The integer quantum Hall effect

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Introduction

The quantum Hall effect (QHE) is one of the most remarkable condensed-matter phenomena discovered in the second half of the 20th century, and our talk is mainly about the integer quantum hall effect. The basic results are $\sigma_{xx} = 0$ and $\sigma_{xy} = \nu e^2/h$, in which ν is the integer quantum number. We will show how to get these really beautiful results step by step.

The Landau Level

Let us search for the quantum dynamics in strong B fields of the 2-d material. First, we choose the Landau gauge: $\overrightarrow{A}(\overrightarrow{r}) = xB\hat{\mathrm{y}}$. After separating variables we have the effective one-dimensional Schrödinger equation

$$h_k f_k(x) = \epsilon_k f_k(x) \tag{1}$$

where

$$h_k = \frac{1}{2m}p_x^2 + \frac{1}{2}m\omega_c^2(x + kl^2)^2 \tag{2}$$

which is simply a one-dimensional displaced harmonic oscillator, so we have an entire family of energy eigenvalues

$$\epsilon_{kn} = (n + \frac{1}{2})\hbar\omega_c \tag{3}$$

which are Landau levels. And the total number of states in each Landau level is then

$$N = \frac{L}{2\pi}\int_o^{L_x/l^2} dk = \frac{L_x L_y}{2\pi l^2} = N_\phi \tag{4}$$

where $N_\phi = \frac{BL_xL_y}{\phi_0}$.

IQHE Edge States

We can consider the problem of electrons confined in a Hall bar of finite width by a non-uniform electric field $V(x)$.If we assume that the system still has translation symmetry in the y direction, the solution to the Schrödinger equation must still be of the form

$$\psi(x, y) = \frac{1}{\sqrt{L_y}}e^{iky}f_k(x) \tag{5}$$

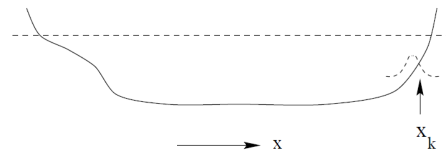

Figure 1: *Illustration of a smooth confining potential which varies only in the x direction.*

We see that the group velocity $\overrightarrow{v_k} = \frac{1}{\hbar}\frac{\partial\epsilon_k}{\partial k}\hat{y}$ has the opposite sign on the two edges of the sample.This means that in the ground state there are edge currents of opposite sign flowing in the sample. The semi-classical interpretation of these currents is that they represent 'skipping orbits' in which the circular cyclotron motion is interrupted by collisions with the walls at the edges as illustrated in fig.2.

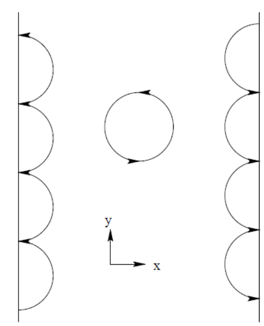

Figure 2: *Semi-classical view of skipping orbits at the fermi level at the two edges of the sample where the confining electric field causes $E \times B$ drift.*

Quantized Conductance

To calculate this current we have to add up the group velocities of all the occupied states

$$I = -\frac{e}{L_y}\int_{-\infty}^{+\infty} dk \frac{L_y}{2\pi}\frac{1}{\hbar}\frac{\partial\epsilon_k}{\partial k}n_k \tag{6}$$

Assuming zero temperature and noting that the integrand is a perfect derivative, we have

$$I = -\frac{e}{h}\int_{\mu_R}^{\mu_L} d\epsilon = -\frac{e}{h}[\mu_L - \mu_R] \tag{7}$$

Borrowing from the Landauer formulation of transport, the Hall voltage drop corresponds to a chemical potential difference between two edges

$$(-e)V_H = (-e)[V_R - V_L] = [\mu_L - \mu_R] \tag{8}$$

Hence,

$$I = \nu \frac{e^2}{h} V_H \tag{9}$$

where we have now allowed for the possibility that ν different Landau levels are occupied in the bulk and hence there are ν separate edge channels contributing to the current. Using the fact that the current flows at right angles to the voltage drop we have the desired results

$$\sigma_{xx} = 0, \quad \sigma_{xy} = \nu \frac{e^2}{h} \tag{10}$$

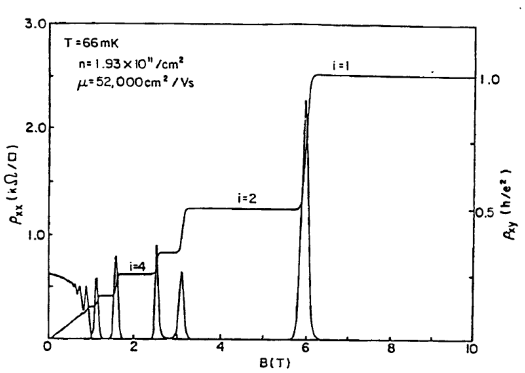

Figure 3: *The experiment plot of ρ_{xy} and ρ_{xx} vs magnetic field B*

Why we have plateaus

In the second section, we know that the total number of states in each Landau level is $N_\phi = \frac{BL_xL_y}{\phi_0}$. So, as the magnetic field B increases or decreases, we will have quasi-holes or quasi-particles in the system. In the absence of disorder, these quasi-holes or quasi-particles will contribute to the current and we will have linear dependence of σ_{xy} vs B. But we have disorder in the system, the important disorder will localize these quasi-holes or quasi-particles in a finite change of magnetic field B, so we will have plateaus in the presence of disorder.

References

[1] Steven M. Girvin, *The Quantum Hall Effect: Novel Excitations and Broken Symmetries* (Springer Verlag and Les Editions de Physique, 1999)

[2] A. Karlhede, S. A. Kivelson and S. L. Sondhi *The Quantum Hall Effect: The Article* (World Scientific, 1993)