Glass Phases in Disordered Superfluids and Superconductors

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Colloquium in honor of Heiner Müller-Krumbhaar

PRL **100**, 060402 (2008)
EPL **85**, 30002 (2009)
PR B 80, 104515 (2009)

Introduction

BEC : finite part of atoms in the state with minimal energy.

Examples: Superfluid ^{4}He, laser cooled atoms in a trap, SCs, excitons in semiconductors, BEC of spin waves

Disorder: Superfluid He in porous media (J.D. Reppy et al '92)
Cold atoms in speckle potential (R.G. Hulet et al. '08)
Disordered superconducting films (V.F. Gantmakher '09)

➡ Breakdown of superfluidity at strong disorder

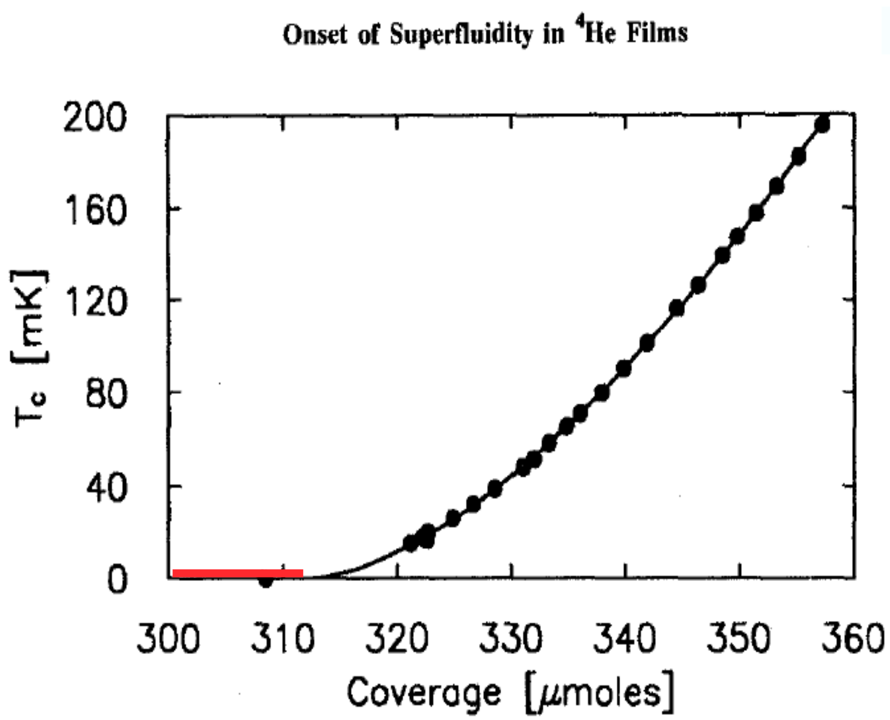
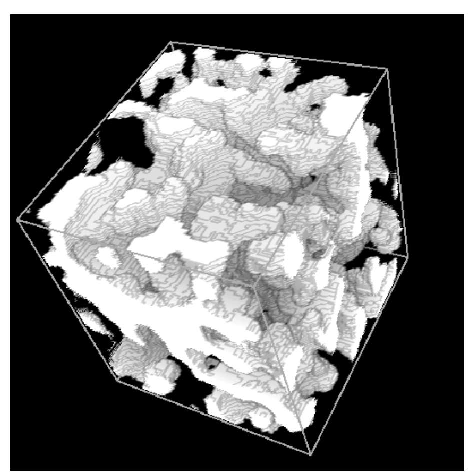
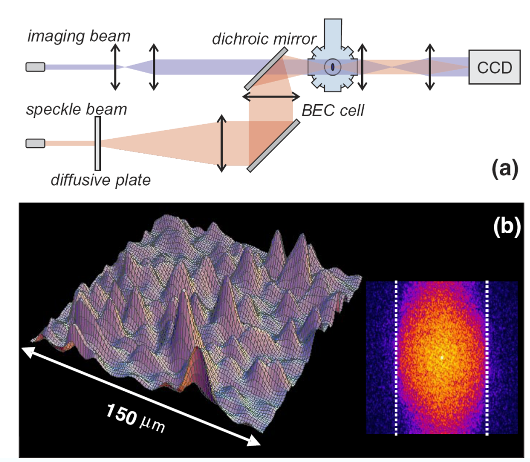
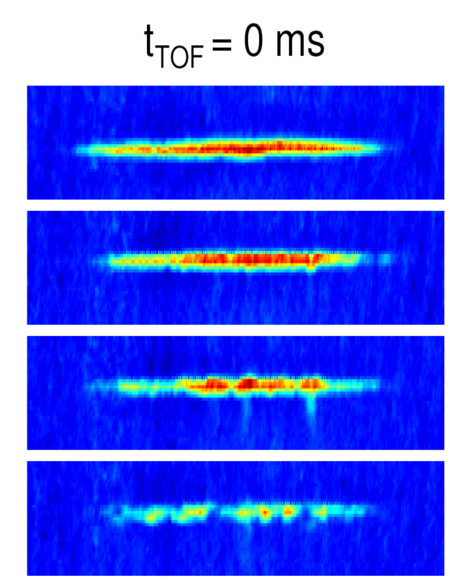


