Chiral spin liquids, network models and fractional quantum Hall states

Erwin Schrödinger Institute, Vienna (September 2014)

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Perimeter

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quantum spin liquids

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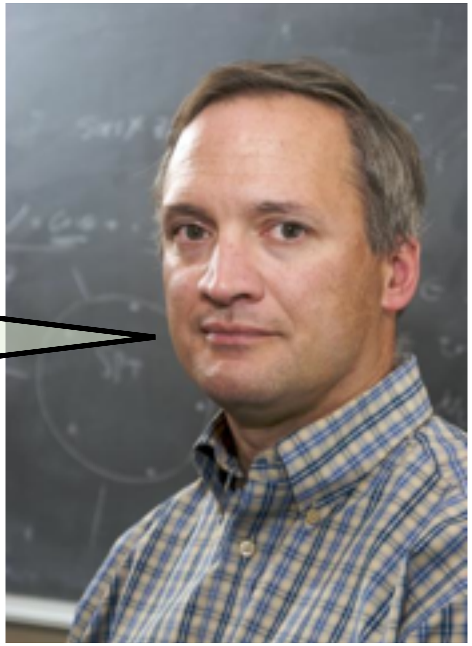
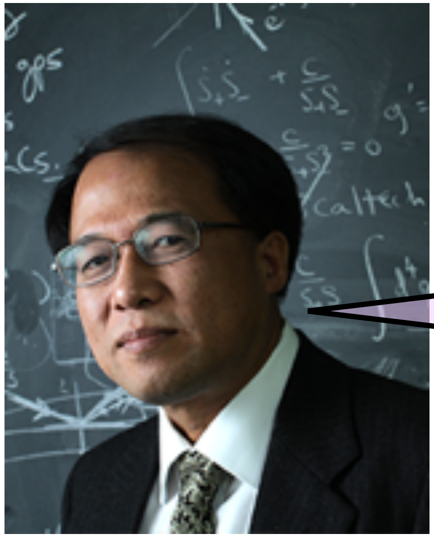
Quantum spin liquids

Quantum spin liquids are exotic ground states of frustrated quantum magnets, in which **local moments** are **highly correlated** but **still fluctuate strongly** down to zero temperature.

Nature **464**, 199 (2010).



Leon Balents



Xiao-Gang Wen

Quantum spin liquids are **long-ranged entangled** states with **fractionalized excitations**. Some of them exhibit **intrinsic topological order** – very much like the fractional quantum Hall states.

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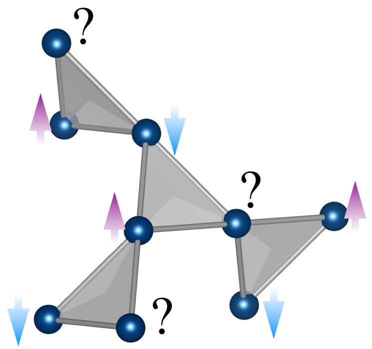
Where do we find them?

Herbertsmithite

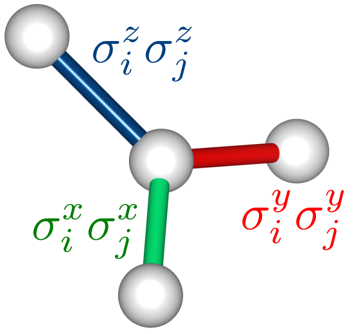
Volborthite

hyperkagome $Na_4Ir_3O_8$

What features are we looking for in these materials?



geometric frustration



exchange frustration

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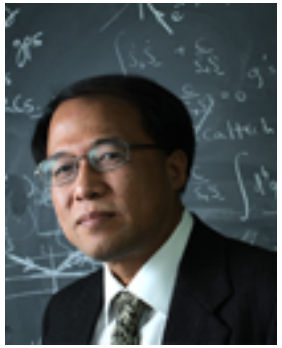
How many are there?

Classification scheme



three spatial dimensions

spatial dimensions — two spatial dimensions

energy spectrum — gapped | gapless

topological spin liquids

gapless spin liquids

entanglement — $S = aL - \gamma$

singular points e.g. Dirac cones

spinon surface, Bose surface

time-reversal symmetry				
	yes	**quantum double models** Levin-Wen / Kitaev models	**Kitaev model** (honeycomb)	**“Bose metals”** (Motrunich, Sheng & Fisher)
	no	**chiral spin liquids** Kalmeyer & Laughlin ✓	✗	✓

Kagome model
this talk

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How do we talk about spin liquids?

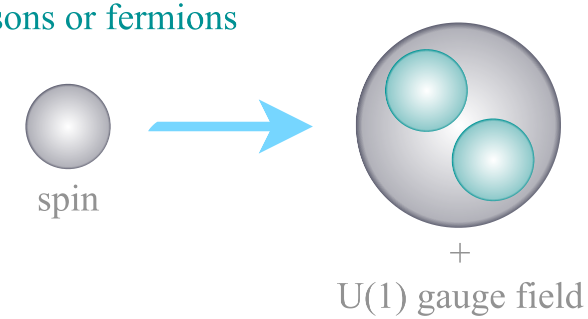
Schwinger representation

U(1) spin liquids

$c_1^\dagger c_1 + c_2^\dagger c_2 = 2S$
bosons or fermions

$$S^\gamma = \frac{1}{2} c_\alpha^\dagger \sigma_{\alpha,\beta}^\gamma c_\beta$$

Pauli matrices

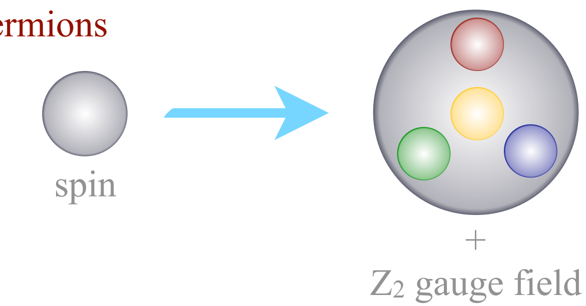


Majorana fermions

Z_2 spin liquids

Majorana fermions

$$S^\gamma = i a^\gamma c$$



→ Maria's & Xiao-Liang's talk

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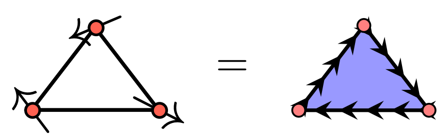
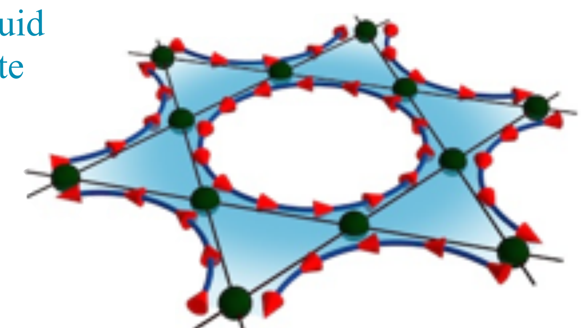
network models

chiral spin liquids

topological liquid
with edge state





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chiral spin liquids

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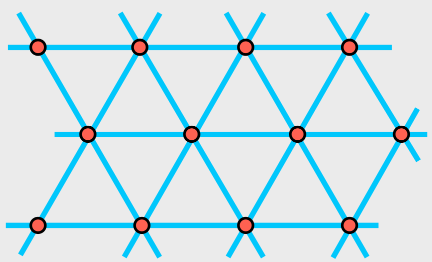
Chiral spin liquids (CSL)

The original proposal

Kalmeyer & Laughlin (1987)

Ground state of triangular lattice
Heisenberg antiferromagnet
conjectured to be **ν=1/2 Laughlin state**

$$H = \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j$$



Wen, Zee & Wilzek (1989); Baskaran (1989)

Scalar spin chirality as order parameter
for chiral spin states

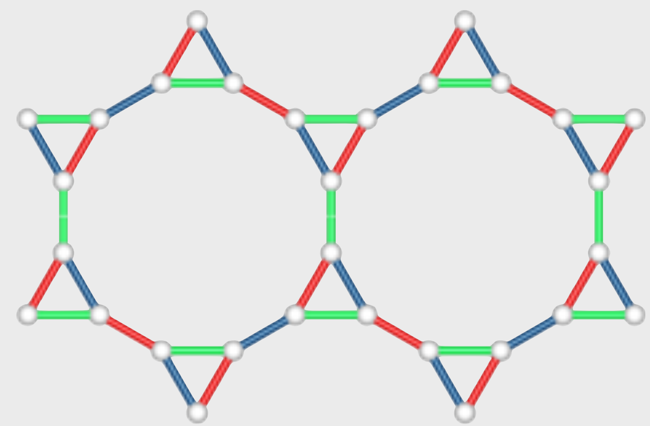
$$\chi_{ijk} = \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k)$$

A recent revival

Exact parent Hamiltonians

Yao & Kivelson (2007)
decorated honeycomb lattice

Schroeter, Thomale, Kapit, Greiter (2007-2012)
long-range interactions



Topological flat band models

Tang *et al* (2011)
Sun *et al* (2011)
Neupert *et al* (2011)
and others

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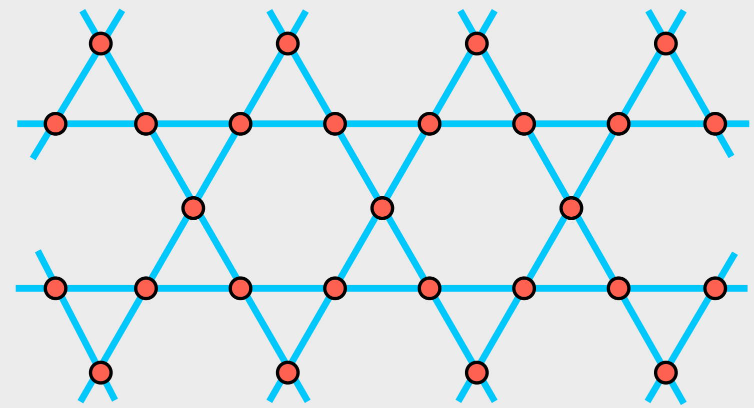
A path to much simpler models

