Interacting Flux Lines in a Random Medium

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We study the continuum field theory for an ensemble of directed lines $\mathbf{r}_i(t)$ in $1 + d'$ dimensions that live in a medium with quenched point disorder and interact via short-range pair forces $g\Psi(\mathbf{r}_i - \mathbf{r}_j)$. In the strong-disorder (or low-temperature) regime, attractive forces generate a bound state with localization length $\xi_\perp \sim |g|^{-\nu_\perp}$; repulsive forces lead to mutual avoidance with a pair distribution function $\mathcal{P}(\mathbf{r}_i - \mathbf{r}_j) \sim |\mathbf{r}_i - \mathbf{r}_j|^\theta$ reminiscent of fermions. In the experimentally important dimension $d' = 2$, we obtain $\nu_\perp \approx 0.8$ and $\theta \approx 0.4$.

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Dirty type-II superconductors are a well-known system with *quenched disorder* [1]. Close to the lower critical field h_{c1}, their magnetic flux lines have an average distance R much larger than the London penetration depth l_0, and can, hence, effectively be regarded as a dilute ensemble of fluctuating 1D objects, also called *directed polymers* [2]. (These are of interest also because of their links to more complicated random systems such as to spin glasses [3], to surface growth [4,5], and to randomly driven hydrodynamics [6].) The statistical properties of the lines differ from those of free, thermally fluctuating lines. Moreover, the disorder modifies their interactions with other objects. For example, the pinning of flux lines by columnar or planar defects has been studied experimentally [7]. Much theoretical work has been concerned with the low-density limit of a *single* line in a disordered medium. Its interaction with a columnar defect, for example, turns out to be *weaker* than in a pure system. A weakly attractive defect localizes the line only up to the borderline dimension $d'_* = 1$; in higher dimensions, the transition to a localized state takes place at finite coupling strength [8,9]. In a pure system, the borderline dimension is $d'_* = 2$ [10].

Mutual interactions between several identical lines in a disordered medium are the subject of this Letter. Somewhat surprisingly, we find such forces to act in a *stronger* way than in a pure system: They lead to *long-ranged* effects on the lines in *any* dimension where the low-temperature behavior of a single line is governed by nonthermal scaling exponents. To obtain our results, we use systematic methods of quantum field theory. Related aspects of this system have been treated by Bethe ansatz [11] and numerically [12] in $d' = 1$, on a hierarchical lattice [13] (these results also indicate the relevance of such forces in higher dimensions), and in a Wilson renormalization group [14,15] (see the remarks below).

Weakly attractive short-ranged forces are shown to localize the lines to a bundle of width $\xi_\perp \sim |g|^{-\nu_\perp}$ for small interaction strength g. In a pure system, the localization would require a finite strength g above the borderline dimension $d'_* = 2$.

Of equal importance are short-ranged repulsive forces; for example, the magnetic interaction between flux lines decays exponentially on the *microscopic* scale l_0. For a dilute system of thermal lines, such forces are important only in $d' = 1$, where they act as an effective constraint on the fluctuations that is equivalent to the Pauli principle: On *mesoscopic* scales $l_0 \ll |\mathbf{r}_i - \mathbf{r}_j| \ll R$, the lines behave like the world lines of *free fermions* characterized by a pair distribution function $\mathcal{P}(\mathbf{r}_i - \mathbf{r}_j) \sim |\mathbf{r}_i - \mathbf{r}_j|^\theta$ with $\theta = 2$ [16–18]. For $d' > 1$, short-ranged forces have no effect on $\mathcal{P}(\mathbf{r}_i - \mathbf{r}_j)$ beyond the scale l_0, i.e., $\theta = 0$. In the presence of disorder, however, we find that $\mathcal{P}(\mathbf{r}_i - \mathbf{r}_j)$ obeys an asymptotic power law also for $d' > 1$, with a new exponent θ that depends on d'. The strong effect of repulsive interactions can be understood qualitatively from the energetic competition with an effective line-line attraction due to the impurities. In almost all realizations of the disorder, there is a unique ground state, i.e., a path of minimal energy [19]. At low temperatures and without direct forces, two lines will share this path with finite probability even in a system of infinite size [12,20] as expressed by Eq. (3) below. Repulsive interactions, however, force one of the lines to a (distant) excited path. In other words, the behavior of several lines with contact interactions in a random medium efficiently probes the statistics of *excitations* for a single line. This property may be used to obtain information on the theoretically important, but still unresolved, question of the *upper critical dimension* of the single-line system (see the discussion at the end of this Letter).

