Critical Depinning in DC- and AC-Fields

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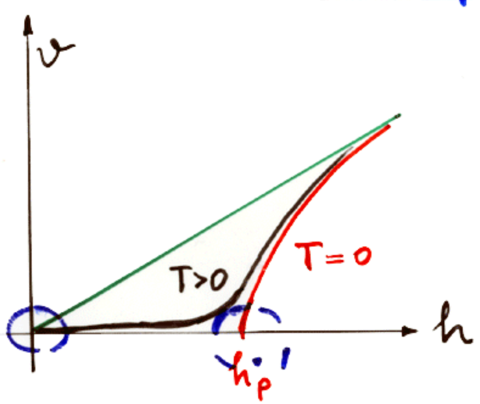
Outline:

- Depinning in DC-Fields, T=0
- Pinning in AC-Fields, T=0 (a poor man‘s approach)
- Thermal Fluctuations

PROBLEM: DRIVEN ELASTIC OBJECT IN RANDOM MEDIA

Examples:

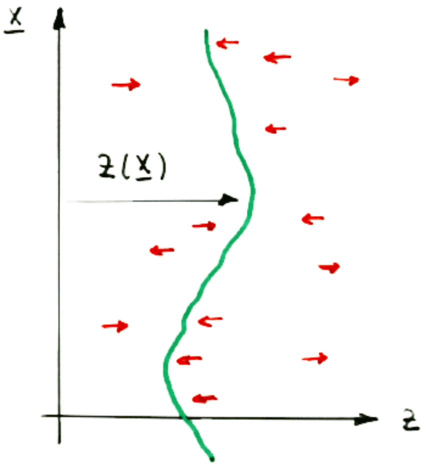


- Domain wall in random magnets → magnetic hysteresis
- Interface between two inmiscible fluids in porous medium (oil/water)
- Flux or dislocation line in impure SC or solid
  Contact line
  ↳ critical current
- Lattices of walls, lines, CDWs, ...

Questions:

- ∃ sharp, reversible transition? h_p (disorder, dimension, ...)
- domain wall self-affine?
  "roughness" $w(t,L) \sim L^{\zeta} f(t/L^{z})$ ζ, z, β?
- $v \propto (h-h_p)^{\beta}$ • oscillating fields?
- influence of thermal fluctuations?

minimal MODEL :



stiffness, random pinning force, $V_z \equiv \frac{\partial V}{\partial z}$

$$\frac{1}{\gamma}\frac{\partial z}{\partial t} = \underset{\downarrow \, = 1}{\Gamma} \, \nabla^2 z - V_z(\underline{x}, z) + h_0 \cos(\omega_0 t)$$

mobility, driving force $h(t)$

$\underline{x} = (x_1, \dots, x_D)$

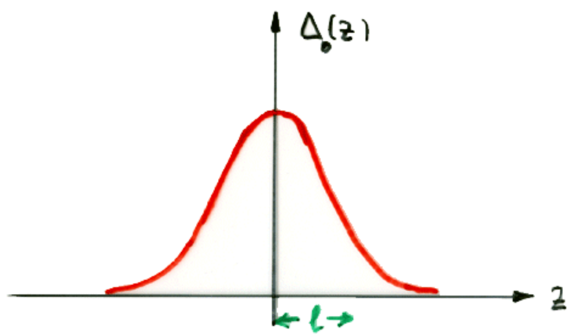
$\exists$ statistical inversion symmetry : $h \to -h$

$\langle \dot{z} \rangle = v \to -v$

Random force :

gaussian

$$\langle V_z(\underline{x}, z) \rangle = 0 \qquad \langle V_z(\underline{x}, z) V_z(\underline{x}', z') \rangle = \delta_\xi(\underline{x} - \underline{x}') \, \Delta(z - z')$$



$$\frac{1}{L^D} \left\langle \left(\int_L d^D x \, V_z(\underline{x}, z) \right)^2 \right\rangle^{1/2} = \frac{\Delta(0)^{1/2}}{L^{D/2}}$$

periodic media : $\Delta_0(z)$ also periodic

RELEVANT LENGTH SCALES:

$$\frac{1}{\gamma}\frac{\partial z}{\partial t} = \underline{\nabla}^2 z - V_z(\underline{x}, \underline{z}) + h_0 \cos(\omega_0 t)$$

(i) Larkin-Length L_p:

$$L_p = |\Delta_0''(0)|^{-\frac{1}{4-D}}$$

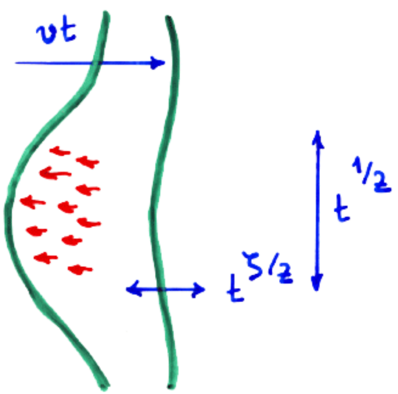
→ depinning threshold $h_p \sim \frac{1}{L_p^2}$

(ii) $\omega_0 = 0$ Correlation Length ξ_v:

$$\boxed{z = vt + u(\underline{x}, t)}$$

(co-moving frame)