Hubbard model

$$H = -\sum_{\langle i,j\rangle,\sigma} \left(t_{ij} c^{\dagger}_{i\sigma} c_{j\sigma} + t^{*}_{ij} c^{\dagger}_{j\sigma} c_{i\sigma} \right) + \frac{h_z}{2} \sum_i (n_{i\uparrow} - n_{i\downarrow}) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

t/U expansion



Kagome lattice
lattice of **corner-sharing** triangles

Spin model

$$H = J_{\mathrm{HB}} \sum_{\langle i,j\rangle} \vec{S}_i \cdot \vec{S}_j + h_z \sum_i S_i^z + J_\chi \sum_{i,j,k\in\triangle} \boxed{\vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k)} + \ldots ,$$

scalar spin chirality

SU(2) preserving
TR breaking

order parameter

interaction term

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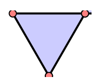
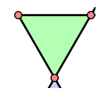
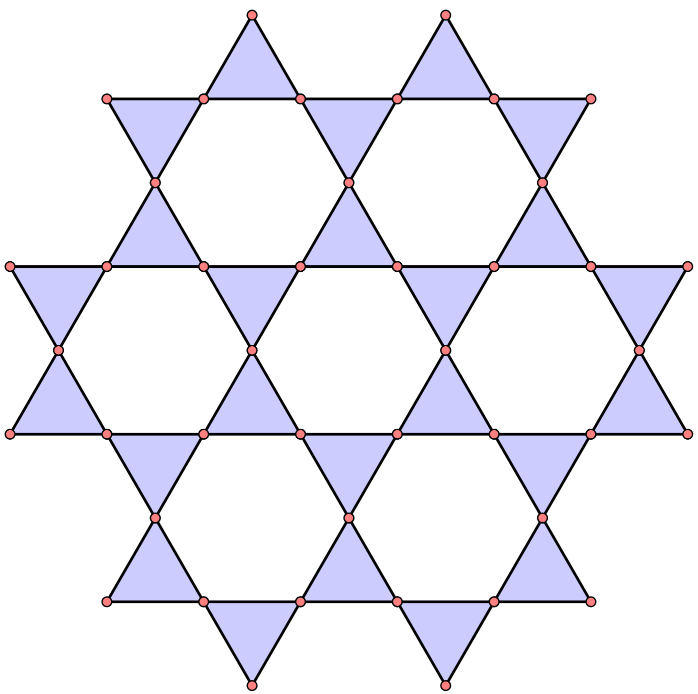
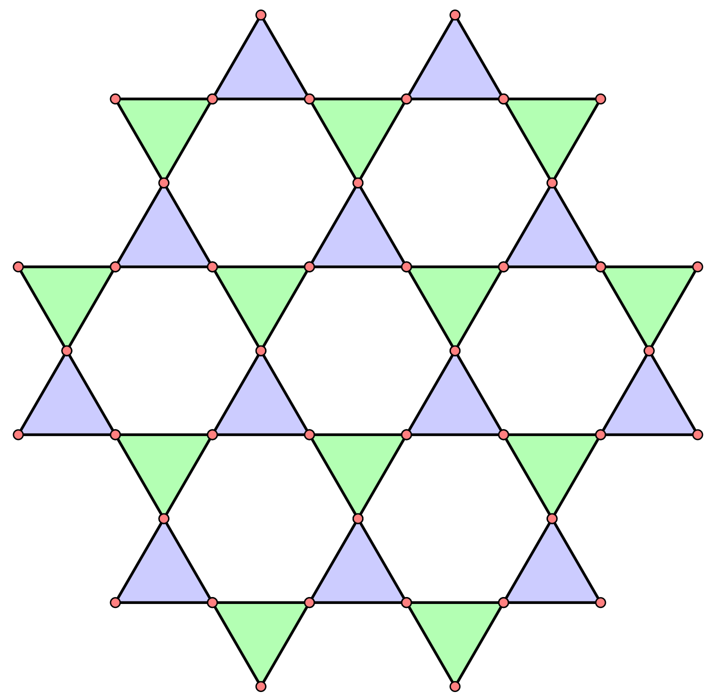
A rather simple model in two variants

$$H = \sum_{\triangle} \chi_{ijk} \pm \sum_{\bigtriangledown} \chi_{ijk}$$

 $= +\chi_{ijk}$  $= -\chi_{ijk}$



homogeneous



staggered

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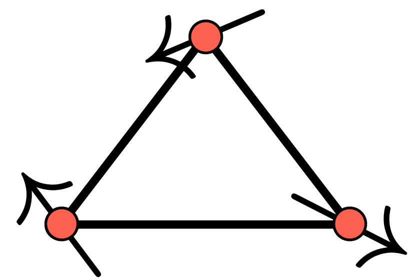
Warm-up: Majorana fermions

$$H = \sum_{\triangle} \chi_{ijk} \pm \sum_{\bigtriangledown} \chi_{ijk}$$

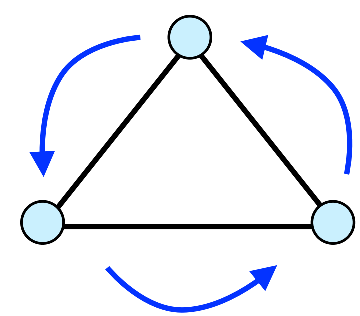
spin degrees of freedom

$$\chi_{ijk} = \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k)$$



spinless fermions / Majorana fermions

$$\begin{aligned}\tilde{\chi}_{ijk} = i[&(c_i^\dagger c_j - c_j^\dagger c_i)+\\ &(c_j^\dagger c_k - c_k^\dagger c_j)+\\ &(c_k^\dagger c_i - c_i^\dagger c_k)]\end{aligned}$$



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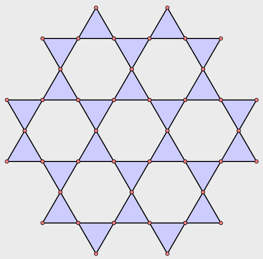
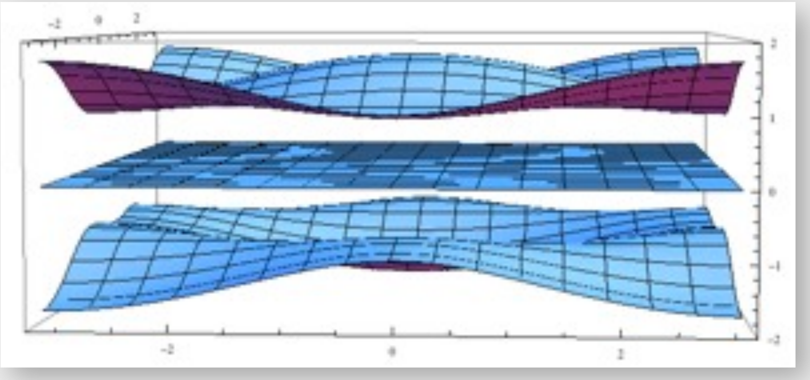
Warm-up: Majorana fermions

homogenous couplings

$$H = \sum \tilde{\chi}_{ijk}$$





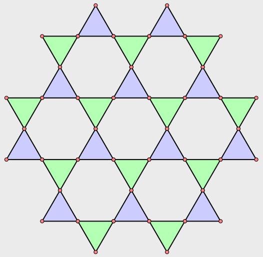
Gapped spectrum

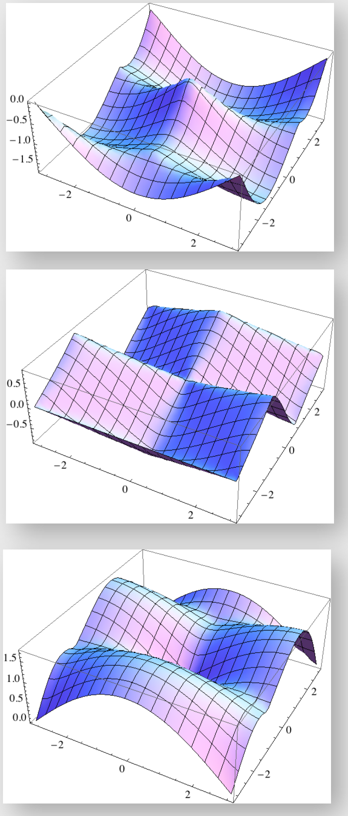
Chern number of the top and bottom bands: $C = \pm 1$

Ohgushi, Murakami, Nagaosa, (2000)

staggered couplings

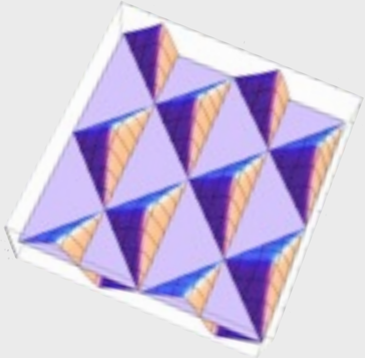
$$H = \sum_{\triangle} \tilde{\chi}_{ijk} - \sum_{\bigtriangledown} \tilde{\chi}_{ijk}$$





Gapless spectrum

Fermi surfaces
= lines in momentum space



Shankar, Burnell & Sondhi, (2009)

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network models

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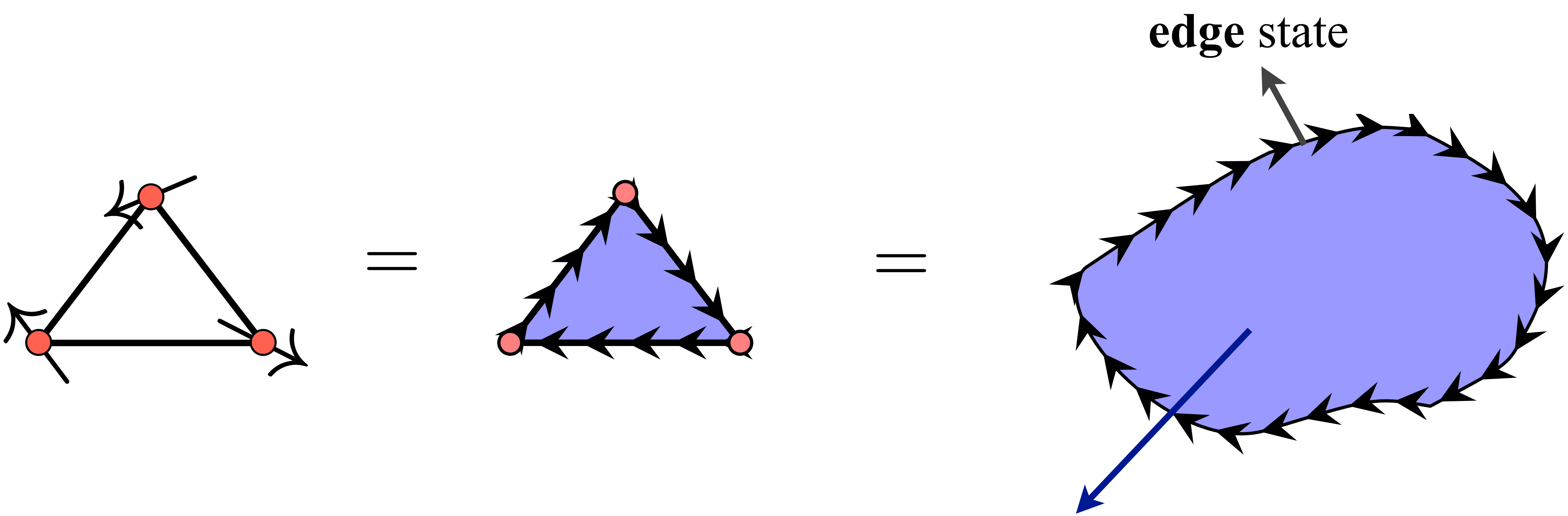
A conceptual perspective



