Week 7: Field theory on curved space-times

	Discussion of outline	Discussion of talk	Your talk
Important dates	before 20.11.2015	before 27.11.2015	04.12.2015

Your seminar talk should roughly cover the following keywords and concepts:

- This presentation is to a large extent based on the previous one. The focus here will be on how to couple various types of matter fields to gravity, especially spinor fields and gauge fields. For the former, one needs to make explicit use of the vielbein formulation of gravity since spin couples to torsion.
- One of the important points in this context is that spinors (as representations of the spin group [the double cover of the Lorentz group]) cannot be defined directly in curved space-time. Instead, one needs to employ the mapping to flat coordinates. The latter is done by means of the vielbein (see Week 05). The vielbein then permits to define "curved γ-matrices" out of the flat ones.
- Clifford algebras associated with vector spaces (in various dimensions) equipped with a quadratic form/scalar product and their representation(s). You may (but don't need to) focus on 2+1D and 3+1D, corresponding to graphene and general relativity.
- The $\mathbb{Z}_2$-grading (even/odd number of elements)
- The associated Pin group as the formal group generated by vectors with "norm" 1. The Spin group as the subgroup of even elements.
- The existence of an isomorphism between Spin(1,3) and SL(2,$\mathbb{C}$) (both have Lie algebra $sl_2 \oplus sl_2$)
- Representations of the spin groups, reducibility, the chirality operator and the (non-)existence of (chiral) Weyl spinors in even (odd) dimensions
- One could also speak about Majorana fermions in this context (not the one of Kitaev!!!) but this might lead too far
- Reminder on the vielbein and the spin connection
- Incorporation of spinors and Yang-Mills fields into general relativity (by means of the action). This mainly refers to the correct definition of the covariant derivative which now involves both the spin connection and the gauge field.
- The action always features $\sqrt{|\det g|}d^nx$, the diffeomorphism invariant volume element
- Physical relevance: Curved graphene sheets, light bending
- Not all space-times allow for globally defined frame fields. If a manifold does, it is said to possess a spin structure. There are certain (cohomological) obstructions which can be mentioned but should not be detailed.
- Give examples of spaces which admit spin structures (see Wikipedia and [1, Chapter 11.7])

Important aspects that should be emphasized:

- The focus should be on the coupling of spinor fields to a curved background
- Sheets of graphene provide a physical example where bending admits an effective description in terms of coupling to a gravitational field [2, 3].

Remarks:

- It is your task to turn the material related to your topic into a coherent story. This requires a detailed examination and understanding of the subject. Merely giving definitions without motivation and without pointing out the bigger picture is not sufficient.
- You will realize that time is rather limited and that you will need to focus on essentials.
- Personally, I am using 6-7 handwritten A4 pages for a 90 minutes lecture. It is recommended to aim at no more than 4-5 pages for your own presentation (and do not try to gain extra space by writing extra small).
- Please emphasize the physical ideas, not the mathematical formalism. Also avoid detailed calculations (except where they add to the conceptual understanding).
- In the two preparatory meetings you will be able to get feedback and assistance by your supervisor before you give your presentation, both on content and style. In order to maximize the benefit of these meetings it is important that you are well prepared.
- In view of the overlap with the previous talk it might be a good idea that you prepare part of the material together. There is also some freedom with regard to how to split the material into two presentations.
- I am also happy if the more applied aspects (e.g. graphene) are given more space than indicated from my list above. Suitable references here might be (parts of!) [4, 2, 3]. In particular, the last one claims to be pedagogical while the second is rather short.
- The story about exploring effects of curved spaces in graphene recently became a research theme by itself. There is thus a vast literature (which I don't know) and therefore it is really important to focus on some simple and basic facts.

References:

- With regard to the spinor stuff I would suggest to use the book of Göckerler and Schücker [1], Chapters 5 and 11. Chapter 5 can be studied and discussed together with Sebastian.
- Wikipedia (to get a quick overview)
  - Clifford algebra
  - Spinor
  - Spin structure
  - Einstein-Cartan theory
  - Cartan formalism (physics)
  - Maxwell's equations in curved spacetime

References

[1] M. Göckeler and T. Schücker, Differential Geometry, Gauge Theories, and Gravity. Cambridge University Press, 1987.

[2] A. Mesaros, D. Sadri and J. Zaanen, *Parallel transport of electrons in graphene parallels gravity*, Phys. Rev. **B82** (Aug., 2010) 073405 [0909.2703].

[3] M. A. H. Vozmediano, M. I. Katsnelson and F. Guinea, *Gauge fields in graphene*, Phys. Rep. **496** (Nov., 2010) 109–148 [1003.5179].

[4] A. Iorio and G. Lambiase, *Quantum field theory in curved graphene spacetimes, Lobachevsky geometry, Weyl symmetry, Hawking effect, and all that*, Phys. Rev. **D90** (July, 2014) 025006 [1308.0265].