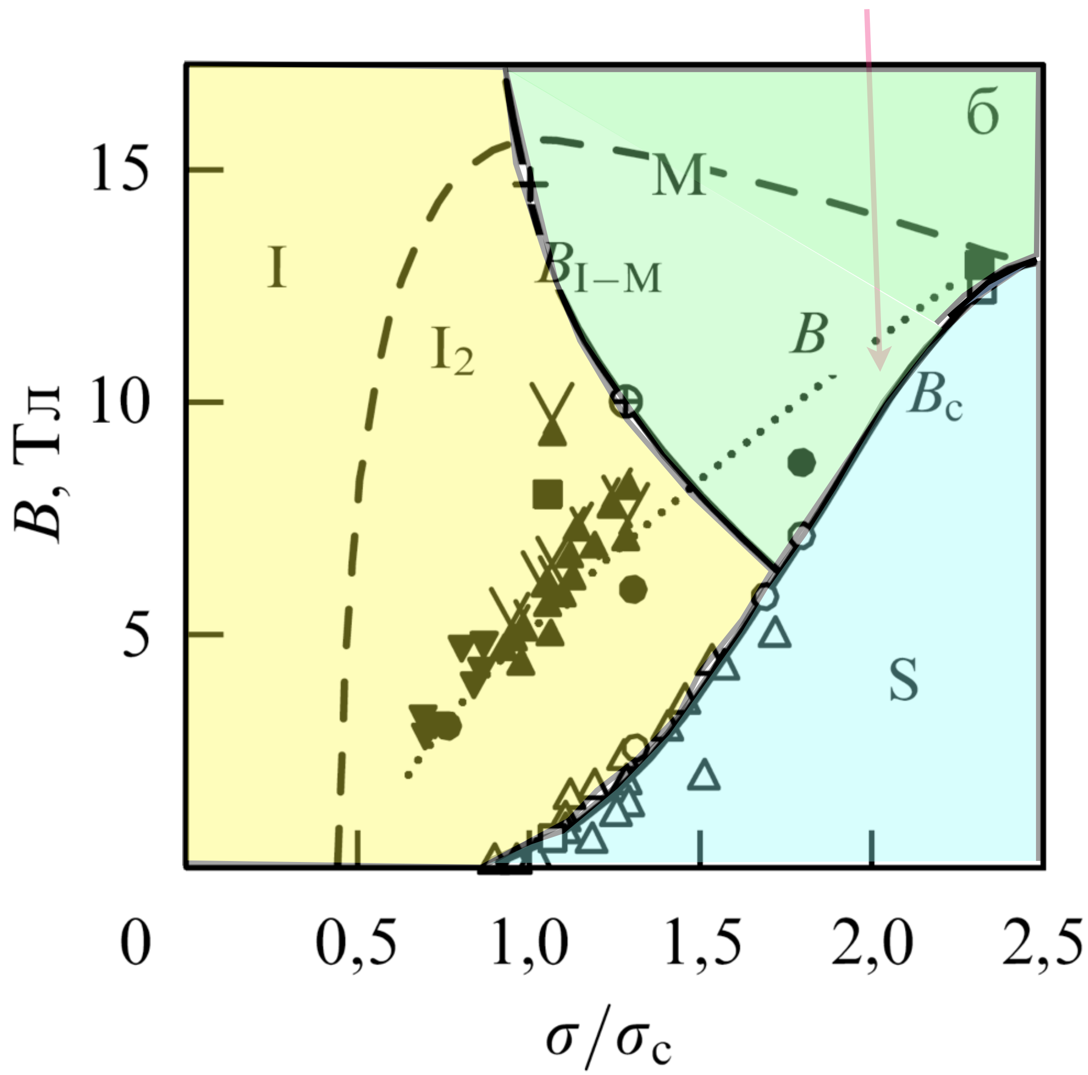
FIG. 3. T_c is shown as a function of the coverage n. The curve is a fit to the power law $A(n-n_c)^w$, where $w = 1.6 \pm 0.1$ and $n_c = 313 \pm 1$ μmoles.