$\langle u \rangle = 0$



Assume self-affine moving wall: $u(L,t) \sim L^{\zeta} \sim t^{\zeta/2}$

→ linear problem if $vt \gg u(L,t) \sim t^{\zeta/2}$

↓ $t \gg v^{-2/(2-\zeta)}$

$$L \gg \xi_v \sim v^{-\frac{1}{2-\zeta}}$$

(iii) $\omega_0 \neq 0$ Diffusion Length L_ω:

$$L_\omega = L_p\left(\frac{\gamma}{\omega_0 L_p^2}\right)^{1/2} = L_p\left(\frac{\omega_p}{\omega_0}\right)^{1/2}$$

$\omega_p = \frac{\gamma}{L_p^2}$

ADIABATIC LIMIT: $\omega_0 \to 0$:

(i) Perturbation theory (Schmid + Hauger, Larkin + Ovchinnikov, ... Feigel'man)

$$\gamma \to \gamma_{eff} = \gamma\left[1 - c_1\left(\frac{\xi_v}{L_p}\right)^{4-D} - c_2\left(\frac{\xi_v}{L_p}\right)^{2(4-D)} + \dots\right]$$

divergent if $v \to 0$

$h_p \sim \Delta_0'(\pm 0) = 0$

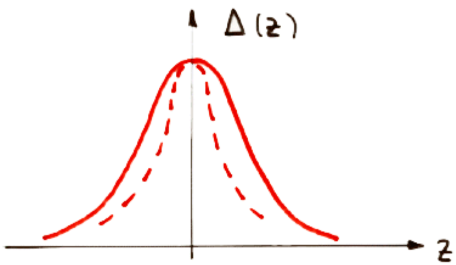
(ii) Renormalization Group

("Parquet"-eqs.)

$$\left[\begin{aligned} \frac{d\gamma_L}{d\ln L} &= c\gamma_L \Delta_L''(0) \cdot L^{\epsilon} \\ \frac{d\Delta_L''(0)}{d\ln L} &= -3c\,\Delta_L''^{2}(0)\,L^{\epsilon} \end{aligned}\right.$$

$\epsilon = 4-D$

Efetov + Larkin '77



$\Downarrow$

$$\Delta_L''(0) = \frac{\Delta_0''(0)}{\underbrace{1 + \frac{3c}{\epsilon}\Delta_0''(0)\left(L^{\epsilon} - a^{\epsilon}\right)}_{\downarrow}}$$

pole at $L \sim L_p$

$\nearrow$ Landau-ghost?

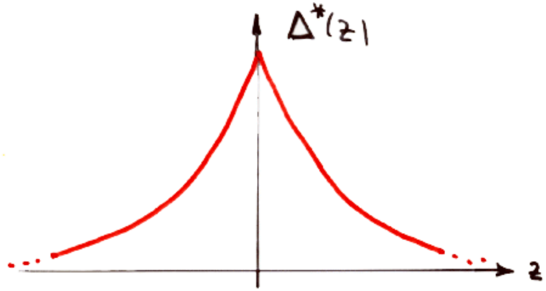
$\searrow$ Appearence of metastability

D. S. Fisher '86

(iii) Functional Renormalization Group

N., Stepanow, Tang, Leschhorn '92

$$\frac{d\,\Delta(z)}{d\ln L} = -cL^{\epsilon}\left[\tfrac{1}{2}\Delta^2(z) - \Delta(z)\,\Delta(0)\right]$$



$\rightarrow$

$$\boxed{\begin{aligned} \Delta_L(z) &\sim L^{2\zeta-\epsilon}\,\Delta^*\left(\tfrac{z}{\ell}\left(\tfrac{L}{L_p}\right)^{-\zeta}\right) \\ \gamma_L &\sim \gamma\left(L/L_p\right)^{2-z} \end{aligned}}$$

$\rightarrow h_p \sim -\Delta'(\pm 0) \neq 0$

$\epsilon = 4 - D$

$\rightarrow v \sim (h - h_p)^{\beta}$

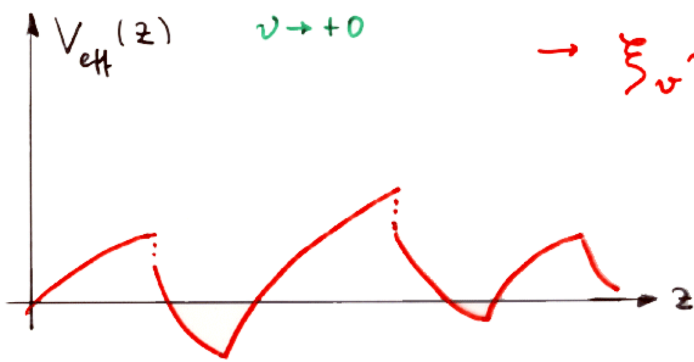
$\beta = 1 - \frac{\epsilon}{9} - 0.040123\,\epsilon^2$

$\rightarrow$ domain wall self-affine

superdiffusive

$z = 2 - \frac{2}{9}\epsilon - 0.04321\,\epsilon^2$

$\zeta = \frac{\epsilon}{3}\left(1 + 0.14331\,\epsilon^2\right)$



$\rightarrow \xi_v \sim (h - h_p)^{-\nu}$

Chauve, Le Doussal, Wiese '01

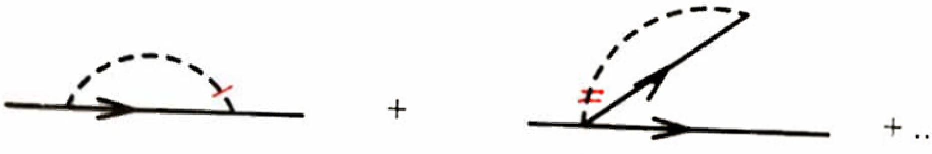
$$\boxed{\nu = \frac{\beta}{z-\zeta} = \frac{1}{2-\zeta} \geqslant \frac{2}{D+\zeta}}$$

