# *Highway Traffic*

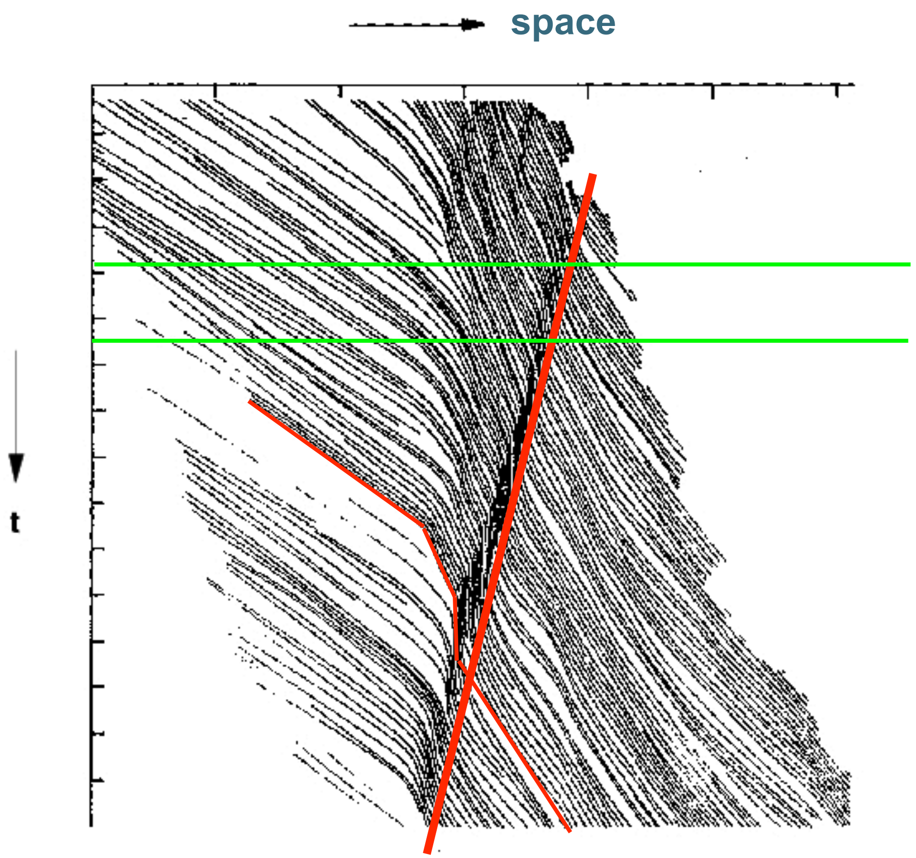
# Introduction

- Traffic = macroscopic system of interacting particles (driven or self-driven)
- Nonequilibrium physics:

  Driven systems far from equilibrium
- Collective phenomena ➡ physics!

# *Empirical Results*

# Spontaneous Jam Formation




jam velocity:

-15 km/h (universal!)

Phantom jams

start-stop-waves

⇨ interesting collective phenomena

# Experiment

(Tokai TV)

# Experiment

Experiment for WDR television, 2006

# Empirical data

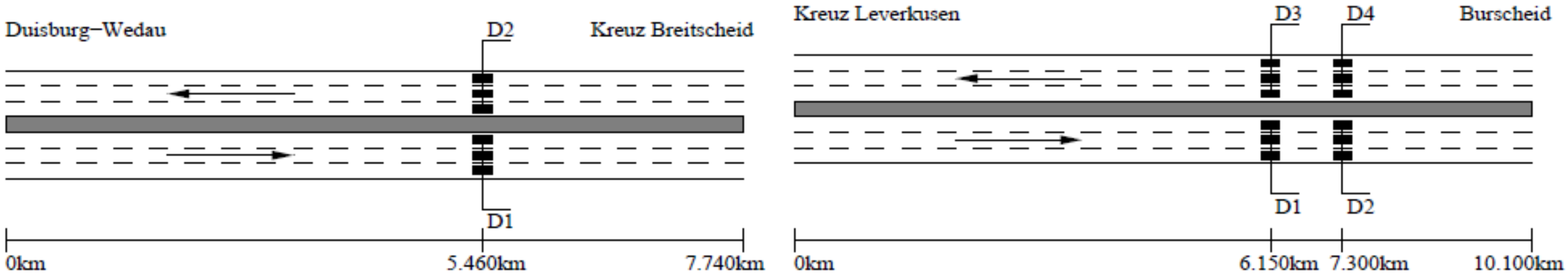
- Inductive loops integrated in the lane
- All measured quantities are typically time averages
- Some inductive loops provide single vehicle data

- Aerial photography
- All measured quantities are spatial averages, especially velocity and density


- Floating cars: Data taken from special cars inside the flow
- All measured quantities are time and spatial averages
- Density is calculated from mean distance

# Empirical data

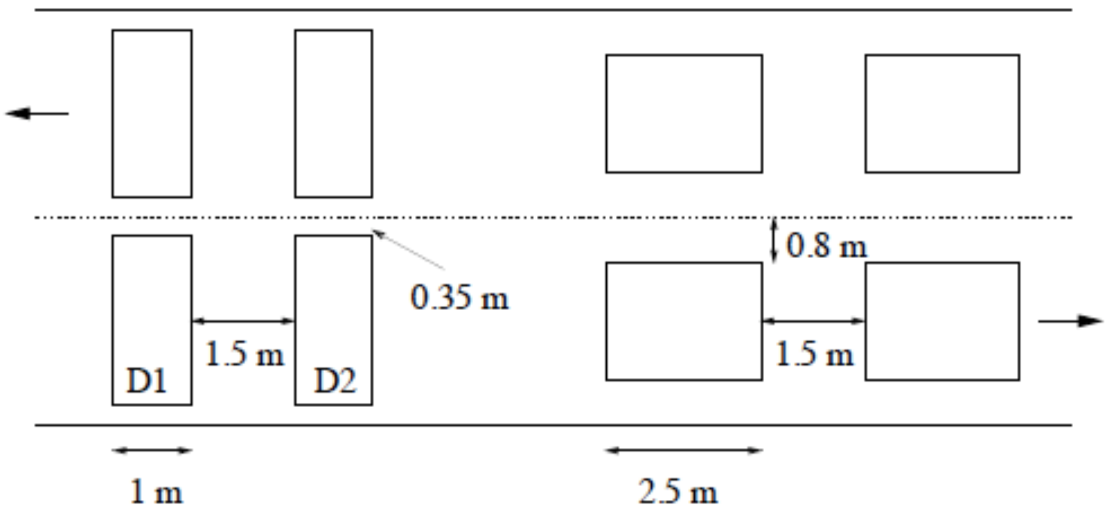
**Single vehicle measurement:** detect each vehicle, no aggregation




- Time (in hundredth seconds) at which the vehicle reaches the detector
- Lane of the vehicle
- Type of the vehicle (truck, car)
- Velocity in km/h (lower bound = 10 km/h)
- Length in cm with an accuracy of 1 cm