On a mesoscopic scale, a system of p lines in a random medium is described by an effective continuum Hamiltonian

$$'\mathcal{H} = \sum_{i=1}^{p} \int dt \left\{ \frac{1}{2} \left(\frac{d\mathbf{r}_i}{dt} \right)^2 - \eta(\mathbf{r}_i, t) \right\} + g \sum_{i<j} \int dt\, \Psi_{ij}(t) . \quad (1)$$

Here $\mathbf{r}_i(t)$ denotes the displacement vectors of the lines in d' transversal dimensions as a function of the longitudinal

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"timelike" coordinate t. All lines are subject to the same random potential $\eta(\mathbf{r}, t)$ which models the quenched point defects. It is assumed to be Gauss distributed with $\overline{\eta(\mathbf{r}, t)} = 0$ and $\overline{\eta(\mathbf{r}, t)\eta(\mathbf{r}', t')} = 2\sigma^2 \delta^{d'}(\mathbf{r} - \mathbf{r}')\delta(t - t')$. Averages over disorder are denoted by an overbar and thermal averages by brackets $\langle \cdots \rangle$. The direct pair interaction, which is assumed to be short ranged, is described by the continuum fields $\Psi_{ij}(t) = \delta^{d'}(\mathbf{r}_i(t) - \mathbf{r}_j(t))$. The pair distribution function $\mathcal{P}(\mathbf{r}_1 - \mathbf{r}_2)$ in the dilute limit $|\mathbf{r}_1 - \mathbf{r}_2| \ll R$ may then be obtained by neglecting the direct interaction with all other lines, i.e., from the restricted Hamiltonian (1) for these two lines ($p = 2$) in a system of transversal size R.

For $g = 0$, the two lines feel only the random potential, and the free energy of the system is just twice the free energy of a single line. The continuum field theory for this system serves as the starting point for a systematic renormalized perturbation theory in the line-line coupling g. The analysis is complicated by the fact that even the "free" theory ($g = 0$) has non-Gaussian multipoint correlations due to the quenched averaging. Nevertheless, we are able to obtain, solely in terms of single-line properties, the scaling dimension and the form of the operator product expansion of the pair interaction field $\Psi_{12}(t)$ [see (3) and (4) below]. These determine the renormalization group equations for the interaction strength to leading order, and hence the phase diagram of the system [21].

