DISORDER AND DIAGRAMS

Kim Alina Forche
kforche@smail.uni-koeln.de

I. Introduction

Diagrammatic perturbation theory is a useful method to solve the problem of electrons propagating in a disordered system. A one-particle-system can be described by an electron travelling from one point to another being scatterd at impurities on its way. The aim is to find the impurity-averaged Green function for this process where over all possible impurity configurations is averaged.

II. Replica Trick

The replica trick can be used to calculate the disorder average of an observable. The idea is to consider R identical copies of the original system. For an observable obtained by differentiation of the free energy, the disorder average is therefore:

$$\langle O \rangle = -\frac{\delta}{\delta J} \langle \ln Z[J] \rangle = -\frac{\delta}{\delta J} \lim_{R \to 0} \frac{1}{R} \langle Z^R \rangle$$

where J is the source field and Z^R is the replicated partition function. By assuming that the various impurities are independent of each other and can be described by a short ranged Gaussian distribution, the disorder average of Z^R in a free system gives:

$$\langle Z^R[J] \rangle_{dis} = \int \mathcal{D}(\bar{\psi}, \psi) e^{-\sum_{a=1}^{R} S_{cl} - \sum_{a,b=1}^{R} S_{dis}}$$

with $S_{cl}[\psi^a, \bar{\psi}^a, J]$ the action of the clean system and the disorder action

$$S_{dis} = -\frac{\gamma^2}{2} \sum_{mn} \int d^d r \bar{\psi}_m^a(r) \psi_m^a(r) \bar{\psi}_n^b(r) \psi_n^b(r)$$

where a and b are the replica indices and γ the interaction strength. From this also follows that:

$$\langle O \rangle_{dis} = \lim_{R \to 0} \frac{1}{R} \sum_{a=0}^{R} \langle O(\bar{\psi}^a, \psi^a) \rangle_{\psi}$$

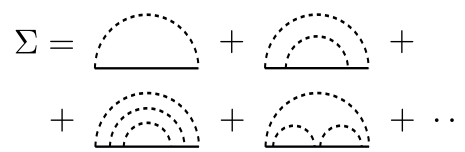
where $\langle ... \rangle_{\psi}$ is the functional average including the disorder action.

III. Green Function And Diagrams

The previous equation can be applied to the Green function, the propagator of the electron in an diordered environment:

$$\langle G_{p,p',n} \rangle_{dis} = \lim_{R \to 0} \frac{1}{R} \sum_{a=0}^{R} \langle O(\psi_{n,p}^a, \bar{\psi}_{n,p}^a) \rangle_{\psi} \delta_{p,p'}$$

Using diagrammatic perturbation theory and applying the disorder average so that diagrams that give no or only a small contribution drop out, the self-energy can be written as:



The total Green function can then be obtained from the Dyson equation:

$$G = G_0 + G_0 \Sigma G \to G = \frac{1}{(G^0)^{-1} - \Sigma}$$

By inserting the Ansatz $Im\Sigma_{p,n} = -\frac{1}{2\tau} sgn(\omega_n)$ with the Matsubara frequency ω_n into the Dyson equation, the final result for the impurity-averaged Green function will be:

$$\langle G \rangle_{dis} = \frac{1}{i\omega_n + E_F - \frac{p^2}{2m} + \frac{i}{2\tau} sgn(\omega_n)}$$

and in spatial coordinates

$$\langle G(x, y; \tau) \rangle_{dis} = G_{cl}(x, y; \tau) e^{-\frac{|x-y|}{2l}}$$

where $G_{cl}(x, y; \tau)$ is the Green function of the clean system without impurities and l is the elastic mean free path. This shows that the amplitude of the electron in an disordered environment is exponentially suppressed. The same methods can be used to calculate higher correlation functions.

References

[1] Altland and Simons, *Condensed Matter Field Theory*, Cambridge University Press, second edition, 2010