# Detectors

2-loop detectors




velocity: $v_n = \frac{d_D}{t_{D_2} - t_{D_1}}$

temporal headway: $\Delta t_n = t_{D_1,n} - t_{D_1,n-1}.$

spatial headway: $\Delta x_n = v_n \Delta t_n - l_{F,n}$

# Density

Problem: determination of density from local measurements

- from time of occupation $t_{B,n}$: $\tilde{\rho} = \frac{1}{T}\sum_{n=1}^{N} t_{B,n}$ (occupancy)

  (N cars passing in time T)

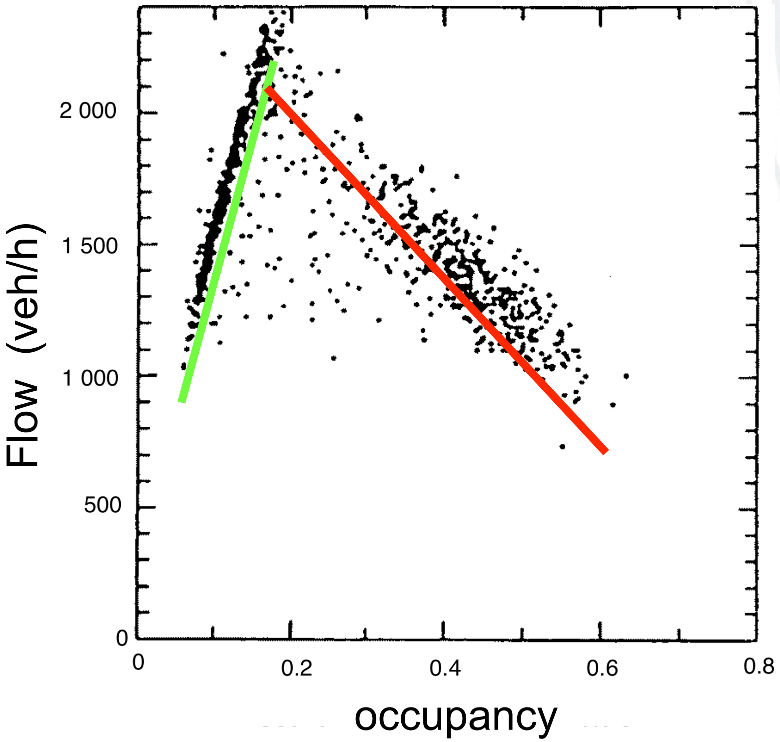
  density: $\rho = \tilde{\rho}\rho_{\max}$,

- from hydrodynamic relation $J=\rho v$: $v = \frac{1}{N}\sum_{n=1}^{N} v_n$.

generically: density is underestimated!

# Fundamental diagram

## Relation: current (flow) vs. density

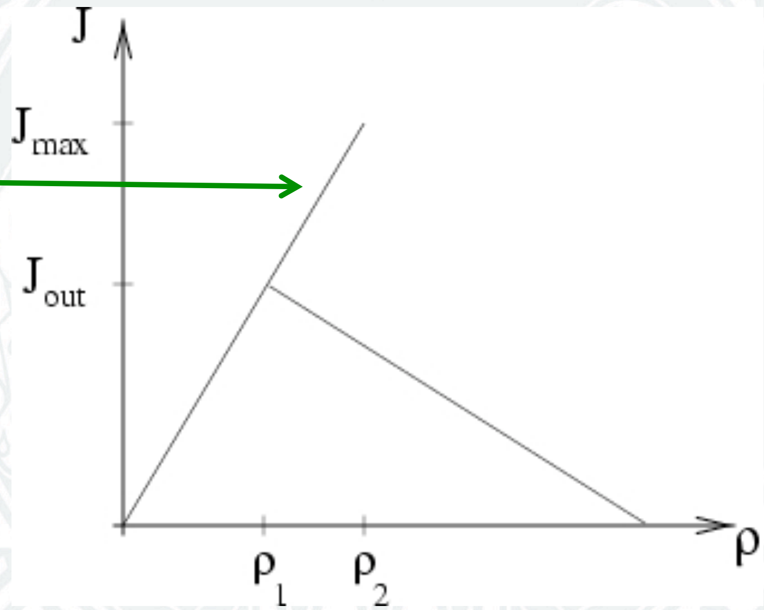



free flow

congested flow (jams)

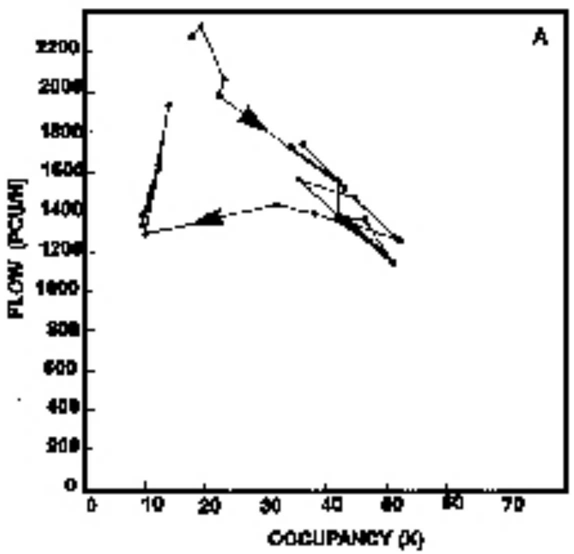
## more detailed features?