spin model
on lattice

network model
of edge states

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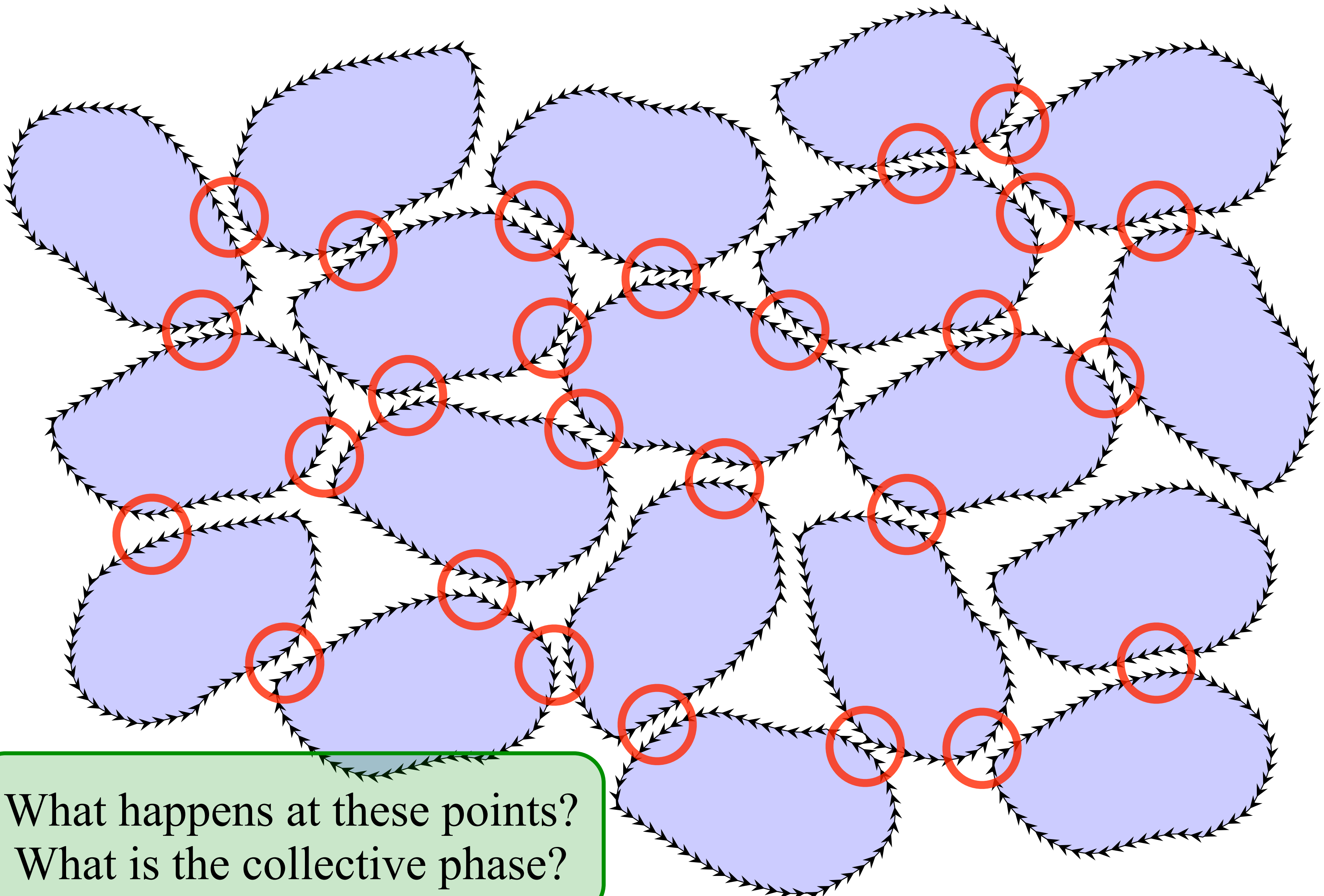
Network of edge states



What happens at these points?
What is the collective phase?

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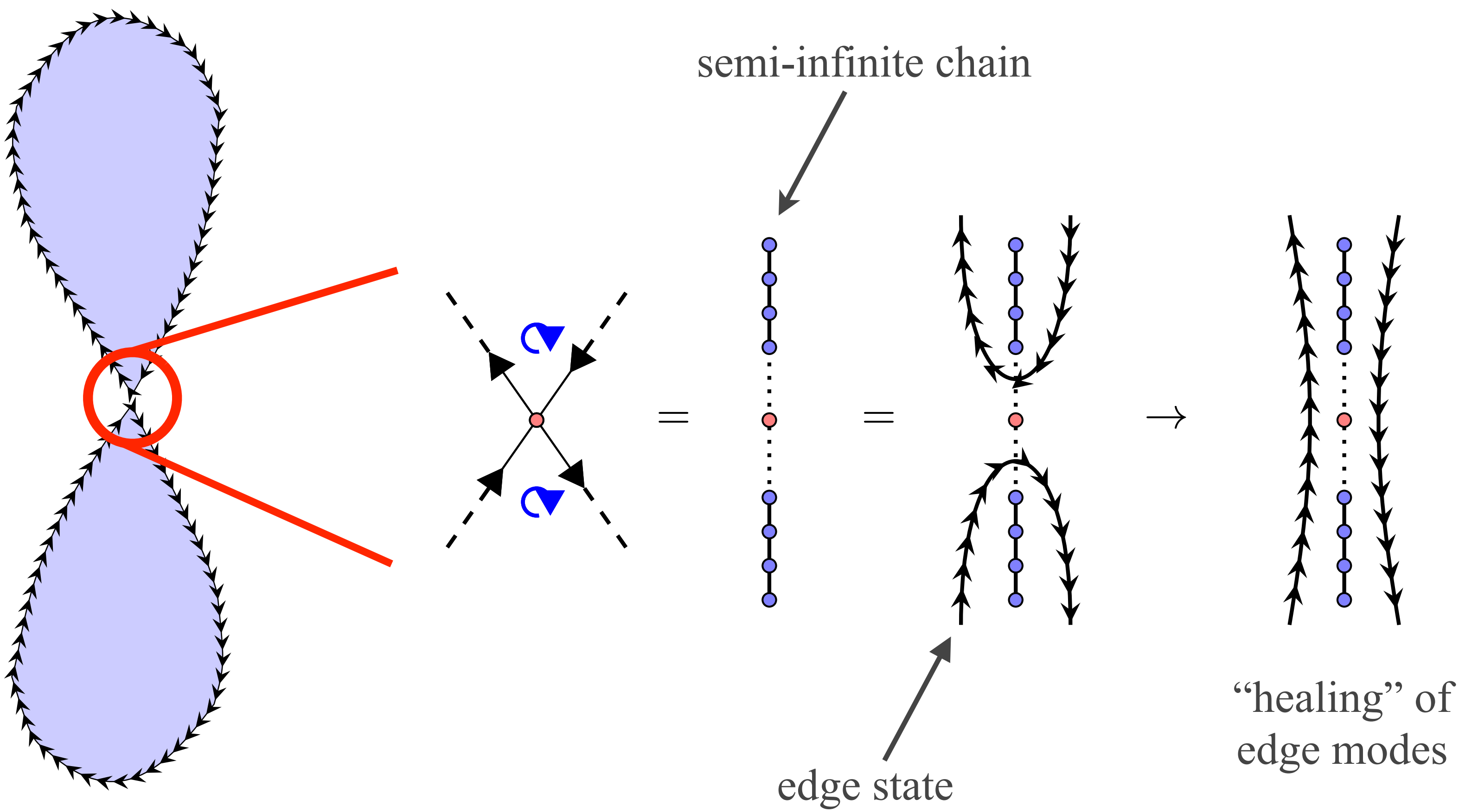
How to join two puddles?

The physics of joining two puddles
is an incarnation of the **2-channel Kondo physics**



Eggert & Affleck (1992)
Kane & Fischer (1992)

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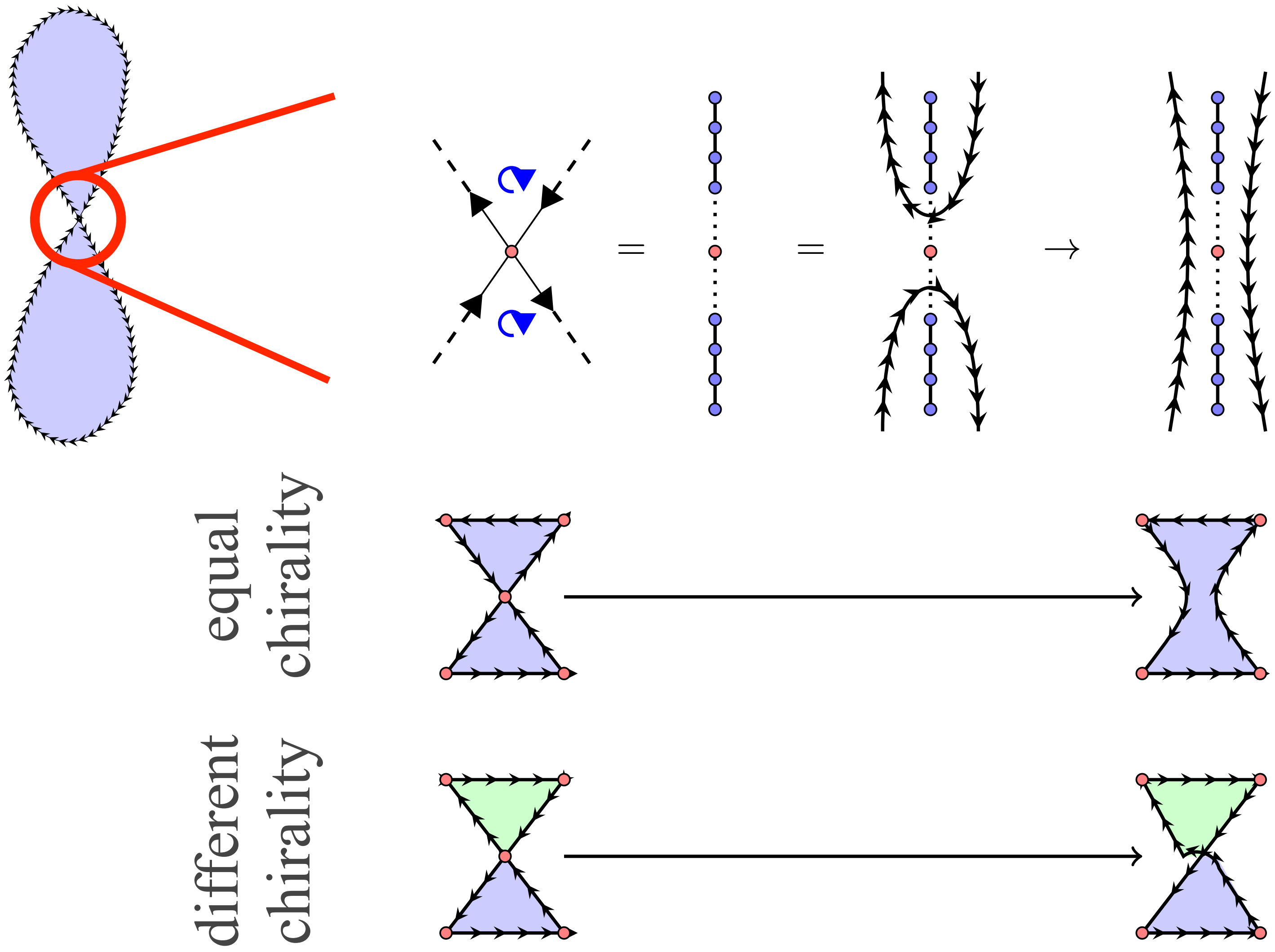
Two triangles



For both cases: **one edge state** remains after “healing”

