Disordered Quantum Wires*

Thomas Nattermann
Universität zu Köln

Thierry Giamarchi
Pierre Le Doussal
Bernd Rosenow
Sergey Malinin
Michel Fogler

Geneva
Paris
Köln/Paris
Köln/Detroit
UCSD

*e=h=k_B=1
- Electrons in 3 dimensions
- Electrons in 1 dimension: what is different?
- Electrons in 1 dimension: experiments
- Quantum wire as a chain of quantum dots
- Long wires: Rare events
- Conclusions
1. Electrons in three dimensions

Non-interacting electrons: mass $m$, mean spacing $a$, Planck’s constant $\hbar$

$\rightarrow$ energy scale $\hbar^2/ma^2 \sim E_F$

$T \ll E_F$: Pauli principle determines pressure $\rightarrow p \sim \hbar^2/ma^{d+2}$

Compressibility*: $\kappa = -a^{-2d} \partial \ln V / \partial p \sim 1/(E_F a^d)$

Specific heat: $C \sim T/E_F$
1. Electrons in three dimensions

**Interacting spinless electrons:**

charge $e \rightarrow$ strength of interaction $\sim (e^2/a) / E_F \sim a/a_B$

but energy and momentum conservation reduce phase space

$\Rightarrow$ Landau's Fermi fluid of quasi-particles

with finite lifetime $\hbar/\tau \sim E_F (T/E_F)^2$

**Disordered spinless electrons:**

Metal insulator transition
2. Electrons in one dimension - what is different?

Electrons cannot avoid each other → Landau’s picture breaks down

⇒ density wave excitations: plasmons

(a) 1D clean wire: Luttinger liquid

\[ G = \frac{dJ}{dV} = \frac{\sigma}{L} = (K) \frac{e^2}{h} \]

(b) + single impurity: K<1: impurity relevant,

\[ G \sim (\max (T, \text{eV}))^{2/K-2} \]

Kane & Fisher ‘92, Furusaki & Nagaosa

K>1: impurity irrelevant
3. Electrons in one dimension: Experiments

- **MoSe Nanowires**  
  Venkataraman, PRL (2006)

\[
\frac{J}{T^{\alpha+1}} \sim \max \left( \frac{V}{T}, \frac{V^{\beta+1}}{T^{\alpha+1}} \right)
\]

**FIG. 1** (color online). (a) Structural model of a 7-chain MoSe nanowire along with the triangular Mo$_3$Se$_3$ unit cell. (b) and (c) AFM height images of MoSe nanowires between two Au electrodes. The wire heights are 7.2 nm and 12.0 nm, respectively. Scale bar = 500 nm.

short wires (L \sim 1 \mu m):

“Temperature” Exponent (a) is close to “Voltage” Exponent (b)

Agrees with the conventional “Luttinger-liquid” picture with

\[\alpha = \beta = 2/K - 2\]
3. Electrons in one dimension: Experiments

- **Multiwall carbon nanotubes**

![AFM images of junctions formed between two MWNTs: (a) end-bulk junction and (b) end-end junction. The arrows indicate the position of the junctions.](image)

**conductance**

\[ J \sim e^{-\left(\frac{T_0}{T}\right)^{1/2}} \]
3. Electrons in one dimension: Experiments

**Variable Range** Hopping conduction in polydiacetylene single crystals

A. N. Aleshin*

*School of Physics and Condensed Matter Research Institute, Seoul National University, Seoul 151-747, Korea
and A. F. Ioffe Physical-Technical Institute, Russian Academy of Sciences, St. Petersburg 194021, Russia

J. Y. Lee, S. W. Chu, S. W. Lee, B. Kim, S. J. Ahn, and Y. W. Park

*School of Physics and Condensed Matter Research Institute, Seoul National University, Seoul 151-747, Korea

<table>
<thead>
<tr>
<th>$T_0$(K)</th>
<th>$E_0$(V/cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2.57 \times 10^3$</td>
<td>$4.9 \times 10^5$</td>
</tr>
<tr>
<td>$2.47 \times 10^3$</td>
<td>$3.2 \times 10^5$</td>
</tr>
<tr>
<td>$4.72 \times 10^3$</td>
<td>$1.1 \times 10^6$</td>
</tr>
</tbody>
</table>

