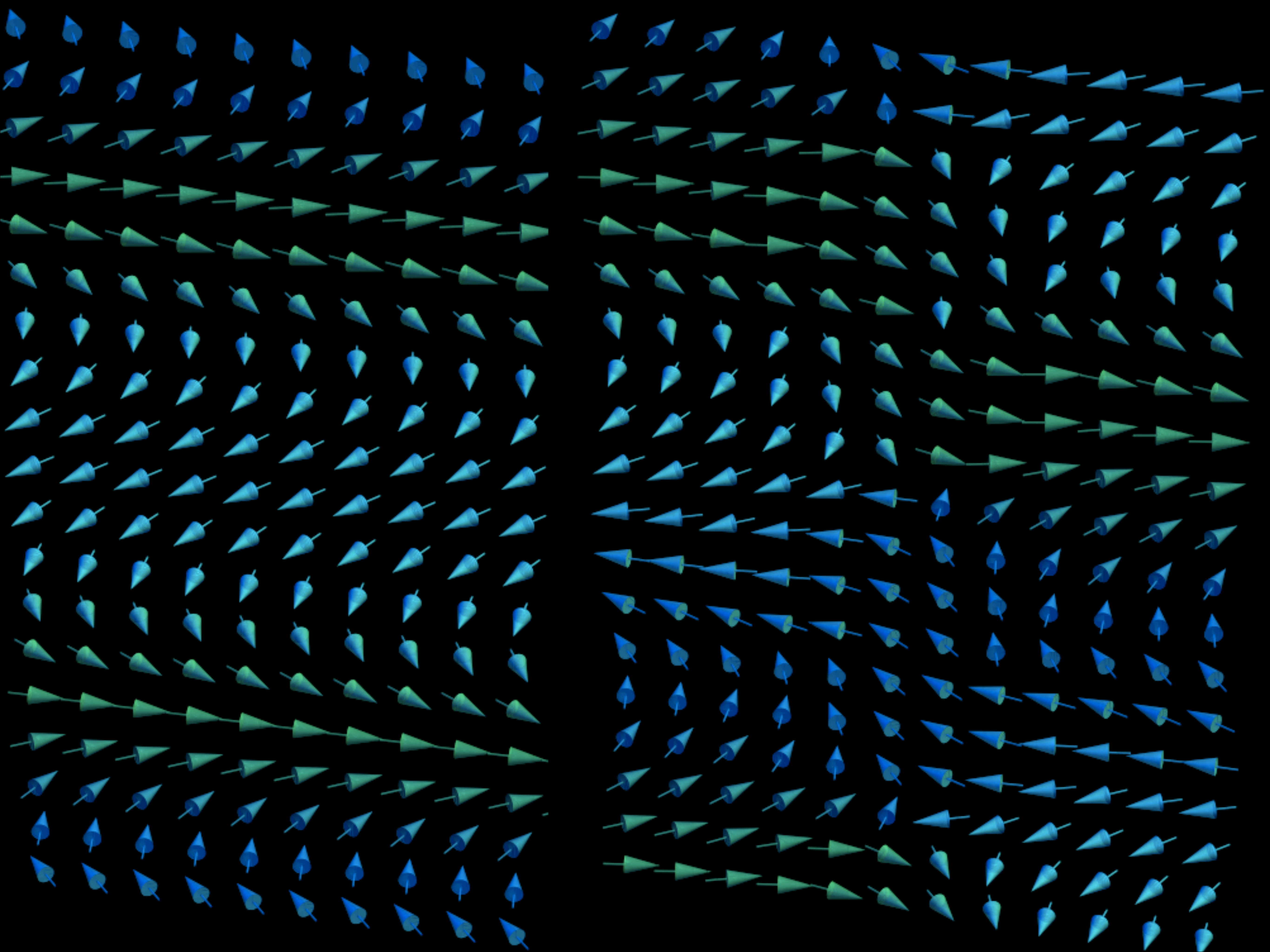


Motivation

(iii) wall in yz-plane similar to Neel wall: magnetization rotates around axis in the wall