"Harris" – Argument for sharp transition

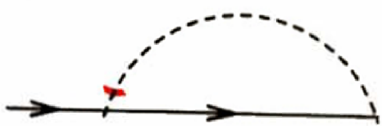
In co-moving frame:

$$\frac{1}{\gamma}\frac{\partial u}{\partial t} = \underline{\nabla}^2 u - V_z(\underline{x}, vt+u) + h - \frac{v}{\gamma}$$

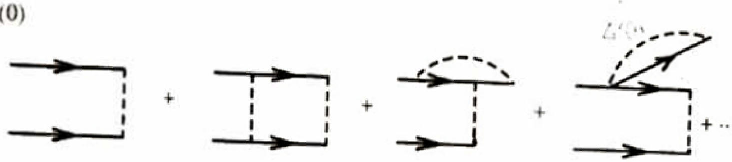
Corrections to propagator:



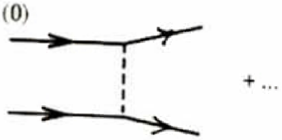
Correction to velocity:



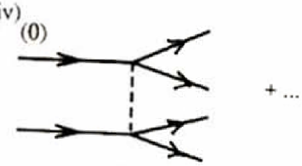
$\Delta(0)$



$\Delta''(0)$



$\Delta^{(iv)}(0)$

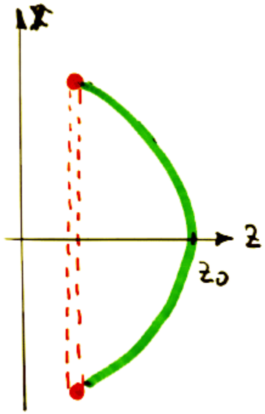


$\vdots$

Wall pinned at the boundary:

$$\frac{1}{\gamma}\frac{\partial z}{\partial t} = 0 = \underline{\nabla}^2 z - V_z(\underline{x}, z) + h$$

$$z(\underline{x})\Big|_{\underline{x} = \underline{x}_{boundary}} = 0$$



pure system : $z(\underline{x}) = z_0 - \frac{h}{2D}\underline{x}^2$

impure system : $z(\underline{x}) = z_0 - \frac{h-h_c}{2D}\underline{x}^2$ $\quad h > h_c$

correlated region : $\xi^5 \sim \xi^{-1/\nu + 2}$

CDWs

Thermal Fluctuations:

Expectation: $v \neq 0$ at $h = h_p$ if $T > 0$

Scaling: $v(h,T) \sim (h-h_p)^{\beta}\, \tilde{\Phi}\left((h-h_p)^{\tau}/T\right)$

$\sim T^{\beta/\tau}$ at $h = h_p$

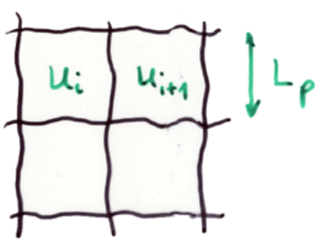
D. S. Fisher
A. Middleton

τ ?

$$\frac{1}{\gamma}\frac{\partial z}{\partial t} = \underline{\nabla}^2 z + h - V_z(\underline{x}, u) + \zeta_{th}(\underline{x}, t)$$

irrelevant at large scales

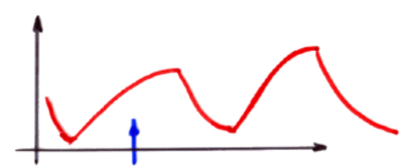
$h \leq h_p$: All degrees of freedom depinned apart from those on scale L_p : u_i



$$\mathcal{H}_{eff} = \sum_i \left\{ V_i(u_i) - h u_i - \chi_i (h-h_p) u_i \right\}$$

MF coupling between domains

$V_i(u)$



$\approx h_p u \left(1 - \frac{u^2}{3u_0^2}\right)$ $\rightarrow$ $E_B \sim (h_p - h)^{3/2}$