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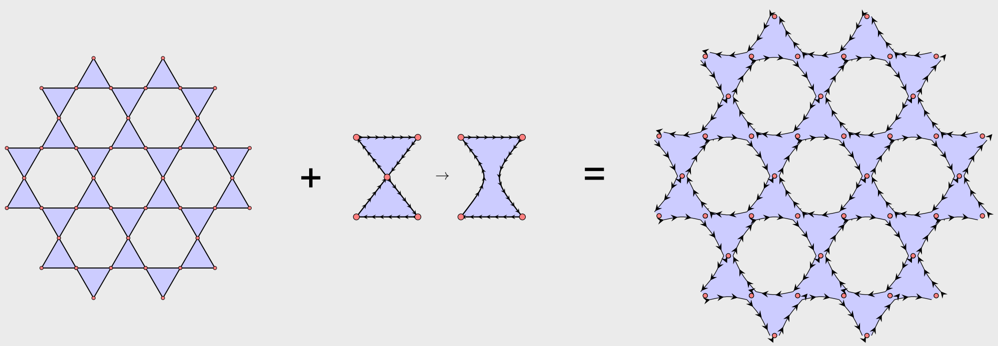
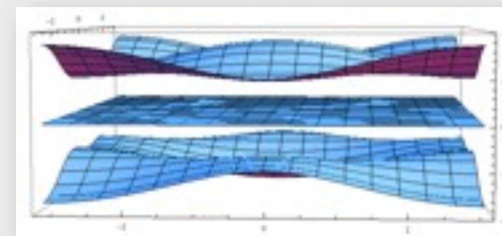
Network models

homogenous couplings

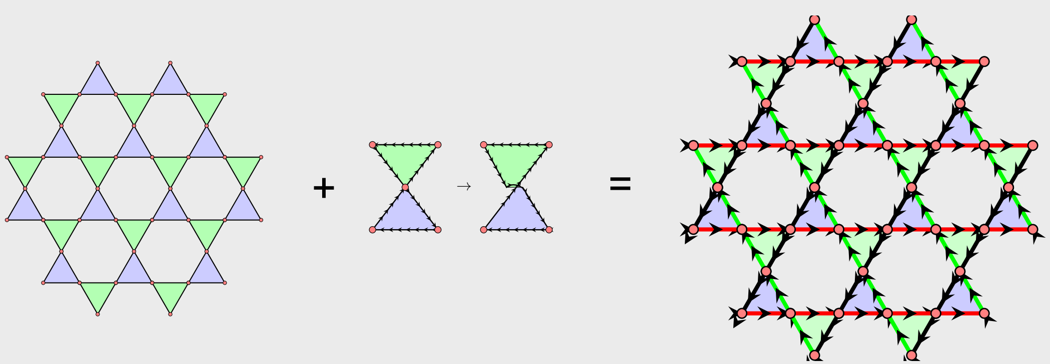


Gapped bulk phase
with edge state running around outer perimeter.

⟶ **topological phase**

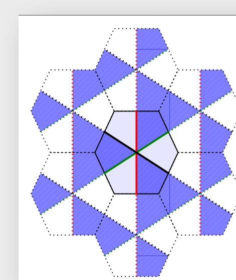


staggered couplings



Extended gapless edge states along chains, which remain uncoupled

⟶ **gapless bulk phase**



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numerical results

Can we repeat the same with spins?

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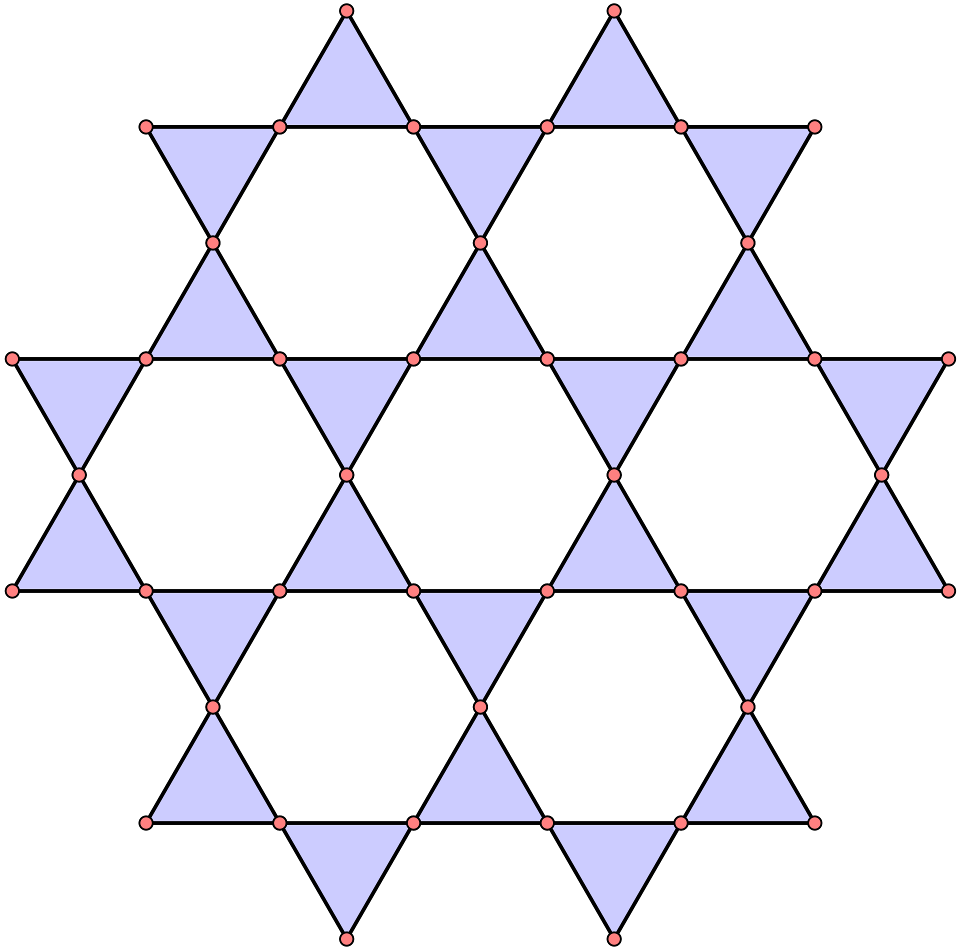
topological phase – chiral spin liquid



homogeneous

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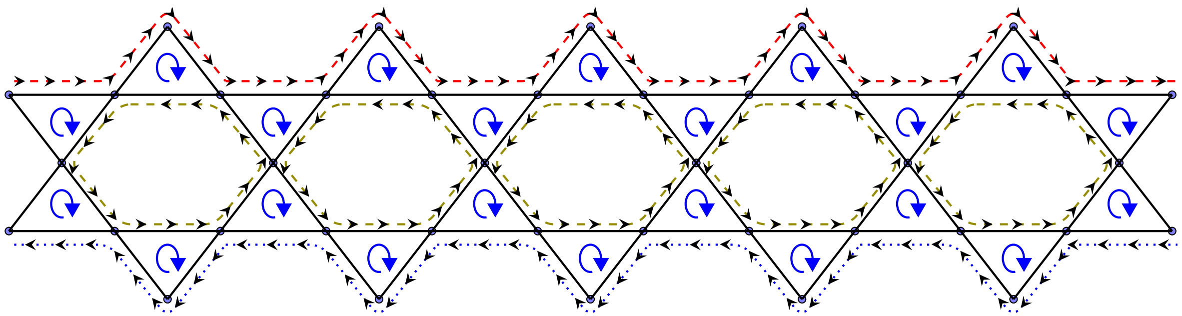
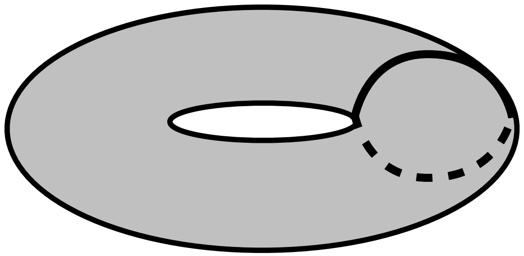
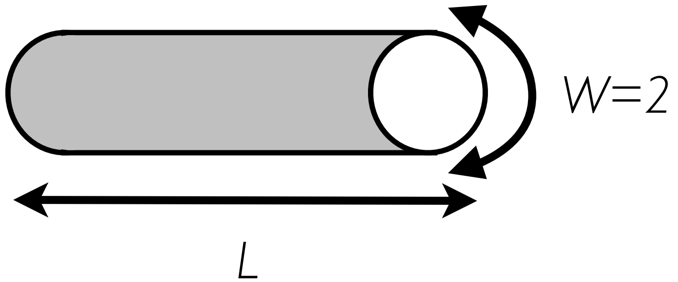
topological phase – thin systems



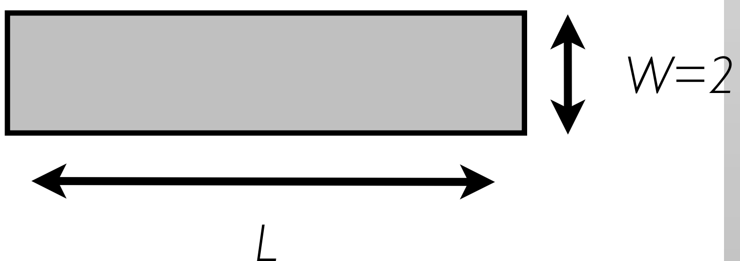
torus



cylinder



strip



- *2xL* cylinders (DMRG & ED) and tori (ED)
  - DMRG with up to 2400 states
  - ED up to 32 sites (2x5x3+2)

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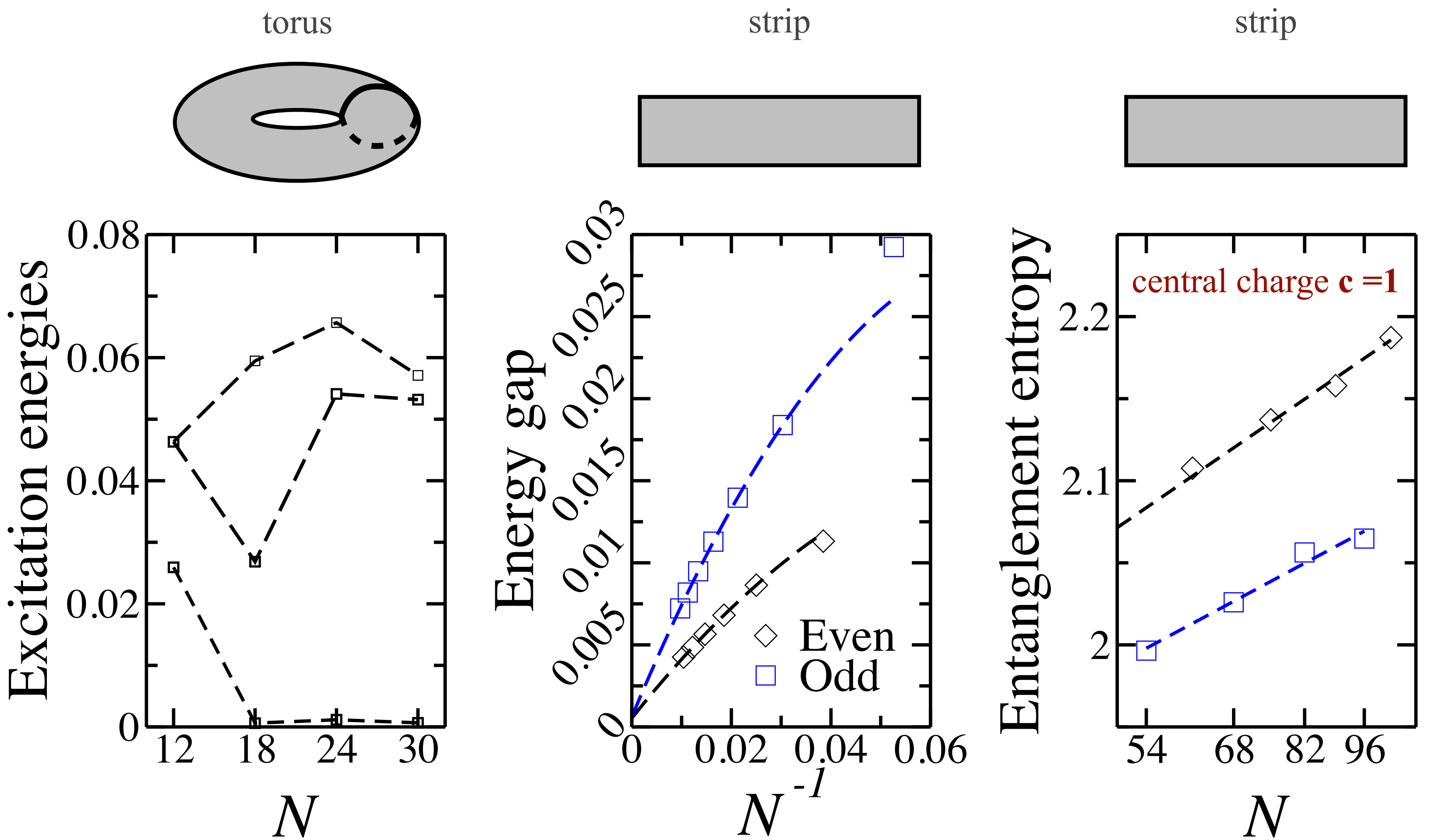
topological phase – energy & entanglement