The large-scale behavior of a single line is generated by the sample-to-sample fluctuations of the ground state paths and defines two important exponents. The *roughness exponent* ζ characterizes the mean transversal excursions of the line, given, e.g., by the two-point function $\overline{\langle [\mathbf{r}(t) - \mathbf{r}(t')]^2 \rangle} \sim |t - t'|^{2\zeta}$. The exponent $-\omega$ is the anomalous dimension of the *disorder-averaged free energy* $\overline{F} = \beta^{-1} \overline{\ln \mathrm{Tr} \exp(-\beta \mathcal{H})}$, whose universal part has the scaling form $\overline{F}(T, R) \sim T^{\omega} \mathcal{F}(R/T^{\zeta})$ in a finite system of transversal size R and longitudinal size T [22]. The two exponents are related by a "tilt" symmetry of the system, $\omega = 2\zeta - 1$ (see, e.g., [5,23]). In low dimensions, ζ is always larger than in the case of thermal fluctuations, namely, $\zeta = 2/3$ for $d' = 1$ and $\zeta \approx 0.62$ for $d' = 2$. For $d' > 2$, a phase transition appears at a finite temperature; in the high-temperature phase, the system is asymptotically thermal, i.e., $\zeta = 1/2$ and $\omega = 0$ (see, e.g., [5], and references therein). Whether a finite upper critical dimension $d'_>$ exists, such that for $d' \geq d'_>$ the thermal exponents govern also the low-temperature phase, is controversial; some workers believe $d'_> \approx 4$ [24]. In the continuum theory (1) with $g = 0$, the large-scale regime is reached in a crossover from the Gaussian theory with characteristic longitudinal length $\tilde{\xi}_\parallel = \beta(\sigma^2 \beta^3)^{-2/(2-d')}$. We have discussed the renormalized continuum field theory for this regime in Ref. [9]. Its construction involves a reparametrization of the transversal displacement and of the free energy, $\mathbf{r} \to \beta^{1/2} \tilde{\xi}_\parallel^{\omega/2} \mathbf{r}$ and $\overline{F} \to \beta \tilde{\xi}_\parallel^{\omega} \overline{F}$, such that the renormalized variables remain finite in the limit $\tilde{\xi}_\parallel \to 0$ (i.e., $\beta^{-1} \to 0$ or $\sigma^2 \to \infty$). The additional reparametrization $\Psi_{12} \to \beta^{-d'/2} \tilde{\xi}_\parallel^{d'/2} \Psi_{12}$ and $g \to \beta^{1+d'/2} \tilde{\xi}_\parallel^{\omega - d'/2} g$ keeps all correlation functions of the pair field free of singularities in this limit, as we will see explicitly below. This is accompanied by a temperature reparametrization; the renormalized temperature $\beta^{-1} \sim \tilde{\xi}_\parallel^{\omega}$ is an irrelevant scaling variable of dimension $-\omega$.

With this renormalized theory for $g = 0$ at hand, the effect of pair interactions may now be expanded in powers of g. For example, the perturbation series for the free energy density $\overline{f} \equiv \lim_{T \to \infty} \overline{F}/T$ in a system of transversal size $R \equiv L^{\zeta}$,

$$\overline{f(g, L)} - \overline{f(0, L)} = -\beta^{-1} \sum_{N=1}^{\infty} \frac{(-\beta g)^N}{N!} \int dt_2 \cdots dt_N \times \overline{\langle \Psi_{12}(0) \Psi_{12}(t_2) \cdots \Psi_{12}(t_N) \rangle^c}, \tag{2}$$

involves integrals over connected correlation functions of the pair field Ψ_{12} evaluated at $g = 0$. We are, hence, led to study these correlation functions. While they are finite by construction, the "time" integrations generate new singularities that have to be handled by an additional renormalization. The one-point function $\overline{\langle \Psi_{12}(t) \rangle}$ gives the probability density that the two lines intersect at time t, averaged over thermal and disorder fluctuations. We will show below that for $R^{1/\zeta} \gg \tilde{\xi}_\parallel$,

$$\overline{\langle \Psi_{12}(t) \rangle} = bR^0, \tag{3}$$

where b is a constant independent of $\tilde{\xi}_\parallel$. Hence $\Psi_{12}(t)$ can be regarded as a scaling field of dimension zero. The higher connected correlation functions of the local pair field Ψ_{12} can be shown to obey the *operator algebra*

$$\Psi_{12}(t)\Psi_{12}(t') = c\beta^{-1}|t - t'|^{-\omega} \Psi_{12}(t') + \cdots \tag{4}$$

with a coefficient $c > 0$, which is valid as an asymptotic identity inserted into any such correlation function $\overline{\langle \cdots \Psi_{12}(t) \Psi_{12}(t') \cdots \rangle^c}$ for $t \to t'$. This type of operator algebra is familiar from Ref. [9] (where the reader is referred to for a more detailed discussion): The field Ψ_{12} couples to itself, but the leading singularity in (4) involves a correction-to-scaling exponent related to the irrelevant coupling constant β^{-1}. It is instructive to compare (3) and (4) with the case of thermally fluctuating lines ($\eta = 0$) [18]. The intersection probability is then just the inverse transversal volume, $\langle \Psi_{12}(t) \rangle = R^{-d}$. The operator algebra $\Psi_{12}(t)\Psi_{12}(t') \sim |t - t'|^{-d/2} \Psi_{12}(t') + \cdots$ has a stronger singularity than in the random case, originating from the return probability of the Gaussian paths r_1 and r_2 to each other after a "time" $|t - t'|$.