$\tau = \frac{3}{2}$

Depinning in AC-Fields

$$h(t) = h_o \cos(\omega_o t)$$

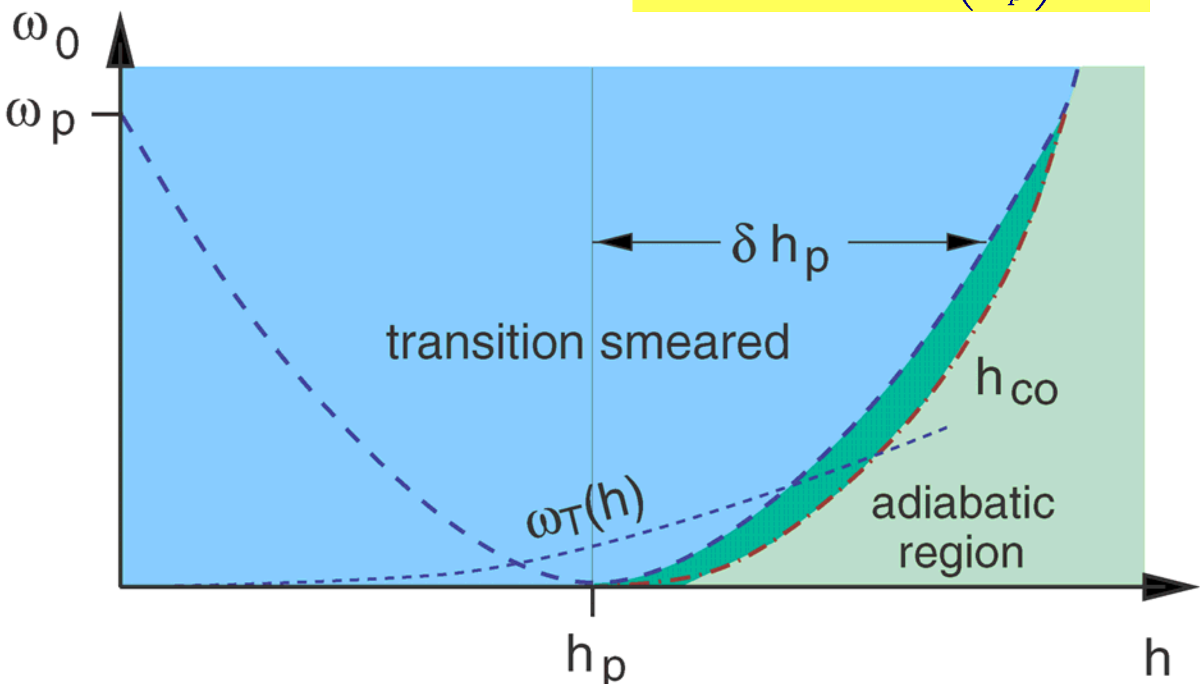
$\omega_0 \neq 0$: Pertubations spread only over $L_\omega = L_p \left(\frac{\omega_p}{\omega_0}\right)^{1/z}$, $\omega_p = \gamma / L_p^2$

→ no sharp depinning transition

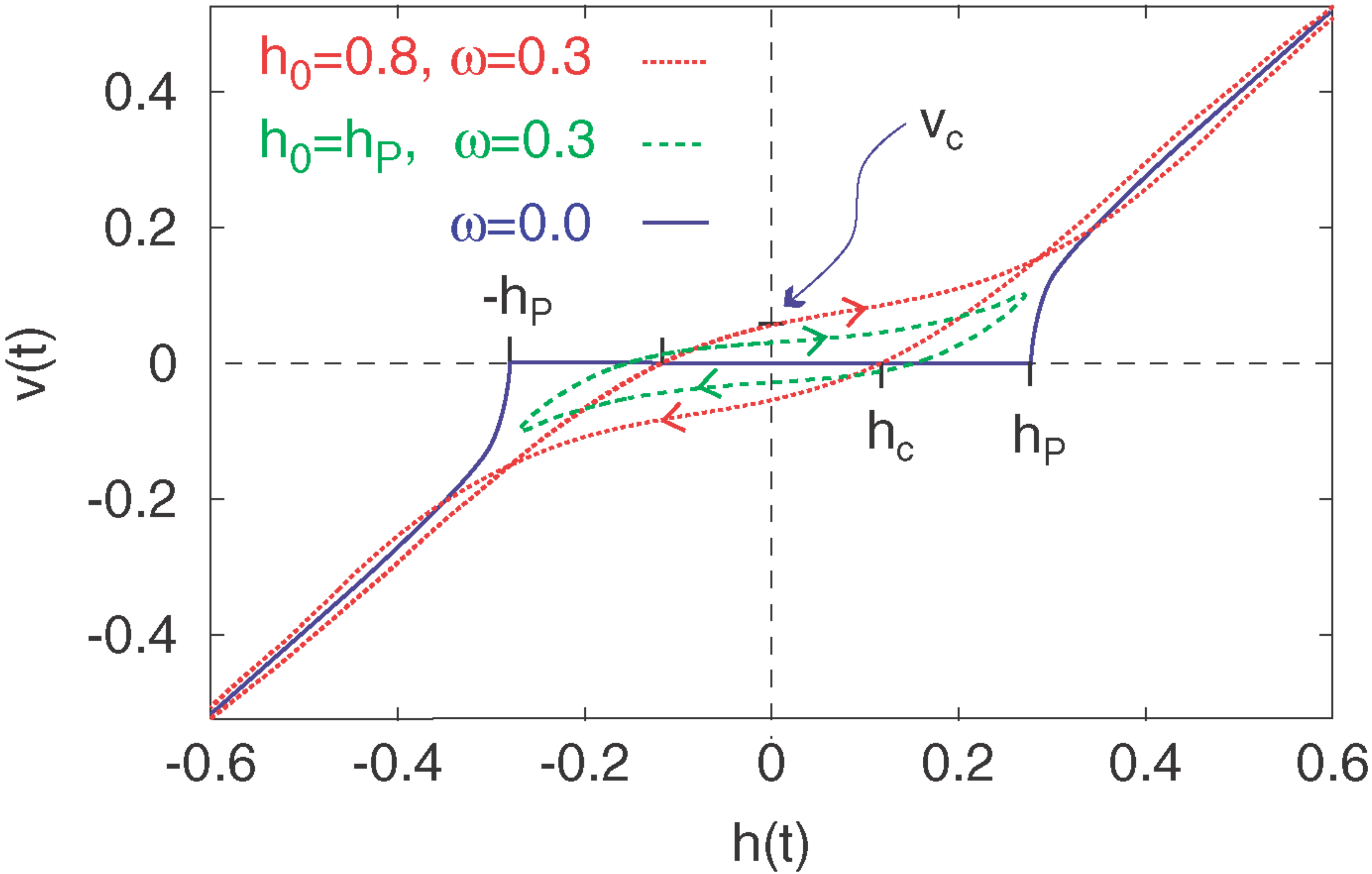
$$\frac{\delta h_p}{h_p} = \left(\frac{L_p}{L_\omega}\right)^{(D+\zeta)/2} = \left(\frac{\omega_0}{\omega_p}\right)^{(D+\zeta)/2z}$$

