Lösungen Blatt 7

1.) Komplexe Zahlen

1.1)

$$\begin{cases} i = 0 + 1 \cdot i \\ i = e^{i\frac{\pi}{2}} \end{cases} \qquad \begin{cases} i^2 = -1 = -1 + 0 \cdot i \\ i^2 = e^{i\pi} \end{cases}$$

$$\begin{cases} i^3 = -i = 0 - 1 \cdot i \\ i^3 = e^{i\frac{3\pi}{2}} \end{cases} \qquad \begin{cases} i^4 = i^2 \cdot i^2 = 1 = 1 + 0 \cdot i \\ i^4 = 1 \cdot e^{i0} = 1 \cdot e^{i2\pi} \end{cases}$$

$$\begin{cases} i^5 = i^4 \cdot i = i = 0 + 1 \cdot i \\ i^5 = e^{i\frac{\pi}{2}} = e^{i\frac{5\pi}{2}} \end{cases} \qquad \begin{cases} \frac{1}{i} = -i = 0 - 1 \cdot i \\ \frac{1}{i} = e^{i\frac{3\pi}{2}} \end{cases}$$

$$\begin{cases} 1 + i = 1 + 1 \cdot i \\ 1 + i = \sqrt{2} \cdot e^{i\frac{\pi}{4}} \end{cases} \qquad \begin{cases} 1 - i = 1 - 1 \cdot i \\ 1 - i = \sqrt{2} \cdot e^{-i\frac{\pi}{4}} \end{cases}$$

$$\begin{cases} (1+i)^2 = 2i = 0 + 2 \cdot i \\ (1+i)^2 = 2 \cdot e^{i\frac{\pi}{2}} \end{cases} \qquad \begin{cases} \frac{5}{4-3i} = 4 + 3i \\ \frac{5}{4-3i} = 5 \cdot e^{i\varphi} \end{cases}$$

$$\varphi = \arccos\left(\tfrac{4}{5}\right) \approx 0.64 \text{ rad}$$

1.2)

$$z_1 + z_2 = -5 - 3i + 1 + i = -4 - 2i$$

$$z_1 - z_2 = -5 - 3i - 1 - i = -6 - 4i$$

$$z_1 \cdot z_2 = (-5 - 3i)(1 + i) = -5 - 5i - 3i + 3 = -2 - 8i$$

$$\frac{z_1}{z_2} = \frac{-5-3i}{1+i} = \frac{(-5-3i)(1-i)}{2} = \frac{-8+2i}{2} = -4+i$$

1.3)

$$z^2 + 3z + \tfrac{9}{2} = 0$$

$$\Rightarrow z_{1/2} = -\frac{3}{2} \pm \sqrt{\frac{9}{4} - \frac{9}{2}}$$

$$= -\frac{3}{2} \pm \sqrt{-\frac{9}{4}}$$

$$= -\frac{3}{2} \pm i \cdot \frac{3}{2} = \frac{3}{2}(-1 \pm i)$$

1.4)

$$\frac{1+i}{1-i} = \frac{(1+i)^2}{2} = \frac{2i}{2} = i$$

$$(\,i\,)^* = -i$$

1.5)

$$z = -1-i$$

$$z^2 = (-1-i)(-1-i) = (1+i)^2 = 2i = 2 \cdot e^{i\,\pi/2}$$

$$\left(\sqrt[4]{z}\right)_k = 2^{\frac{1}{4}} \cdot e^{i\left(\frac{\frac{\pi}{2}+2k\pi}{2}\right)}, \quad k \in \{0,1\}$$

$$\left(\sqrt[4]{z}\right)_0 = 2^{\frac{1}{4}} \cdot e^{i\,\pi/4}$$

$$\left(\sqrt[4]{z}\right)_1 = 2^{\frac{1}{4}} \cdot e^{i\,\frac{5\pi}{4}}$$

1.6)

$\sqrt{(-1)\cdot(-1)} \neq \sqrt{-1}\cdot\sqrt{-1}$,

da $\sqrt{-1} = \pm i$ aber $\sqrt{1} = 1$

2.) Differentialgleichungen

a) $\dot{x}(t) = i\omega x_0 e^{i(\omega t+\varphi)}$

$\ddot{x}(t) = -\omega^2 x_0 e^{i(\omega t+\varphi)} = -\omega^2 x(t)$

$\Rightarrow m\ddot{x}(t) = -m\omega^2 x(t) \Rightarrow$ Lsg. wenn $D = m\omega^2$

b) $\rightarrow m\ddot{x}(t) = -Dx(t) + \Gamma\dot{x}(t)$

Amplitude x_0 kann nicht konstant sein)

$x_0 \rightarrow x_0(t)$