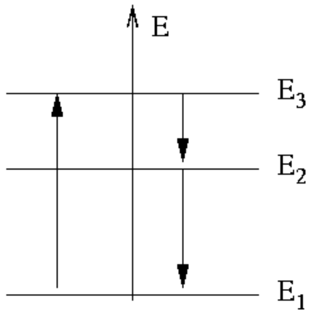
Nonequilibrium Physics: Problem Sheet 5

www.thp.uni-koeln.de/∼as/noneq13.html

8. Master equation for atomic transitions

We consider the transitions between three energy levels in an atom as shown in the figure. The transition rate corresponding to the transition from level m to level n is denoted by $w(m \to n)$.



a) Write down the master equation for the probability $P(n,t)$ to find the electron in state n.

b) Write the master equation as a matrix equation $\dot{\vec{P}}(t) = \mathbf{M} \cdot \vec{P}(t)$ by introducing the vector $\vec{P}(t) = (P(1,t), P(2,t), P(3,t))^t$.

c) Find the stationary state.

d) Is the principle of detailed balance obeyed?

e) Determine the eigenvalues of **M** and the relaxation time.

f) Generalize the result in c) to a process with cyclic transitions between n levels ($E_n \to E_{n-1} \to \ldots \to E_1 \to E_n$).

9. Pairwise balance

In the lecture we have seen that detailed balance is sufficient for a stationary state. However, it is not necessary.

a) Show that the condition of pairwise balance

$$\forall i, j \exists_1 k \quad W(C_i \to C_j)P(C_i) = W(C_k \to C_i)P(C_k) \tag{1}$$

(where $\exists_1$ means "exists exactly one") is also sufficient for a stationary state.

b) Show that detailed balance is a special case of pairwise balance.

c) Give a simple example for pairwise balance, e.g. by a system which can be in three different states.

d) The random walk with hopping rates p to the right and q to the left satisfies detailed balance only for $p = q$. Show that it satisfies pairwise balance for $p \neq q$.

e) Determine the stationary state of the random walker from the pairwise balance condition!