→ cross-over to adiabatiic limit if $L_\omega \approx \xi_v$:

$$h_{co} - h_p \approx h_p \left(\frac{\omega_0}{\omega_p}\right)^{1/\nu z}$$



Velocity hysteresis in D=1

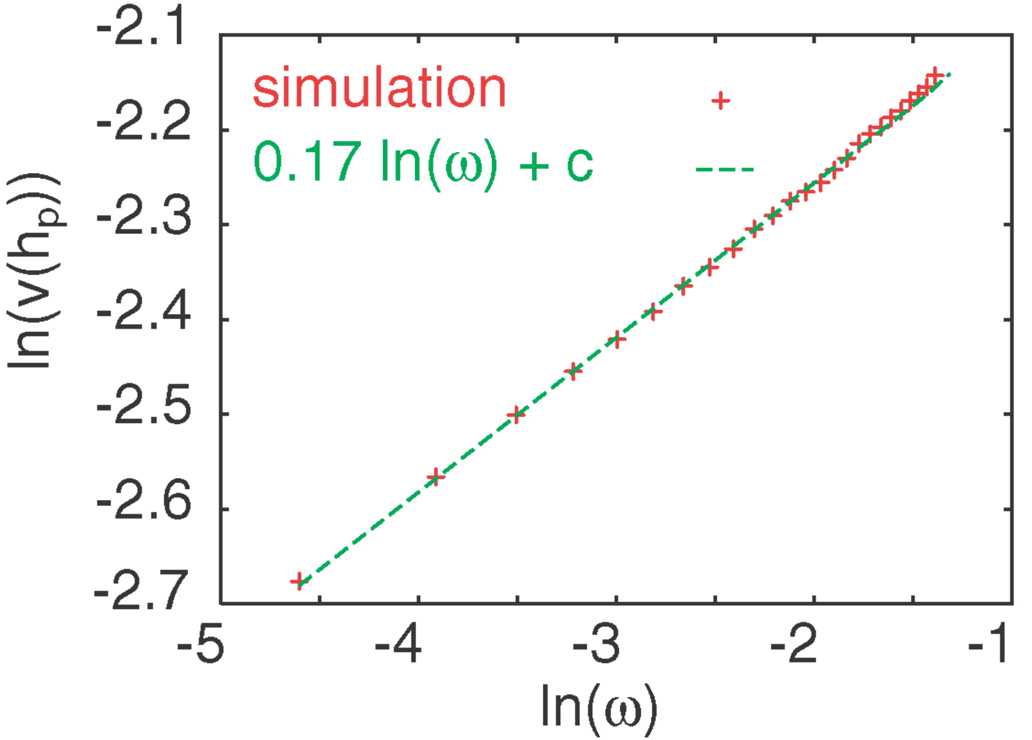


Scaling-Ansatz

$$\omega_0 \ll \omega_p = \gamma / L_p^2$$

$$v(h(t)) = \omega_p l \left(\frac{h(t)-h_p}{h_p}\right)^{\beta} \phi_{\pm}\left[\underbrace{\left(\frac{h(t)-h_p}{h_p}\right)^{-\nu}\left(\frac{\omega_0}{\omega_p}\right)^{1/z}}_{\xi_v / L_\omega}\right]$$