# Metastable States




**metastable high-flow states**

**hysteresis**



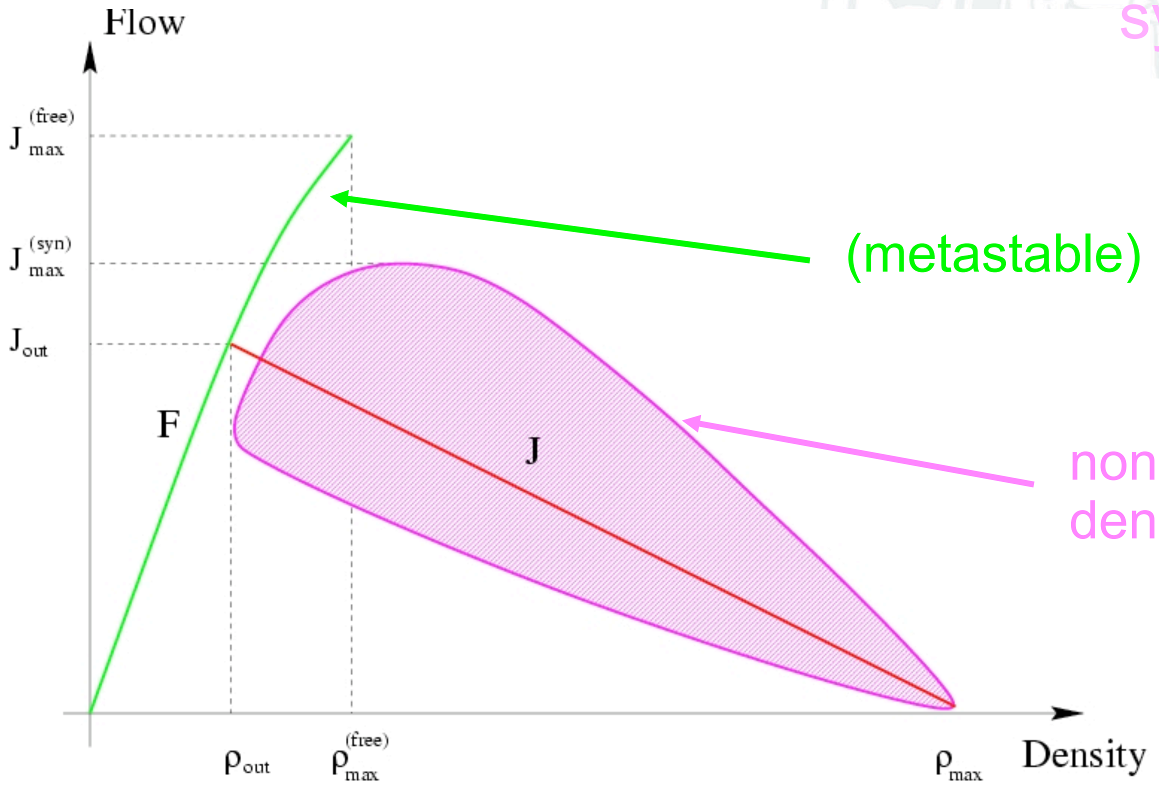
# 3-Phase Theory

3 phases

free flow

(wide) jams

synchronized traffic




(metastable) high-flow states

non-unique flow-density relation

(Kerner 1997)

# Synchronized Flow

New phase of traffic flow (Kerner – Rehborn 1996)

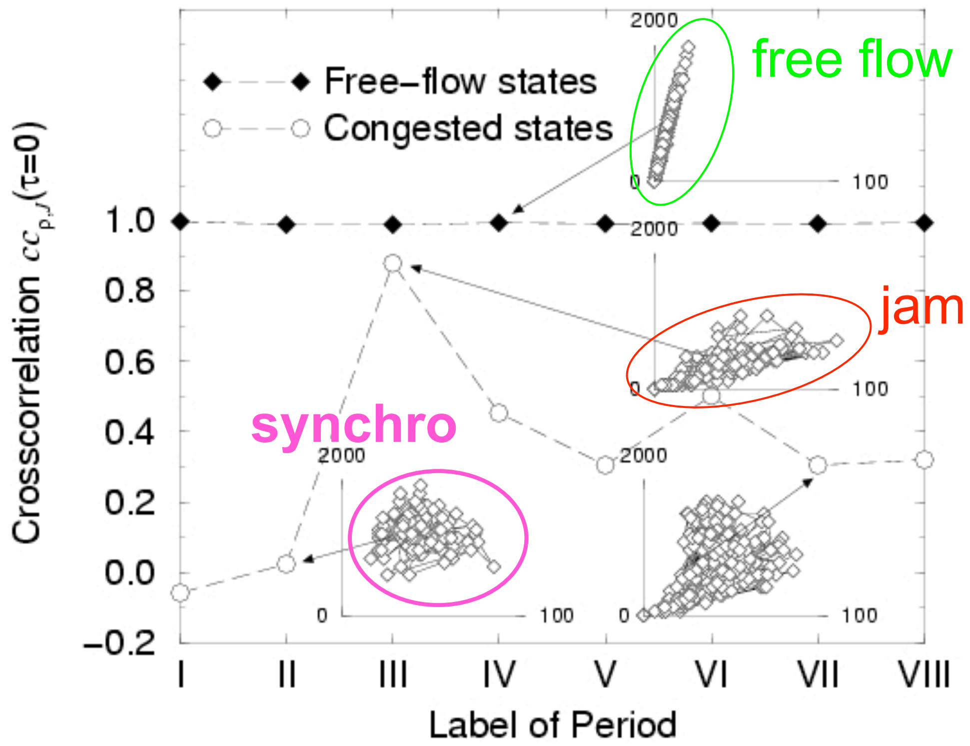
States of

- high density and relatively large flow
- velocity smaller than in free flow
- small variance of velocity (bunching)
- similar velocities on different lanes (synchronization)
- time series of flow looks „irregular“
- no functional relation between flow and density
- typically observed close to ramps

# Synchronized traffic

**Cross-correlation function:**

$$cc_{J\rho} = \frac{1}{\sqrt{\sigma(J)\sigma(\rho)}}[\langle J(t)\rho(t+\tau)\rangle - \langle J(t)\rangle\langle\rho(t+\tau)\rangle]$$




free flow, jam: $cc_{\rho,J}(\tau) \approx 1$

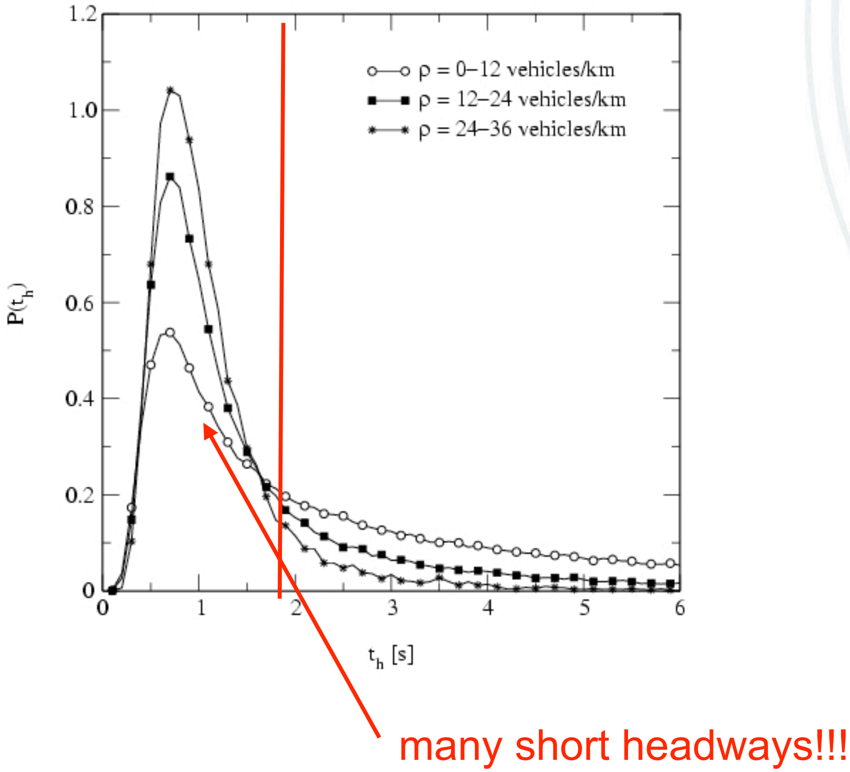
synchronized traffic:

$$cc_{\rho,J}(\tau) \approx 0$$

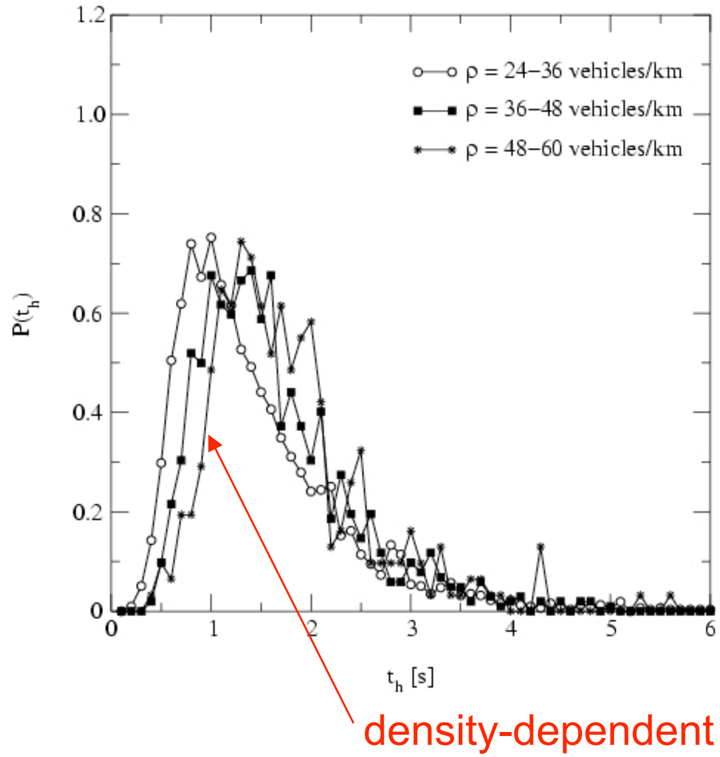
Objective criterion for classification of traffic phases

# Time Headway

free flow




synchronized traffic



# Traffic Models

# Modelling approaches

Classification of models:

- **description**: microscopic ↔ macroscopic
- **dynamics**: stochastic ↔ deterministic
- **variables**: discrete ↔ continuous
- **interactions**: rule-based ↔ force-based
- **fidelity**: high ↔ low
- **concept**: heuristic ↔ first principles

# Modelling approaches

Various modelling approaches:

- hydrodynamic
- gas-kinetic
- force-based
- cellular automata

macroscopic

microscopic

# Modelling approaches

## microscopic:

- Model equations for each vehicle i at time t
- Variables are position $x_i(t)$, velocity $v_i(t)$ and acceleration $a_i(t)$
- Flow quantities are calculated building mean values
- **Examples:**
  - Optimal velocity models
  - Car-following models
  - Cellular automata

## macroscopic:

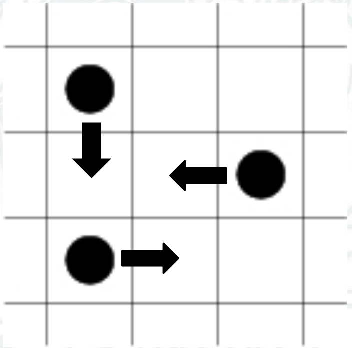
- Model equations for the macroscopic flow, similar to hydrodynamics
- Basis is continuity equation (= car conservation)
- Additional dynamic equation for mean velocity v(x, t)
- Requires empirical flow density relation as input
- **Examples:**
  - Lighthill-Whitham model
  - Kerner-Konhäuser model

## mesoscopic:

- Using queuing or renewal theory, cluster or aggregation theory and gas kinetic theory
- quantity of interest: f (x, v, t), where f (x, v, t)dxdv is the probability to find a car between x and x + dx with velocity between v and v + dv at time t
- **Examples:**
  - Gas-kinetic models

# Cellular Automata

Cellular automata (CA) are discrete in
- space
- time
- state variable (e.g. occupancy, velocity)



- often: stochastic dynamics

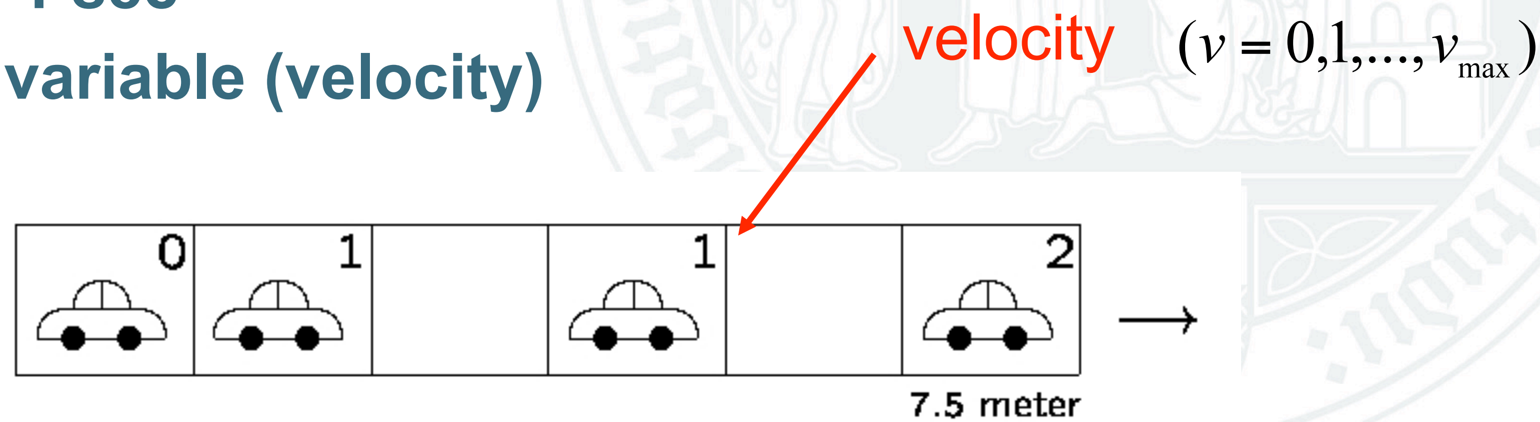
Advantages:
- efficient implementation for large-scale computer simulations
- intuitive rule-based definition of dynamics

# Modelling of Traffic Flow

# Cellular Automata Models

Discrete in

- Space: 7.5m
- Time: 1 sec
- State variable (velocity)




dynamics: Nagel – Schreckenberg (1992)

# Update Rules

Rules (Nagel, Schreckenberg 1992)

1) **Acceleration:** $v_j \rightarrow \min(v_j + 1, v_{max})$

2) **Braking:** $v_j \rightarrow \min(v_j, d_j)$ ($d_j$ = # empty cells in front of car j)