$J \sim e^{-\left(E_0/E\right)^{1/2}}$

The charge transport in polydiacetylene quasi-1D single crystals (PDA-PTS) has been studied as a function of temperature, electric and magnetic fields. In the Ohmic regime the temperature dependence of the resistivity, $\rho(T)$, is characteristic of hopping conduction with a crossover at $T<50$ K from activated $\rho(T) = \rho_0 \exp \left(\frac{E_a}{k_B T}\right)$, with $E_a \sim 13–19$ meV to variable-range hopping transport $\rho(T) = \rho_0 \exp \left(\frac{T_0}{T}\right)^p$, with $p \sim 0.65–0.70$. At modest electric fields the resistivity depends as $\rho(E, T) = \rho(0, T) \exp \left(-eE/L/k_B T\right)$, where the characteristic hopping length changes as $L \sim T^m$ with $m \sim 0.5$ at $T > 50$ K and $m \sim 0.75$ at $T < 50$ K. At high electric fields the low temperature current becomes temperature independent and follows: $I(E) = I_0 \exp \left(-\left(E_0/E\right)^{0.5}\right)$, which corresponds to the regime of activation-free phonon-emission-assisted hopping.
3. Electrons in one dimension: Experiments

- **Polymer nanofibers**

  long wires: 10μm

- **Electrons in one dimension: Experiments**

  \[ I = \frac{T^{-\alpha+1}}{a} \]

  \[ V = (T^{-\alpha+1}) \]

  \[ J / T^{\alpha+1} \sim \max (V/T, V^{\beta+1}/T^{\alpha+1}) \]

  Aleshin et al., PRL (2004)

  **Voltage** / **Temperature**

  **Disagrees** with the conventional Luttinger-liquid picture

  **“temperature” exponent exceeds “voltage” exponent!**
3. Electrons in one dimension: Experiments

**Observed Power-Law Exponents**

<table>
<thead>
<tr>
<th>Sample</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>2.8</td>
<td>5.5</td>
<td>7.2</td>
<td>5.6</td>
<td>5.0</td>
<td>4.1</td>
</tr>
<tr>
<td>$\beta$</td>
<td>1.5</td>
<td>3.8</td>
<td>4.7</td>
<td>1.0</td>
<td>1.1</td>
<td>1.8</td>
</tr>
</tbody>
</table>

$L \sim 10 \, \mu m$ polymers  
Aleshin et al.,  
PRL (2004)

<table>
<thead>
<tr>
<th>Sample</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>2.3</td>
<td>3.4</td>
<td>4.5</td>
<td>4.6</td>
</tr>
<tr>
<td>$\beta$</td>
<td>1.3</td>
<td>3.4</td>
<td>2.8</td>
<td>2.0</td>
</tr>
</tbody>
</table>

$L \sim 100 \, \mu m$ InSb wires  
Zaitsev-Zotov et al.,  
JPCM (2000)

- In long wires, T-exponent exceeds V-exponent: $\alpha > \beta$
- Exponents are sample-dependent
4. Quantum wire as a chain of quantum dots

"quantum dots" with integer number of electrons

Model:

Adding or removing a charge costs energy!

strong impurities, randomly (Poissonian) distributed
V=0: classical ground state (K<<1)

integer number $q_i$ of electrons between impurities $i$ and $i+1$

$$q_i = [Q_i]_G, \quad Q_i = a_i (2\pi k_F + \mu C) \text{ background charge}$$

excited states: change number of electrons by $n_i$

$$\rightarrow \text{energy change} \quad E_i(n_i) = \Delta_i \left\{ (1/2)n_i^2 + n_i(q_i-Q_i) \right\}$$

$$\Delta_i = 1/(Ca_i) \quad \text{charging energy of quantum dot}$$

$C$: capacitance/unit length

$$a_i = x_{i+1} - x_i$$

$$\Delta = 1/Ca$$
Transport from dot \( j \) to dot \( k \) by activation and co-tunneling

\[
J_{jk} \sim e^{-E_{jk}/T} e^{-s|k-j|} \sinh \left( \frac{(\xi_k - \xi_j)/T}{\mu_j} \right)
\]

local electro-chemical potential \( \xi_j = Fx_j - \mu_j \)

tunneling transparency \( s = 2(K^{-1} - 1) \ln(ak_F) \) Larkin, Lee '78
Transport from dot $j$ to dot $k$ by activation and co-tunneling

$$J_{jk} \sim e^{-E_{jk}/T} \ e^{-s|k-j|} \ sinh \left( \frac{(\xi_k - \xi_j)}{T} \right)$$

**Ohmic regime**

typical energy mismatch $E_{jk} \sim \Delta / |k-j|$ \quad $|k-j| \sim (\Delta / Ts)^{1/2} \equiv x_{VRH}/a$

$$\Rightarrow \quad J \sim e^{(s\Delta / T)^{1/2}} F, \quad Ts << \Delta \quad VRH$$

