Scaling Theory of Localization

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1 Introduction

In the semi-classical Drude model of conduction, where the electrons are assumed to bounce between impurities in a fully classical way, the diffusive zigzag motion will merely be reduced by increasing the amount of disorder, furthermore the zigzag motion itself is locally not influenced by the extent of the material. Therefore, the conductivity will neither (i) vanish suddenly with an increasing strength of disorder, nor (ii) is there an explicit dependency on the extent of the sample. Yet both phenomena occur when a material is highly disordered, because the wave nature of the electron then becomes important. In the limit of high disorder, the electrons localize as standing waves due to backscattering in the disordered ion-potential. The diffusion then is governed by tunneling processes, which give rise for a strong dependence on the sample size and the disorder strength. The scaling theory now, aims to describe the behaviour of the scaling function, that describes the scaling behaviour of the conductance characteristically.

2 Weak-localization and Anderson-localization

Due to backscattering processes, it is possible that electrons pass the same point twice. To take first quantum mechanical corrections into account, one has to consider the propagation along the path in the opposite direction. This in conclusion leads to a constructive interference at the starting point, that means an enhanced probability to find the elctron there again. The correction therefore, decreases the conductivity. By explicit calculations one finds a dependence of the deviation $\frac{\delta\sigma}{\sigma_0}$ on the sidelength L of a d-dimensional hypercubic sample, i.e. $\frac{\delta\sigma}{\sigma_0} = \frac{\delta\sigma}{\sigma_0}(L,d)$. These first corrections are called *weak localization*.

Based on a Tight-Binding model, where the disorder is simulated by a random potential, it turns out that, for a certain disorder strength, the electrons begin to localize as standing waves. Therefore, the diffusion is given by tunneling processes, consequently we assume an exponentially decrease with the sidelength L of the material

$$G = G_0 exp(-L/\eta) \tag{1}$$

with a the localization length η. This correction is called *strong localization* or *Anderson localization*.

3 Scaling Theory of Localization

Consider a hypercubic sample of size L^d in dimension $d = 1,2,3$ with sidelength L and cross section $A = L^{d-1}$. Starting with the familiar description of conduction, we find for the conductance G

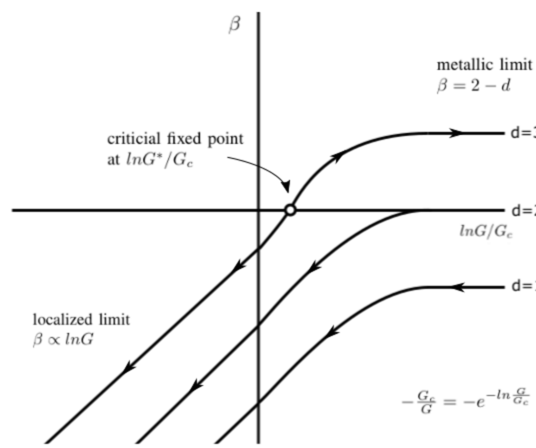
$$G = \sigma_0 \frac{A}{L} = \sigma_0 L^{d-2} \tag{2}$$

with the conductivity σ_0. To scale the sample size, add up b^d further hypercubes with an equally amount of cubes in each spatial direction. To proceed in our attempt to describe the change in the scaling behaviour, we have to define a characteristic entity. According to the *scaling hypothesis of localization* this entity just depends on the conductance G and not explicity on L, that is underpinned by a deeper physical principle, that cannot be discussed here in a proper way. Heuristically, the following definition of the scaling function can be motivated by the fact that in a low disordered material, i.e. with $G \neq G(L)$, the scaling is determined by the exponent of L. Assuming that a change of the scaling behaviour again appears in the exponent of L we define

$$\beta(G(L)) = \frac{dlnG}{dnL} \tag{3}$$

In the metallic limit with low disorder, one finds (i) $\beta = d-2 = \beta_0^d$. In case of weak localization, G is still assumed to be high, therefore suppose that β can be calculated as a perturbation series in G^{-1}, that means (ii) $\beta(G) = d-2-G_c/G + O(G^{-2}) \approx \beta_0^d - exp(-lnG/G_c)$. In case of high disorder, Anderson localization leads to (iii) $\beta(G) = lnG/G_c$. With knowledge of (i)-(iii) and the assumption of continuity and a simple behaviour, we can draw a Graph. Furthermore, due to the change in sign, we find a fixed point for $d = 3$ for a certain strength of disorder and a flow depending on L.

Fig. 1 The scaling function of conduction. For $d = 3$ we have a fixed point and a metallic behavior, that is not the case for $d = 1,2$



References

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