3) **Randomization:** $v_j \rightarrow v_j - 1$ (with probability p)

4) **Motion:** $x_j \rightarrow x_j + v_j$

# Example

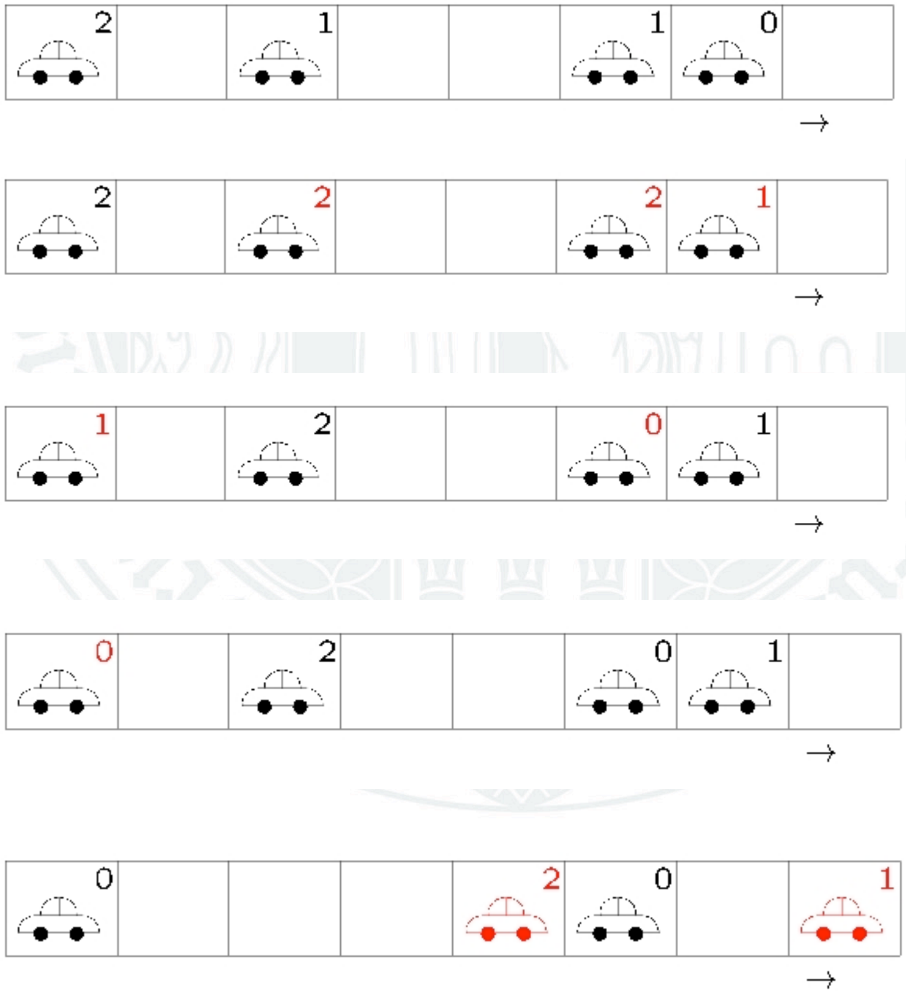
Configuration at time t:

Acceleration ($v_{max} = 2$):

Braking:

Randomization ($p = 1/3$):

Motion (state at time t+1):



# Interpretation of the Rules

1) **Acceleration**: Drivers want to move as fast as possible (or allowed)

2) **Braking**: no accidents

3) **Randomization**:
    a) overreactions at braking
    b) delayed acceleration
    c) psychological effects (fluctuations in driving)
    d) road conditions

4) **Driving**: Motion of cars

# Realistic Parameter Values

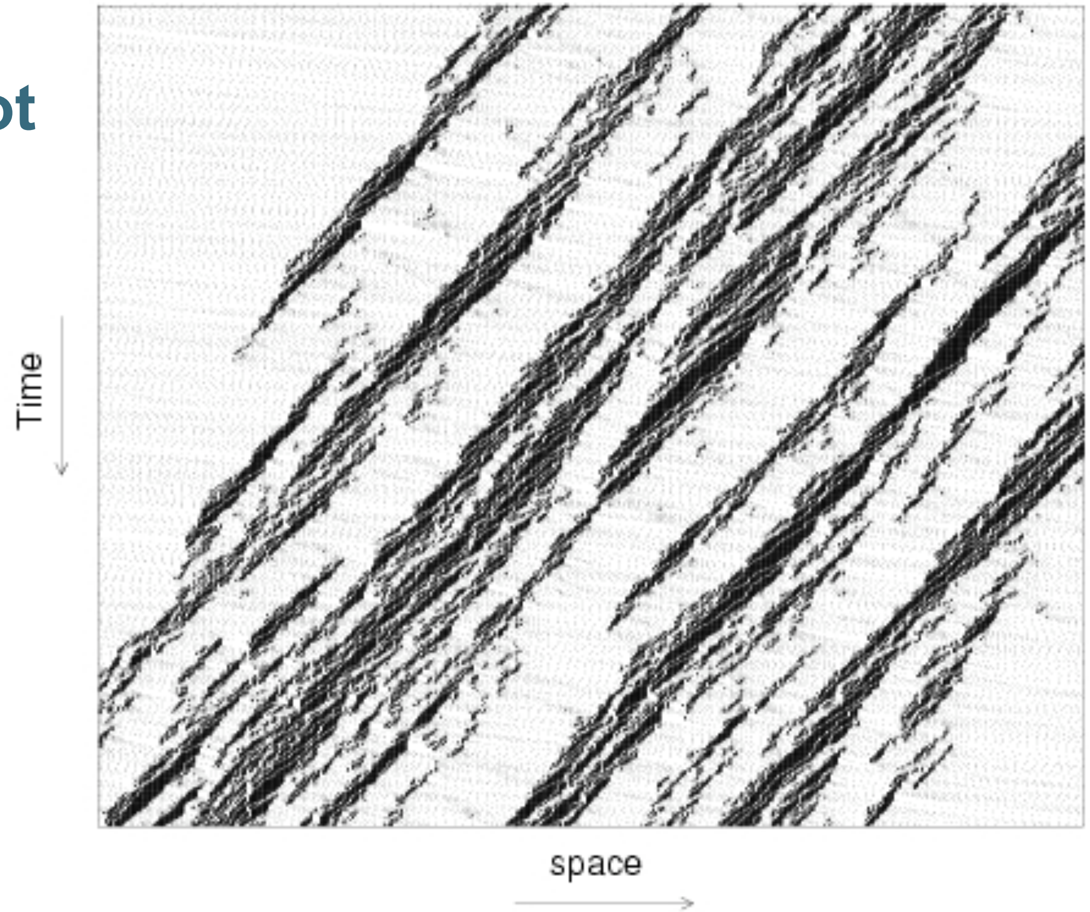
- Standard choice: $v_{max}=5$, $p=0.5$
- Free velocity: 120 km/h $\cong$ 4.5 cells/timestep
- Space discretization:

| |
|---|
| 1 cell $\cong$ 7.5 m |
| 1 timestep $\cong$ 1 sec |

- Reasonable: order of reaction time (smallest relevant timescale)