**Non-ohmic regime**

typical $|k-j|aF \sim \Delta / |k-j|$ \quad $|k-j| \sim (\Delta / aF)^{1/2}$

$$\Rightarrow \quad J \sim e^{s(\Delta / aF)^{1/2}}, \quad aF << \Delta \quad VRH$$
Tunneling action dominated by spreading of charge:

\[ S_{\text{tun}} \sim \int d\tau \ E(\tau) \sim \int d\tau \ (C \ x(\tau))^{-1} \sim K^{-1} \ln \left( \frac{E_{\text{initial}}}{E_{\text{final}}} \right) \]

\[ E_{\text{initial}} = \frac{k_F}{C} \]

\[ E_{\text{final}} = \max (\Delta, T, V) \]

Large Voltage/temperature: power laws

Larkin and Lee '78
Tunneling action dominated by spreading of charge:

\[ S_{\text{tun}} \sim \int d\tau \ E(\tau) \sim \int d\tau \ (C \ x(\tau))^{-1} \sim -K^{-1} \ln \left( \frac{E_{\text{final}}}{E_{\text{initial}}} \right) \]

\[ E_{\text{initial}} = \frac{k_F}{C} \]

\[ E_{\text{final}} = \max (\Delta, T, V) \]

\[ s_{\text{eff}} \approx 2K^{-1} \ln \left[ a k_F / \max (1, T/\Delta, V/\Delta) \right] \approx s - 2K^{-1} \ln (\max(T, V)) \text{ if } \max(T, V) > \Delta \]

\[ \rightarrow T > \Delta : \ J_{k,k+1} \sim \left( \frac{\max(T, V)}{\Delta} \right)^{2/K} \]

Kane-Fisher 1992

Large Voltage/temperature: power laws

Larkin and Lee '78
Results from typical dots:
(short wires)

\[ T_{1,cr} = Ke^s\Delta \]

\[ J/F \sim e^{-s(T^{2K^{-1}})^{1/2}} \]

\[ J \sim \exp \{-s(\Delta/Fa)^{1/2}\} \]

\[ \xi_{loc} \sim a/s \]

Instanton hits boundary

single impurity tunneling

many impurity tunneling

VRH
**Strong and weak pinning**

$T > T_{1,cr}$: single impurity weak

$T < T_2$: collective effects

$u < k_F$: SCHA $u \rightarrow u_{eff}$

---

**Giamarchi, 2004: "Quantum Physics in One Dimension"**

![Diagram](image.png)

- **Strong Pinning**:
  - $k_F(a_F)^{K-1}$
  - $K\Delta a/\xi = T_{loc}$
  - $T^{2-2K(T)}$
  - $T_2 \sim K\Delta$

- **Weak Pinning**:
  - $T^{2-2K}$
  - $T^{1/k}\rho - 1$

---

Hongkong, May 2006  International Conference on the Frontiers of Nonlinear and Complex Systems  18
4. Long wires: rare events

- So far considered: typical quantum dots with $a_i \approx a$
- Now: consider regions with many narrow dots with $a_i \ll a$

\[ P(\varepsilon) = 1 - \left(\frac{\varepsilon}{\Delta}\right) \left(1 - e^{-\Delta/\varepsilon}\right) \sim \frac{\Delta}{2\varepsilon}, \quad \varepsilon \rightarrow \infty \]

Probability that gap larger than $\varepsilon$
4. Long wires: rare events

- So far considered: typical quantum dots with $a_i \approx a$
- Now: consider regions with many narrow dots with $a_i << a$

Probability that gap larger than $\delta$

$$P_\varepsilon(\varepsilon) = 1 - (\varepsilon/\Delta)(1 - e^{-\Delta/\varepsilon}) \sim \Delta/(2\varepsilon)$$

$\varepsilon \to \infty$
Wire: non-overlapping breaks + low resistance connecting pieces

\[ R = L \int P_u(u) R_0 e^u \, du \]

\( P_u(u) : \) prob. /unit length that break resistance at least \( e^u \)

\( I_{jk} \sim e^{-s|j-k|} e^{-E_{jk}/T} \sinh \left( \frac{(\xi_j - \xi_k)}{T} \right) \quad \xi_j = Fx_j - \mu_j \)