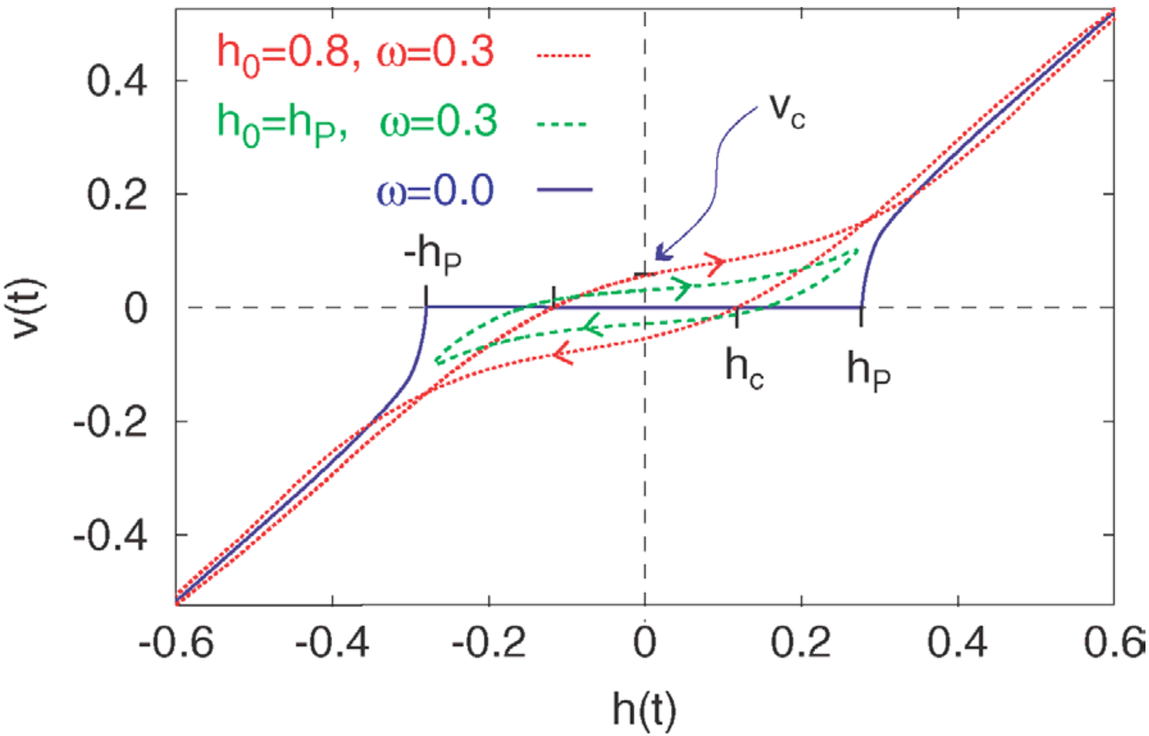
$$v(h_p) \sim \left(\frac{\omega_0}{\omega_p}\right)^{\beta/\nu z}$$



Chauve et al: $\beta/\nu z = 0.19$

Scaling

$$h_c \approx h_p \left(1 - c_- \left(\frac{\omega_0}{\omega_p}\right)^{1/\nu z}\right)$$
$$v_c \approx \omega_p l \left(\frac{\omega_0}{\omega_p}\right)^{\beta/\nu z}$$



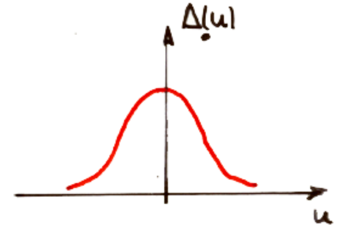
RENORMALIZED PERTURBATION THEORY

lowest order p.t.

$v = \langle \dot{u} \rangle$

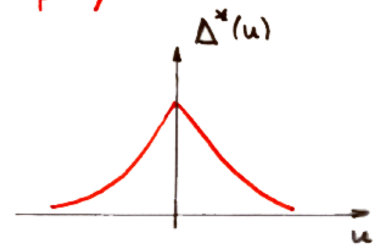
$$\frac{1}{\gamma} v(t) = h(t) + \int_0^\infty dt' \int_p e^{-C\gamma p^2 t'} \Delta'\left(\int_{t-t'}^{t} v(t'')dt'' \right) \equiv h(t) + \tau(t)$$

(i) $\omega_0 = 0$, $v \to 0+$ $\qquad 0 = h_p + \Delta'(0+)$ const.



functional

RG: $p > \xi_0^{-1}$: $\gamma \to \gamma(p)$, $\Delta_0(u) \to \Delta_p(u) \sim p^{4-D-2\zeta} \Delta^*\left(\frac{u}{\ell}(pL_p)^\zeta\right)$



$\Delta_p'(0) \neq 0 \longrightarrow h_p \neq 0$

(ii) $0 < \omega_0 < \omega_p$

inner hysteresis : $h(t) = 0$

$\dot{h} > 0 \quad \to \quad \tau(t) > 0$

$\dot{h} < 0 \quad \to \quad \tau(t) < 0$

(ii) $0 < \omega_0 < \omega_p$:

System still critical if $L \ll \min(\xi_v, L_\omega)$

→ use renormalized parameters of adiabatic case on these scales

→ fluctuations negligible on larger scales

Problem: time-dependent cut-off since all terms between $t, \ldots, t - 1/\omega_0$ appear.