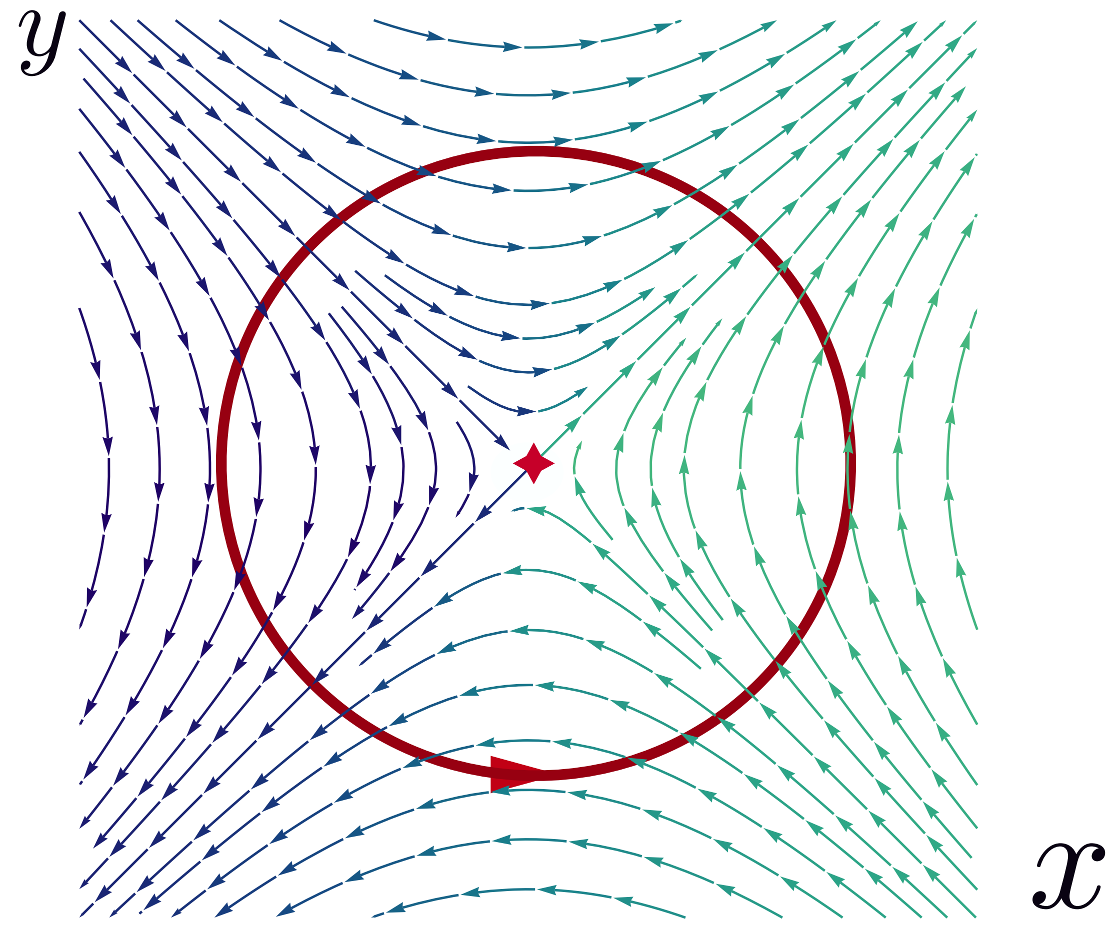


$$Q = 2\pi\gamma\chi\boldsymbol{m}^2$$



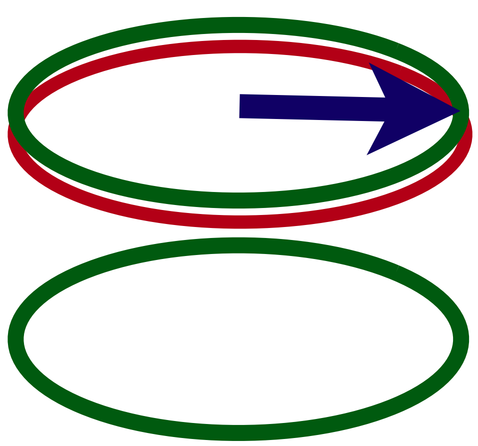
Defects: vortex line parallel to helix

real space



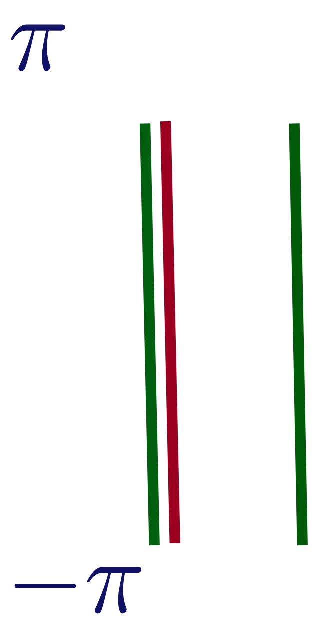
$$\varepsilon_{\|} \approx 2\pi J \ln L$$

$$n_w = \oint_{\mathcal{C}} d\varphi / 2\pi = -1$$



degeneracy space

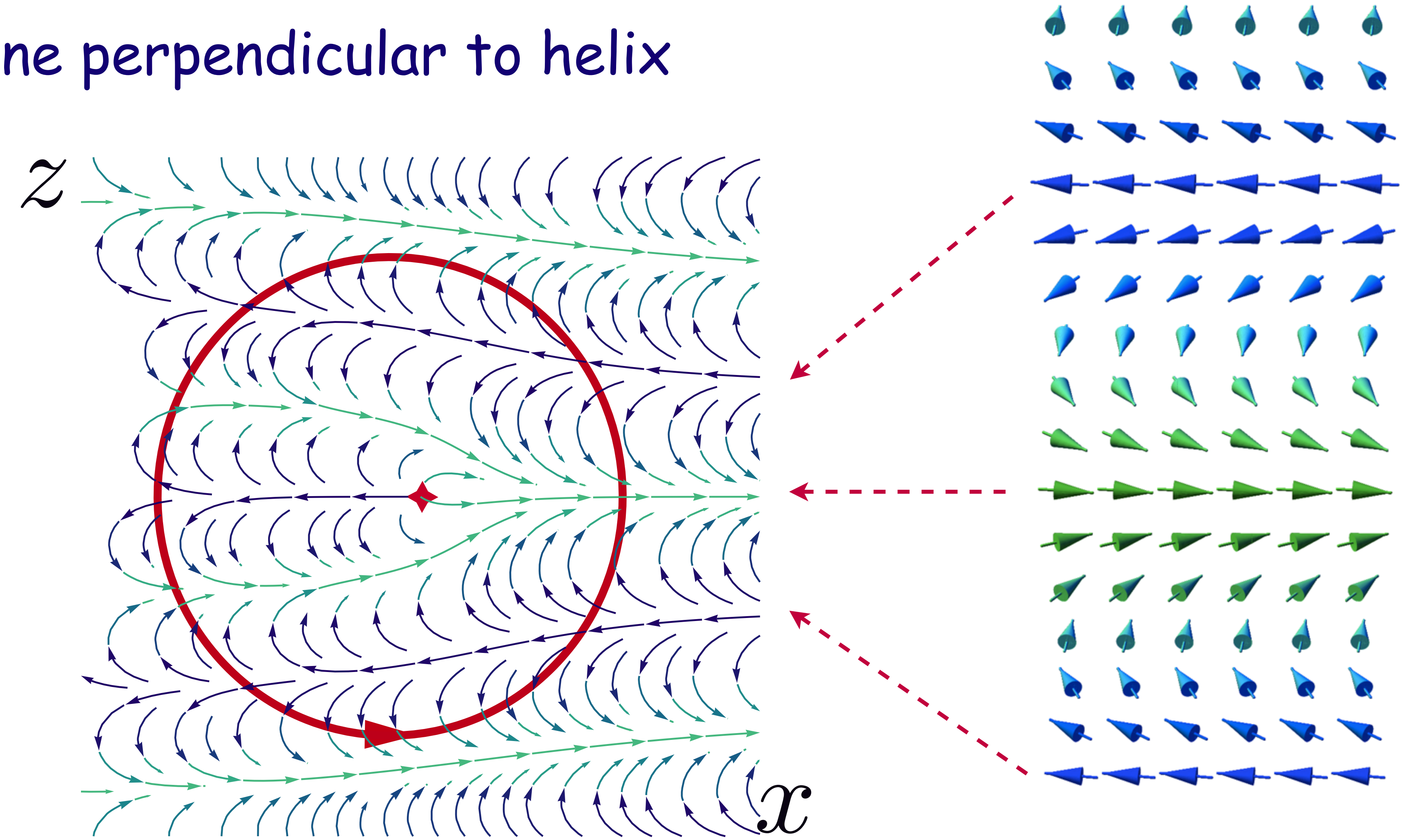
$$\mathcal{R} = \mathcal{S}^1 \times \mathcal{S}^0$$



