

Doppelpendel mit Reibung ($g=|=l$)

In[1]=

```
draw[{p1_, p2_}] := Module[{r1, r2}, r1 = {Sin[p1], -Cos[p1]};
  r2 = r1 + {Sin[p2], -Cos[p2]};
  Graphics[{AbsolutePointSize[10.2], Point[{0, 0}],
    AbsolutePointSize[20.2], AbsoluteThickness[3], Line[{{0, 0}, r1]},
    Line[{r1, r2}], {Blue, Point[r1]}, {Red, Point[r2]}]]]
```

In[2]=

```
ClearAll[equations, eq1, eq2];
equations[m1_, m2_,  $\gamma$ 1_,  $\gamma$ 2_] =
Module[{r1, r2, tt, eq1, eq2}, r1[tt_] = {Sin[ $\phi$ 1[tt]], -Cos[ $\phi$ 1[tt]]};
  r2[tt_] = r1[tt_] + {Sin[ $\phi$ 2[tt]], -Cos[ $\phi$ 2[tt]]};
  lagrangian =  $\frac{1}{2}$  m1 r1'[t].r1'[t] +
     $\frac{1}{2}$  m2 r2'[t].r2'[t] - m1 r1[t][[2]] - m2 r2[t][[2]] // FullSimplify;
  eq1 = D[D[lagrangian,  $\phi$ 1'[t]], t] == D[lagrangian,  $\phi$ 1[t]] -
     $\gamma$ 1 r1'[t].D[r1[t],  $\phi$ 1[t]] -  $\gamma$ 2 r2'[t].D[r2[t],  $\phi$ 1[t]] // FullSimplify;
  eq2 = D[D[lagrangian,  $\phi$ 2'[t]], t] == D[lagrangian,  $\phi$ 2[t]] -
     $\gamma$ 1 r1'[t].D[r1[t],  $\phi$ 2[t]] -  $\gamma$ 2 r2'[t].D[r2[t],  $\phi$ 2[t]] // FullSimplify;
  {eq1, eq2}];
```

In[3]=

```
equations[m1, m2,  $\gamma$ 1,  $\gamma$ 2]
```

Out[3]= $\left\{ \left(\gamma_1 + \gamma_2 \right) \phi_1'[t] + \gamma_2 \cos[\phi_1[t] - \phi_2[t]] \phi_2'[t] + m_2 \sin[\phi_1[t] - \phi_2[t]] \phi_2'[t]^2 + \right.$
 $\left. \left(m_1 + m_2 \right) \left(\sin[\phi_1[t]] + \phi_1''[t] \right) + m_2 \cos[\phi_1[t] - \phi_2[t]] \phi_2''[t] = 0, \right.$
 $\left. \gamma_2 \cos[\phi_1[t] - \phi_2[t]] \phi_1'[t] + \gamma_2 \phi_2'[t] + \right.$
 $\left. m_2 \left(\sin[\phi_2[t]] + \cos[\phi_1[t] - \phi_2[t]] \phi_1''[t] + \phi_2''[t] \right) = m_2 \sin[\phi_1[t] - \phi_2[t]] \phi_1'[t]^2 \right\}$