# Spontaneous jam formation in NaSch model

## space-time plot



# Simulation of NaSch Model

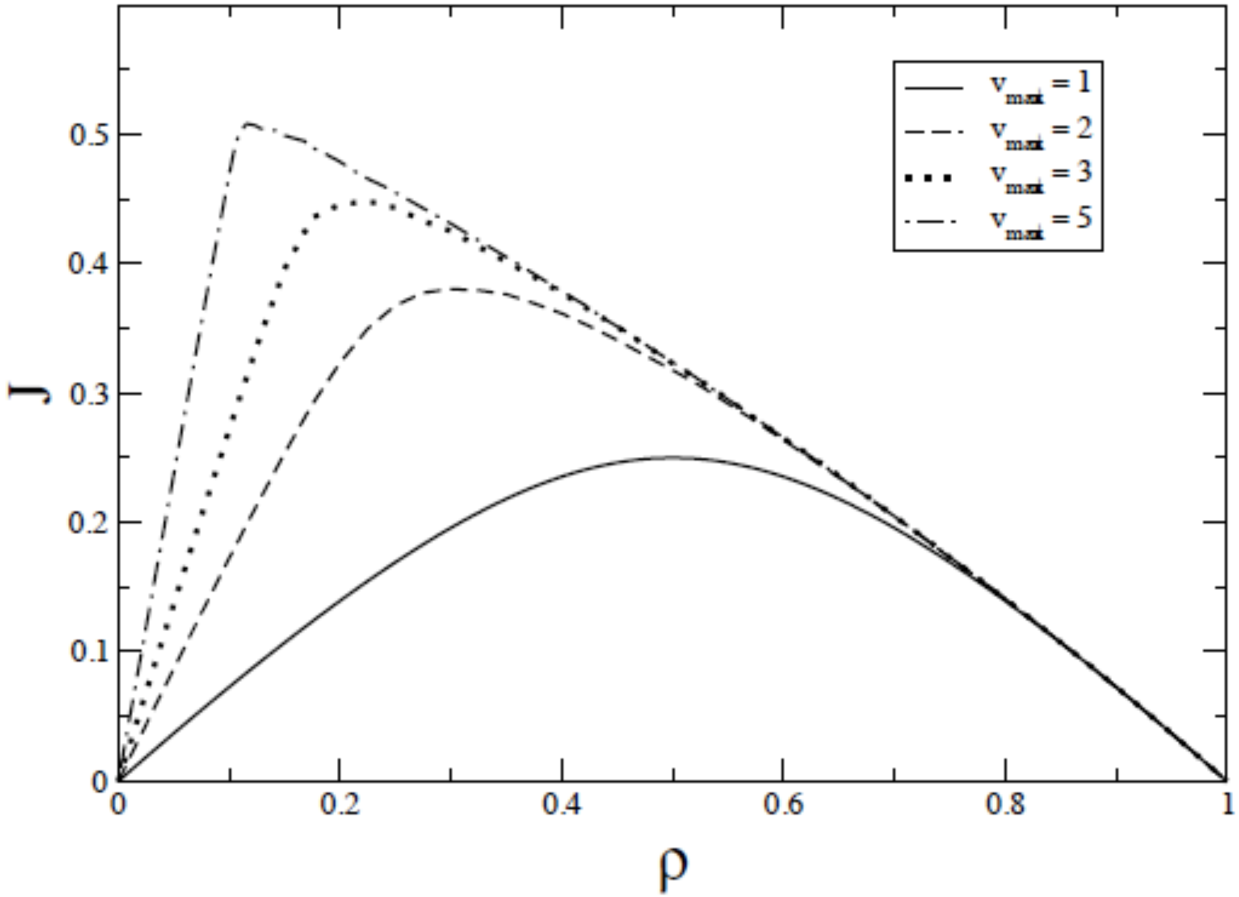
- Reproduces **structure of traffic on highways**
  - Fundamental diagram
  - Spontaneous jam formation

- **Minimal model**: all 4 rules are needed

- **Order** of rules important

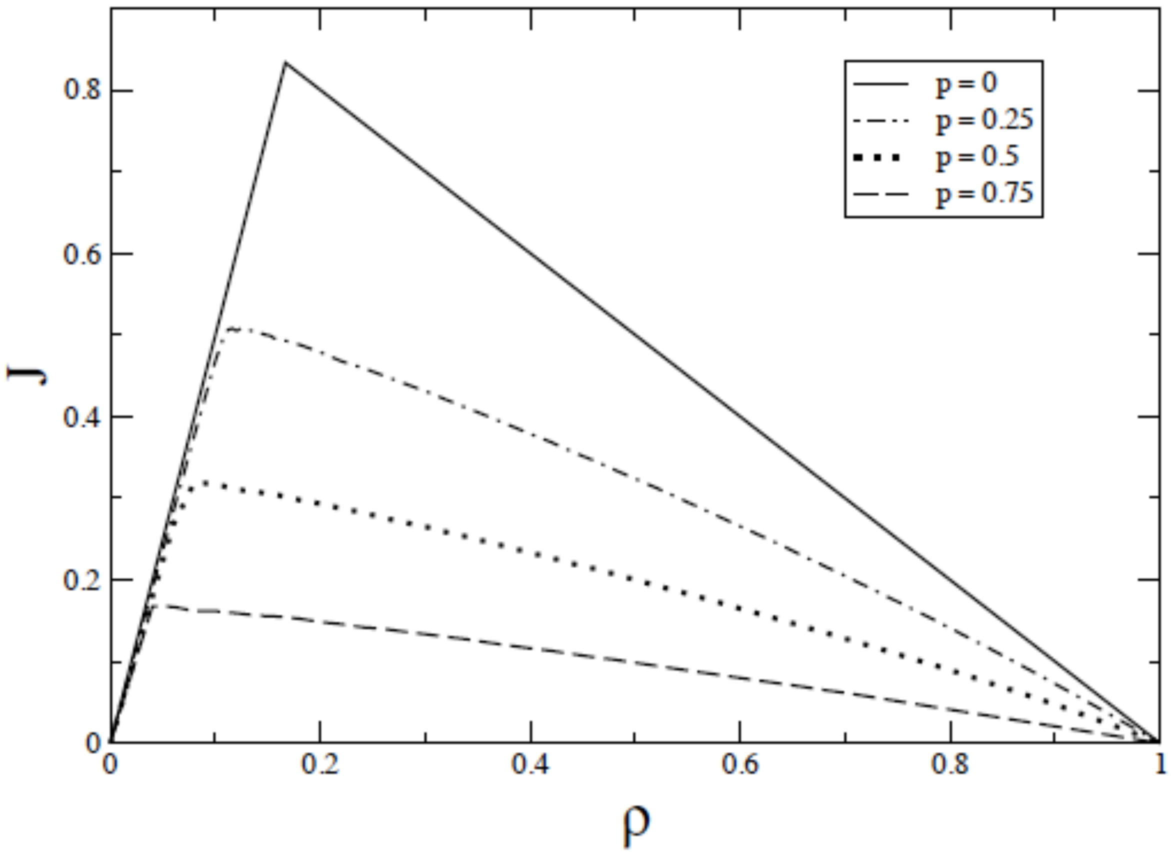
- Simple as traffic model, but rather complex as stochastic model

