Fractional Quantum Hall Effect (FQHE)

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Introduction

In 1980, Klaus von Klitzing discoverd that[1], the Hall conductance is quantized ($\Sigma_H = \nu e^2/h$), where ν is an integer. Subsequently, 2 years later, Tsui, Strömer and Gossard[2] discovered that there exists some rational values of ν around which Hall plateaus can be centered. In 1983, Lauglhin[3] came along and gave a theoretical explanation for the existence of Fractional values. In this talk, we will discuss the phenomena of FQHE and some fascinating properties due to existence of FQHE.

Laughlin's Wave Function

As we know, Integer Quantum Hall Effect (IQHE) can be understood in the independent electron picture. In this framework, its not possible to explain FQHE, since the existence of peaks at fractional fillings indicate that extended states be only partially filled which would immediately lead to non-zero value of Σ_{xx}. Thus, understanding the FQHE requires for the account of electron-electron interactions.

Laughlin guessed a variational ansatz [3] for the ground state wave function of the FQHE state based on generalization of numerical solutions to small number of electrons. In the Landau gauge ($A_x = -By$), the wave function for the lowest Landau level can be written as,

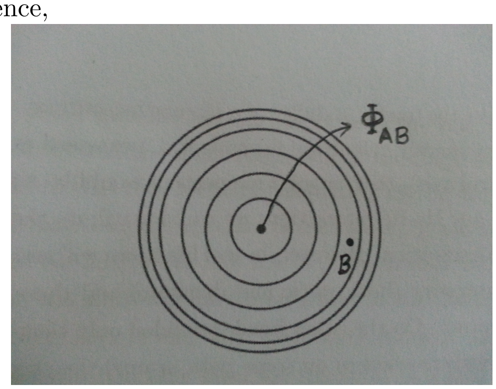
$$\psi(z_1, z_2, ...z_N) = \prod_{j<k}(z_j - z_k)^q exp-(|z_i|^2/4l_M^2)\,; \quad q\,odd \tag{1}$$

where $z \equiv x + iy, \quad l_M^2 \equiv \hbar/eB$

Note:
1. Here, $\nu = \frac{1}{q}$ is the filling fraction of the states.
2. For case $q = 1$, the Laughlin wave function just represents the GSWF for non-interacting electons that we had seen in the case of IQHE

Fractional Quantization

[4] Now let us try to derive the fractional quantization of Hall conductance using gauge arguments given beautifully by Laughlin himself.
Consider a Corbino-disk geometry as shown in the figure below (disk with a circular hole in its center) with an uniform magnetic field B perpendicular to its surface, plus a Aharonov-Bohm (AB) $\Phi_{AB}(t)$ flux through its centre. Now we slowly vary Φ_{AB} over a time $t^{'}$ by an amount q flux quanta. Due to this change, q orbits will have moved out through the outer edge of the disk and q in through the inner edge. Since the filling factor is $1/q$, in the end a single electron would have moved across the edges of the disk. Hence,



$$current, I = e/t' \quad voltage, V = q\Phi_0/T$$
$$Hall\,conductance, \quad \Sigma_H = I/V = e^2/hq$$

Thus the Hall conductance is fractionally quantized, as observed experimentally.

Fractional Charge

As discussed in the previous section, now let us change the AB flux not by by q flux quantum, but by one flux quanta in the one of area of the disk. We know that, this change leads to increase of l-value of outermost Landau orbit by one. Physically, we can interpret this as electron from each orbit has moved to next orbit creating a "quasi hole" at the origin[1] ! It can be shown that, the effective charge of this quasi-hole is given by,

$$e^* = -e/q \tag{2}$$

Intuitively , we can think that, by decreasing the flux by one quanta, we would create a "quasi-electron" of charge

[1] Origin corresponds to the point the disk where the flux is changed

$e^* = +e/q$.
These fractionally charged excitations has been experimentally observed. It is important here to note that, these fractional charge (say 1/3, $q = 3$) doesn't mean that, we have split electron into 3 pieces. Instead, these fractional charges are observed with respect to a background. Here, the Laughlin's Fractional Quantum hall states form the background.

Fractional Statistics

[5] Fractional statistics, perhaps most fundamentally reflects the two-dimensionality of the physical system under consideration. The statistics of the identical-particles is defined by the phase change of the wavefunction when two particles interchange their positions.
In 3-D, the space is doubly connected, i.e, when the particles are interchanged twice, they return to initial state. Thus the phase change $e^{i\alpha}$ is equal to one, which implies $\alpha = 0 \, or \, \pi$ corresponding to fermions or bosons respectively.
In 2-D, the space is infinitely connected, or the twice interchange of position of 2 particles do not give us back the original wavefunction. As a consequence, α is arbitrary (anyons). It can be shown that, for the Laughlin quasi-particles,

$$\alpha = q\pi \tag{3}$$
$$where, \;\; q \, is \, the \, filling \, fraction.$$

Thus a Lauglihn quasi-particles is indeed an "anyon"

References

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