Skyrmions in quantum Hall liquids

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Introduction

This talk as part of a series of talks about topological aspects in condensed matter physics deals with excitations in quantum Hall liquids. In the discussion of these excitations, which are Skyrmions, we will discover that topology plays an important role. Furthermore we will see that Skyrmions combine almost everything we have encountered so far. After a short introduction of the setting we will study possible excitations by considering a simple example of a Skyrmion. Also we will see that Skyrmions carry charge and that they are topologically stable.

Setting

In the quantum Hall effect we study the physics of a two dimensional electron gas (2DEG) under the influence of a strong magnetic field $\vec{B} \parallel \hat{z}$. Due to the Zeeman-effect one would expect the spin dynamics completely frozen out. In this talk we want consider a different problem in which we vary Zeeman energy $g\mu_B B$, this can theoretically be done by varying the effective gyromagnetic factor g. One finds that for $g \geq 0$the ground state does not depend on g, but the nature of quasiparticle excitations will change and as $g \to 0$ they will become Skyrmions. This is however not a completely theoretical work, but one can find that the effective g-factor indeed varies, e.g. in GaAs at filling factor $\nu = 1$ due to spin-orbit scattering the coupling to the magnetic field is reduced which reduces the effective g-factor.

Topological excitations

The order parameter describing the system is the magnetization $\vec{m}(\vec{r})$, where $\vec{r}$ varies in the $\mathbb{R}^2$ (our 2DEG) and $\vec{m}$ lies on the sphere S^2.
The ground state (GS), as already mentioned independent of g, is the purely ferromagnetic state where all spins are pointing up.
To excite the GS we want to flip a spin, but we also take into account the desire of the spins to be parallel. That means we smoothly vary our magnetization and create a smooth topological defect which will not be a singularity.(Fig1)

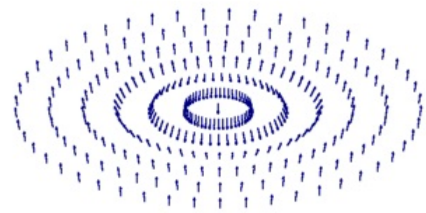

Figure1: Isolated Skyrmion, magnetization varies smoothly towards the flipped spin in the center (http://www.mpq.univ-paris7.fr/spip.php?article205&lang=en)

To study the effect of such a texture in the magnetization on the physics we consider an electron moving through with a velocity $\dot{x}$. The spin of the electron wants to align with the local magnetization $\vec{m}$ and therefore the spin of the electron will change $\dot{me}^\mu = \triangledown m \cdot \dot{x}^\mu$ (we used here m for the local magnetization and for the spin of the electron). The Lagrangian describing this system is

$$L = -\frac{e}{c}\dot{x}^\mu A^\mu + \hbar S \dot{m}^\nu M^\nu(m) \tag{1}$$

where A^μ is the usual vector potential and M^ν a 'vector potential' for the spin (read [1] 1.10.4 for more information). This can be rewritten as

$$L = -\frac{e}{c}\dot{x}^\mu (A^\mu + a^\mu) \tag{2}$$

with the additional 'berry connection'
$a^\mu = -\frac{\phi_0}{2\pi} S \frac{\partial m^\nu}{\partial x^\mu} \cdot M^\nu(m)$. That means the electron sees the spin texture as an additional berry connection and therefore picks up an extra berry phase. Calculating this yields

$$b = -\frac{\phi_0}{8\pi}\epsilon^{\alpha\beta}\langle \vec{m}, \partial_{x^\alpha}\vec{m} \times \partial_{x^\beta}\vec{m}\rangle \tag{3}$$

$$b = -\phi_0 \rho_{top} \tag{4}$$

As we have seen in a previous talk, adding flux corresponds to adding charge and we see that the total charge carried by the Skyrmion is $Q = -\frac{\sigma_{xy}\phi_0}{c} Q_{top} = \nu e Q_{top}$ with $Q_{top} = \int \rho_{top} d^2 r$.
We have seen how these excitations carry charge, but now we want to establish that the Skyrmions are indeed topologically stable. To see this we need to study Q_{top} and see that this topological charge is an integer.
In the first talk we encountered the winding number of a map $f : S^1 \to S^1$ which measured how many times the map f wrapped around the circle. Here the magnetization $\vec{m}$ defines a map $m : \mathbb{R}^2 \to S^2$(Fig2)

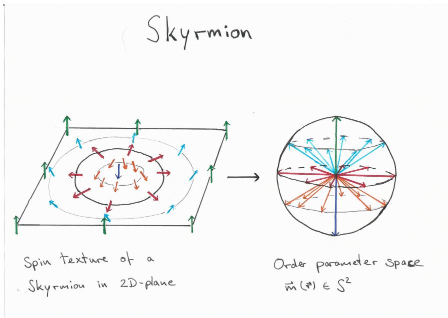

Figure2: Illustration of the mapping defined by the magnetization $\vec{m}$

Since we want all spins far away from the center of the Skyrmion to point up one can also interpret this as a map $m : S^2 \to S^2$ which is similar 'quantized' as the one-dimensional case.

$$Q_{top} = \frac{1}{4\pi} \int \frac{1}{2} \epsilon^{\alpha\beta} \langle \vec{m}, \partial_{x^\alpha} \vec{m} \times \partial_{x^\beta} \vec{m} \rangle \tag{5}$$

Taking a closer look at this one can see that Q_{top} measures how many times $\vec{m}$ wraps the plane/sphere around the sphere, by measuring the volume of the image of $\vec{m}$ and dividing it by the volume of the unit sphere.
(remark: Knowing the language of differential forms one can calculate that $Q_{top} = \frac{1}{\int_{S^2} \omega} \int_{S^2} m^* \omega$, where $\omega = xdy \wedge dz + ydz \wedge dx + zdx \wedge dy$ is the volume form on S^2.)
Considering for example the map illustrated in (Fig2) the topological charge $Q_{top} = 1$ since (Fig2) is nothing else than a picture of the identity on the sphere.
The above example was only a simple one, the System of course can exhibit much more complicated structures consisting of much more complicated Skyrmions or even of several Skyrmions. And we see that these Skyrmions can not be continuously deformed into the purely ferromagnetic state since they are characterized by the topological charge Q_{top} which makes them topologically stable excitations.

References

[1] Steven M. Girvin, The Quantum Hall Effect: Novel Excitations and Broken Symmetries (Springer Verlag and Les Editions de Physique, 1999)

[2] S.L. Sondhi, A. Karlhede, S.A. Kivelson, Skyrmions and the crossover from the integer to fractional quantum Hall effect at small Zeeman energies (Physical Review B, Volume 47 nr 24, 15 June 1993)