Majorana Fermions in solid state systems feature non-abelian statistics, i.e. braiding not only changes the phase but leads to different quantum states. Therefore an application as qubits seems possible.

Kitaev chain

A toy tight binding Hamiltonian for a p-wave superconductor reads

\[ H = \sum_{j=1}^{L} \left[ -\mu a_j^\dagger a_j - t \left( a_{j+1}^\dagger a_j + h.c. \right) + \left( \Delta a_{j+1}^\dagger a_j^\dagger + h.c. \right) \right] \]

Introduce Majorana operators \( \gamma_{2j-1} = a_j + i a_j^\dagger \) and \( \gamma_{2j} = -i a_j + a_j^\dagger \) with \( \{ \gamma_l, \gamma_m \} = 2\delta_{lm} \) and \( \gamma_j = \gamma_j^\dagger \) Easy to solve in cases

1. \( \Delta = t = 0, \mu < 0 \)
   - \( H = -\frac{\Delta}{2} \sum_{j=1}^{L} \gamma_{2j-1} \gamma_{2j} \)
   - Majoranas on same site pair
   - Unique unoccupied groundstate

2. \( \Delta = t \neq 0, \mu = 0 \)
   - \( H = i\Delta \sum_{j=1}^{L-1} \gamma_{2j} \gamma_{2j+1} \)
   - Majoranas on neighbouring sites pair
   - \( \gamma_1 \) and \( \gamma_{2L} \) are unpaired. Combine to highly non-local fermion \( d_M = \frac{1}{2} (\gamma_1 + i \gamma_{2L}) \)
   - groundstate two-fold degenerate: \( \left| 0 \right> \) and \( \left| 1 \right> = \left| d_M^\dagger |0> \right) \)

Associated with the transition is a topological invariant \( \nu = s_{0x}s_{y} \) where \( s_{0x} \) is the sign of the kinetic energy at momentum \( 0/\pi \), i.e \( \nu \) is -1 for an odd number of band crossings in half the Brillouin zone and can only changes when the gap closes. For parameters in the range \( |\mu| < 2t \), the topological phase is realized with MFs no longer sharply localized to the ends, but decaying exponentially and overlapping, giving rise to an interaction

\[ H_{\text{int}} = \frac{i}{2} t' \gamma' \gamma'' \]

with \( t' \propto \exp(-L/\xi) \) Therfor states split, but only exponentially \( \rightarrow \) neglectable. Realisation is difficult, because no p-wave-SC is available and fermions must appear spin-less.

Physical Realization

Solution: Semiconductor/s-wave-SC heterostructure can mimic p-wave-SC through Spin-Orbit coupling.

\[ H = \int dx \psi_\alpha^\dagger(x) \left\{ -\frac{\partial^2}{2m} - \mu - i\alpha \partial_x \sigma_y + V \sigma_x \right\} \psi_\beta(x) + H_{\text{SC}} \]

with

\[ H_{\text{SC}} = \int dx \left\{ \Delta \psi_\uparrow \psi_\downarrow + h.c. \right\} \]

Spin-Orbit- and Zeemann-coupling split bands and give rise to an effective spinless regime for \( |\mu| < V \). The Hamiltonian then maps to Kitaev’s model and the topological phase occurs at \( V > \sqrt{\Delta^2 + \mu^2} \).

Experiments

Two main effects should be visible. Measuring the DoS via tunnel spectroscopy should reveal a zero-energy-peak. A junction between two superconductors via a topological SC exhibits a fractional Josephson current.

References