Defects: vortex line perpendicular to helix

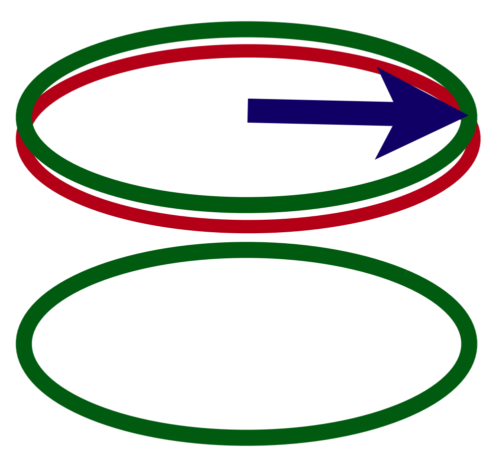
real space

FeGe, Tokura et al. ,08



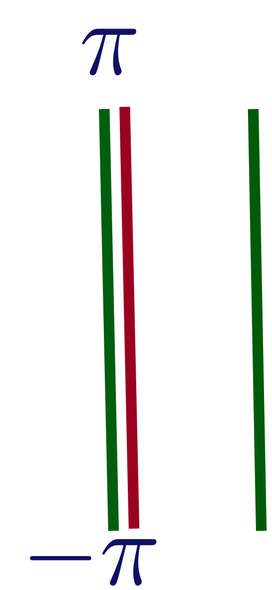
$$\varepsilon_{\perp} \approx 2\pi J \left\{ q \ln\left(\frac{Lq}{\pi}\right) + \frac{\sqrt{5}}{16a} \ln^{1/2}\left(\frac{\pi}{qa}\right) \right\}$$