To establish Eq. (3), it is useful to study the pair distribution function $\mathcal{P}(\mathbf{d}) \equiv \overline{\langle \delta^{d'}(\mathbf{r}_1(t) - \mathbf{r}_2(t) - \mathbf{d}) \rangle}$, i.e., the probability density that the two lines have the relative displacement $\mathbf{d}$. We find that for $g = 0$ and in the limit

$R \to \infty$, $\mathcal{P}$ has a scaling form that depends only on $|\mathbf{d}|$ and $\tilde{\xi}_\parallel$ with the asymptotics [25]

$$\mathcal{P}(\mathbf{d}) \sim \begin{cases} \tilde{\xi}_\parallel^{\omega} |\mathbf{d}|^{-(d'\zeta+\omega)/\zeta} & \text{for } |\mathbf{d}| \gg \tilde{\xi}_\parallel^{\zeta}, \\ \tilde{\xi}_\parallel^{-d'\zeta} & \text{for } |\mathbf{d}| \ll \tilde{\xi}_\parallel^{\zeta}. \end{cases} \quad (5)$$

Recall that $\tilde{\xi}_\parallel^{\omega}$ is the renormalized temperature β^{-1}. Hence with finite probability the two lines share a common "tube" of width $\tilde{\xi}_\parallel^{\zeta}$ (i.e., $|\mathbf{d}| < \tilde{\xi}_\parallel^{\zeta}$), but they do make large excursions whose probabilities decay with a power of $|\mathbf{d}|$ such that all positive integer moments $\int d^{d'}\mathbf{d} |\mathbf{d}|^k \mathcal{P}(\mathbf{d})$ diverge for $R \to \infty$ [26]. [This is what distinguishes (5) from a bound state probability distribution, where large separations are exponentially suppressed and all moments remain finite.] The expectation value of the pair potential Ψ_{12} at $g = 0$ involves an integral over the potential function times $\mathcal{P}(\mathbf{d})$. If Ψ_{12} has a microscopic range l_0, this integral does not depend on R which is just (3). For $l_0 \ll \tilde{\xi}_\parallel^{\zeta}$, the integral develops a singularity $\tilde{\xi}_\parallel^{-d'\zeta}$ that we have absorbed into the definition of the *renormalized* field Ψ_{12}.

By analogy with the case of thermal lines, the form of the operator algebra (4) is already obvious: The leading short-distance singularity between intersections $\Psi_{12}(t)$ and $\Psi_{12}(t')$ should be given by the intersection field $\Psi_{12}(t)$ itself, times a modified return probability of the random paths to each other. To calculate that singularity, we exploit the exact mapping [15] of the polymer system defined by (1) to a generalized Kardar-Parisi-Zhang equation (4): If $Z(\mathbf{r}_i, \mathbf{r}_2, t)$ denotes the restricted partition sum over all two-line configurations with fixed end points $(\mathbf{r}_1, \mathbf{r}_2, t)$, the field $h(\mathbf{r}_1, \mathbf{r}_2, t) = \beta^{-1} \ln Z(\mathbf{r}_1, \mathbf{r}_2, t)$ obeys

$$\partial_t h = \nu \sum_{i=1}^{2} \nabla_{\mathbf{r}_i}^2 h + \frac{\lambda}{2} \sum_{i=1}^{2} (\nabla_{\mathbf{r}_i} h)^2 + \sum_{i=1}^{2} \eta(\mathbf{r}_i, t) - g\delta^{d'}(\mathbf{r}_1 - \mathbf{r}_2). \quad (6)$$