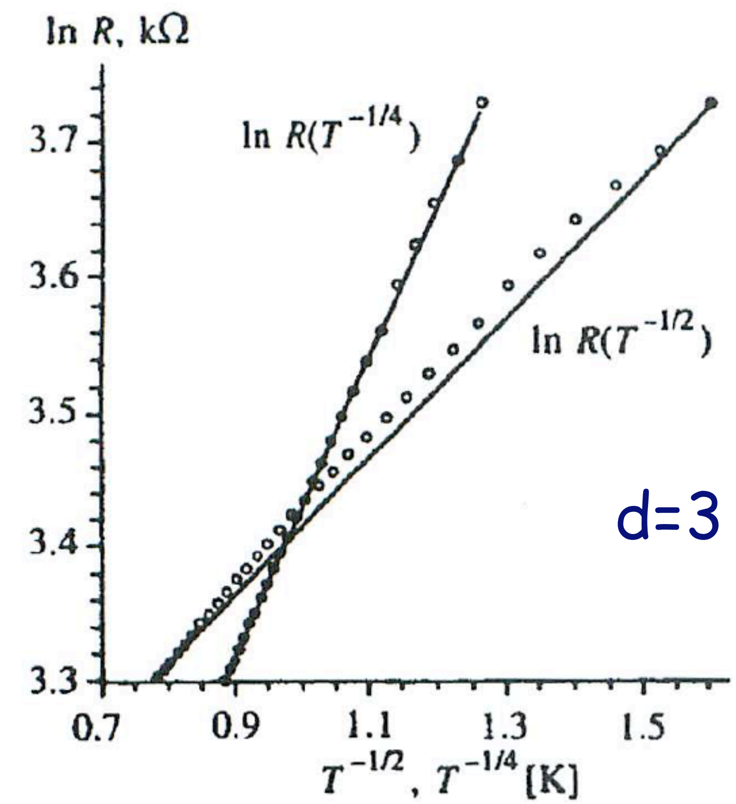




Superconductor to insulator transition of InO, TiN, Bi, Be, high-Tc materials



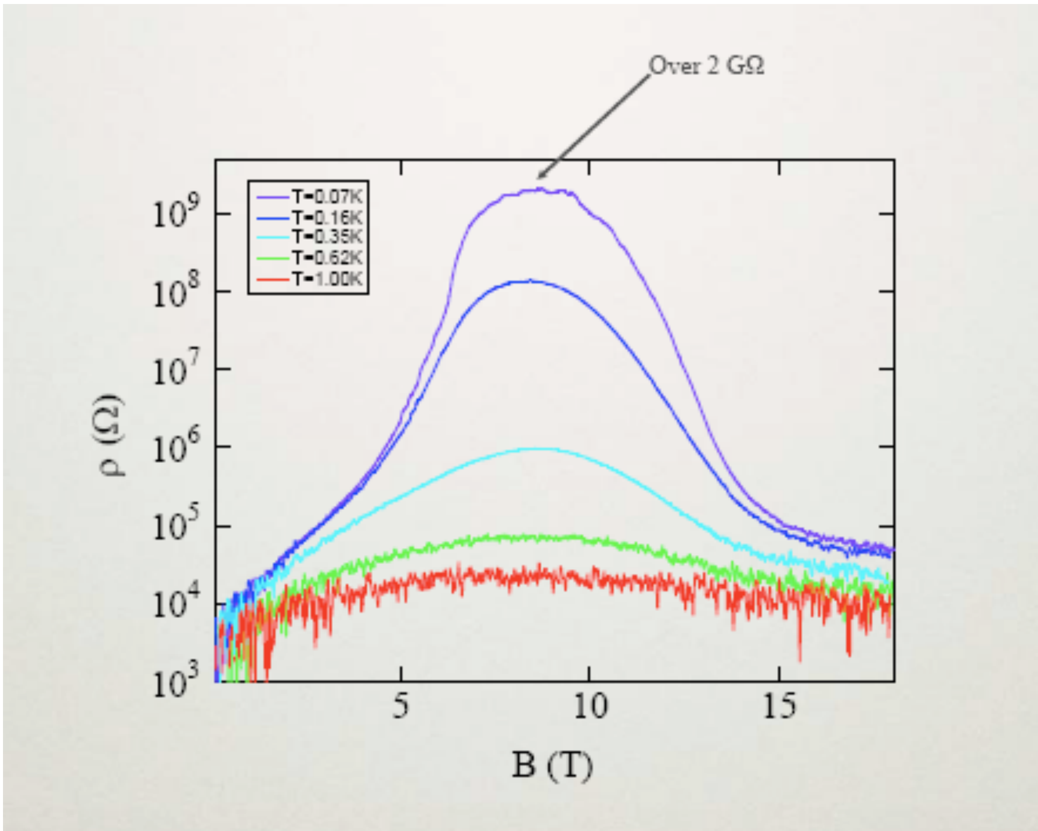
Gantmakher et al. 2010



d=3 VRH

Common believe:
Cooper pairs survive in insulating phase!

Giant negative magnetic resistance



Part 1: superfluids

Classical particles

$$p = Tn$$

Fermions

$$p = E_F n$$

interaction

disorder

Fermi liquid

Localization

Ioffe-Regel $nL_c^3 \leq 1$

many body localization

Bosons

$$p = \frac{T}{\lambda_T^3} \qquad n_c = n - \frac{1}{\lambda_T^3}$$

interaction

disorder

Superfluid

Non-ergodic state

$\boxed{\text{many body localization of bosons ?}}$

Minimal model

scattering length

$$\mathcal{H} = \int d^3x \Psi^\dagger \left(-\frac{\hbar^2}{2m} \left[\nabla^2 + 4\pi a \Psi^\dagger \Psi \right] + U(\mathbf{x}) \right) \Psi \qquad n = \Psi^\dagger \Psi$$

Random potential

$$\langle U(\mathbf{x}) \rangle = 0 \qquad \langle U(\mathbf{x}) U(\mathbf{x}') \rangle = \kappa^2 \delta(\mathbf{x} - \mathbf{x}')$$

Larkin length: mean free path or extension of localized states

$$\boxed{L_c \sim \frac{\hbar^4}{m^2 \kappa^2}}$$

Two problems:
(i) architecture of the random energy landscape
(ii) fill the potential wells

a=0: Density of states, search for the optimal fluctuation of random potential

$$\nu(E) = \int DU\ Tr\,\delta(E-\hat{H})e^{-\int d^3 r U^2/2\kappa^2}$$

$$\sim \exp\left[-\int d^3 r U^2/2\kappa^2 + \lambda(E - min_\Psi\langle\Psi|H|\Psi\rangle)\right]$$

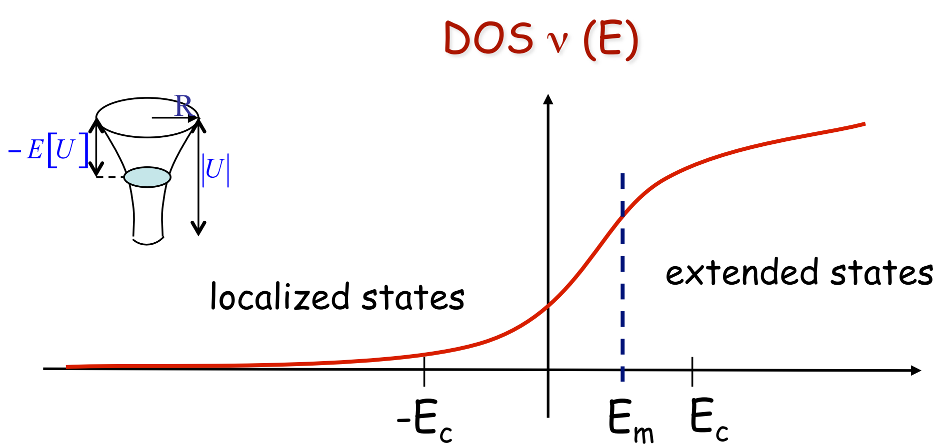
$$\boxed{\begin{array}{l} \rightarrow U(\mathbf{r}) = -\lambda\kappa^2|\Psi|^2 \\ \rightarrow \hat{H}\Psi = E\Psi \qquad \rightarrow \Psi \end{array}}$$

non-linear Schroedinger equation

I.M. Lifshitz '66,
Zittartz and Langer '66,
Halperin and Lax,'66
Cardy '78

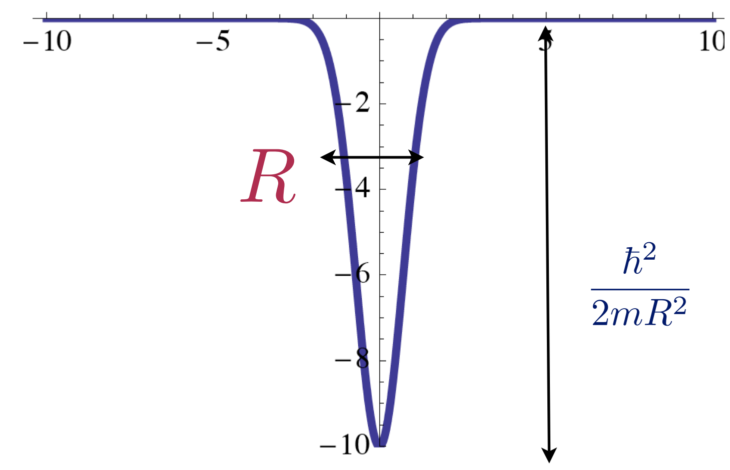
simplification $\Psi(\mathbf{r}) \sim e^{-r^2/2R^2}$ $\qquad \langle\Psi|\hat{H}|\Psi\rangle(R,\lambda) = E \quad \rightarrow \lambda(E,R)$

$$\int d^3 r \frac{U^2}{2\kappa^2} = \Phi(R,\lambda(E,R)) \quad \boxed{\rightarrow min_R \rightarrow E = E(R)}$$