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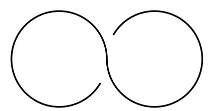
topological phase – modular matrices

Cincio & Vidal (2012)
Zhang, Grover, Turner, Oshikawa and Vishwanath (2012)

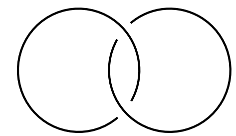
Modular matrices — ***T*-matrix** — ***S*-matrix**

$$T_{ab} = e^{-i\frac{2\pi}{24}c}\delta_{ab}d_a\Theta_a$$

with twist factor $\Theta_a =$



$$S_{ab} = \frac{1}{\mathcal{D}}$$



Modular matrices for **v=1/2 Laughlin state**

$$T = e^{-i\frac{2\pi}{24}}\begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix} \qquad S = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$$

Modular matrices for **chiral spin liquid**

$$T = e^{-i\frac{2\pi}{24}0.988}\begin{bmatrix}1 & 0\\ 0 & i\cdot e^{-i0.0021\pi}\end{bmatrix} \qquad S = \frac{1}{\sqrt{2}}\begin{bmatrix}0.996 & 0.995\\ 0.996 & -0.994e^{-i0.0019\pi}\end{bmatrix}$$

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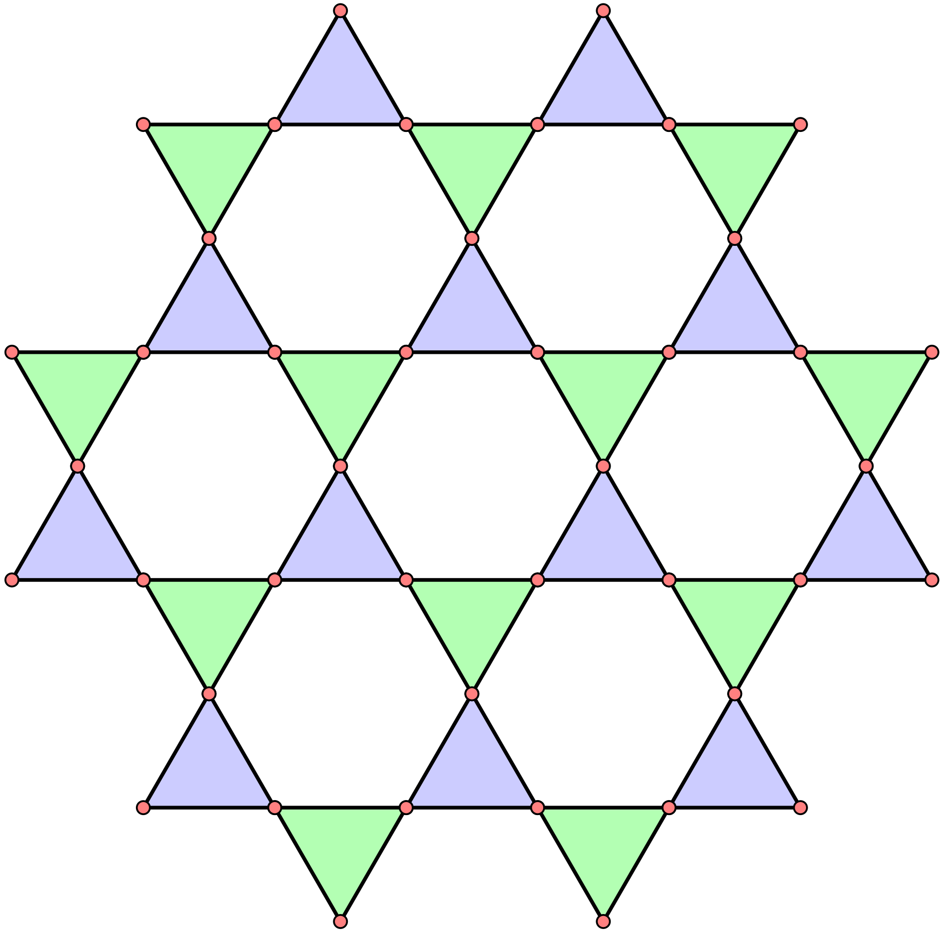
gapless phase – staggered chiralities



staggered

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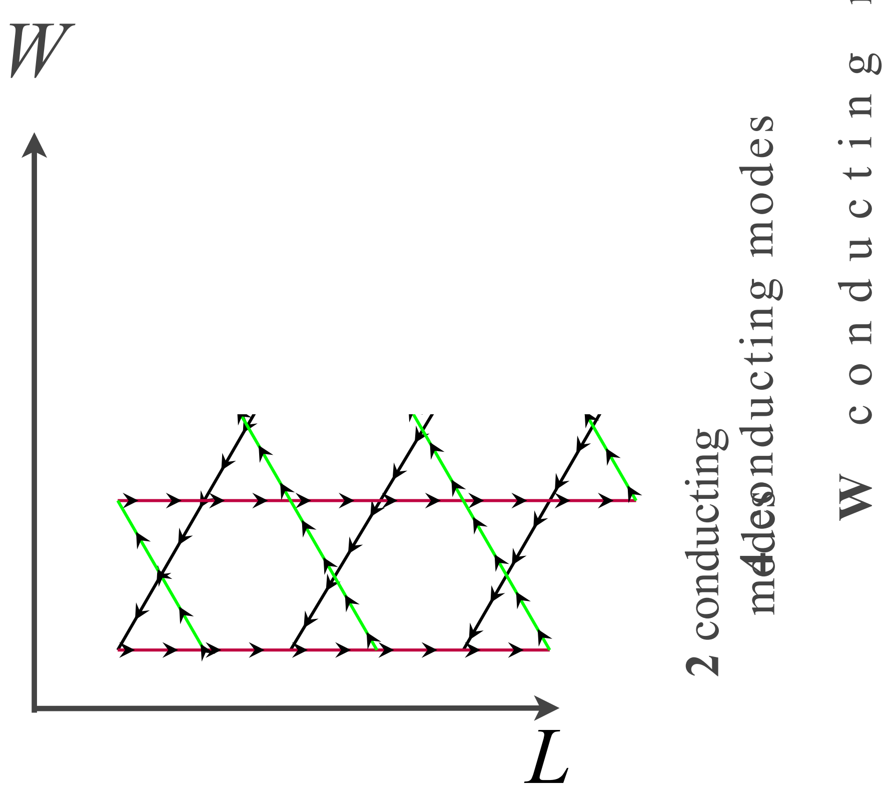
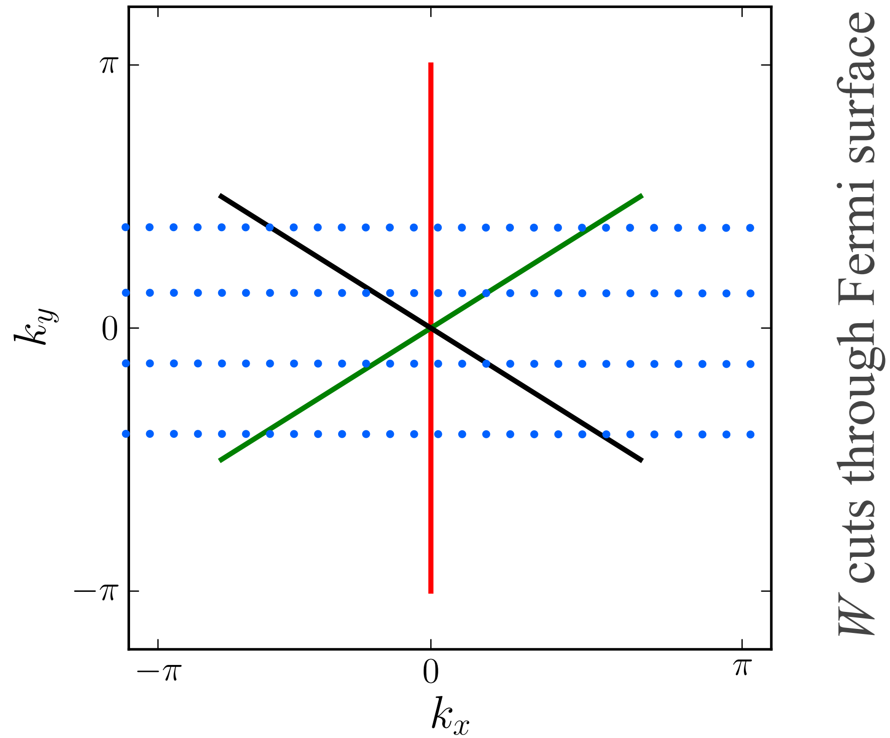
gapless phase – quasi-1d predecessors





width W of "ladder system"

~ # of k_y points

~ number of gapless modes

$$S \sim L \log L$$

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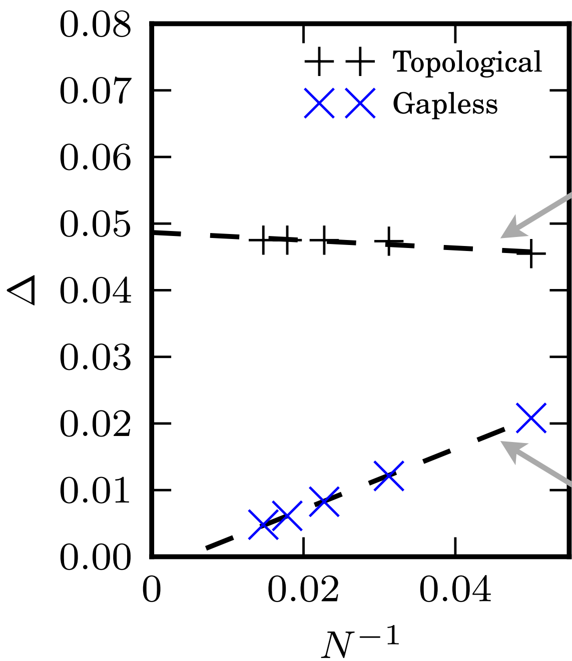
gapless phase – entanglement



uniform chiralities

staggered chiralities

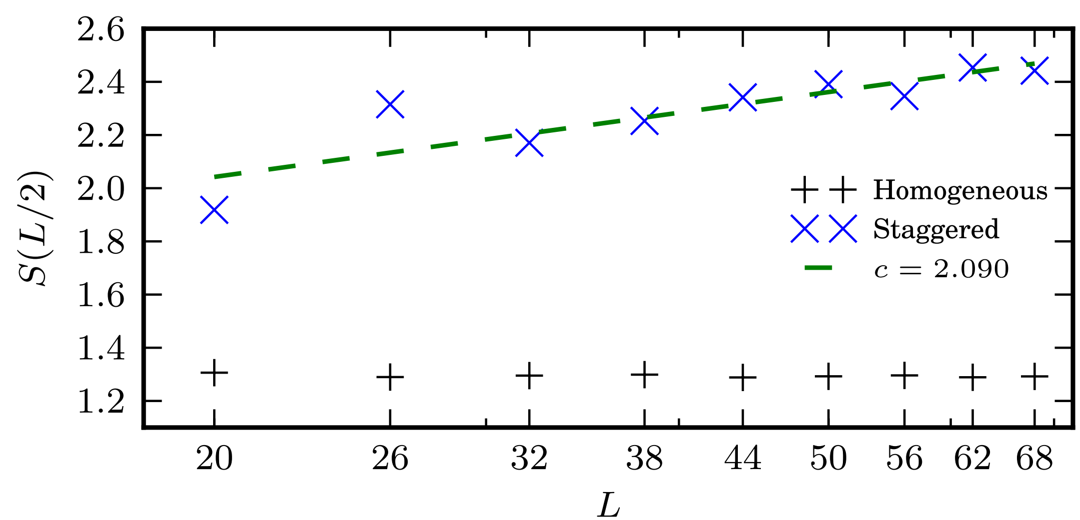
energy gap vanishes as *1/L* for staggered chiralities

gapless phase

entanglement entropy
(at the center of the system)