# Fundamental Diagram (p=0.25)




no particle-hole symmetry for $v_{max}>1$

# Fundamental Diagram ($v_{max}$=5)




**No particle-hole symmetry**

# VDR model

Modified NaSch model: **VDR model** (velocity-dependent randomization)

- Step 0: determine randomization $p=p(v(t))$

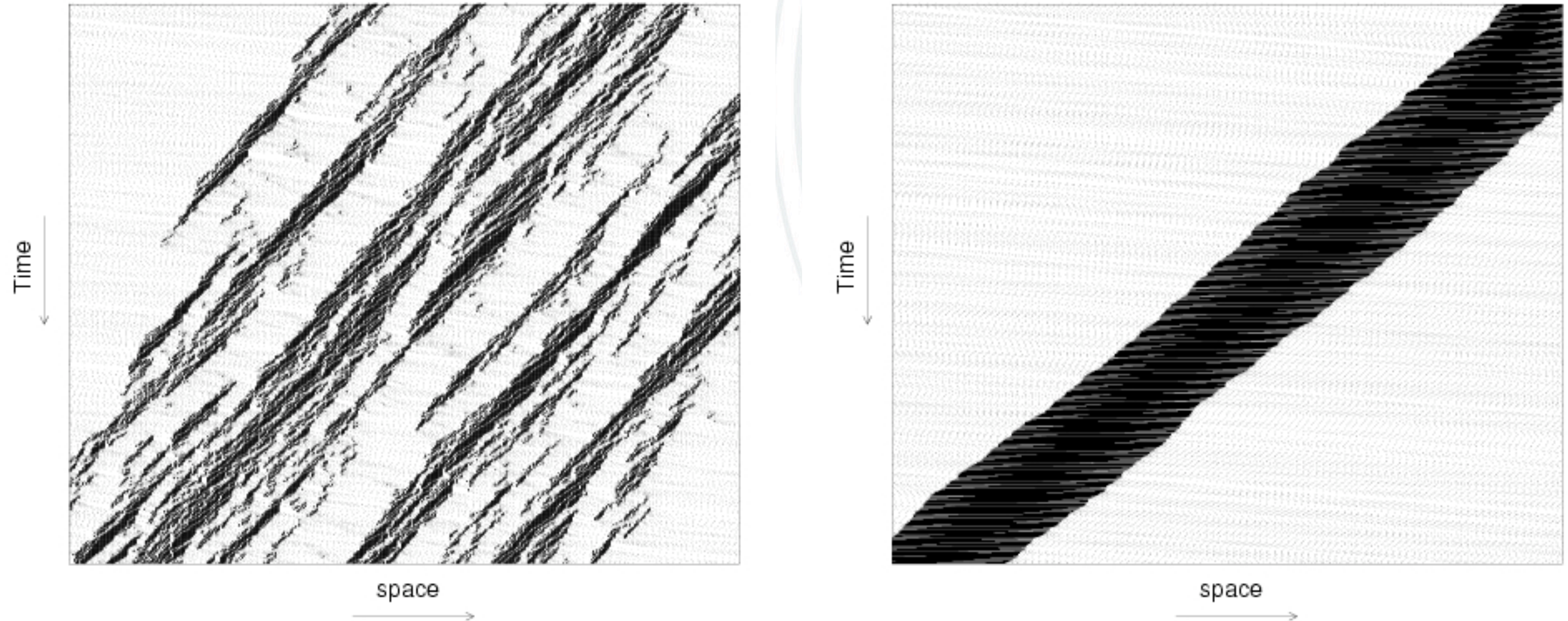
$$p(v) = \begin{cases} p_0 & \text{if } v = 0 \\ p & \text{if } v > 0 \end{cases} \qquad \text{with } p_0 > p$$

**Slow-to-start rule**

# Jam Structure

NaSch model

VDR model




VDR-model: phase separation

Jam stabilized by $J_{out} < J_{max}$

# Brake-light model

- **Nagel-Schreckenberg model**
  1. **acceleration**
  2. **braking**
  3. **randomization**
  4. **motion**

- **plus:**
  - **slow-to-start rule**
  - **velocity anticipation**
  - **brake lights**
  - **interaction horizon**
  - **smaller cells**
  - **...**

**brake-light model**

a.k.a.

**comfortable driving model**

➡ good agreement with single-vehicle data

(Knospe-Santen-Schadschneider-Schreckenberg 2000)

# More realistic CA models

### NaSch model

- free-flow + jammed regime
- spontaneous jamming

slow-to-start rule

### VDR model

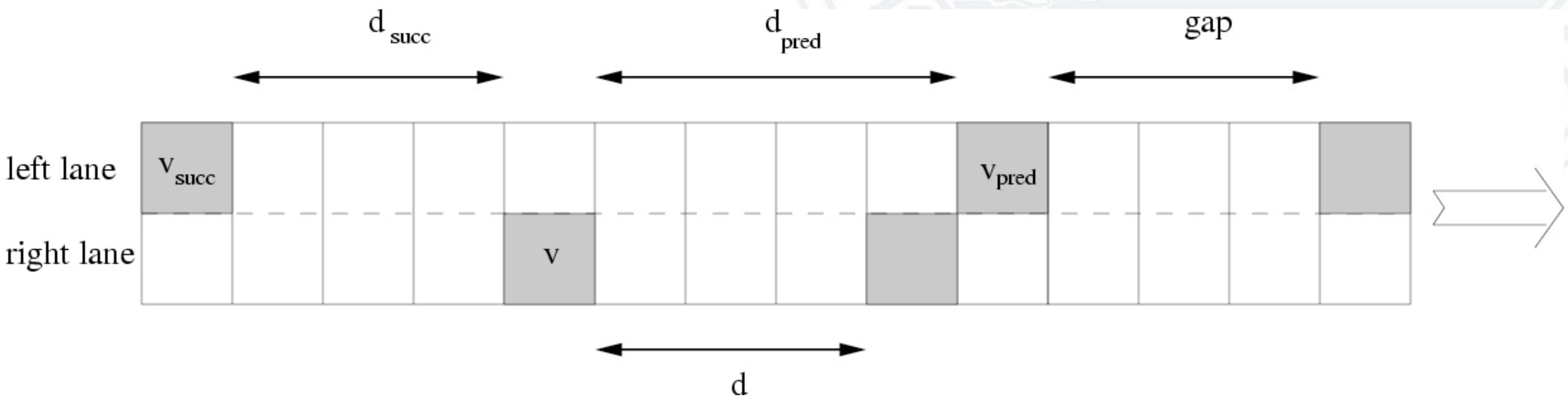
- metastable high-flow states
- hysteresis
- capacity drop

- anticipation
- comfortable driving
- mechanical restrictions

- synchronized flow
- short headways
- OV function
- ...