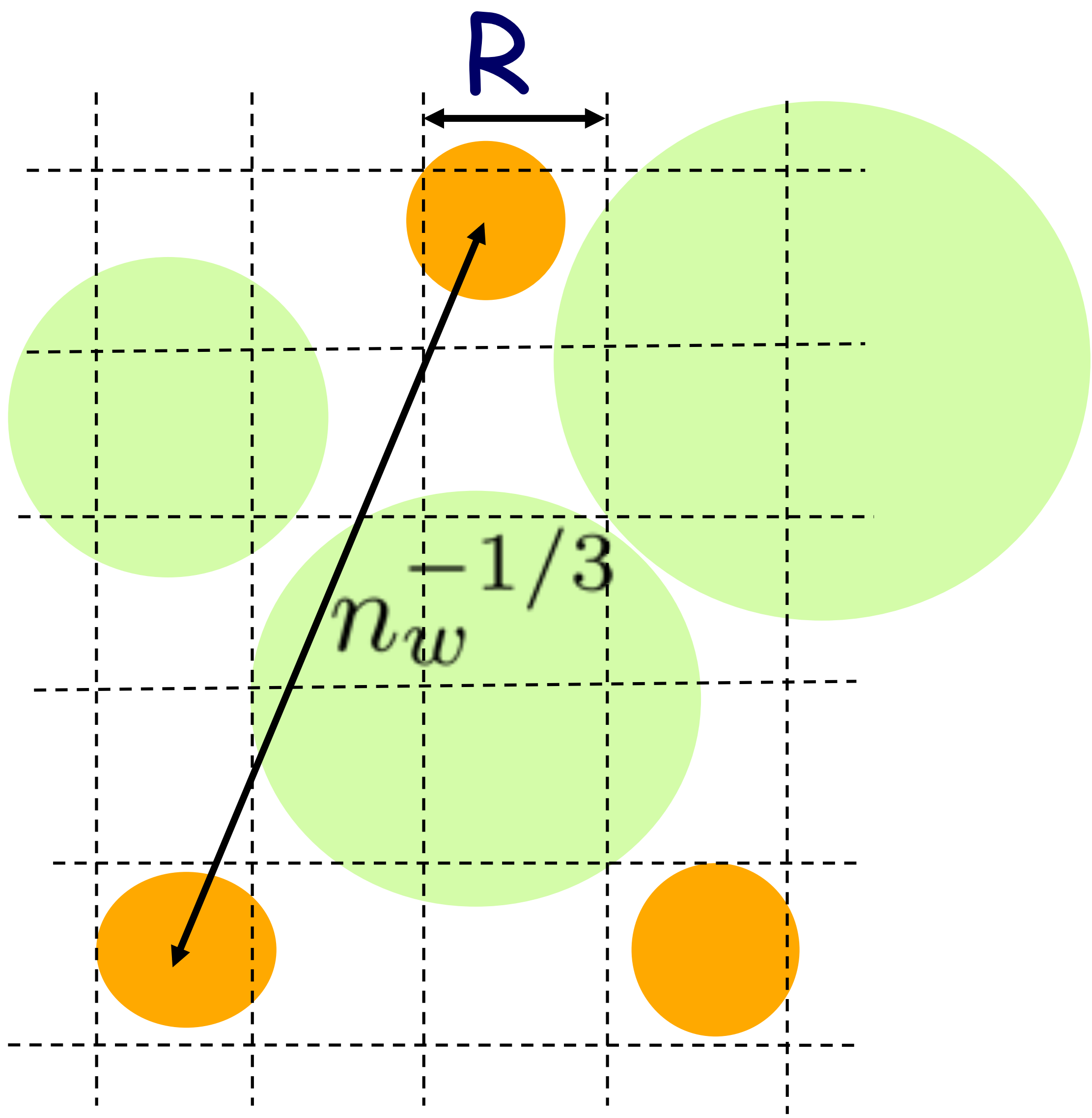
$$\boxed{\rightarrow \nu(E) \sim e^{-\Phi(R)}}$$

$$\boxed{d=3:}\quad E_b \approx -\frac{\hbar^2}{2mR^2}, \quad U \sim E_b e^{-r^2/R^2}, \quad \nu \sim e^{-L_c/R}$$



density of well with radius smaller than R:

$$\boxed{n_w(R) = \int_0^R dR\,\nu(R) \sim \frac{L_c}{R^4} e^{-L_c/R}}$$



Tunneling amplitude t(R) between wells with radius < R :

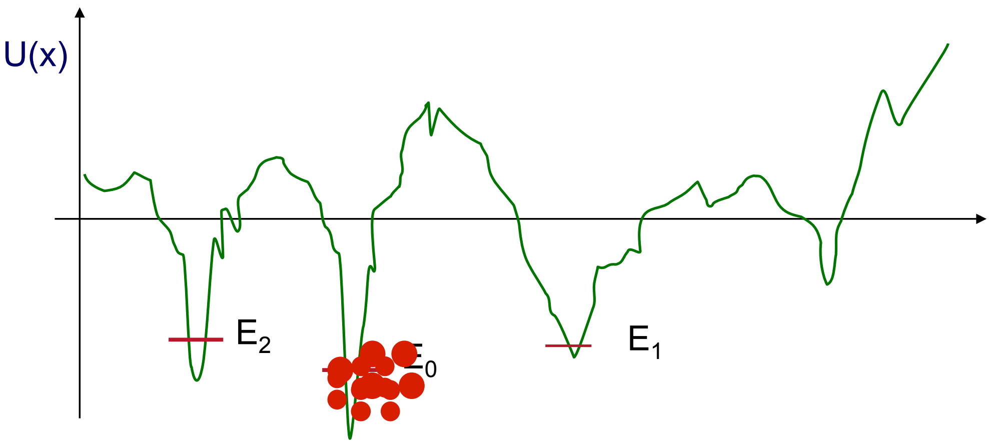
$$t(R) = \exp\left(-\frac{1}{\hbar}\int |p|dr\right)$$

$$\frac{1}{\hbar}\int |p|dr \approx n_w^{-1/3}/R \sim e^{L_c/3R}$$

$$\boxed{t(R) \sim e^{-\left(\frac{R}{L_c}e^{L_c/R}\right)^{1/3}}}$$

Filling wells with particles (T=0)

