Computational Many-Body Physics

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SS 2020

Sheet 1 - please submit your solutions via e-mail to Chae-Yeun Park until Monday, May 4, 2020, 12:00.

Exercise 1: Rule N

(9 points)

a) Write a code which calculates the first 50 generations of rule 30, starting from the configuration with all $z_i(t = 0) = 0$, $i = 1, \ldots, 120$, except for $z_{60}(t = 0) = 1$. The code should produce a plot showing the full time evolution of the configurations (see, for example, the wikipedia article on rule 30). (5 points)

b) Calculate the time dependence of the number of cells with $z_i = 1$: (2 points)

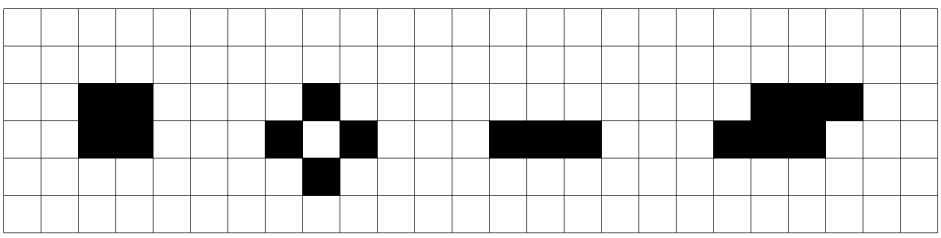
$$n(t) = \sum_i z_i(t) \ , \quad t = 1, \ldots, 50 \ .$$

c) Which of the 256 possible rules reproduces exactly any given configuration? (2 points)

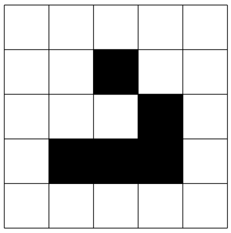
Exercise 2: Game of Life

(11 points)

a) Write a code which simulates Conway's Game of Life on a 20×20 grid with periodic boundary conditions. Check your code with a few simple configurations, such as the ones shown in the figure: (7 points)

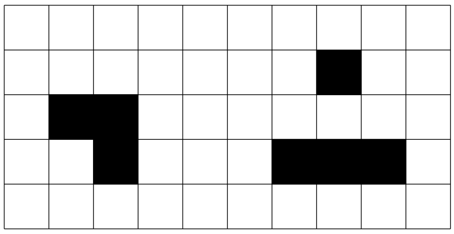


b) The following pattern ("glider") translates across the 2d grid along the diagonal.



Visualize the evolution of the glider for the same grid as in part a). (2 points)
Note: If you have difficulties with the animation of this time development, a code which calculates the configuration after a given number of time steps is sufficient.

c) The following pattern ("diehard") disappears after 130 generations.



Calculate the number of live cells, $n(t) = \sum_i z_i(t)$, for $t = 0, \ldots 135$.
(2 points)

Exercise 3: TASEP

(8 points)

As shown in the lecture, the TASEP with parallel update corresponds to rule 184. Here we consider the TASEP with $N = 50$ sites and periodic boundary conditions.

a) Choose a random starting configuration with M particles ($M \approx 25$) and calculate the average flow for $N_t = 100$ time steps. The flow is defined here as the number of particles per unit of time transferred from site N to site 1.
(5 points)

b) Calculate the fundamental diagram (flow versus density) for this model for $0 \leq \rho \leq 1$ ($\rho = M/N$). Starting from a single configuration for each value of ρ is sufficient here, but the quality of the data improves when the flow is averaged over many starting configurations. (3 points)