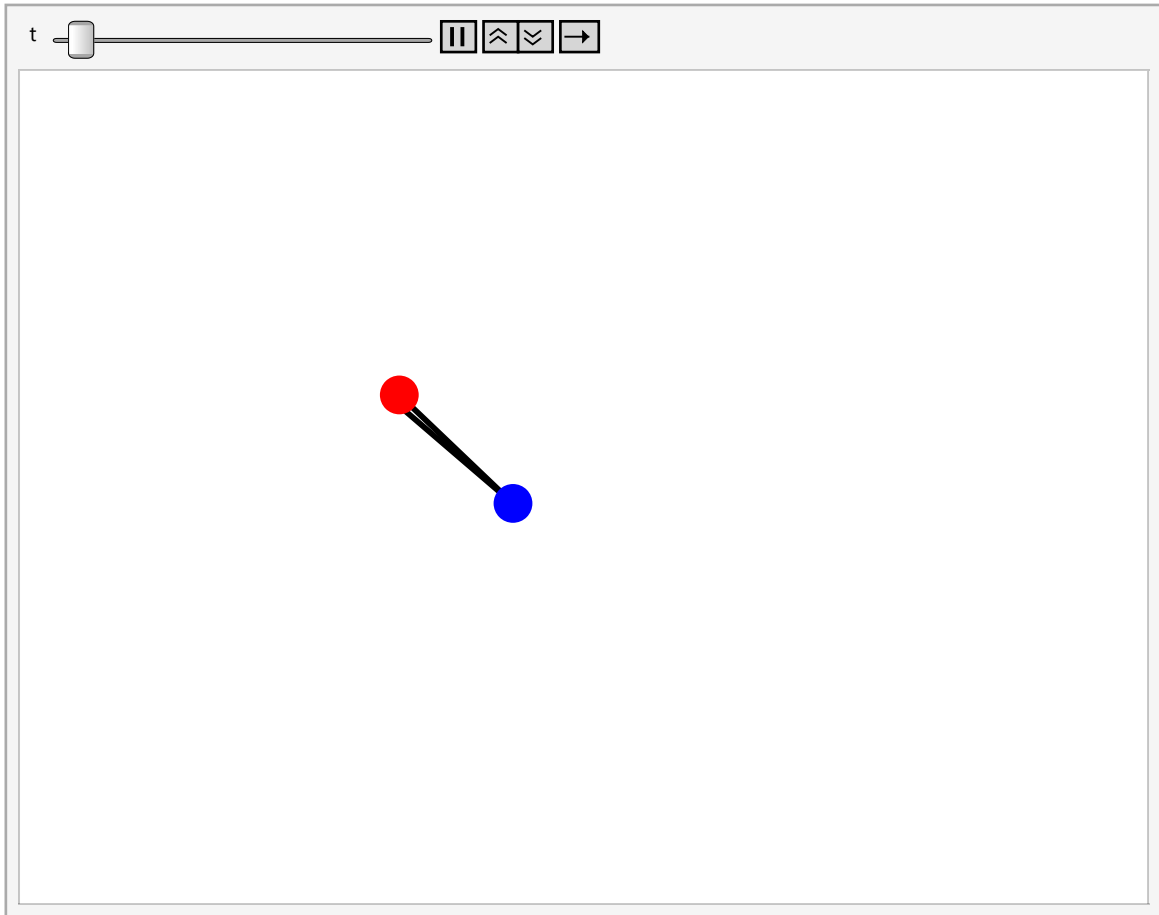
In[4]=

```
ClearAll[solve0, solve, energy, eq1, eq2];
solve0[m1_, m2_,  $\gamma$ 1_,  $\gamma$ 2_,  $\phi$ 10_,  $\phi$ 20_] := solve0[m1, m2,  $\gamma$ 1,  $\gamma$ 2,  $\phi$ 10,  $\phi$ 20] =
Module[{tt, eq1, eq2}, {eq1, eq2} = equations[m1, m2,  $\gamma$ 1,  $\gamma$ 2] /. t -> tt;
  NDSolve[{eq1, eq2,  $\phi$ 1[0] ==  $\phi$ 10,  $\phi$ 2[0] ==  $\phi$ 20,  $\phi$ 1'[0] == 0,  $\phi$ 2'[0] == 0},
    { $\phi$ 1,  $\phi$ 2}, {tt, 0, 40}][[1]]
solve[m1_, m2_,  $\gamma$ 1_,  $\gamma$ 2_,  $\phi$ 10_,  $\phi$ 20_][t_] := { $\phi$ 1[t],  $\phi$ 2[t]} /.
  solve0[m1, m2,  $\gamma$ 1,  $\gamma$ 2,  $\phi$ 10,  $\phi$ 20]

energy[m1_, m2_,  $\gamma$ 1_,  $\gamma$ 2_,  $\phi$ 10_,  $\phi$ 20_][t_] :=
Module[{r1, r2, s = solve0[m1, m2,  $\gamma$ 1,  $\gamma$ 2,  $\phi$ 10,  $\phi$ 20], tt},
  r1[tt_] = {Sin[ $\phi$ 1[tt]], -Cos[ $\phi$ 1[tt]]} /. s;
  r2[tt_] = r1[tt_] + {Sin[ $\phi$ 2[tt]], -Cos[ $\phi$ 2[tt]]} /. s;
   $\frac{1}{2}$  m1 r1'[t].r1'[t] +  $\frac{1}{2}$  m2 r2'[t].r2'[t] + m1 r1[t][[2]] + m2 r2[t][[2]]]
```

```
In[19]:= Animate[Show[draw[solve[10, 1, .1, .1, Pi/2, Pi/2][t]],
  PlotRange -> {{-2.2, 2.2}, {-2.2, 1}}], {t, 0, 40, .1}]
```

Out[19]=



Beispiel : $m_1 = 10$, $m_2 = 1$, ändere Reibung (stark bei erstem, letztem, keinen)

```
In[20]:= Plot[energy[10, 1, 0.1, 0.1, Pi/2, Pi/2][t], {t, 0, 40}, PlotRange -> All]
```

Out[20]=