(i) no interaction (a=0): all particles in ground state

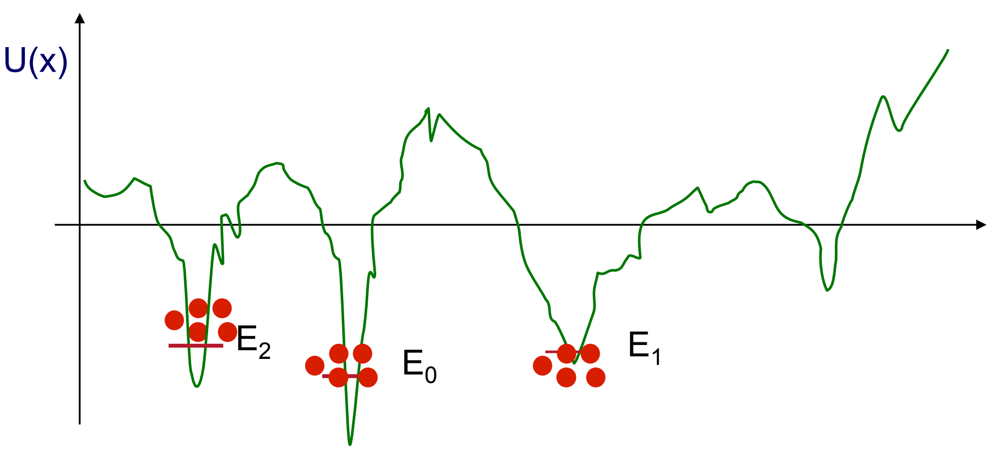


(ii) non-zero interaction (a>0):

density of particles in well of radius R: $\frac{n}{n_w(R)R^3} \sim \frac{R}{L_c} n e^{L_c/R}$

$$\mathcal{H} = \int d^3x \Psi^\dagger \left(-\frac{\hbar^2}{2m} \left[\nabla^2 + 4\pi a \Psi^\dagger \Psi \right] + U(\mathbf{x}) \right) \Psi$$

$$\Rightarrow E(R,n) \approx \frac{\hbar^2}{2m} \left(-\frac{1}{R^2} + 4\pi a n \frac{R}{L_c} e^{L_c/R} \right)$$



$$\Rightarrow R(n) = \frac{L_c}{\ln(n_c/n)}, \quad n_c \approx (3L_c^2 a)^{-1} \quad E_b(n) \approx -\frac{\hbar^2}{2mL_c^2} \ln(n_c/n)$$

Preliminary conclusions

$\Rightarrow$ At $n \ll n_c$ Bose gas decays into fragments,
particle density in fragments each of density $n_c \sim 1/(aL_c^2)$

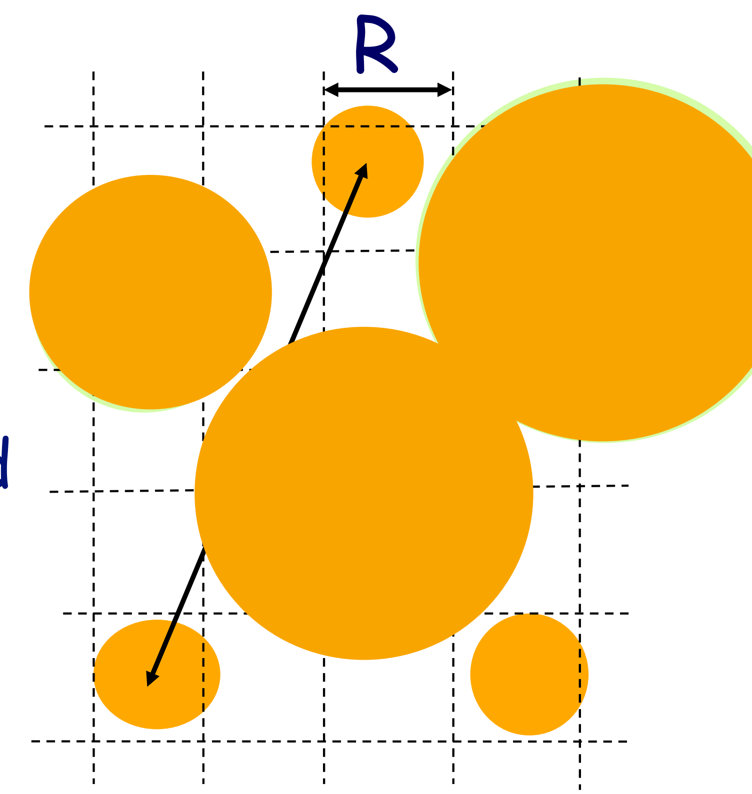
$\Rightarrow$ tunneling exponentially suppressed: $t(n) \sim e^{-c(n_c/n)^{1/3}}$

$\Rightarrow$ particle number in fragments $\sim L_c/a \gg 1$ well defined

$\Rightarrow$ phase uncertain, no phase coherence $\Rightarrow$ no superfluidity

$\Rightarrow$ finite compressibility „Bose glass"

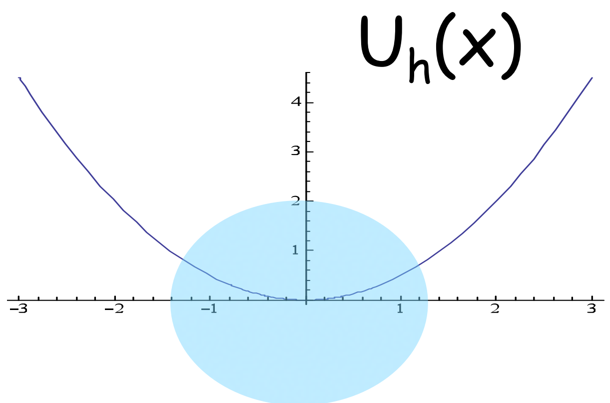
$\Rightarrow$ charged bosons VRH $\boxed{R(T) \sim e^{(T_0/T)^{1/4}}, \quad T_0 = E_c n_c/n}$



For $n \approx n_c$ i.e. fragments merge $\rightarrow$ transition to superfluid

Bosons in traps (uncorrelated disorder)

