Splitting Kramers degeneracy with superconducting phase difference

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Plan

Using phase difference in a Josephson junction as a means of breaking time reversal symmetry.

- What does ‘breaking time reversal’ mean?
- Why it won’t work.
- How to make it work (and why 3 is much better than 2)?

Time reversal breaking in a mesoscopic JJ

Several manifestations:

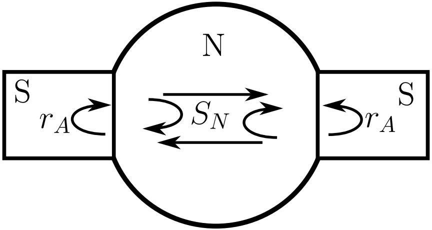
- Splitting of Kramer's degeneracy
  (Chtchelkatchev&Nazarov, Béri&Bardarson&Beenakker)
- Closing of the induced gap
- Protected zero energy level crossings (switches in the ground state fermion parity)

$$\mathcal{P} = \mathrm{Pf}(iH)$$

- Spectral peak in the DOS (Ivanov, Altland&Bagrets)

$$\rho(E) = \rho_0 \left(1 + \frac{\sin(2\pi E/\delta)}{2\pi E/\delta}\right)$$

Setup and formalism



Scattering matrices of electrons and holes:

$$S_h(-E) = S_e^*(E)$$

Andreev reflection matrix:

$$r_A = ie^{i\phi_i}$$

Bound state condition:

$$S_e(E) r_A S_h(E) r_A^* \psi = e^{-2i \arccos(E/\Delta)} \psi$$

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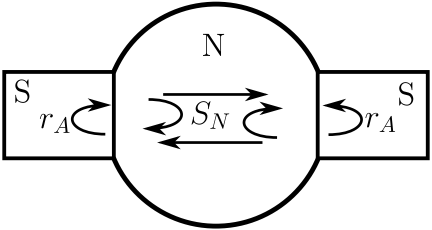

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Short junction limit

$$S(E) \approx S(0)$$

Lowest density of Andreev states, strongest effect phase difference on a single state.

Due to unitarity and time reversal symmetry of S the energies are given by $E_n = \pm\Delta\sqrt{1 - T_n \sin^2(\phi/2)}$ (Beenakker)

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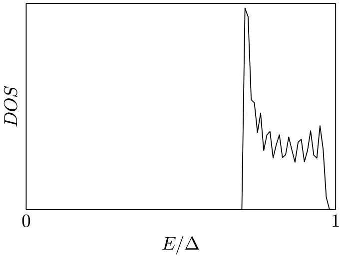

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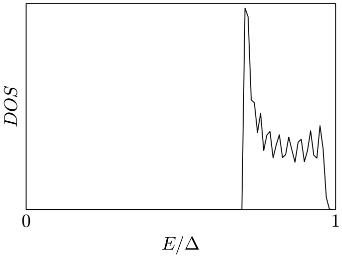

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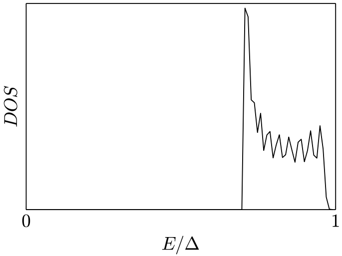

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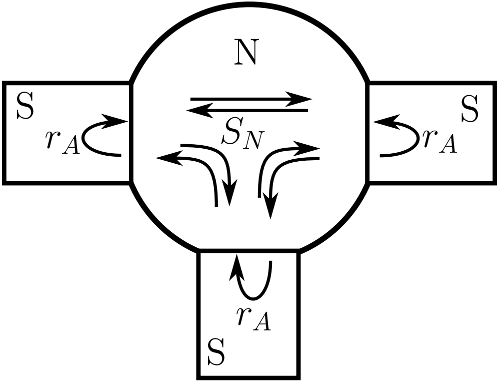
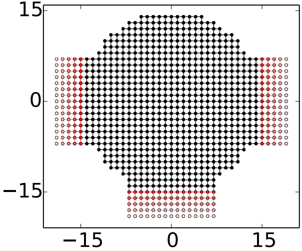
A big improvement



All the special properties of the spectrum originate from the small number of leads!

