# Week 14: Wilson loops and knot invariants

| | Discussion of outline | Discussion of talk | Your talk |
|---|---|---|---|
| Important dates | before 22.1.2016 | before 29.1.2016 | 5.2.2016 |

**Your seminar talk should roughly cover the following keywords and concepts:**

- What are knots and links?
- The concept of a knot polynomial as a topological invariant (is there a simple way to tell two knots apart?)
- Pictures of knots in the plane, skein relations, invariance under Reidemeister moves etc.
- Examples: Kauffman bracket and Jones polynomial
- The claimed relation to (quantum) Chern-Simons theory, identification of parameters
- Wilson loops as natural gauge invariant observables
  - Non-locality
  - Why are they gauge invariant?
  - Dependence on representations
  - Independence on curves except for topology (linking)
- Links and the associated partition function (see Witten)
- Symmetries of the partition function under reversal of orientation, change of representation etc. (see Witten, page 356)
- A U(1) gauge group gives rise to linking numbers
- Self-linking needs regularization in terms of framing (see Witten)
- Knot polynomial interpretation for SU(N) [two-variable Jones polynomial] and SO(N) [Kauffman polynomial] Chern-Simons theory when evaluating Wilson loops for the fundamental representation
- Potentially: Vassiliev invariants from perturbative Chern-Simons theory

**Important aspects that should be emphasized:**

- There are no reasonable local gauge invariant observables in Chern-Simons theory (Witten says on page 354 that they would spoil general covariance since their definition requires a metric)
- Note the quantum renormalization from $k$ to $k+N$ in SU(N) Chern-Simons theory

**Remarks:**

- It is your task to turn the material related to your topic into a coherent story. This requires a detailed examination and understanding of the subject. Merely giving definitions without motivation and without pointing out the bigger picture is not sufficient.
- You will realize that time is rather limited and that you will need to focus on essentials.
- Personally, I am using 6-7 handwritten A4 pages for a 90 minutes lecture. It is recommended to aim at no more than 4-5 pages for your own presentation (and do not try to gain extra space by writing extra small).
- Please emphasize the physical ideas, not the mathematical formalism. Also avoid detailed calculations (except where they add to the conceptual understanding).
- In the two preparatory meetings you will be able to get feedback and assistance by your supervisor before you give your presentation, both on content and style. In order to maximize the benefit of these meetings it is important that you are well prepared.

**References:**

- There are some useful remarks in the review [1], see Sections III.C
- Another rather accessible exposition is the appendix at pages 724ff in the book of Kauffman [2] (this is generally a nice book).
- A brief summary of knot invariants is contained in a little paper by Kauffman [3].
- If you have access to it you can check out Chapter II.5 in the book of Baez and Muniain [4]
- The original reference about the quantization of Chern-Simons theory and the relation to knot invariants is [5]. However, this goes way beyond the scope of our seminar.
- Wikipedia (to get a quick overview)
  - Kauffman polynomial
  - Jones polynomial
  - Chern-Simons theory

# References

[1] C. Nayak, S. H. Simon, A. Stern, M. Freedman and S. Das Sarma, *Non-Abelian anyons and topological quantum computation*, Reviews of Modern Physics **80** (July, 2008) 1083–1159 [0707.1889].

[2] L. Kauffman, Knots and Physics. World Scientific, 3rd ed., 2001.

[3] L. H. Kauffman, *State models and the Jones polynomial*, Topology **26** (1987), no. 3 395–407.

[4] J. Baez and J. Muniain, Gauge fields, knots and gravity. World Scientific, 1994.

[5] E. Witten, *Quantum field theory and the Jones polynomial*, Comm. Math. Phys. **121** (Sept., 1989) 351–399.