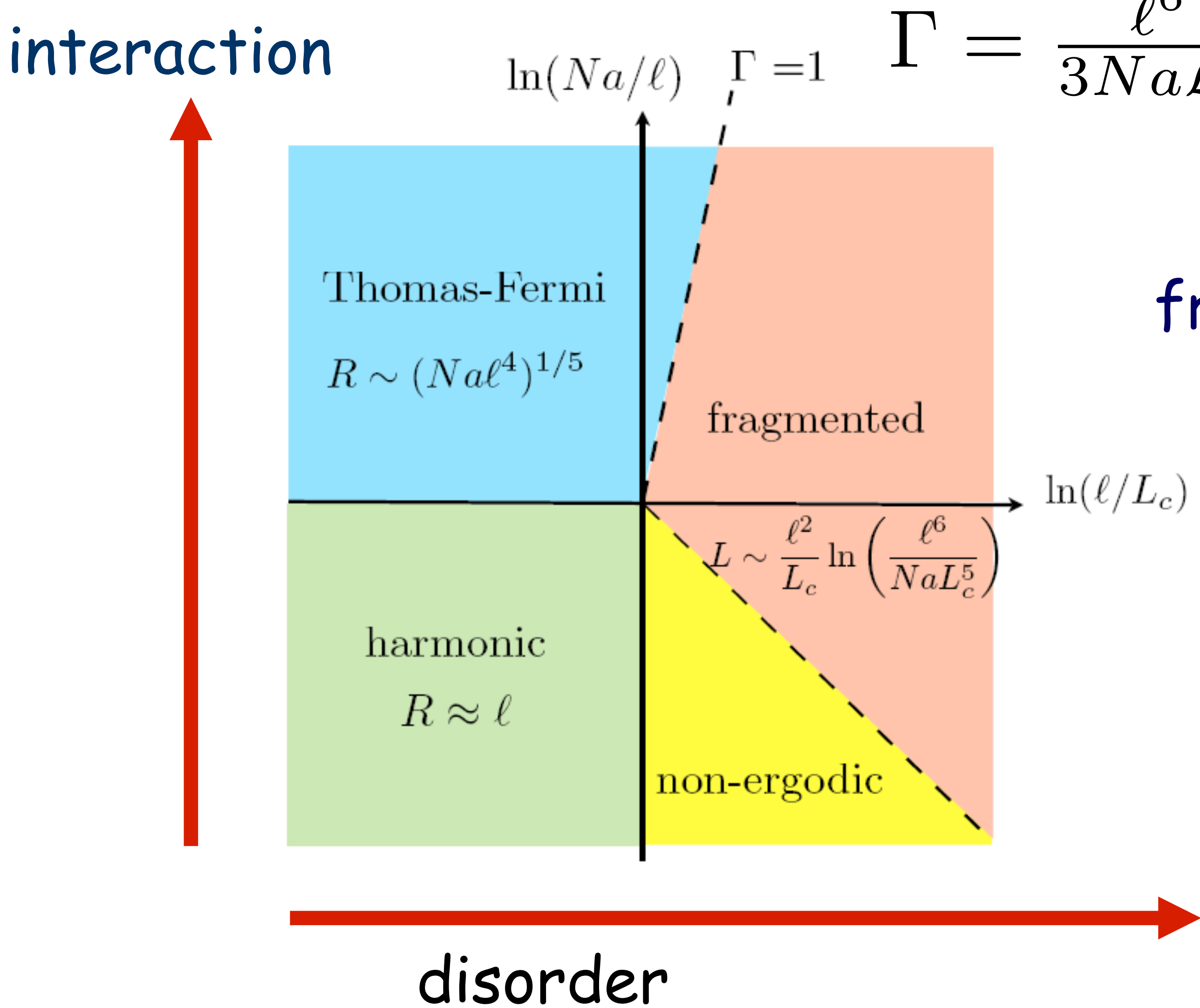
oscillator length $\ell = (\hbar/m\omega)^{1/2}$ ($\approx$ 1000nm), $\hbar\omega \approx$ nK



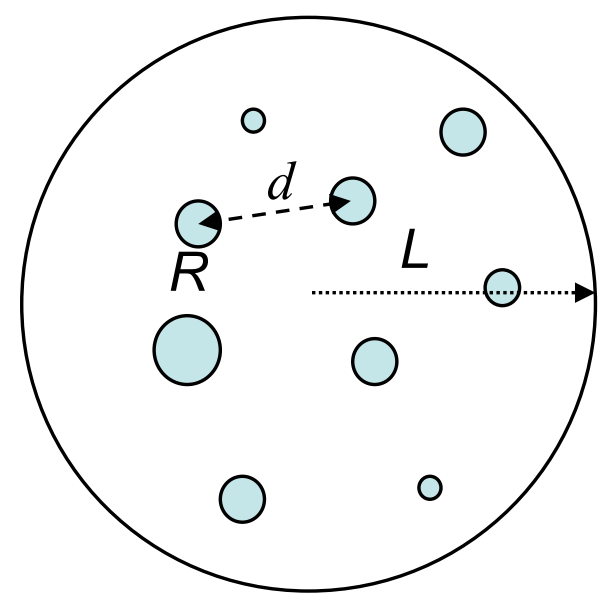
$$\mu(R) = -\frac{\hbar^2}{2mR^2} + E_{int}(R) + \frac{\hbar^2}{2m}\frac{R^2}{\ell^4}$$