From (6), one constructs in a standard way the generating functional of the dynamic correlation functions [27] which are denoted by $\langle\langle \cdots \rangle\rangle$. This functional integral involves the field h and an auxiliary field $\tilde{h}$. Insertions of the auxiliary field generate response functions which are directly related to *connected* polymer averages [9],

$$\beta^{m-1}\overline{\langle \delta^{d'}(\mathbf{r}_1(t_1) - s_1)\delta^{d'}(\mathbf{r}_2(t_1) - \mathbf{s}_1') \cdots \boldsymbol{\delta}^{d'}(\mathbf{r}_1(t_m) - \mathbf{s}_m)\delta^{d'}(\mathbf{r}_2(t_m) - \mathbf{s}'_m)\rangle^c} = \lim_{T\to\infty} \langle\langle h(\mathbf{r}_1, \mathbf{r}_2, T)\tilde{h}(\mathbf{s}_1, \mathbf{s}'_1, t_1) \cdots \tilde{h}(\mathbf{s}_m, \mathbf{s}'_m, t_m)\rangle\rangle. \quad (7)$$

The pair field $\Psi_{12}(t)$ is therefore mapped onto the dynamic field $\tilde{\psi}(t) \equiv \int d^{d'}\mathbf{r}\tilde{h}(\mathbf{r}, \mathbf{r}, t)$. It is straightforward to show that $\tilde{\psi}(t)$ has dimension ω (with time as the basic scale). Hence its self-coupling has to be of the form $\tilde{\psi}(t)\tilde{\psi}(t') \sim c'|t - t'|^{-\omega}\tilde{\psi}(t) + \cdots$. Transforming back to the polymer system then gives (4).

The operator algebra (4) determines the renormalization group of the perturbation series (2) to one-loop order [9]. Inserting (3) and (4) into (2), we find

$$\overline{f}(g, L) - \overline{f}(0, L) = L^{-\varepsilon}\overline{\langle \Psi_{12} \rangle}\left(u - \frac{c}{\varepsilon}u^2\right) + O(u^3, \varepsilon^0 u^2) \quad (8)$$

in terms of the dimensionless coupling $u \equiv gL^{\varepsilon}$ and the expansion parameter $\varepsilon \equiv 1 - \omega(d')$. Absorbing the pole in ε into a renormalized pair coupling $u_R = Z(u)u$ with $Z(u) = 1 - (c/\varepsilon)u_R + O(u_R^2)$ then yields the flow equation $L\partial_L u_R = \varepsilon u_R - c u_R^2 + O(u_R^3)$. To the given order, this equation has the two fixed points $u_R = 0$ and $u_R^* = (1 - \omega)/c$. In any physical dimension d', one has $\varepsilon > 0$. Hence the trivial fixed point $u_R = 0$ is *unstable*, and the interaction is a *relevant* perturbation. This result is at variance [28] with Ref. [14].

Any *attractive* line-line force ($g < 0$) grows indefinitely in magnitude under the renormalization. The system develops a *bound state* with transversal localization length $\xi_\perp$. As $g \nearrow 0$, the lines unbind continuously, i.e., $\xi_\perp$ diverges as

$$\xi_\perp \sim |g|^{-\nu_\perp} \quad \text{with} \quad \nu_\perp = \frac{\zeta}{\varepsilon} = \frac{1 + \omega}{2(1 - \omega)}. \quad (9)$$

With a *repulsive* interaction ($g > 0$), however, the large-scale behavior of the two-line system is determined by the nontrivial fixed point u_R^*. At this fixed point, the pair field Ψ_{12} acquires the new dimension $x^* = 2(1 - \omega) + O((1 - \omega)^2)$ that determines the large-scale asymptotics of the pair distribution function. For example, the probability of intersection in a system of transversal size R scales as $\overline{\langle \Psi_{12}(t) \rangle} \sim R^{-x^*/\zeta}$. Unlike in the case without pair interactions, the probability of intersection now approaches zero in the limit $R \to \infty$: The lines avoid each other completely. Moreover, for $R \gg |\mathbf{r}_1 - \mathbf{r}_2| \gg \xi_\perp(g)$ the pair distribution function behaves as

$$\mathcal{P}(\mathbf{r}_1 - \mathbf{r}_2) \sim |\mathbf{r}_1 - \mathbf{r}_2|^{\theta} \quad (10)$$

with $\theta = x^*/\zeta - d'$ (i.e., $\theta \approx 1$ for $d' = 1$ and $\theta \approx 0.4$ for $d' = 2$), as follows by standard scaling arguments.