$$n_w = \oint_C d\varphi / 2\pi = -1$$

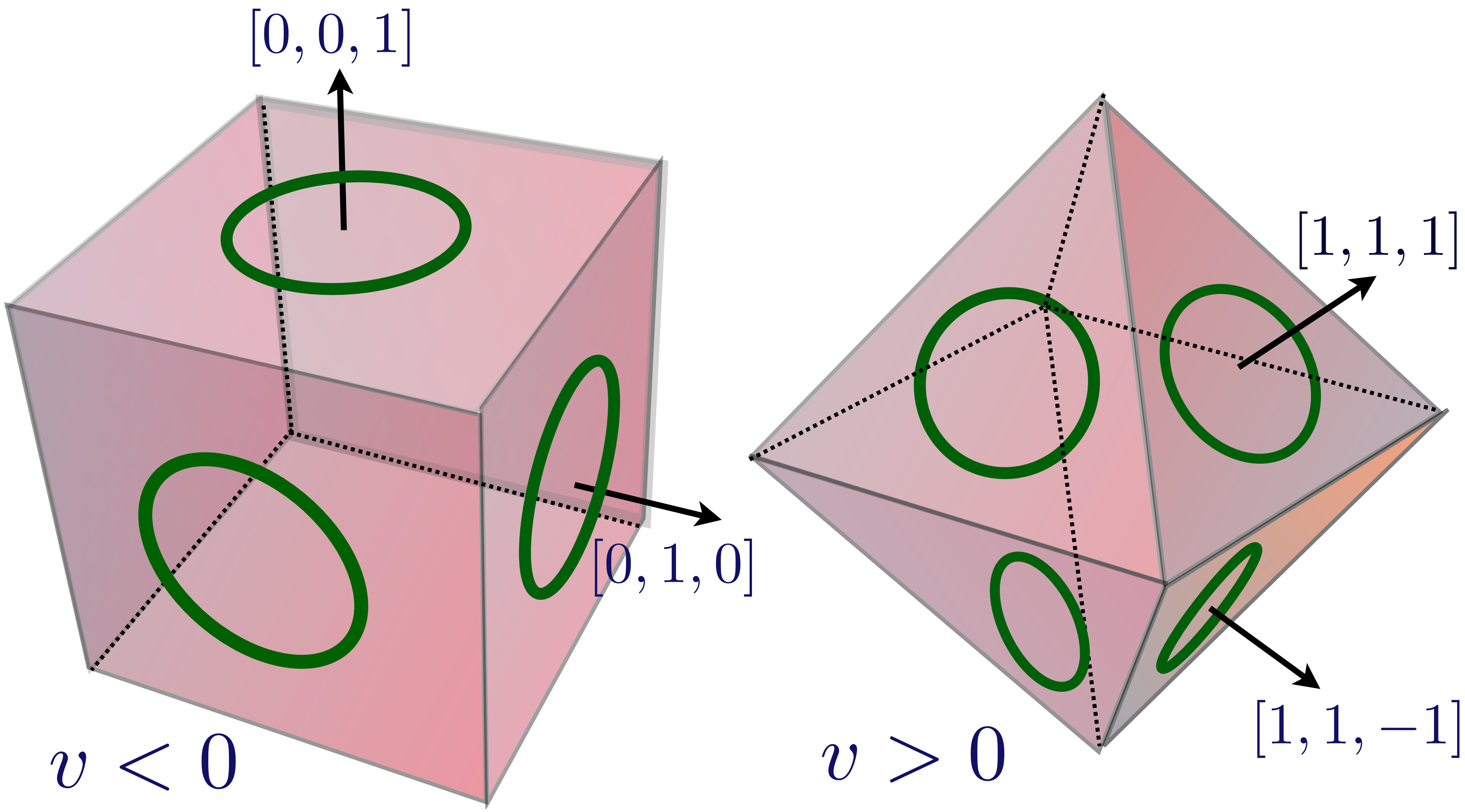


degeneracy space

$$\mathcal{R} = \mathcal{S}^1 \times \mathcal{S}^0$$

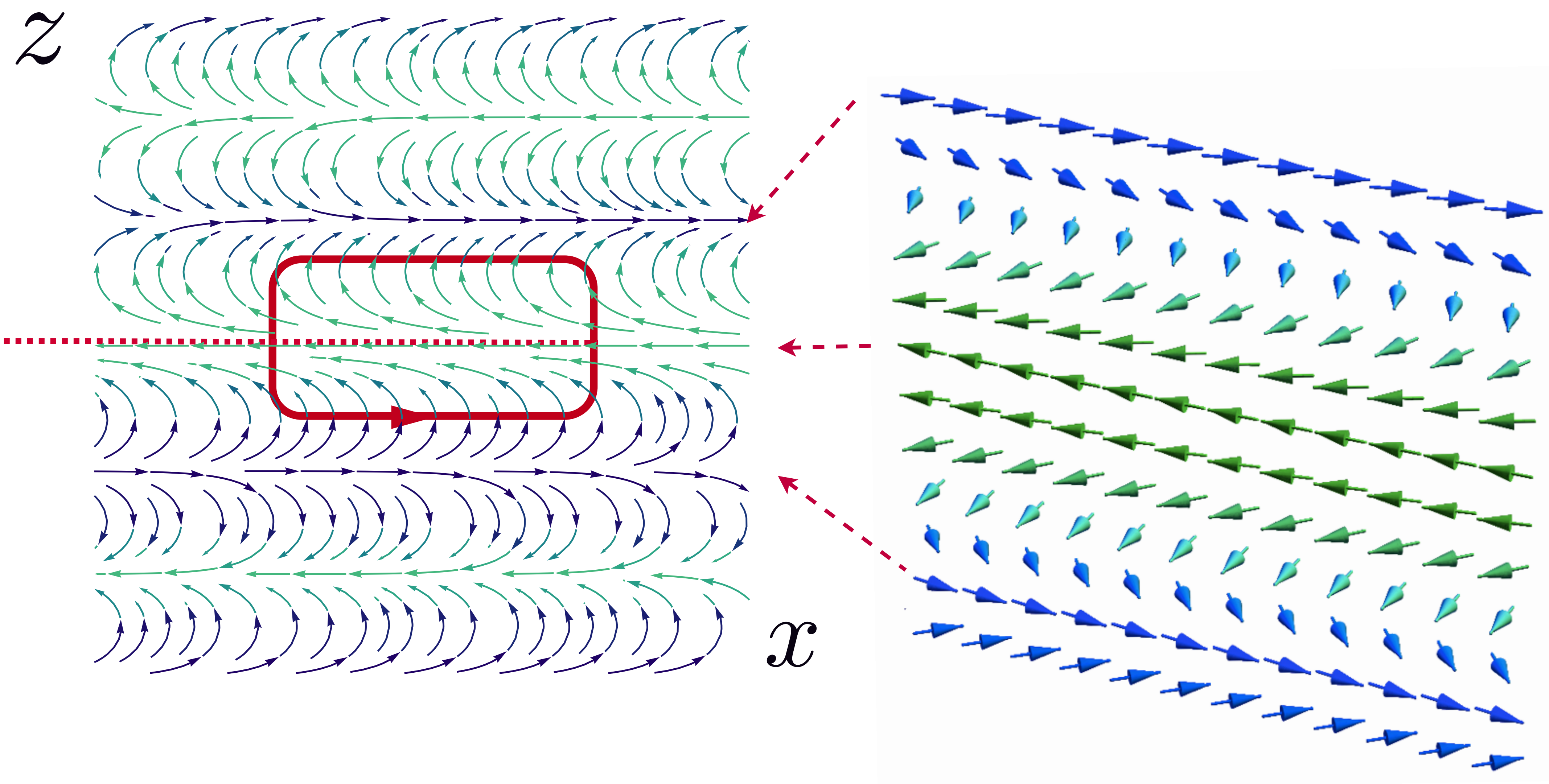


Cubic anisotropy: order parameter space

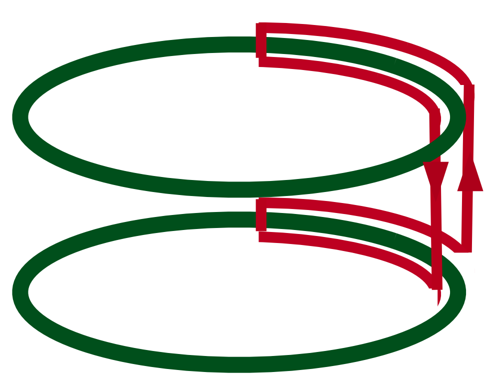


Defects: Hubert domain wall perpendicular to helix

$\sigma \sim Jaq^3$

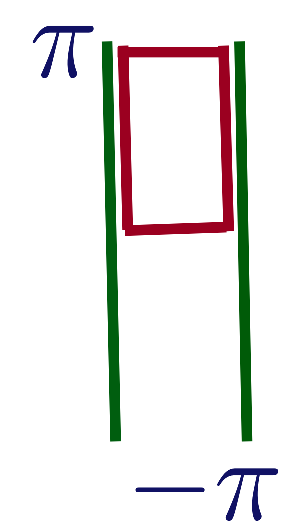


