Computational Many-Body Physics

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SS 2019

Sheet 1 - tutorial of Monday, April 15, 14:00

Exercise 1: Rule N

a) Write a code which calculates the first 50 generations of rule 30, starting from the configuration with all $z_i(t=0) = 0$, $i = 1, \ldots, 120$, except for $z_{60}(t=0) = 1$. The code should produce a plot showing the full time evolution of the configurations (see, for example, the wikipedia article on rule 30).

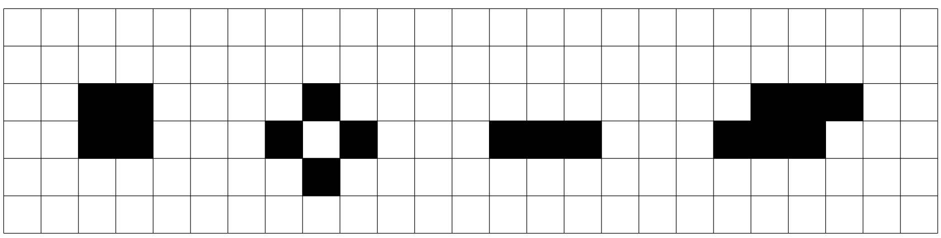
b) Calculate the time dependence of the number of cells with $z_i = 1$:

$$n(t) = \sum_i z_i(t) \ , \quad t = 1, \ldots, 50 \ .$$

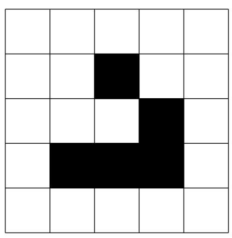
c) Which of the 256 possible rules reproduces exactly any given configuration?

Exercise 2: Game of Life

a) Write a code which simulates Conway's Game of Life on a 20×20 grid with periodic boundary conditions. Check your code with a few simple configurations, such as the ones shown in the figure:

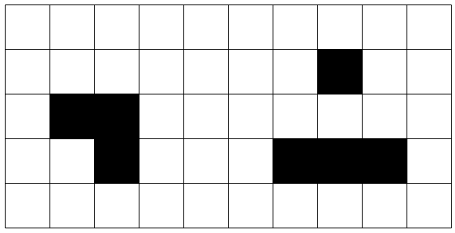


b) The following pattern ("glider") translates across the 2d grid along the diagonal.



Visualize the evolution of the glider for the same grid as in part a).

c) The following pattern ("diehard") disappears after 130 generations.



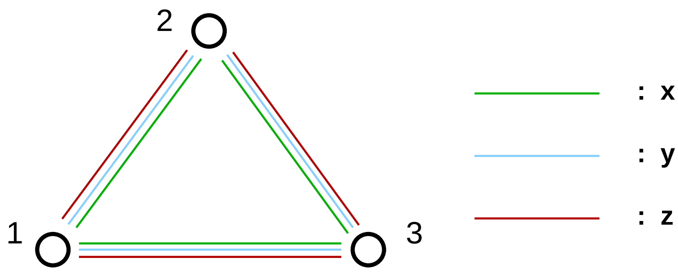
Calculate the number of live cells, $n(t) = \sum_i z_i(t)$, for $t = 0, \ldots 135$.

Exercise 3: Spin-models on a three-site cluster

Consider the following (general) Hamiltonian for a spin-model on a three-site cluster:

$$H = -\sum_{ij\alpha} J_{ij}^{\alpha} S_i^{\alpha} S_j^{\alpha} \ ,$$

with $i, j = 1, 2, 3$ ($i < j$ in $\sum_{ij}$) and $\alpha = x, y, z$.



To visualize the model, a colour code for the x, y and z components of the spin-couplings turns out to be useful, see the figure.

a) Rewrite the Hamiltonian using the operators

$$S_i^{\pm} = S_i^x \pm i S_i^y \ , \text{ and } \ S_i^z \ .$$

Now set up (by hand!) the 8×8 Hamilton matrices $\bar{H}$ for the following three special cases:

b) the Ising model, i.e. $J_{ij}^{\alpha} = J\delta_{\alpha z}$,

c) the isotropic Heisenberg model, i.e. $J_{ij}^{\alpha} = J$, and

d) a model with $J_{12}^x = J_{23}^y = J_{31}^z = J$ and all other $J_{ij}^{\alpha} = 0$.

One of the following exercises deals with a numerical algorithm to set up these Hamilton matrices.