We may use these results to obtain the finite-size scaling of the "overlap" $q(T, g) \equiv T^{-1} \int_0^T dt' \overline{\langle \Psi_{12}(t') \rangle}(T, g)$ in a system of *longitudinal* length T and with $R \to \infty$, a quantity that is discussed in the literature [12,13,15]. We have $q(T, g) = q(T, 0)\tilde{Q}(T/\xi_\parallel) = Q(gT^{1-\omega})$, using $q(T, 0) \sim T^0$ by (3) and the scaling of the longitudinal correlation length $\xi_\parallel \sim \xi_\perp^{1/\zeta}$ given by (9). For $d' = 1$, where $1 - \omega = 2/3$, this scaling form agrees with Mézard's conjecture [12] (which is based on numerical simulations) and with Mukherji's standard dynamic renormalization group [15] for Eq. (6). The latter approach, however, gives no information on the strong-coupling phase in any dimension $d' > 1$.

In summary, we have shown that directed lines in a highly disordered medium respond to pair forces in a stronger way than lines with purely thermal fluctuations. In any dimension $d' < d'_>$, they form a bound state with attractive forces; with repulsive forces they avoid each other, as described by the pair distribution function (10). The reason for this strong effect of short-ranged interactions is that when they are absent, the disorder forces the lines to cluster in the vicinity of a *unique* path of minimal energy. Our results, hence, provide an experimentally and numerically accessible consequence of the clustering. However, the pair distribution function (5) shows a *singular broadening* for fixed $\tilde{\xi}_\parallel$ and $R \to \infty$ as d' approaches the upper critical dimension $d'_>$ (i.e., $\omega \searrow 0$): The width $\Delta(\eta)$ defined by $\int_{|\mathbf{d}|<\Delta} d^{d'}\mathbf{d}\,\mathcal{P}(\mathbf{d}) = \eta$ diverges as $\Delta(\eta) \sim 1/\omega$ for any $0 < \eta < 1$, and accordingly $\overline{\langle \Psi_{12} \rangle} \sim \omega$. If $d'_>$ is finite, we, hence, expect that for $d' > d'_>$ the lines no longer cluster, but exploit *multiple* near-minimal paths even at low temperature, as in a glassy state. This leads to a modification of Eq. (3), $\overline{\langle \Psi_{12} \rangle} \sim R^{-d'}$; weakly attractive pair forces should then no longer generate a bound state. This could be useful to determine $d'_>$ numerically in a way that is less hampered by finite-size effects than the existing simulations of the Kardar-Parisi-Shang equation [29].

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[25] They key point in deriving (5) (see also the discussion in [21]) is to rewrite $\mathcal{P}(\mathbf{d})$ for $g = 0$ and $\mathbf{d} \neq 0$,

$$\begin{aligned}\mathcal{P}(\mathbf{d}) &= \int d^{d'}\mathbf{s}\overline{\langle \delta^{d'}(\mathbf{r}_1(t) - \mathbf{s})\delta^{d'}(\mathbf{r}_2(t) - \mathbf{s} - \mathbf{d})\rangle} \\ &= \int d^{d'}\mathbf{s}\overline{\langle \Phi(t,\mathbf{s})\rangle_1 \langle \Phi(t,\mathbf{s}+\mathbf{d})\rangle_1} \\ &= -\int d^{d'}\mathbf{s}\overline{\langle \Phi(t,\mathbf{s})\Phi(t,\mathbf{s}+\mathbf{d})\rangle_1^c}\end{aligned}$$

in terms of the properties of a *single* line, with $\Phi(t,\mathbf{s}) \equiv \delta^{d'}(\mathbf{r}(t) - \mathbf{s})$ and single line averages marked by the subscript 1. Here we have exploited the fact that without pair forces the two lines are independent in any realization of the disorder, and that the full correlator $\overline{\langle \Phi(t,\mathbf{s})\Phi(t,\mathbf{s}+\mathbf{d})\rangle_1}$ vanishes since a directed line has a single-valued path $r(t)$.

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