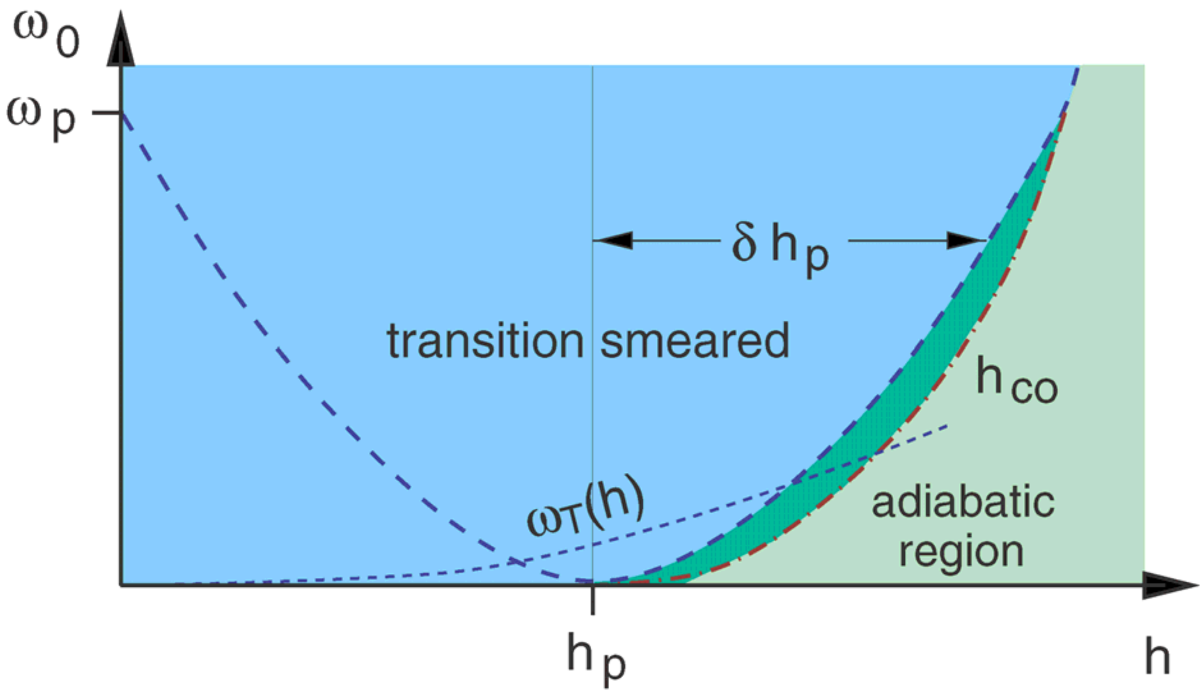
→ restrict to region $|h| \leq h_{co}$ i.e. $\xi_v > L_\omega$

→ approximate expression for $\phi_-(x) = c_- + x$, $c_- = \ldots$

Scaling

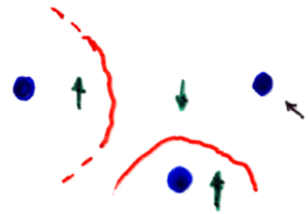
$$v(h,T) \sim (h-h_p)^{\beta}\phi_{\pm}\left[\frac{(\omega_0/\omega_p)^{1/z}}{(h-h_p)^{\nu}}\frac{T}{(h-h_p)^{\tau}}\right]$$

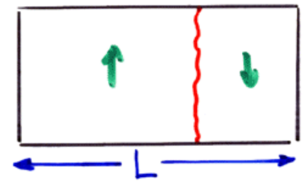
$$v(h_p) \sim T^{\nu z/\tau} \text{ if } \omega_o < \omega_p(T/T_p)^{\nu z/\tau} \equiv \omega_T(h_p)$$