$$\Gamma = \frac{\ell^6}{3Na\mathcal{L}^5}$$

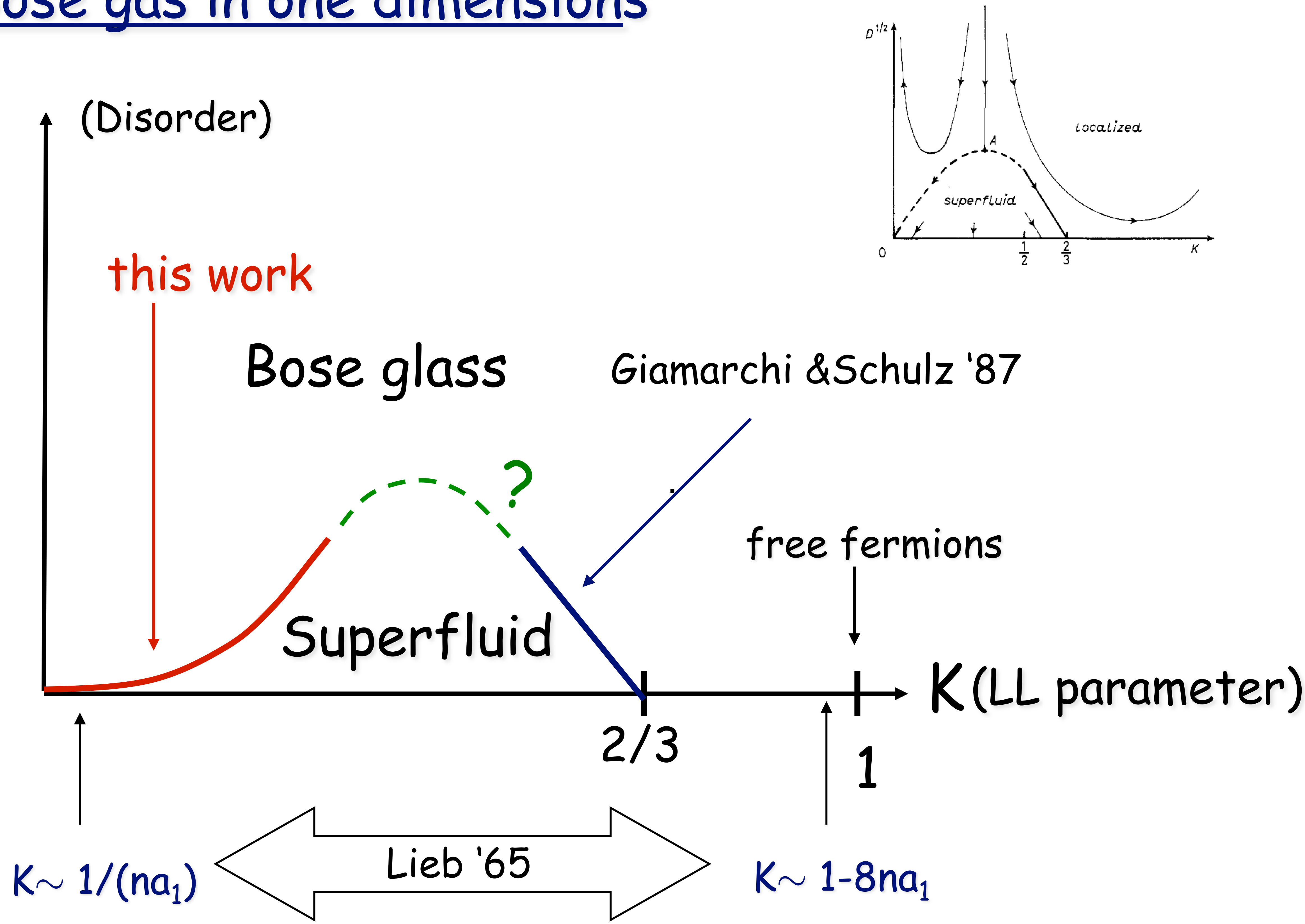
$$n(r) = n_c \left(\frac{n}{n_c}\right)^{\sqrt{1+r^2/r_F^2}},$$



fragmented state



Bose gas in one dimensions



Part 2: superconductors

Main difference: Particles are charged -> Coulomb interaction large in single well

$$\ell_c = \frac{\hbar^2}{me^2} < L_c$$

-> one particle per well, screening of Coulomb interaction on larger scales

$$\hat{\mathcal{H}} = \frac{\hbar^2}{2m_k}[-\nabla^2 + \left(\frac{e_k}{2\hbar c}\right)^2 (\mathbf{r}\times\mathbf{B})^2] + U_k(\mathbf{r}).$$

Length scales

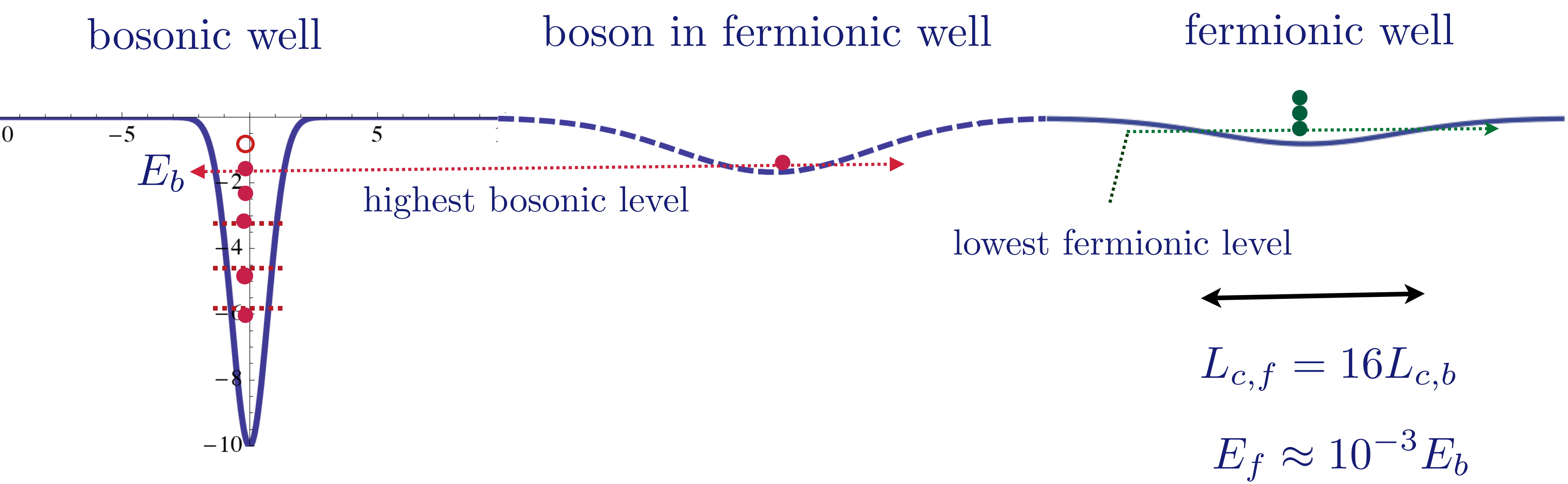
magnetic length $\ell_B = \sqrt{\frac{\hbar c}{eB}}$