- gapped state: $S \sim const$
- gapless state: $S \sim log(L)$

Fit yields expected **c = 2**



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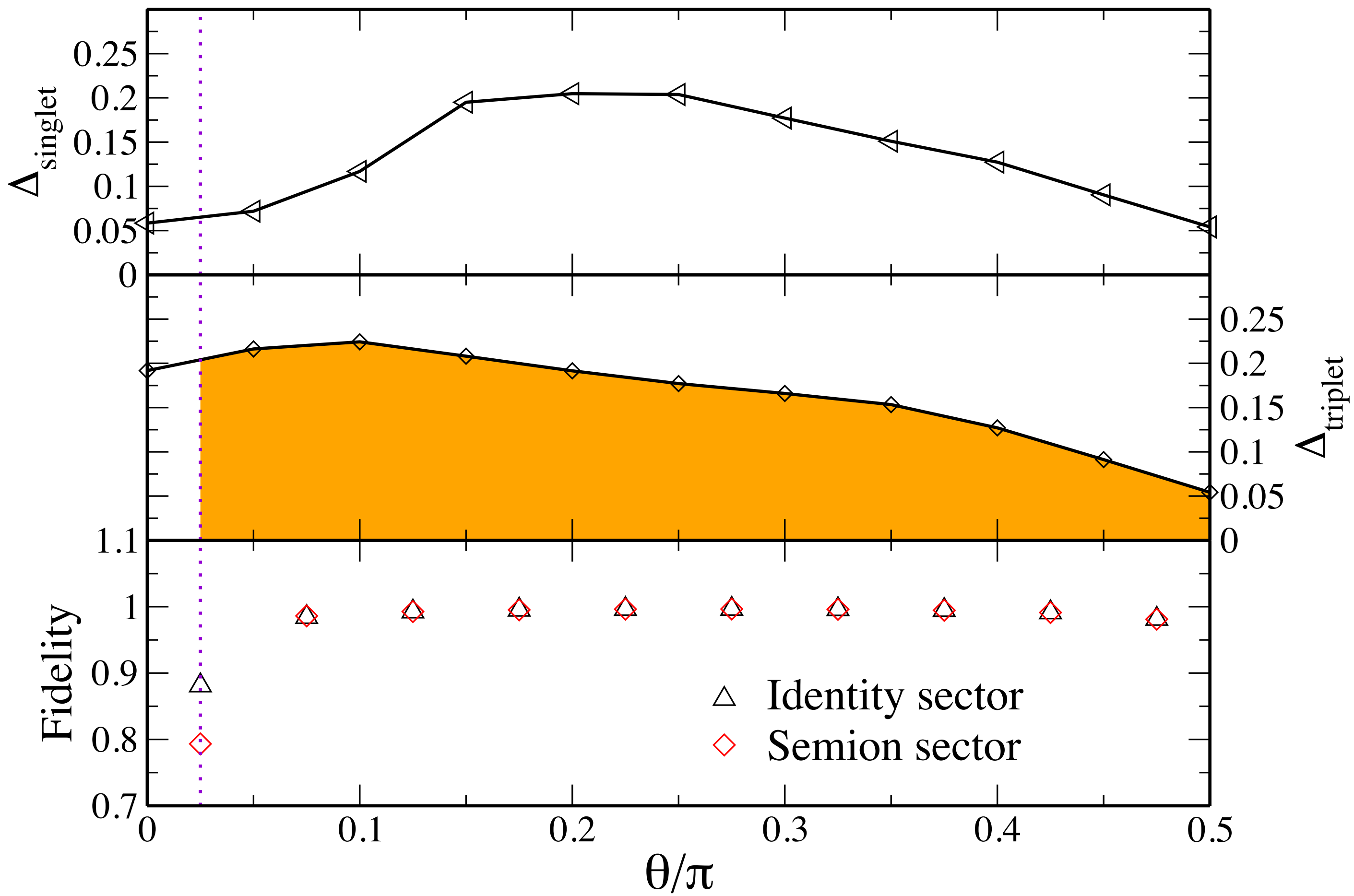
Back to the Heisenberg model

$$H = J_{\mathrm{HB}} \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j + J_\chi \sum_{i,j,k \in \triangle} \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k)$$

$$\tan\theta = J_{\mathrm{HB}} / J_\chi$$



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beyond kagome

Cocoon Tower, Tokyo

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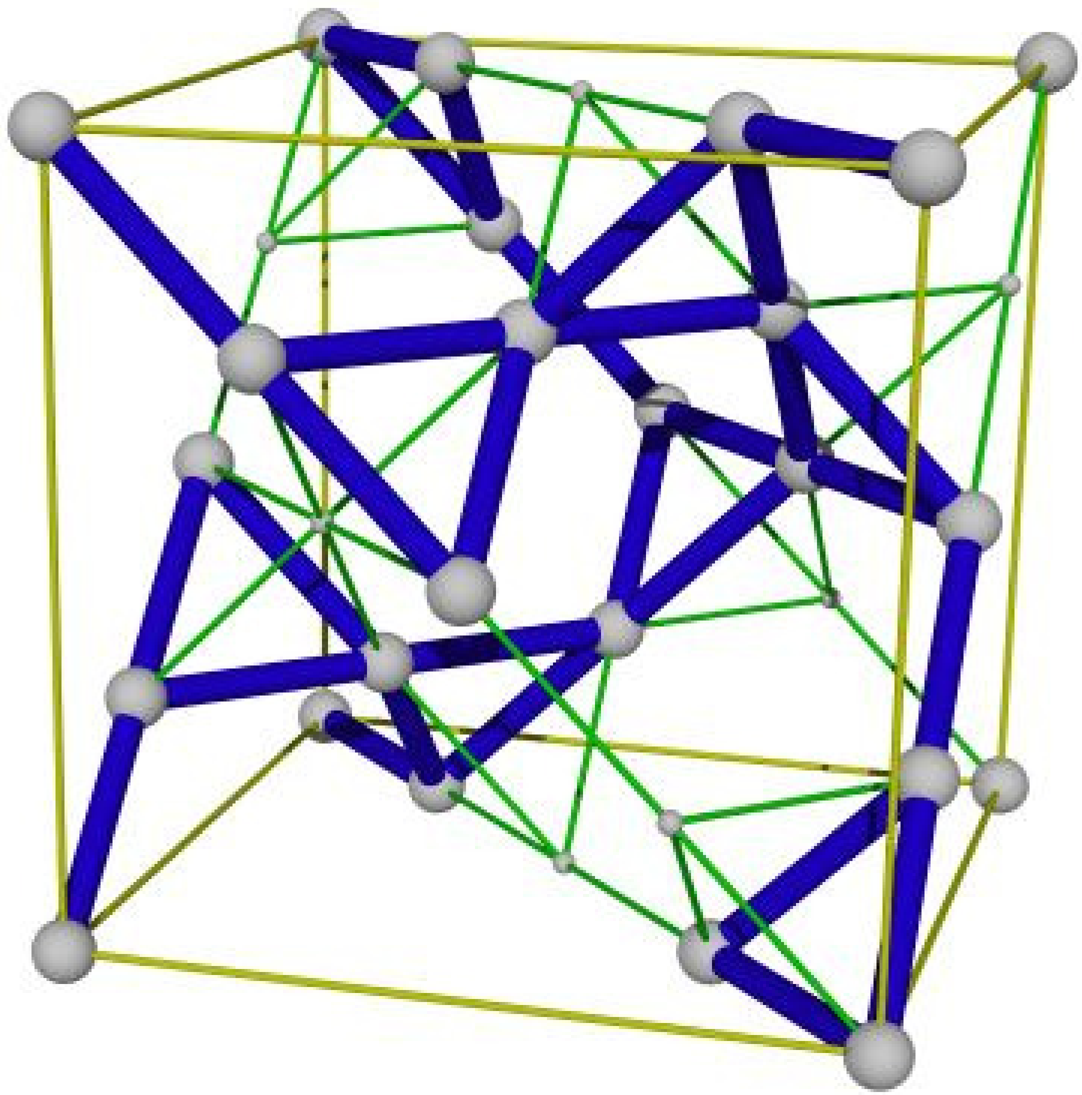
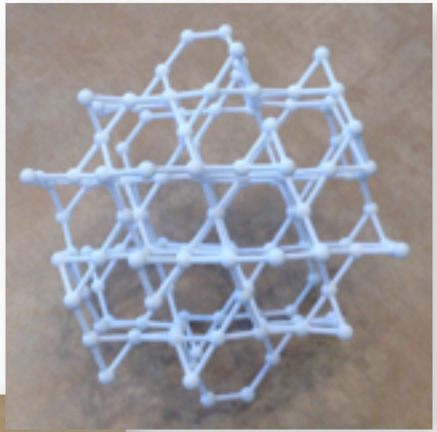
Can this be generalized to 3 dimensions?



hyperkagome lattice



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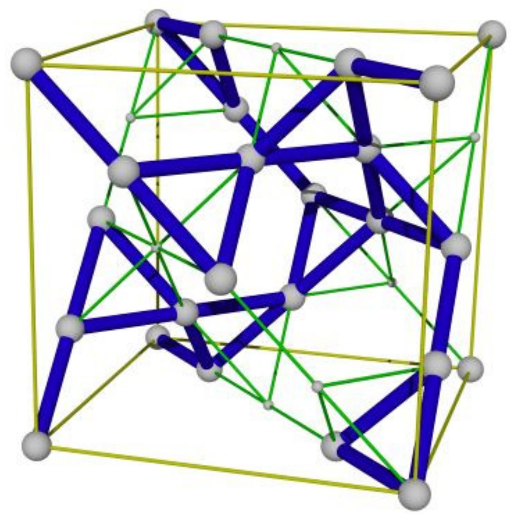
Can this be generalized to 3 dimensions?

