Prof. Dr. A. Schadschneider
Dr. R. Marathe

Nonequilibrium Physics: Problem Sheet 2

www.thp.uni-koeln.de/∼as/noneq13.html

3. Fokker-Planck equation

a) Show that the Fokker-Planck equation can be interpreted as an equation of continuity in velocity space, i.e.

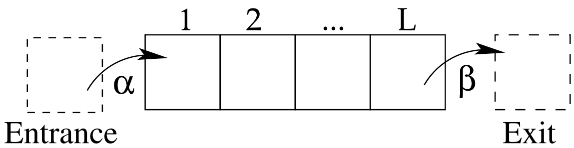
$$\frac{\partial P}{\partial t} + \frac{\partial j_v}{\partial v} = 0 \,. \tag{1}$$

Determine the form of the current j_v.

b) Perform the limit $\gamma \to \infty$ and show that the Fokker-Planck equation takes the form of a diffusion equation in v-space. Determine the corresponding diffusion coefficient D_v.

4. Queuing Problems

We consider a queue e.g. of people waiting in line at a ticket counter. New persons arrive at rate α at the end of the queue whereas people are serviced (and leave the queue) with rate β. The state of the system can be described by its length L where each person has the unit length $l = 1$.



a) The master equation for the system is given by

$$\frac{dP_L}{dt} = \sum_{L'} \left[W(L' \to L) P_{L'} - W(L \to L') P_L \right] \tag{2}$$

where P_L is the probability for finding the system in state L and $W(L \to L')$ is the transition rate from state L to L'. Compute the matrix elements $W(L \to L')$ in terms of α and β.

b) Now we assume that we are in the stationary state, i.e. the probability P_L becomes time independent and the left-hand side of (2) is equal to zero. Show that $P_L = \gamma^L P_0$ where γ only depends on α and β.
Hint: Rearrange (2) to obtain a recursive equation for P_L and distinguish between the cases $L = 0$ and $L > 0$.

c) Calculate P_0 from the normalization condition $\sum_{L=0}^{\infty} P_L = 1$. Which condition (for α and β) has to be fullfilled that P_L is normalisable?

d) Draw a phase diagram in the α-β plane in the stationary limit $t \to \infty$. Which "phases" can be distinguished?