Magnetic Monopoles in Spin Ice

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Frustration

Frustrated systems are an essential ingredient to spin ice. In the systems that we consider here, frustration arises directly from the lattice geometry (as opposed to the Kitaev honeycomb model, where frustration is due to different, competing bond types). A simple example for this 'geometrical frustration' is the Ising model on a triangular lattice, where there is a sixfold ground state degeneracy in just a single triangle (fig. 1a).

Nearest Neighbor Spin Ice

In Spin Ice, there are two fundamentally different contributions to spin-spin interaction. On the one hand, there is a short-range Ising-like interaction. On the other hand, long-range dipole-dipole interaction becomes important in the experimentally realized (dipolar) spin ice.
If one neglects long-range interactions, the system is described by the nearest-neighbor spin ice Hamiltonian

$$H_{\mathrm{nn}} = -J \sum_{\langle i,j \rangle} s_i s_j \hat{z}_i \cdot \hat{z}_j \quad , \tag{1}$$

with $s_i, s_j \in \{-1, +1\}$ binary spins on a pyrochlore lattice and $\hat{z}_i, \hat{z}_j$ unit vectors pointing from a tetrahedron's center towards the spins s_i and s_j, respectively. (fig. 1b).

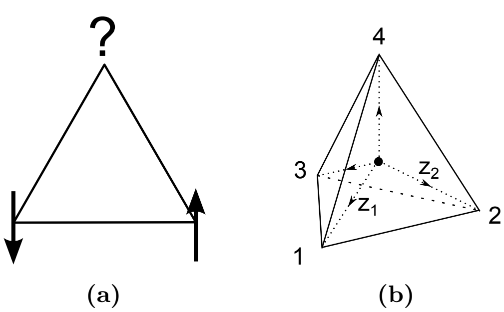


Figure 1: *(a) Geometrical frustration of antiferromagnetic Ising spins on a triangular lattice. (b) Definition of $\hat{z}_i$-axes in a tetrahedron.*

Defining the total spin of a tetrahedron $\triangle$, $L_\triangle := s_{1,\triangle} + s_{2,\triangle} + s_{3,\triangle} + s_{4,\triangle}$, and using $\hat{z}_i \cdot \hat{z}_j = -\frac{1}{3}$ in every tetrahedron, we can rewrite the Hamiltonian as

$$H_{\mathrm{nn}} = \frac{J}{3} \sum_{\triangle} L_\triangle^2 \quad , \tag{2}$$

which reproduces the Hamiltonian 1 up to an irrelevant constant. Choosing $J > 0$, the ground state requires $L_\triangle = 0$ in every tetrahedron. This is known as the **ice rule**.

Dipolar Spin Ice

In dipolar spin ice, experimentally realized for example in $Dy_2Ti_2O_7$, the large spins of the rare-earth element are forced into Ising-like states by the crystal field, but they also carry a large magnetic moment. If we turn on these magnetic dipole-dipole interactions, the Hamiltonian picks up an additional term,

$$H = H_{\mathrm{nn}} + D \sum_{i,j} \frac{s_i s_j \hat{z}_i \cdot \hat{z}_j}{|\vec{r}_{ij}|^3} - \frac{3 s_i s_j \left(\hat{z}_i \cdot \vec{r}_{ij}\right)\left(\hat{z}_j \cdot \vec{r}_{ij}\right)}{|\vec{r}_{ij}|^5}. \tag{3}$$

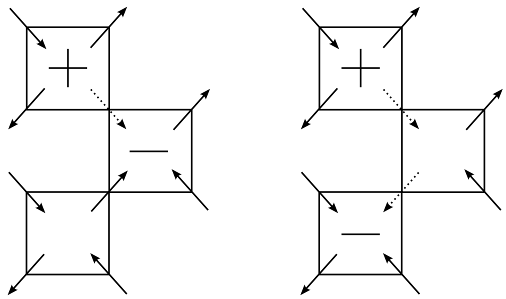
It turns out, that, despite this long-range dipole-dipole interaction, the system is still governed by the ice rule. One way to see this is by replacing the dipoles by pairs of monopoles [1], separated by a distance d, such that the monopoles always sit in the center of a tetrahedron and their signs define the orientation of the replaced dipole. Instead of dipole-dipole interaction, one ends up with interactions of the form

$$U(R_{\alpha\beta}) = \begin{cases} \frac{\mu_0}{4\pi} \frac{Q_\alpha Q_\beta}{R_{\alpha\beta}} & \text{if } R_{\alpha\beta} \neq 0 \\ u_0 Q_\alpha^2 & \text{else} \end{cases} \quad , \tag{4}$$

where $R_{\alpha\beta}$ is the distance between two tetrahedrons α and β, and Q_α is the total magnetic charge in tetrahedron α. One can show that u_0 is large enough to imply $Q_\alpha = 0$ for every tetrahedron in the ground state, which corresponds to the ice rule.

Magnetic Monopoles

Up to now, we have only considered the ground state of spin ice. But at finite temperatures, defects are present. In the setting of spin ice, a 'defect' refers to a tetrahedron that violates the ice rule. These defective tetrahedrons come in pairs and each carry a net magnetic charge (fig. 2a).



(a) **(b)**

Figure 2: *(a) A pair of defective tetrahedrons is created by flipping one spin – inverting the dotted spin restores the ice rules. (b) A defective tetrahedron is moved by an additional spin flip.*

The defective tetrahedrons can be moved around by flipping additional spins (fig. 2b), without growing domain walls (as would happen in a regular magnetic material) – They are effectively deconfined.

Topology

If we average the spins in real space to form a continuous vector field $\vec{P}$, the ice rule translates into $\mathrm{div}\vec{P} = 0$. For any closed surface S, the flux through this surface equals the volume integral over $\mathrm{div}\vec{P}$, i.e. it depends on the number of enclosed monopoles. Although spins are locally disordered, they are still in topological order. The topological order parameter is the net magnetic charge, which is present in the (sub-)system.

Further Reading

A very thorough introduction to spin ice, also discussing experimental signatures, is given in reference [2]. A more general approach with focus on the Coulomb phase, also very profound, is given in [3]. The idea of explaining dipolar spin ice by substituting dipoles with pairs of monopoles is introduced in [1], with detailed calculations provided in the supplementary material.

References

[1] C. Castelnovo, R. Moessner, and S. L. Sondhi. Magnetic Monopoles in Spin Ice. *Nature*, 451:42–45, Jan 2008.

[2] M. J. P. Gingras. Spin Ice. *arXiv*, 0903.2772v1, Mar 2009.

[3] Christopher L. Henley. The Coulomb Phase in Frustrated Systems. *Annu. Rev. Condens. Matter Phys.*, 2010.1:179–210, 2010.