(a) Ohmic break of \( |j-k| \) dots

envelope: \( E_i(\pm 1) \geq \varepsilon_i^\pm \rightarrow s|j-k|+ \max(\varepsilon_j^\pm, \varepsilon_k^\pm)/T \geq u \)

simplification: rectangular break: \( s|j-k| \approx u, \quad \varepsilon_j^\pm \approx uT \)

\[
P_u(u) \sim x_{VRH}^{-1} P_\varepsilon(\varepsilon)|j-k| \sim x_{VRH}^{-1} P_\varepsilon(uT)^{u/s} \begin{cases} \ln P_u(u) \sim -u^2T/\Delta s & \text{if } uT \ll \Delta \\ \ln P_u(u) \approx -u/s \ln (2uT/\Delta) & \text{if } uT \gg \Delta \end{cases}
\]
\[ \ln P_u(u) \sim -u^2 T/\Delta \text{ s} \quad \text{if } uT \ll \Delta \]

\[ \ln P_u(u) \approx -u/s \ln (2uT/\Delta) \quad \text{if } uT \gg \Delta \]

\[ \rightarrow \quad \text{infinite wire:} \]

\[ s \ll 1 : \quad \rho \sim \exp \left( \frac{s\Delta}{4T} \right) \]

Raikh Ruzin '89

\[ s \gg 1 : \quad \rho \sim \exp \left( s\Delta e^{(s-1)/2T} \right) \quad \text{new} \]

provided \( L > a \exp \left( e^{s\Delta(s-1)/Ts} \right) \)
→ **infinite wire:**

Low $T$, $s \ll 1$ : $\rho \sim \exp \left( s \Delta / 4T \right)$

High $T$, $s \gg 1$ : $\rho \sim \exp \left( s \Delta e^{(s-1)/2T} \right)$ new

**finite wire:** $P_u(u)(L/x_{VRH}) \approx 1 \rightarrow u_{\text{max}}$

$sT \ln(L/x_{VRH}) \ll \Delta : \quad \rho \sim \exp \left[ s \Delta \ln \left( L/a \right) / T \right]^{1/2} \quad \text{VRH}$

$sT \ln(L/x_{VRH}) \gg \Delta : \quad \rho \sim \exp[s \ln \left( L/x_{VRH} \right) / \ln \left( Ts/\Delta \right)] \sim T^\alpha$

$\alpha = s \ln \left( L/x_{VRH} \right) / \ln^2 \left( Ts/\Delta \right)$

Raikh Ruzin '89
(b) Non-Ohmic breaks of m dots \[ V_b = \xi_j - \xi_k \gg T \]

(main voltage drop across the breaks, but constant \( I = I_0 \ e^{-u} \) current everywhere)

\[ I_{jk} \sim e^{-s|j-k|} e^{-\xi_{jk}/T} \sinh \left( (\xi_j - \xi_k)/T \right) \rightarrow s|j-k| + \max(\xi_j^\pm, \xi_k^\pm)/T - V_b/2T \geq u \]

average electric field: \[ V/L = F = \int dV_b \ V_b P(V_b) = x_{VRH}^{-1} 12Ts \ (Tu/\Delta)^{-u/s} \]

\[ P(V_b) = x_{VRH}^{-1} \left[ P_\varepsilon(uT + V_b/2) \right]^{u/s} = x_{VRH}^{-1} (Tu/\Delta)^{-u/s} e^{-V_b/2Ts} \]

\[ u = -\ln(I/I_0) = s \ln(2Ts/Fx_{VRH})/\ln(Ts/\Delta) \]

\[ I \sim V^\beta, \quad \beta = s/\ln(Ts/\Delta), \quad \alpha/\beta = \ln(L/x_{VRH})/\ln(Ts/\Delta) > 1 \]
Regime diagram

\[ I(V, T) \text{ is a power law in } V \text{ and } T \text{ here} \]

\[ \frac{\Delta}{s \ln L} \text{, } \frac{\Delta}{\ln L} \]

\[ \frac{\Delta L}{l} \text{, } \Delta \]

\[ L = \text{length of the wire} \]

\[ \Delta = \text{typical charging energy} \]

\[ s = \log \text{ of tunneling transparency} \]
Conclusions:

- linear and non-linear conductivity
- field and temperature cross-over between single and many impurity tunneling
- low field and temperature: Mott-Shklovskii-VRH
- larger E,T: Kane-Fisher - power law behavior
- Kane-Fisher “single-dominant-barrier” theory is not valid in long wires that contain many (> 100 ?) impurities
- true power-law exponents exceed the single-barrier ones by a “large” log-factor (perhaps, by 2 or 3 in practice)!
- resistance is controlled by “difficult spots” - dense clusters of impurities
- global weak/strong pinning regime diagram