Kramers degeneracy splitting

Take a Rashba quantum dot with $E \sim E_{SO}$, $R \gtrsim l_{SO}$, and $\lambda \lesssim R$



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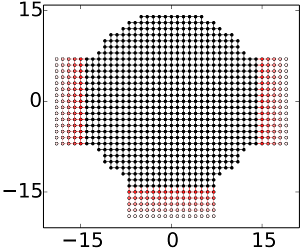

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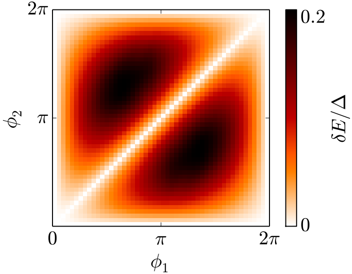
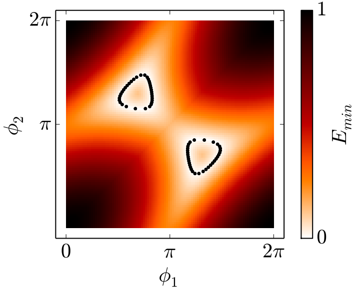


✓Kramers degeneracy is strongly broken.

Protected level crossings

Once again, try a random quantum dot:



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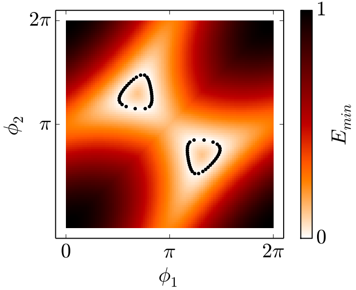

✓Level crossings are allowed.

Protected level crossings

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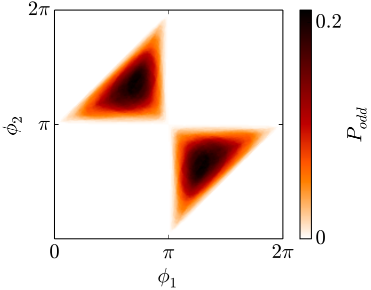

No: the gap may only close when all the clockwise phase differences are smaller (or larger) than π.

(Note that this result holds for any junction)

Proof

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5. The necessary and sufficient condition for existence of a unitary S:
$$\exists \psi, \psi' : \langle\psi|\ r_A\ |\psi\rangle + \langle\psi'|\ r_A\ |\psi'\rangle = \frac{2|E|}{\Delta}\ e^{i\chi}\ \langle\psi'|\psi\rangle\ .$$

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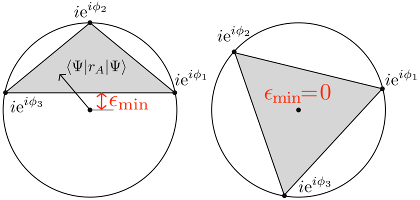
2. Graphical solution:



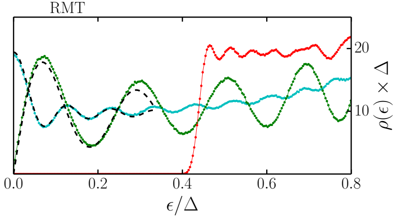
no vortex　　　discrete vortex

3. The lower bound on the gap:
$$E \geq \Delta \min_{i,j} \cos \frac{\phi_i - \phi_j}{2}$$

Gap closing and the spectral peak

Both phenomena are visible

✓In the ensemble (averaging over random antisymmetric S)



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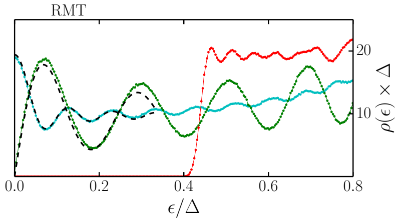
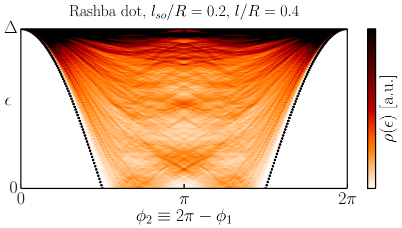
Both phenomena are visible

✓In the ensemble (averaging over random antisymmetric S)

✓In a single realization (averaging over chemical potential)





Conclusions

- Superconducting phase difference can strongly break time reversal symmetry in a Josephson junction.
- This requires more than two superconducting leads.
- Spin degeneracy is split by a large fraction of Δ.
- The induced superconducting gap only closes in a a finite subregion of the phase space.

Conclusions

Thank you all.
The end.