$n_w = \oint_C d\varphi/2\pi = 0$

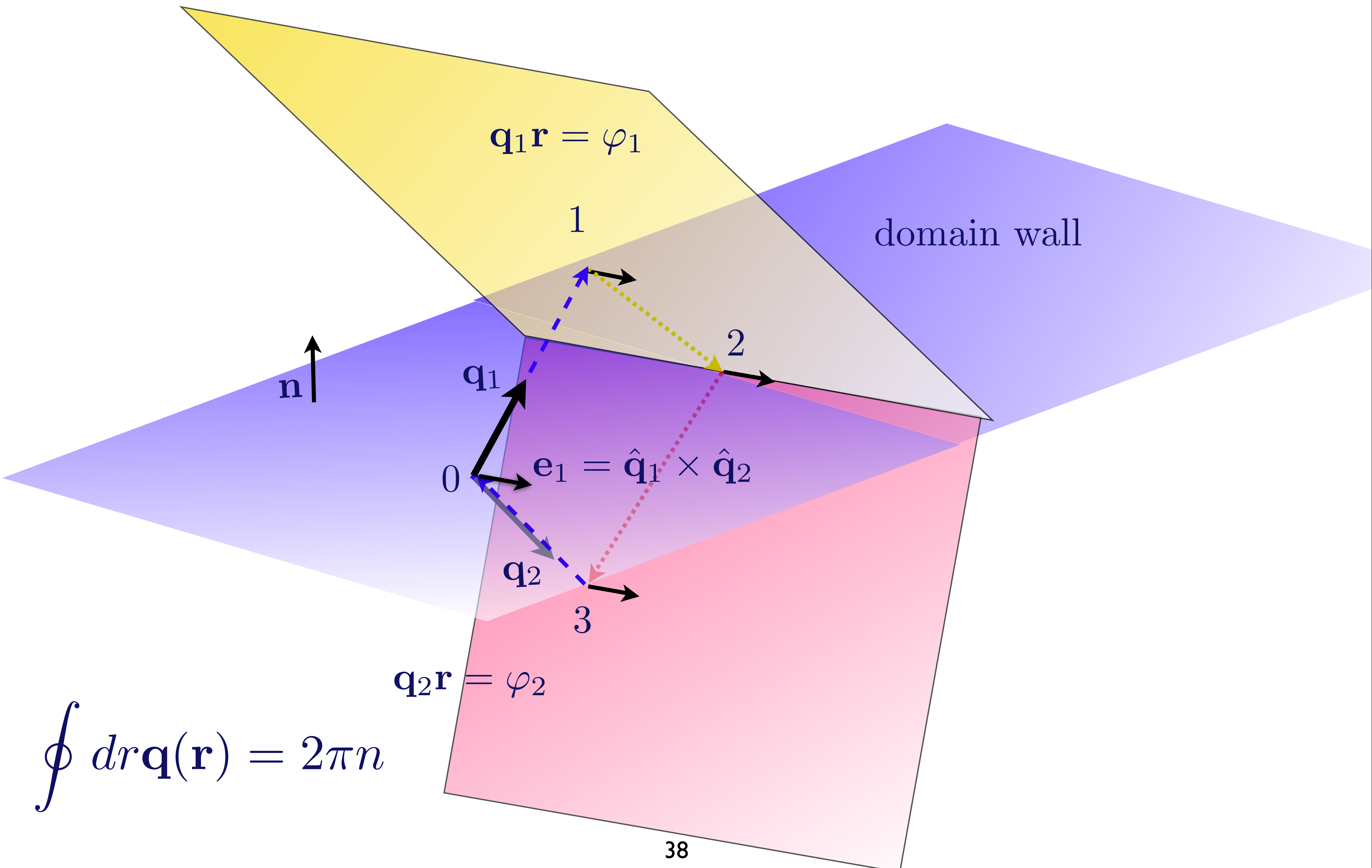


degeneracy space

$\mathcal{R} = \mathcal{S}^1 \times \mathcal{S}^0$

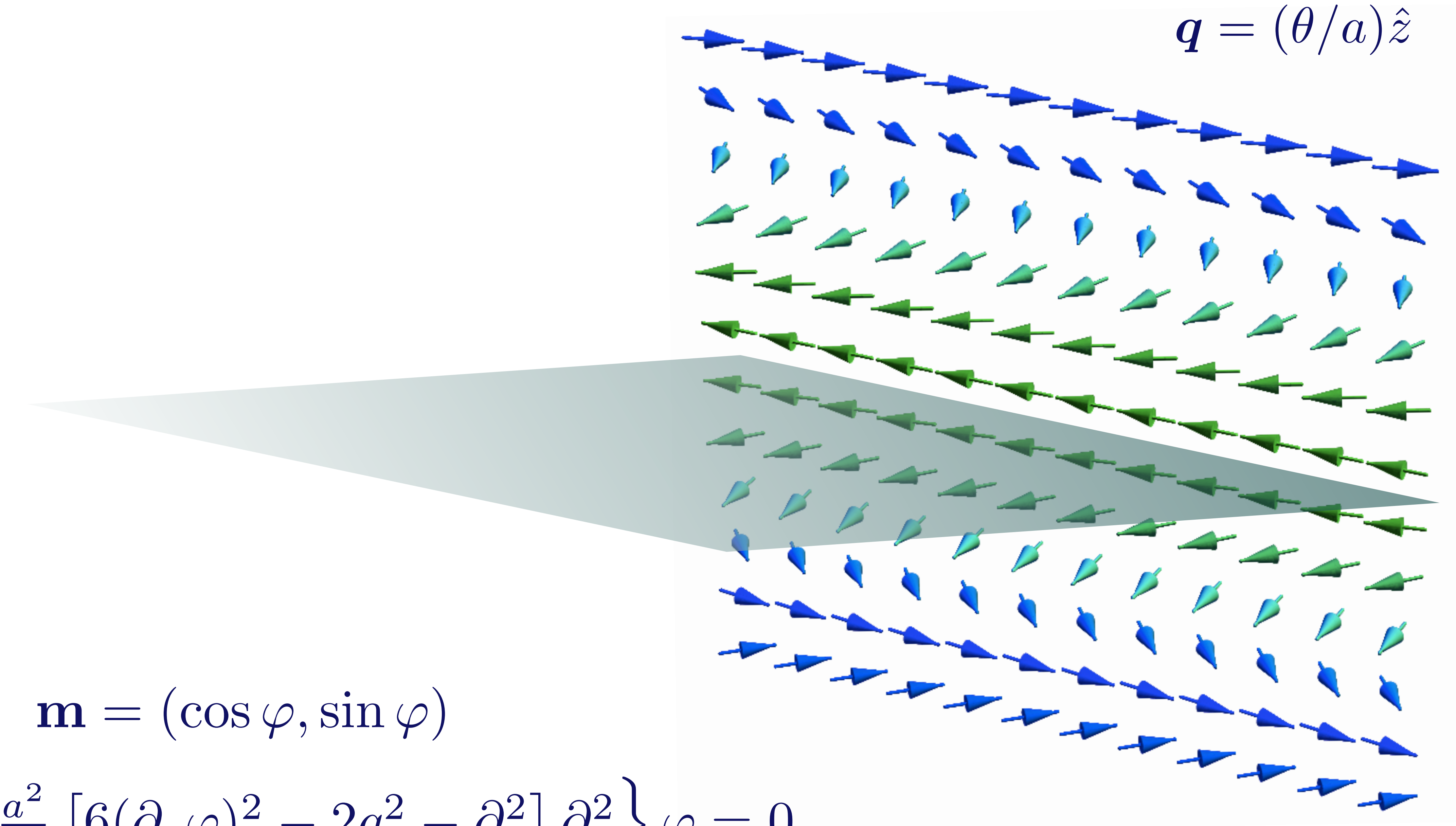


Integration contour



$$\oint dr\mathbf{q}(\mathbf{r}) = 2\pi n$$

Hubert walls (1975):
domain walls perpendicular to helical axis

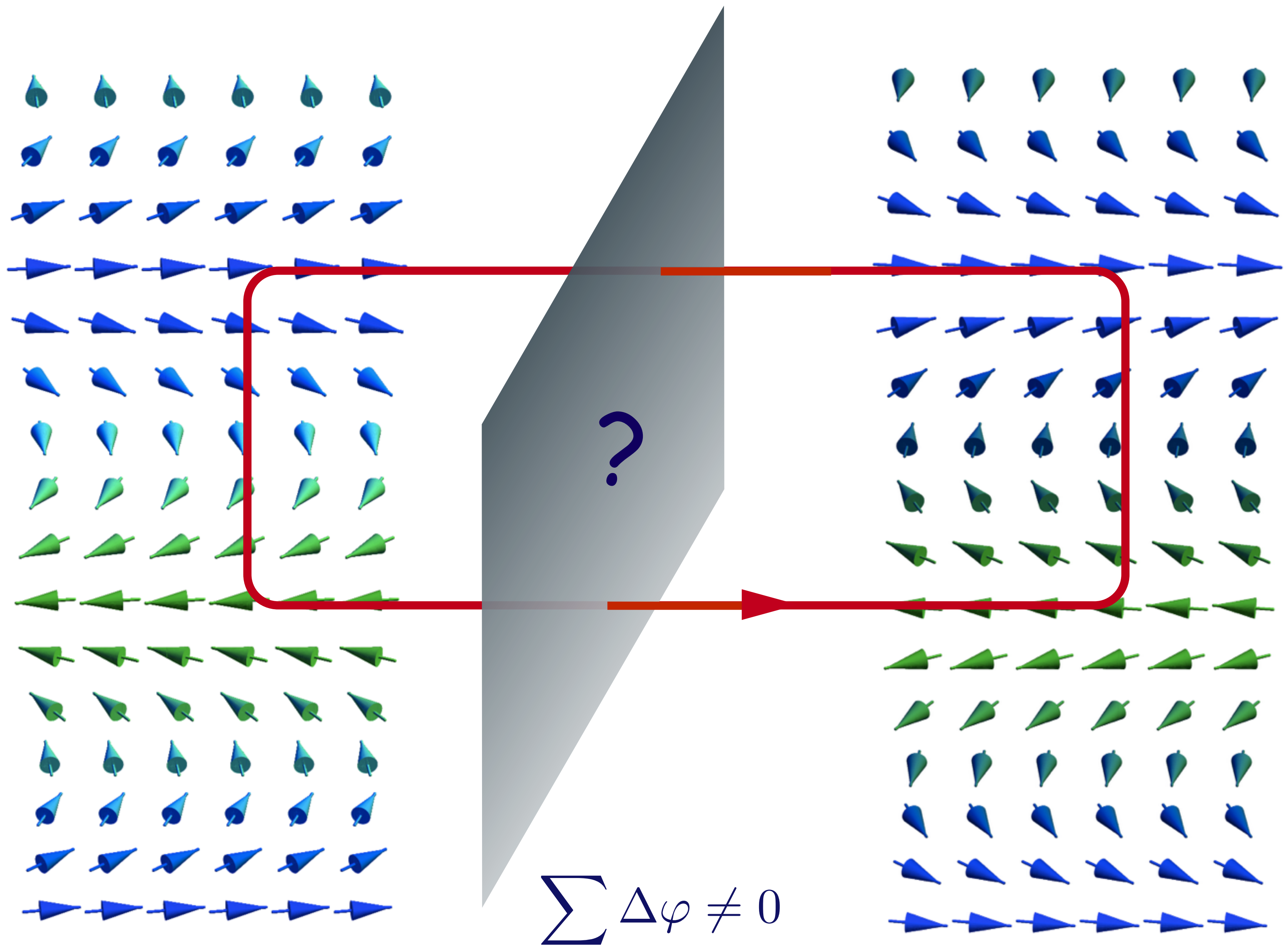


$$\mathbf{m} = (\cos\varphi, \sin\varphi)$$

$$\left\{ \boldsymbol{\nabla}_{\perp}^2 + \tfrac{a^2}{4} \left[6(\partial_z\varphi)^2 - 2q^2 - \partial_z^2 \right] \partial_z^2 \right\} \varphi = 0$$

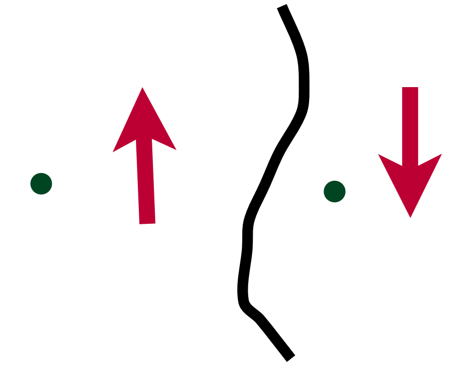
$$\partial_z\varphi \equiv \psi \to \text{domain wall in } \psi^4 \text{ theory}$$

Domain walls parallel to helical axis ?