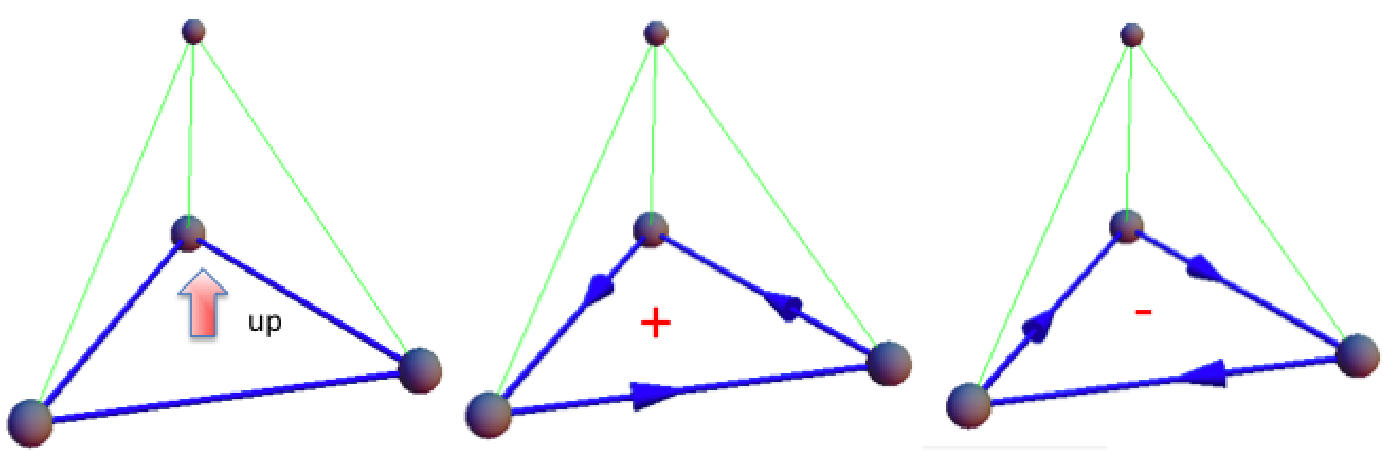


The hyperkagome is a lattice of **corner-sharing** triangles in three dimensions.

The triangles themselves form a **bipartite** lattice (like for the 2D kagome lattice).

The lattice can be viewed as a **site-depleted** pyrochlore lattice.

definition of **chiralities**



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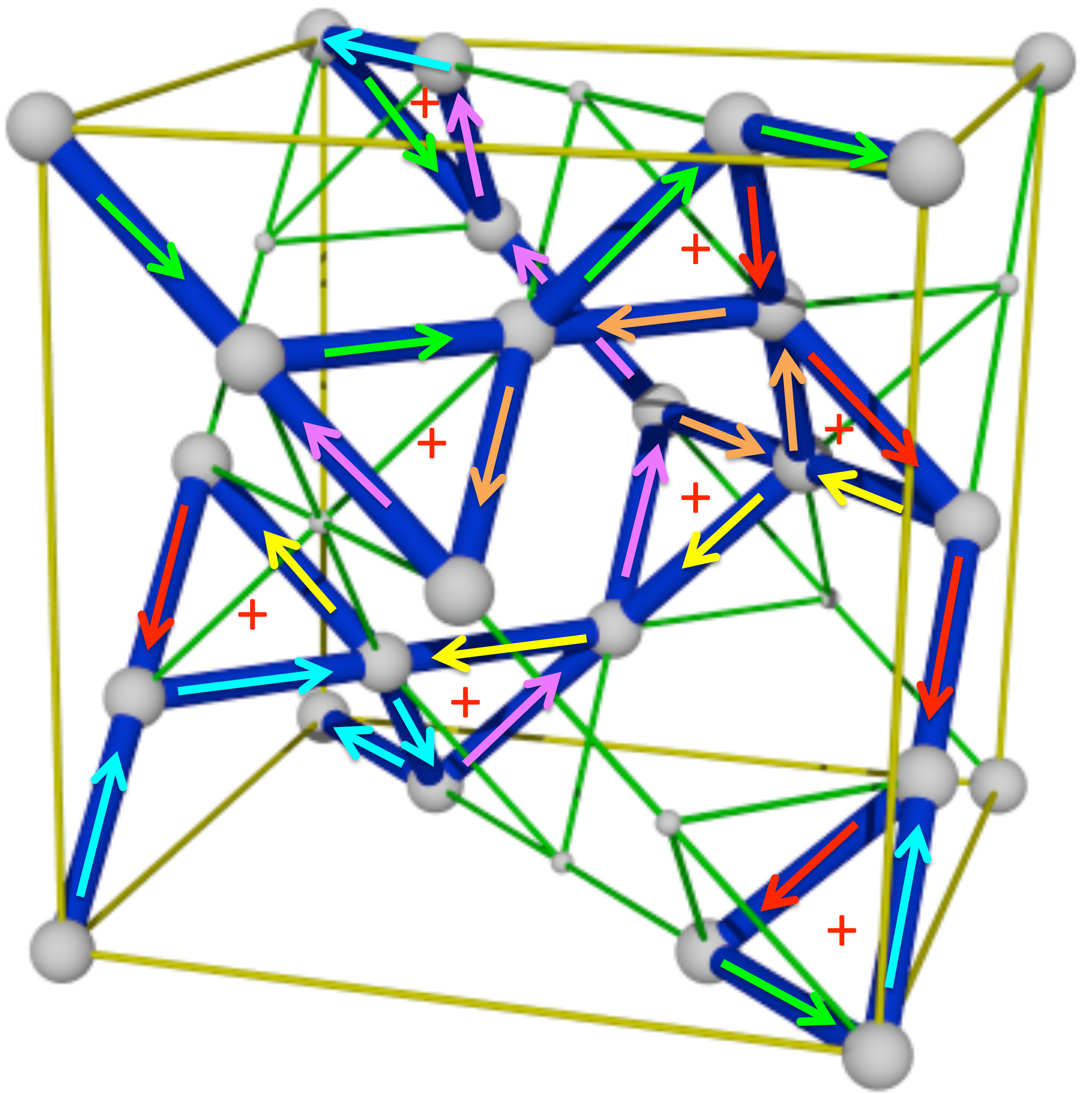
Can this be generalized to 3 dimensions?



Edge network for **uniform** chiralities.

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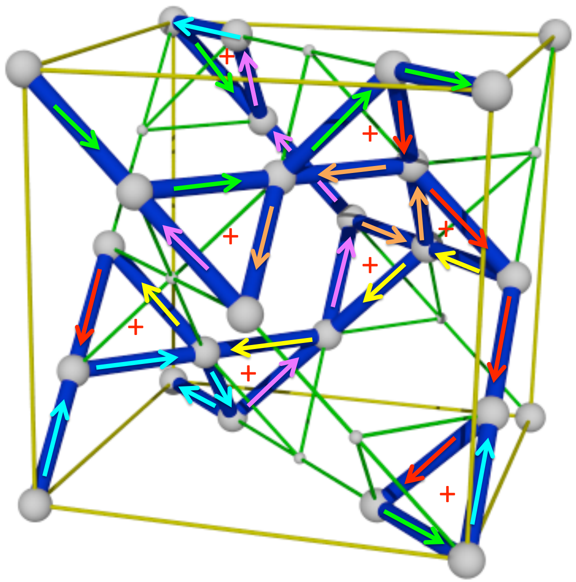
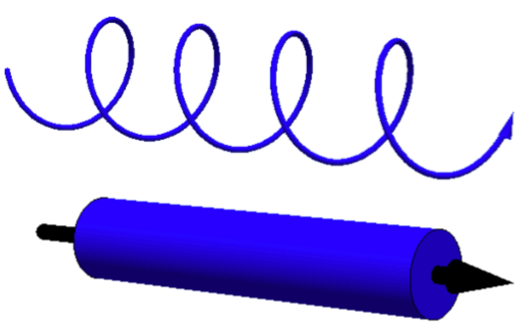
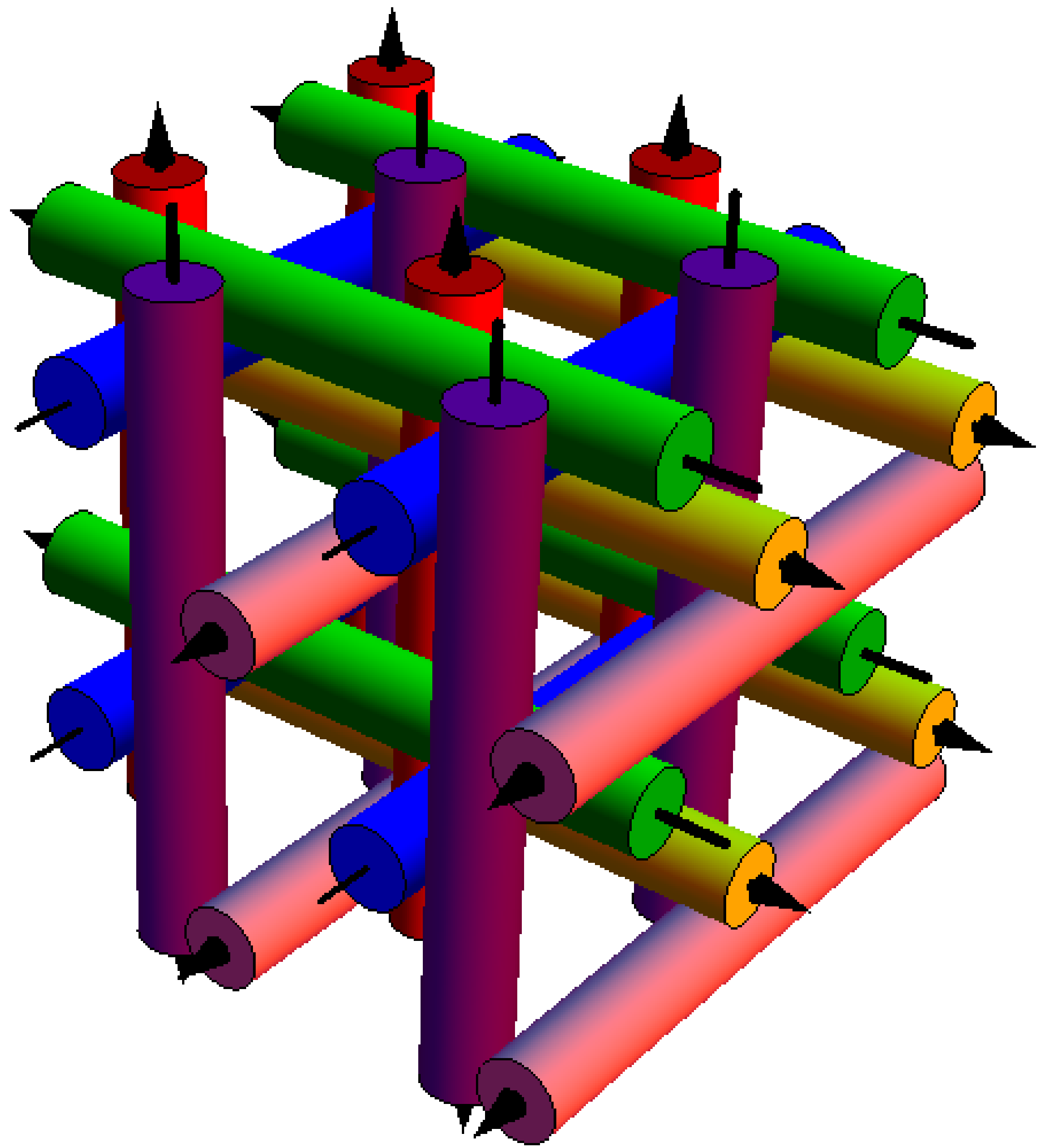
3D – uniform chiralities







