Berry phase in momentum space and Floquet systems

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Abstract

In the talk we explored a simple toy model which possesses different topological phases. Therefore we had to transfer the Berry phase concept to momentum space and introduce the Berry curvature. In the second part we saw an explicit realisation of this toy model via ultracold fermions in an optical lattice. To see that the experimental setup led to the same model, we used floquet theory.

I. Haldane model

We start our considerations with a simple 2D honeycomb lattice with nearest neighbour hopping in the tight binding approximation. The energy dispersion relation shows two points (K and K’) in the first Brillouin zone, where the upper and the lower band touch. To break the inversion symmetry of our model, we introduce a staggered potential +M for the sublattice A and −M for sublattice B. With this potential the dispersion relation is gapped at K and K’. On that basis we insert in our Hamiltonian complex next nearest neighbour hopping $t_2 e^{i\phi}$, thus we break the Time Reversal symmetry. With the two broken symmetries and the (2nd) nearest neighbour connections we obtain the Haldane Hamiltonian $\mathcal{H}$ [1]. We can tune the parameter $(M,\phi)$ in such a way that the gap at K (K’) vanishes. Therefore we obtain two lines in our $(M,\phi)$ parameter space, where the corresponding gaps close (see fig. 1). To get a deeper insight, we analyse the topology of our model. From the Berry connection $A(k)$ (using the eigenstates of $\mathcal{H}$) we can calculate the gauge independent Berry curvature $B(k) = \nabla_k \times A(k)$. The Berry curvature for the first Brillouin zone is also depicted in fig. 1 and you can observe differences in the topology between the regions in our parameter space. To distinguish these regions, we calculate the topological invariant (called Chern number)

$$C = \frac{1}{2\pi} \int_{\text{1st BZ}} d^2k \, B(k_x,k_y).$$

(1)

These distinct topological regions have tremendous consequences on the physical behaviour, like the quantum hall effect (even without external magnetic fields [1, 2]).

II. Experimental realisation

In 2014 this topological model was realised with ultracold fermions in an optical lattice [3]. They used a slightly distorted honey comb lattice to break the inversion symmetry and shook the optical lattice via piezos to obtain a effective Hamiltonian with complex 2nd nearest neighbour hopping. In the talk we saw explicitly that periodic lattice shaking yields the Haldane Hamiltonian. Therefore we had to introduce a solving strategy for time periodic Hamiltonians, the Floquet theory (further readings [4, 5]). In the last part we compared the results from the experiment with our theoretical predictions and we saw that on one hand side our closing lines for K and K’ corresponds to the results and on the other hand side that also the topological properties, due to different chern numbers, was observed in the experiment.

REFERENCES