The Fractional Quantum Hall Effect

1 Important Formulae

We begin by listing the most important formulae of the talk. The magnetic flux quantum is given:

$$\phi_0 = \frac{hc}{e} \tag{1}$$

The magnetic length is:

$$l = \sqrt{\frac{\hbar c}{eB}} \tag{2}$$

A circle of radius $\sqrt{2}l$ then contains one flux quantum, as can be seen by the magnetic field being equal to:

$$B = \frac{\phi_0}{2\pi l^2} \tag{3}$$

The vector potential in the symmetric gauge is given by:

$$\vec{A} = -\frac{1}{2}\vec{r} \times \vec{B}, \quad \vec{r} = (x, y, 0)^T, \quad \vec{B} = -B\hat{z} \tag{4}$$

In the IQHE we derived the Hamiltonian of the problem using semiclassics:

$$\hat{H} = \frac{1}{2m}(\vec{p} + \frac{e}{c}\vec{A})^2 \tag{5}$$

For our symmetric gauge (4), we note that $\vec{p}\cdot\vec{A} = \vec{B}\cdot\vec{L}$ giving us a 2D Harmonic Oscillator up to the L_z term:

$$\hat{H} = \frac{1}{2m}(p_x^2 + p_y^2) + \frac{e^2B^2}{4c^2}(x^2 + y^2) + \frac{2e}{c}BL_z \tag{6}$$

We make a coordinate change:

$$z \equiv \frac{x + iy}{l}, \qquad z = |z|\, e^{i\theta} \tag{7}$$

The solutions to (6) for the Lowest Landau Level(LLL) are given by:

$$\varphi_m \propto z^m e^{-\frac{1}{4}|z|^2}, \quad m \in \mathbb{N}_0 \tag{8}$$

In the talk we derive the unique solution for the filled first Landau Level in the presence of the coulomb repulsion:

$$\Psi[z] = \prod_{i<j}^{N} (z_i - z_j) e^{\sum_{j=1}^{N} |z_j|^2} \tag{9}$$

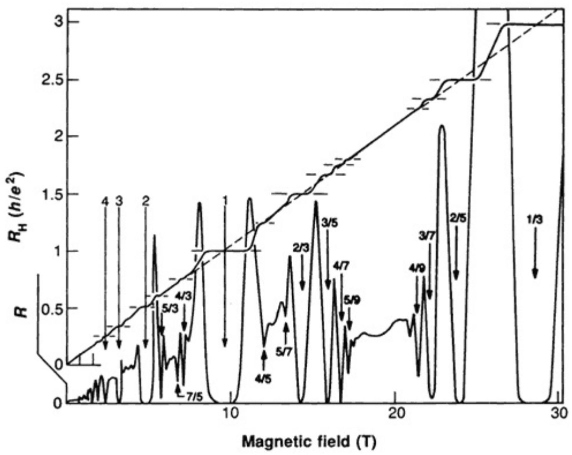
where $[z] = (z_1, z_2, ..., z_N)$. Robert Laughlin's variational wave function for the FQHE problem was then:

$$\Psi[z] = \prod_{i<j}^{N} (z_i - z_j)^m e^{\sum_{j=1}^{N} |z_j|^2} \tag{10}$$

Where m is an odd natural number to preserve antismmetry.

2 Results of the FQHE

The FQHE results in more plateaus appearing in the graph of hall resistance as a function of magnetic field:



The plateaus correspond to a fractional multiples of the quantized hall resistivity: $\rho_{xy} = \frac{p}{q}\frac{h}{e^2}$. The FQHE also gives rise to quasi particles, with fractional charge!

In at least the case of charge $e/3$, these particles have even been experimentally confirmed! An amazing property of these particles is that they do not follow fermi-statistics. They are neither bosons or fermions, but instead anyons, and exchanging them induces a phase $e^{i\phi} \neq \pm 1$!

References

[1] A. Stern, Anyons and the quantum Hall effect - a pedagogical review, 2007

[2] S. Girvin, The Quantum Hall Effect: Novel Excitations and Broken Symmetries, 1998