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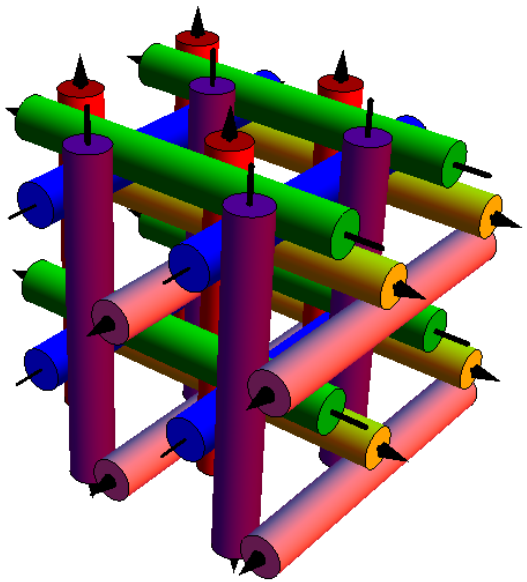
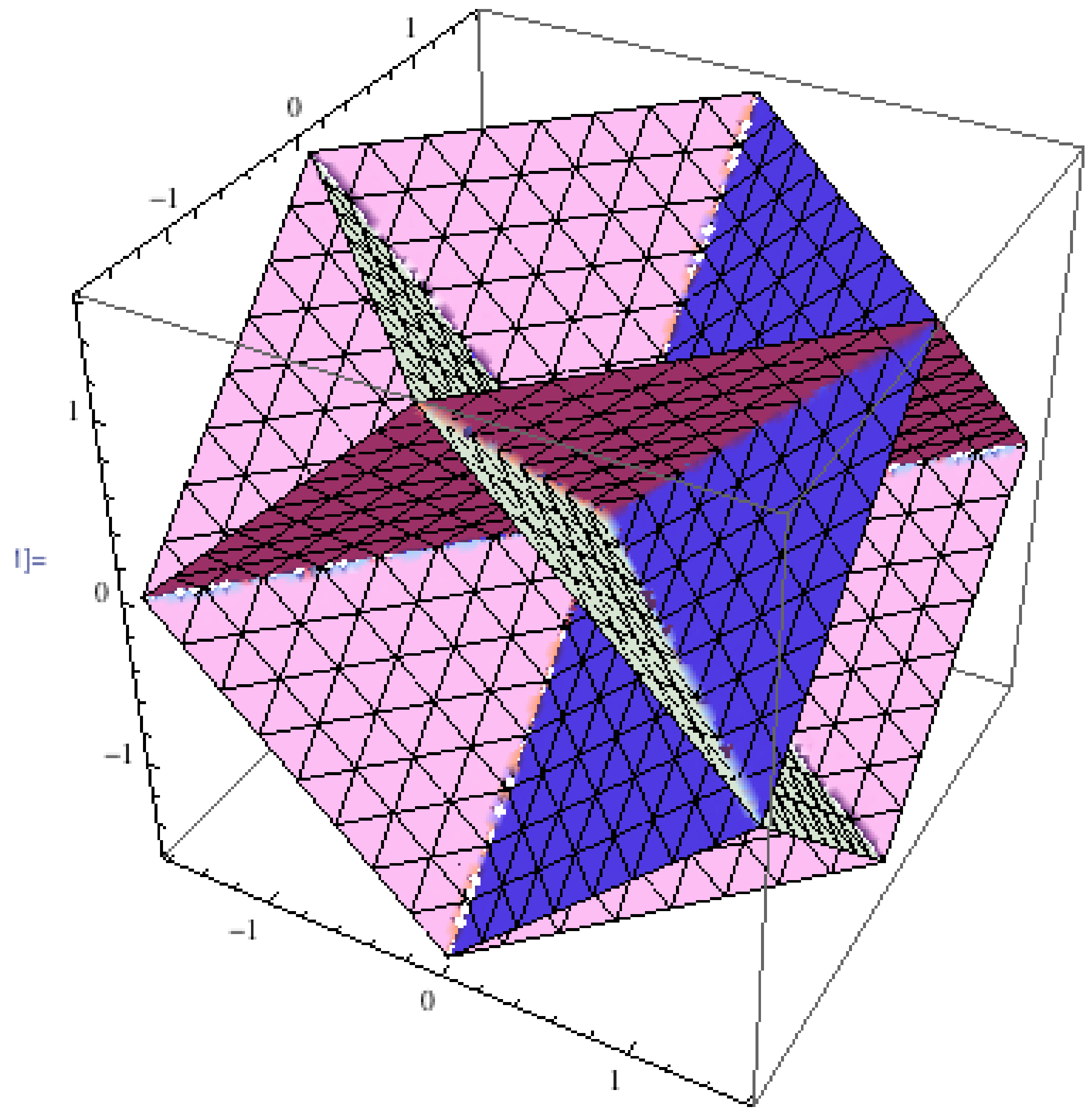
3D – uniform chiralities



real space



momentum space

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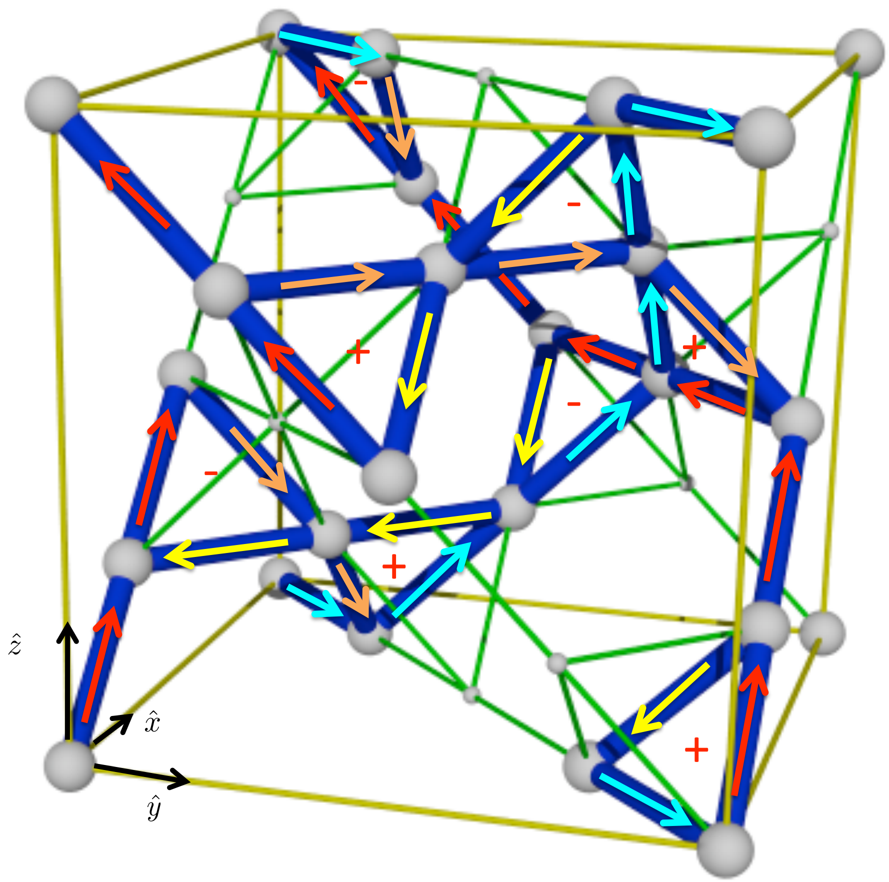
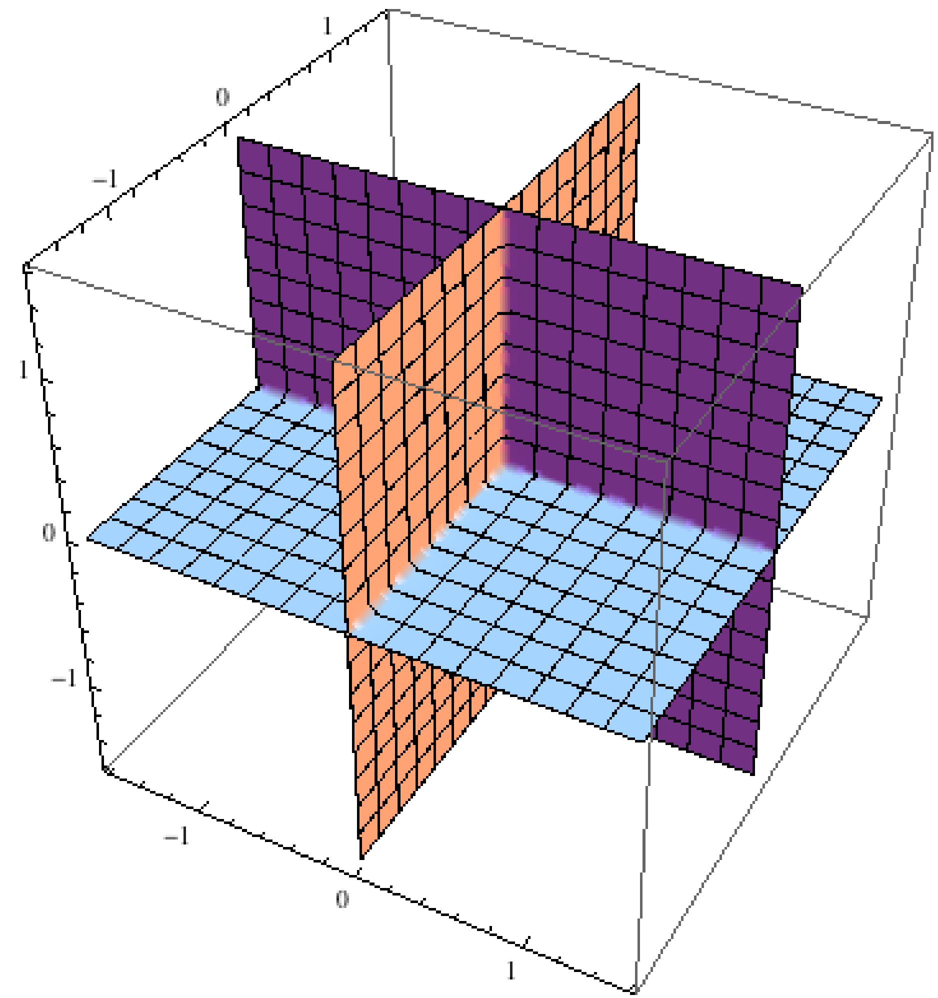
3D – staggered chiralities



real space



momentum space

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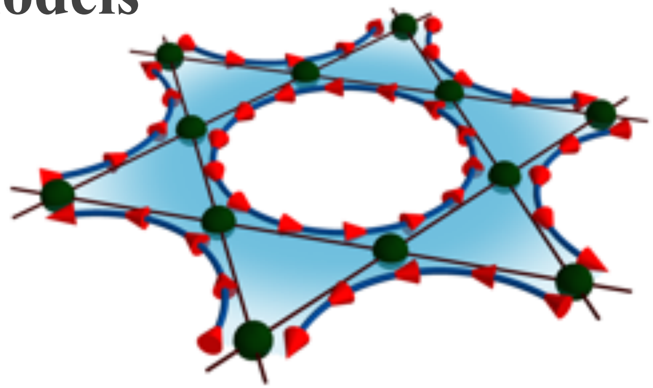
Conclusions & Outlook

- We have developed a **powerful new perspective** on spin liquids rooted in the physics of **network models**



- Prediction & numerical confirmation of
  - a **gapped, chiral spin liquid**
  - a **gapless spin liquid** with gapless excitations on lines in momentum space, including the shape of the Fermi surface
  - identification of **fundamental theory** describing **gapless modes**
  - for a **simple, SU(2)-invariant spin-1/2 model** on the Kagome lattice

arXiv:1401.3017
Nature Comm. in press

arXiv:1303.6963
for more details on the gapless phase

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