coherence length ξ

$L_{c,f} = 16 L_{c,b}$ $\qquad E_f \approx 10^{-3} E_b$ $\qquad k = b, f$

Cooper pair breaking at $\quad E_b - \Delta = 2E_f - g\mu_B B$

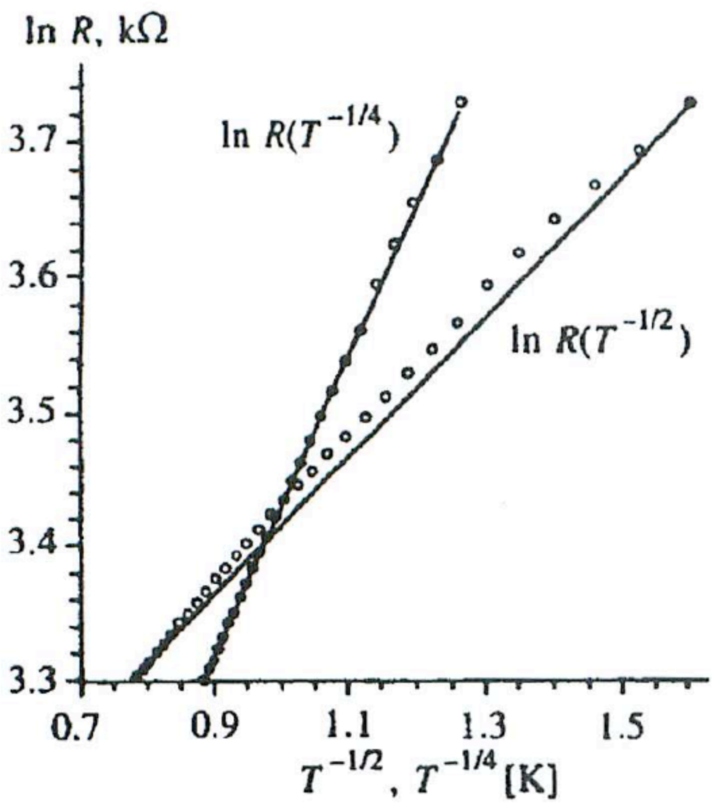
Optimal fluctuation of random potential for bosons and fermions, respectively

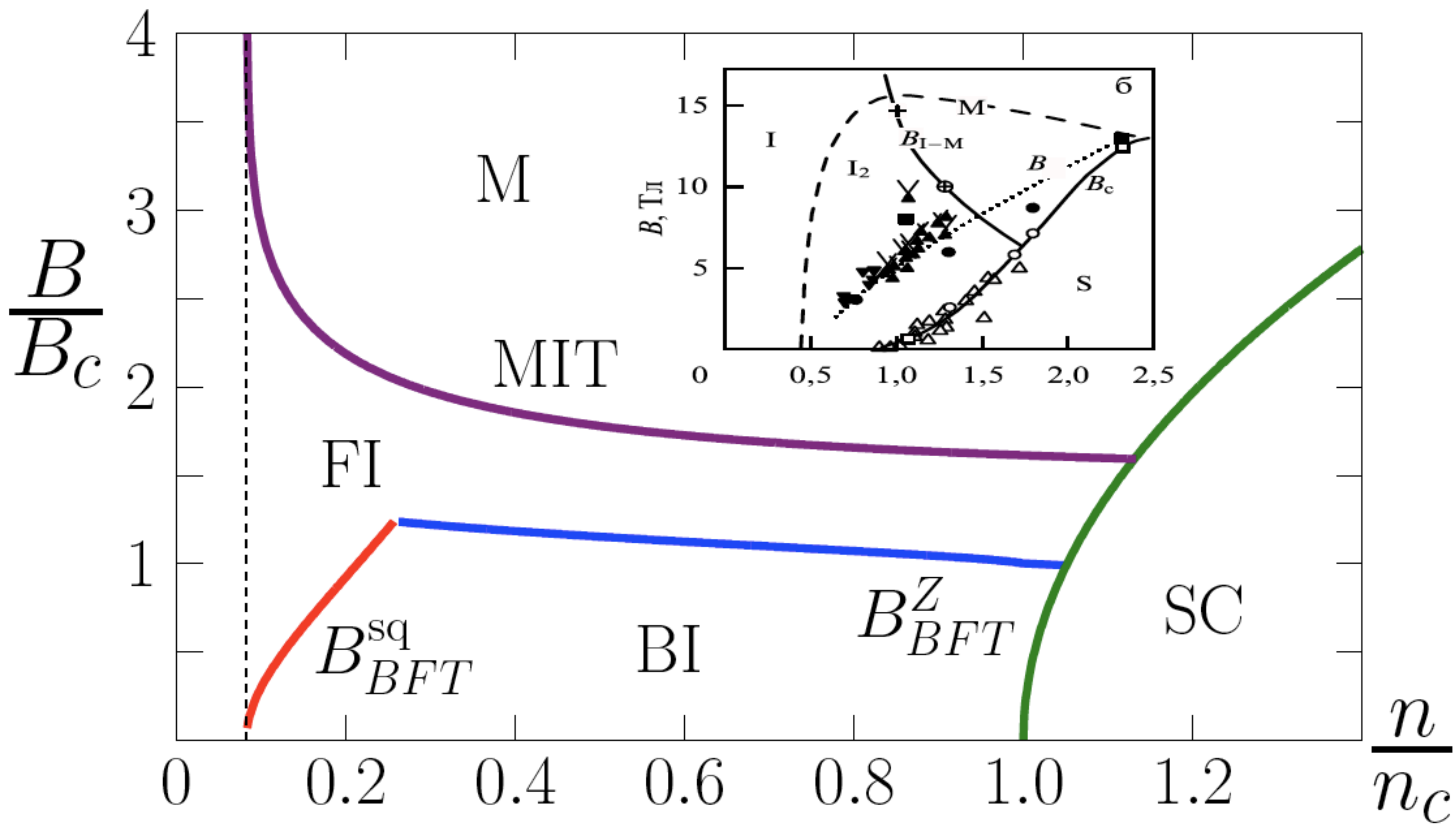


Variable range hopping

$$R(T) \sim e^{(T_0/T)^{1/4}}\ , \quad T_0 = E_c n_c/n$$

$$\frac{T_{0f}}{T_{0b}} \approx \frac{n_b}{n_f}\left(\frac{L_{cb}}{L_{cf}}\right)^{d+2} \sim \left(\frac{L_{cb}}{L_{cf}}\right)^2 = O(10^{-2})$$





$\frac{B}{B_c}$
4
3
2
1
M
MIT
FI
BI
SC
B_{BFT}^{sq}
B_{BFT}^{Z}
0
0.2
0.4
0.6
0.8
1.0
1.2
$\frac{n}{n_c}$
б
M
I
I_2
B_{I-M}
B
B_c
S
B, Тл
15
10
5
0
0,5
1,0
1,5
2,0
2,5