Computational Many-Body Physics

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Sheet 5

Exercise 1: Kitaev clusters

Kitaev clusters are quantum spin models defined by the Hamiltonian

$$H = -J \sum_{ij\alpha} J_{ij}^{\alpha} S_i^{\alpha} S_j^{\alpha} \ ,$$

($i, j = 1, \ldots, N$, $\alpha = x, y, z$) with each spin of the cluster connected to exactly three spins with different couplings (x, y, or z), see section 2.1 in the script. In the following, the couplings J_{ij}^{α} of the Kitaev cluster can be set to 1.

For a given N, various different Kitaev clusters can be constructed (and the number of possible clusters increases with N, of course).

a) Show that, for $N = 6$, one can construct two different Kitaev clusters which have a different spectrum of eigenenergies.

One of the Kitaev clusters that can be constructed for $N = 8$ can be visualized as a cube, with the eight spins of the cluster corresponding to the corners of the cube, and the twelve links corresponding to the edges. This particular Kitaev cluster has a non-degenerate ground state.

b) Show that the spin correlations in the ground state of the Kitaev cube are only non-zero for nearest neighbour sites (this turns out to be a general property of all Kitaev clusters).

Exercise 2: Entanglement entropy for the one-dimensional Heisenberg model

Here we consider the one-dimensional Heisenberg model with open boundary conditions:

$$H = -\sum_{i=1}^{N-1} J_i \, \vec{S}_i \cdot \vec{S}_{i+1} \ .$$

For an even number of sites N and $J_i < 0$ (the anti-ferromagnetic case), the ground-state of this model is non-degenerate. The focus in this exercise is on the entanglement entropy S_e, for which the system is divided in two parts (A and B), with parts A and B comprising sites $1, \ldots, M_A$ and $M_A + 1, \ldots, N$, respectively.

a) Calculate the entanglement entropy for the ground state of the Heisenberg model for $N = 10$ and $J_i = -1$ (the homogeneous case) as a function of M_A.

b) Compare the result from a) with the maximal values of S_e that can be achieved for the entanglement between parts A and B.

c) For $J_i = -1$, $N = 6$, and $M_A = 3$, the entanglement entropy $S_e = 0.711...$ is much lower than the maximal value $S_{e,\mathrm{max}} = 3\ln(2) = 2.079...$. Find a set of J-values (with $-1 \leq J_i < 0$, $i = 1, \ldots, 5$), such that $S_e > 1.5$.