# 2-Lane Traffic

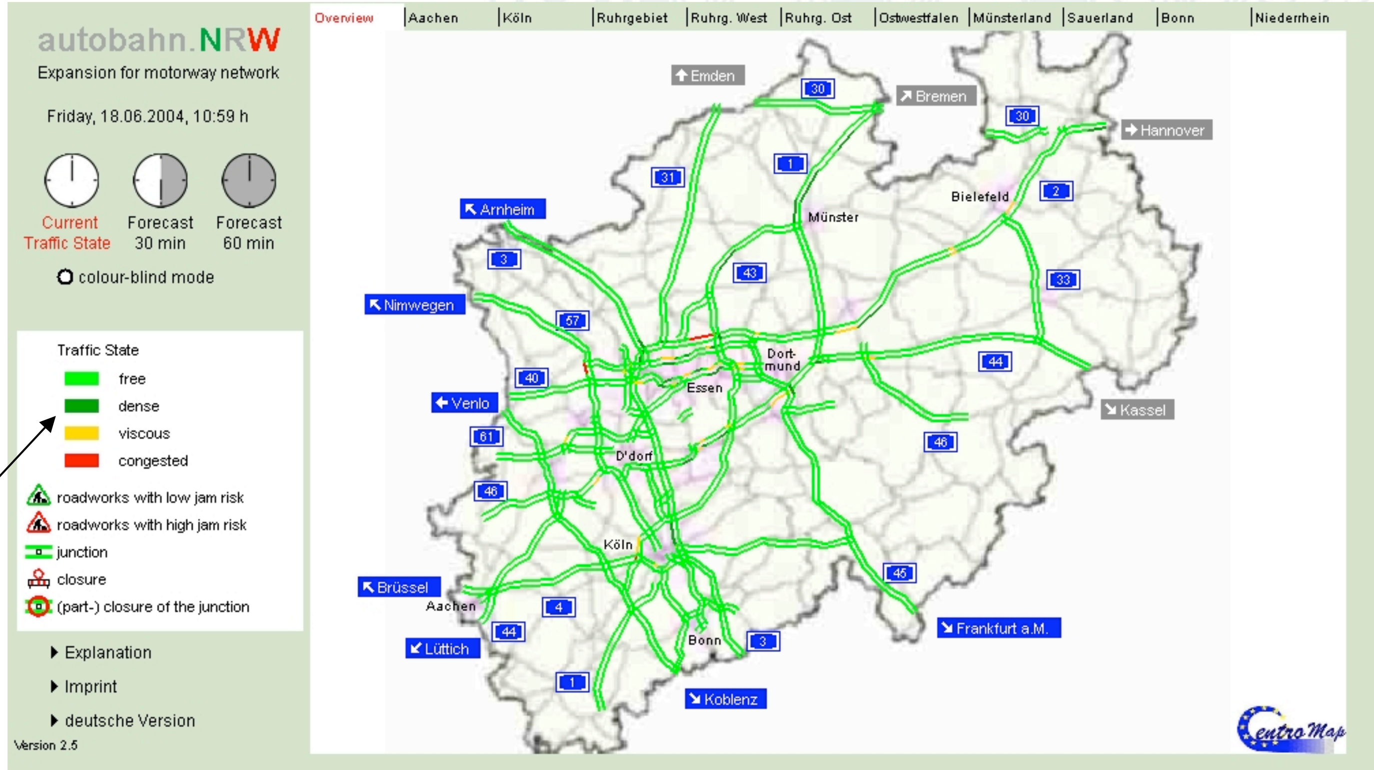
- Rules for lane changes (symmetrical or asymmetrical)
- Incentive Criterion: Situation on other lane is better
- Safety Criterion: Avoid accidents due to lane changes



(www.autobahn.nrw.de)

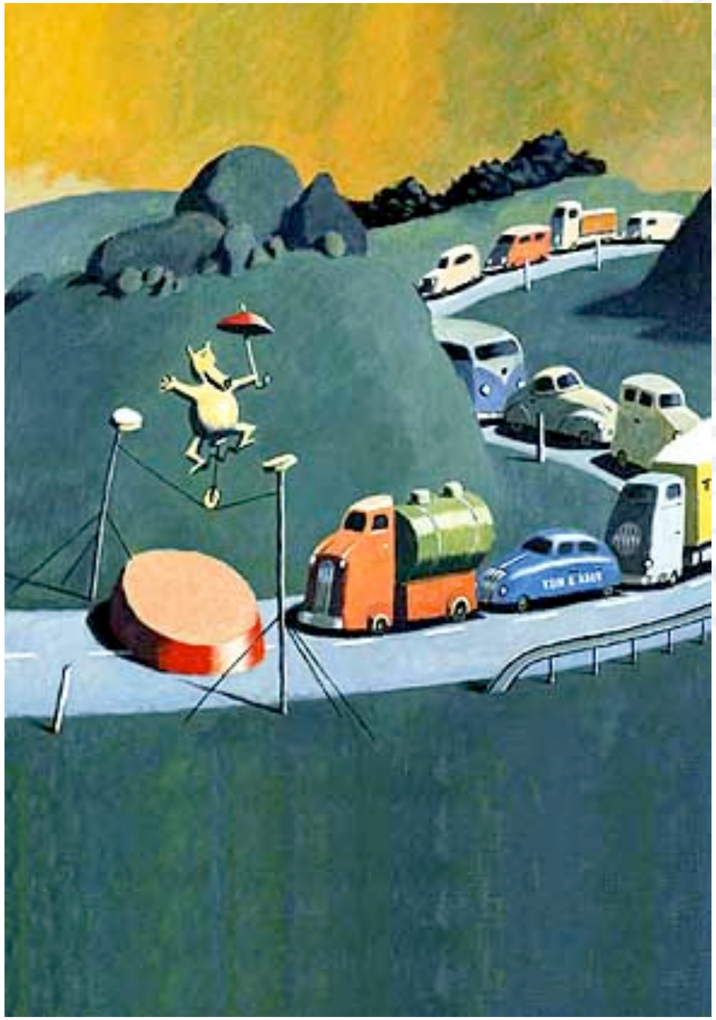
# Traffic Forecasting

- “interpolation” based on online data: **online simulation**
- **30’ and 60’ forecast**




classification into 4 states

# Finally!



Sometimes „spontaneous jam formation“ has a rather simple explanation!

Bernd Pfarr, Die ZEIT