Computational Many-Body Physics

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SS 2020

Sheet 3 - please submit your solutions via e-mail to Chae-Yeun Park until Monday, June 8, 2020, 12:00.

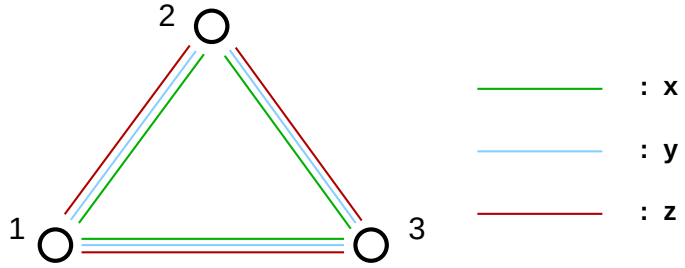
Exercise 1: Spin-models on a three-site cluster

(7 points)

Consider the following (general) Hamiltonian for a spin-model on a three-site cluster:

$$H = -\sum_{ij\alpha} J_{ij}^{\alpha} S_i^{\alpha} S_j^{\alpha} \ ,$$

with $i, j = 1, 2, 3$ ($i < j$ in $\sum_{ij}$) and $\alpha = x, y, z$.



To visualize the model, a colour code for the x, y and z components of the spin-couplings turns out to be useful, see the figure.

a) Rewrite the Hamiltonian using the operators

$$S_i^{\pm} = S_i^x \pm i S_i^y \ , \text{ and } \ S_i^z \ .$$

(1 point)

Now set up (by hand!) the 8×8 Hamilton matrices $\bar{H}$ for the following three special cases:

b) the Ising model, i.e. $J_{ij}^{\alpha} = J\delta_{\alpha z}$, (2 points)

c) the isotropic Heisenberg model, i.e. $J_{ij}^{\alpha} = J$, and (2 points)

d) a model with $J_{12}^x = J_{23}^y = J_{31}^z = J$ and all other $J_{ij}^{\alpha} = 0$. (2 points)

Exercise 2: Hamilton matrices for spin models

(13 points)

Consider a spin model of the form

$$H = -\sum_{ij\alpha} J_{ij}^{\alpha} S_i^{\alpha} S_j^{\alpha} \ ,$$

with $i, j = 1, \ldots, N$ and $\alpha = x, y, z$.

a) Write a program which sets up the Hamilton matrix for a model with arbitrary N and an arbitrary list of couplings $\{J_{ij}^{\alpha}\}$. As discussed in the lecture, the x-, y- and z-links should be treated separately. For each link of the form $-J_{ij}^{\alpha} S_i^{\alpha} S_j^{\alpha}$, the action of $S_i^{\alpha} S_j^{\alpha}$ on the basis state $|n\rangle$ gives another basis state $|l\rangle$ (times a prefactor). With the equation for l given in the lecture, the Hamiltonian matrix can be set up very efficiently. (10 points)

b) Set up the Hamilton matrices for the three models discussed in exercise 1 (the three-site clusters) and calculate the eigenenergies and corresponding eigenstates for each model. (3 points)