Magnets: f $\longrightarrow$ h $= h_0 \sin(\omega t)$

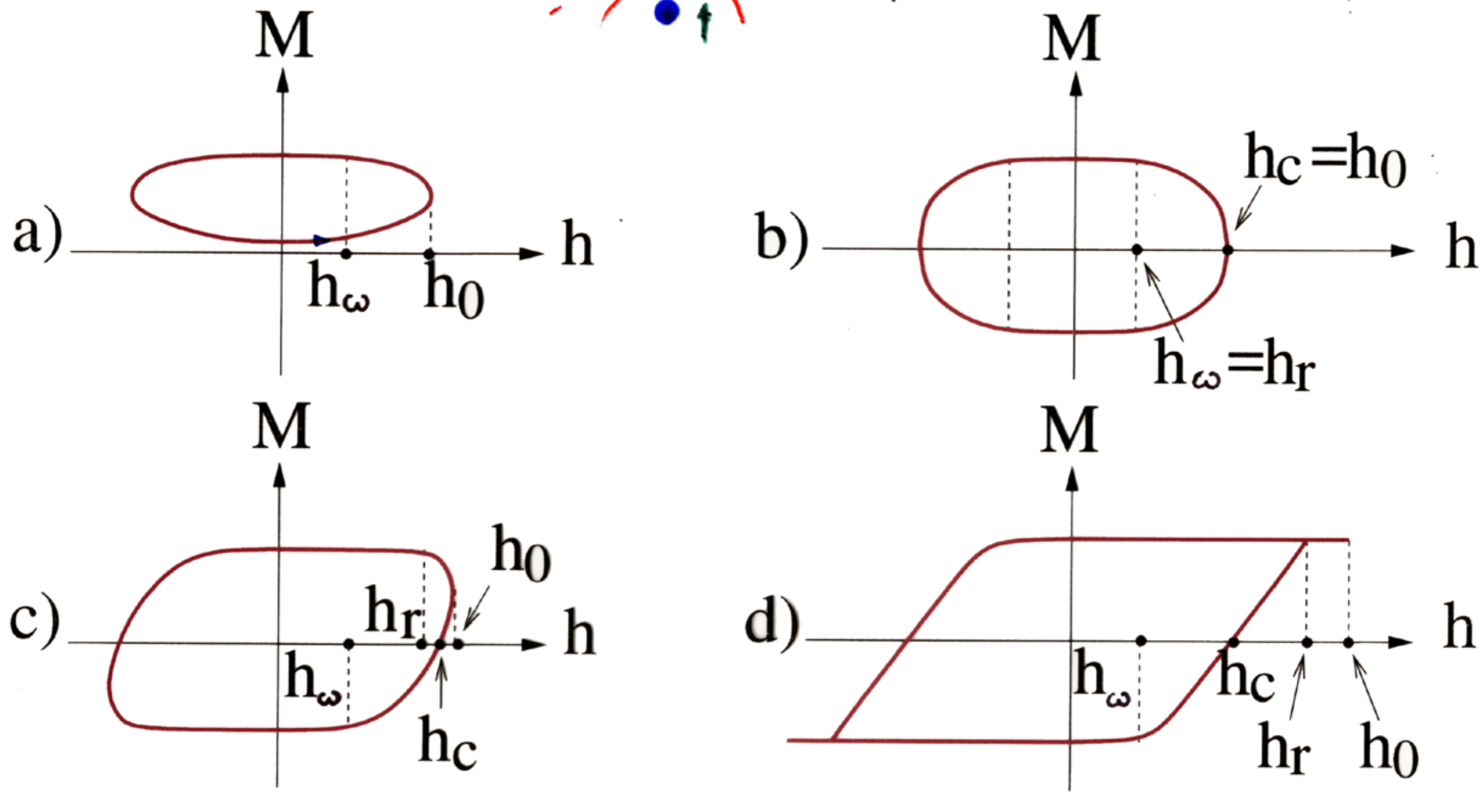
Hysteresis





OR: L = distance between strong pinning centers

$$h_c = h_\omega F\left(\frac{\omega L}{\gamma h_\omega}, \frac{h_0}{h_\omega}\right)$$



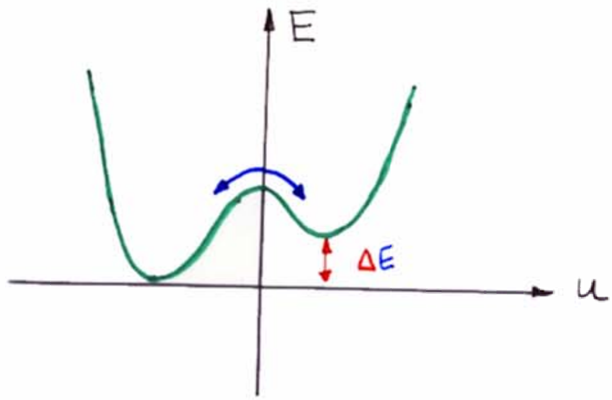
$$h_r = h_\omega F\left(\frac{2\omega L}{\gamma h_\omega}, \frac{h_0}{h_\omega}\right)$$

What happens for $f \ll f_\omega$?

NO global creep, $v \equiv 0$

BUT local oscillations on scales $L \lesssim L_\omega$

MODEL: Two - Level - Systems



Dislocations : Joffe - Vinokur '87

Domain Walls : N., Shapir, Vilfan '90

Vortex Lattices : Koshelev + Vinokur '91

K.H. Fischer + N. '91

Korshunov '01

Jmpedance : $z(\omega) = \rho(\omega) + i\,\ell(\omega)$

e.g. $\ell(\omega) \sim L_\omega^2$

$\rho(\omega) \sim -\omega^2 \frac{d}{d\omega} \ell(\omega)$

similar : susceptibility

Dissipation?

(i) Thermal equilibrium: $n_0(\Delta E) = \dfrac{1}{e^{\Delta E/T} + 1}$ Boltzmann

(ii) External field $f(t)$ disturbs ΔE by $\delta E \simeq f(t) \cdot L^D w(L)$

(iii) $n(\Delta E + \delta E) \approx n_0(\Delta E) + \delta n$

(iv) $\left(\dfrac{\partial}{\partial t} + \dfrac{1}{\tau}\right)\delta n + \dfrac{\partial n_0}{\partial \Delta E} \cdot \dfrac{\partial \delta E}{\partial t} = 0$ $\qquad \tau \propto \omega_p^{-1} e^{E_B(L)/T}$

(v) Dissipated power: on scale L

$$P = -\mathrm{Re}\left\langle \delta n \cdot L^D w \frac{df}{dt} \right\rangle$$

$$\approx \frac{1}{4T}\left[\cosh\frac{\Delta E}{2T}\right]^{-2} \delta E^2 \, \frac{\omega^2 \tau}{1+\omega^2\tau^2}$$

(vi) Total dissipated power:

$$P_{total} \sim \int_{L_p}^{\infty} \frac{dL}{L}\left(\frac{1}{L}\right)^D \frac{\delta E^2(L)}{E_B(L)} \cdot \frac{\omega^2 \tau(L)}{1+\omega^2\tau^2(L)}$$

$$\equiv \frac{1}{2}\,\omega\,\chi''(L,\omega)\, f_\omega^2$$

(vii) Low frequency susceptibility dominated by contributions from L_ω:

$$\chi(T,\omega) \sim \frac{L_p^2}{\gamma}\left[\frac{T}{T_p}\ln\frac{\omega_p}{\omega}\right]^{2/\chi_{eq}}$$

SUMMARY

- There is a sharp, reversible depinning transition in a DC field
- The critical behaviour is described by non-equilibrium scaling laws $\nu = \frac{\beta}{2-\zeta} = \frac{1}{2-\zeta}$
- The dynamics is superdiffusive
- The depinning transition is smeared and exhibits a velocity hysteresis in an AC field
- $v_c \sim \left(\frac{\omega_0}{\omega_p}\right)^{\beta/\nu z}$ $\quad h_c \sim h_p\left(1 - c_-\left(\frac{\omega_0}{\omega_p}\right)^{1/\nu z}\right)$
- Thermal fluctuations lead to an additional smearing if $\omega_0 < \omega_p\left(\frac{T}{T_p}\right)^{\nu z/\theta}$