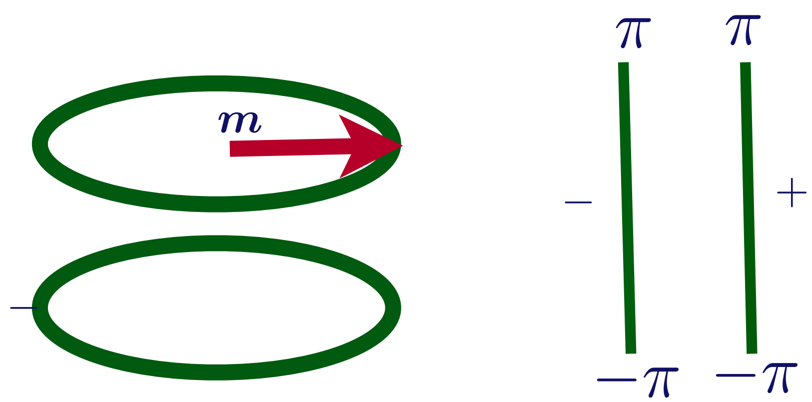
$\boldsymbol{q} = (\theta/a)\hat{z}$ $\qquad$ $\boldsymbol{q} = -(\theta/a)\hat{z}$

Example: Ising domain wall, $\pi_0(\mathcal{R}) = Z_2$



<u>Present</u> <u>case</u>: degeneracy space

$$\mathcal{R} = \mathcal{S}^1 \times \mathcal{S}^0$$



$$\pi_d(\mathcal{R}) = Z\delta_{d,1} + Z_2\delta_{d,0}$$

=> stable defects are <u>vortices</u> and <u>domain walls</u>

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Imaging spiral magnetic domains in Ho metal using circularly polarized Bragg diffraction

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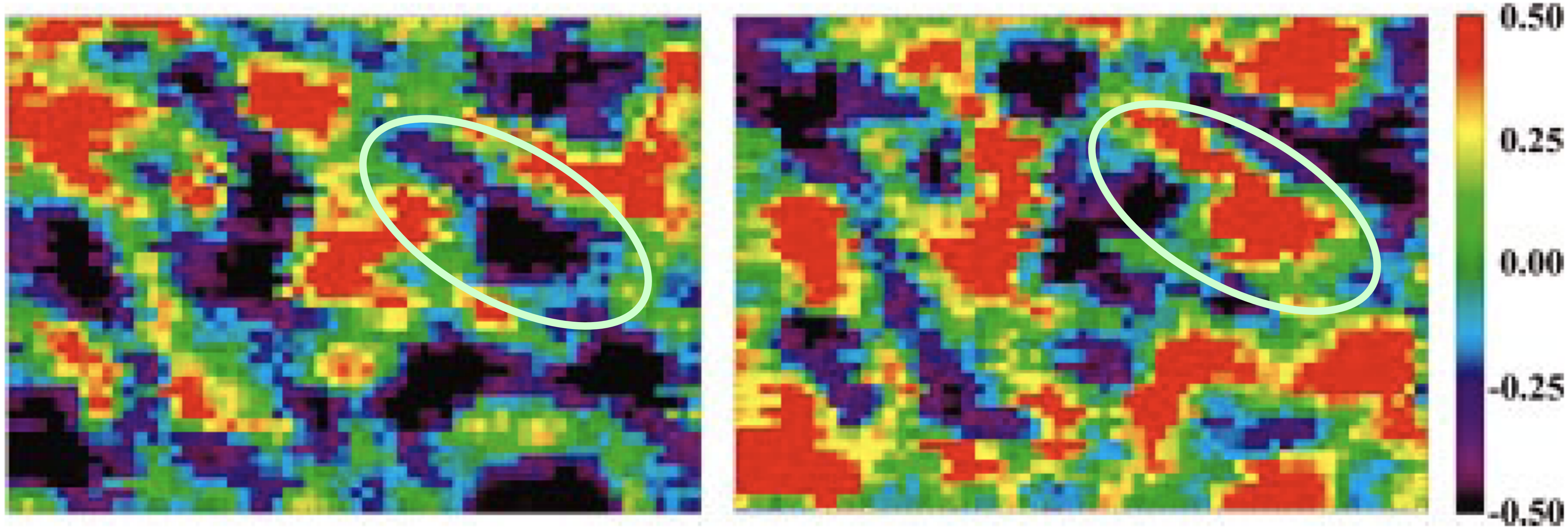
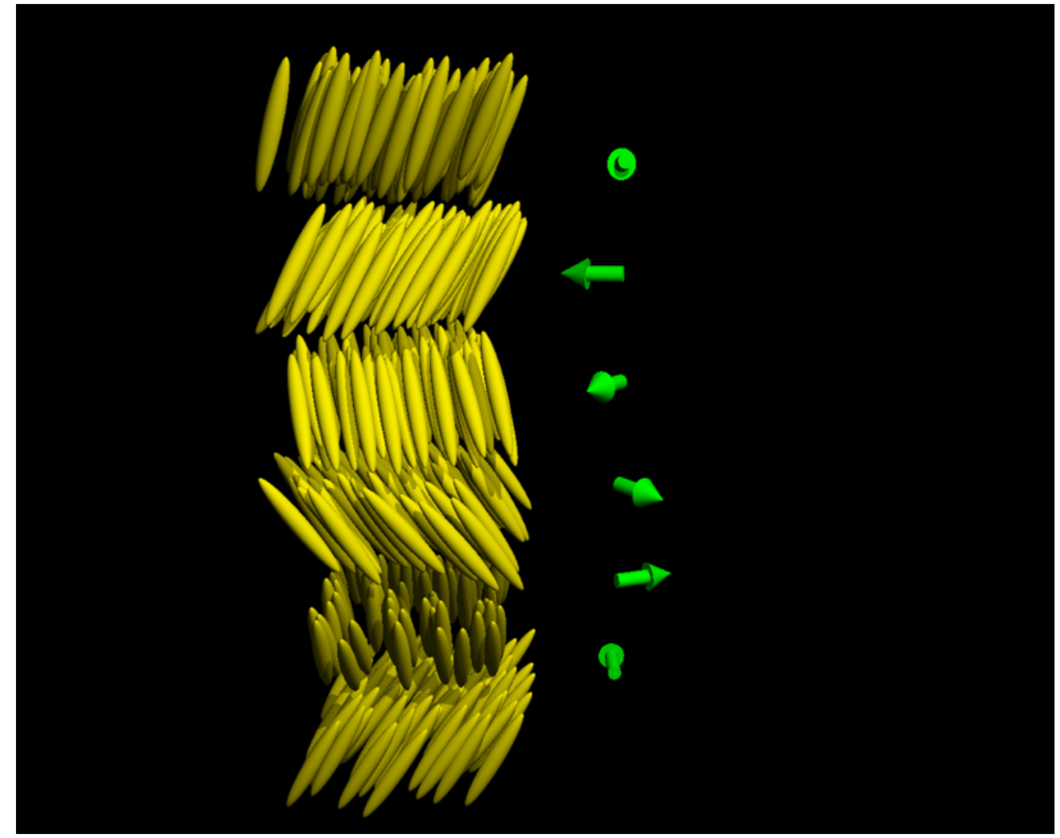
Advanced Photon Source, Argonne National Laboratory, Argonne, Illinois 60439

(Presented on 6 January 2004)

We have used circularly polarized x rays to image the spiral magnetic domain structure in a single crystal of Ho metal. In these structures, the magnetization direction rotates between successive atomic layers forming a helix. At magnetic Bragg diffraction peaks, circularly polarized x rays are sensitive to the handedness of such a helix (i.e., either right or left handed). By reversing the helicity of the incident beam with phase-retarding optics and measuring the difference in the Bragg scattering, contrast between domains of opposing handedness can be obtained. [DOI: 10.1063/1.1688252]

film plane perpendicular to helical axis





$$\langle \mathbf{m}_i \times \mathbf{m}_{i+1} \rangle = 0$$

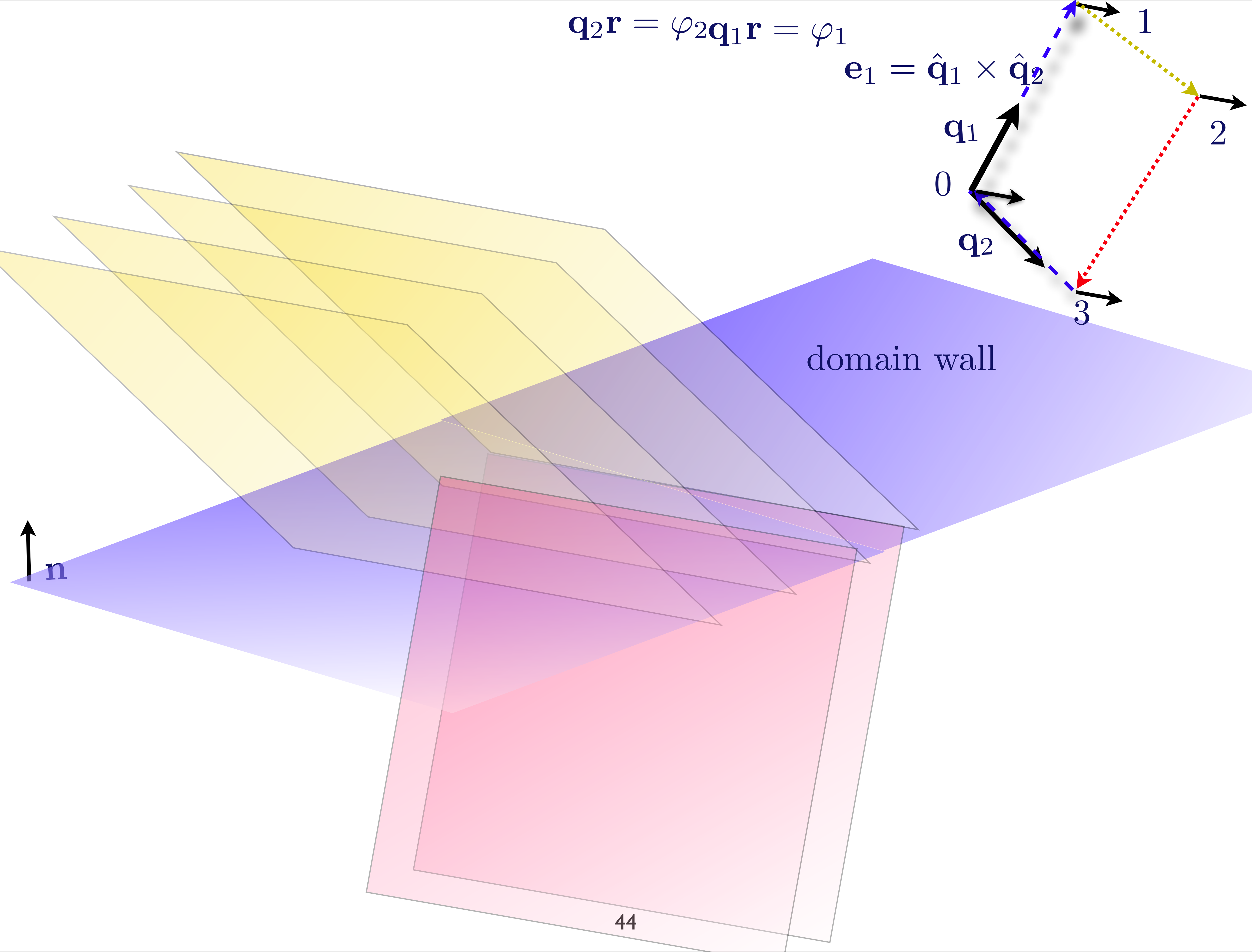
4 August 1972, Volume 177, Number 4047

SCIENCE

More Is Different

Broken symmetry and the nature of the hierarchical structure of science.

P. W. Anderson


$\mathbf{q}_2\mathbf{r} = \varphi_2 \mathbf{q}_1\mathbf{r} = \varphi_1$
$\mathbf{e}_1 = \hat{\mathbf{q}}_1 \times \hat{\mathbf{q}}_2$
1
2
3
0
$\mathbf{q}_1$
$\mathbf{q}_2$
domain wall
$\mathbf{n}$