Topological Kondo effect in Majorana devices

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Overview

Coulomb charging effects on quantum transport in a Majorana device:

„Topological Kondo effect“ with stable non-Fermi liquid behavior *Beri & Cooper, PRL 2012*

- With interactions in the leads: new unstable fixed point
  Altland & Egger, PRL 2013
  Zazunov, Altland & Egger, New J. Phys. 2014
- ‚Majorana quantum impurity spin‘ dynamics near strong coupling *Altland, Beri, Egger & Tsvelik, PRL 2014*
- Non-Fermi liquid manifold: coupling to bulk superconductor *Eriksson, Mora, Zazunov & Egger, PRL 2014*

Majorana bound states

Beenakker, Ann. Rev. Con. Mat. Phys. 2013
Alicea, Rep. Prog. Phys. 2012
Leijnse & Flensberg, Semicond. Sci. Tech. 2012

- Majorana fermions
  - Non-Abelian exchange statistics $\gamma_j = \gamma_j^+ \quad \{\gamma_i, \gamma_j\} = 2\delta_{ij}$
  - Two Majoranas = nonlocal fermion $d = \gamma_1 + i\gamma_2$
  - Occupation of single Majorana ill-defined: $\gamma^+\gamma = \gamma^2 = 1$
  - Count state of Majorana pair $d^+d = 0,1$
- Realizable (for example) as end states of spinless 1D p-wave superconductor (Kitaev chain)
  - Recipe: Proximity coupling of 1D helical wire to s-wave superconductor
  - For long wires: Majorana bound states are zero energy modes

Experimental Majorana signatures

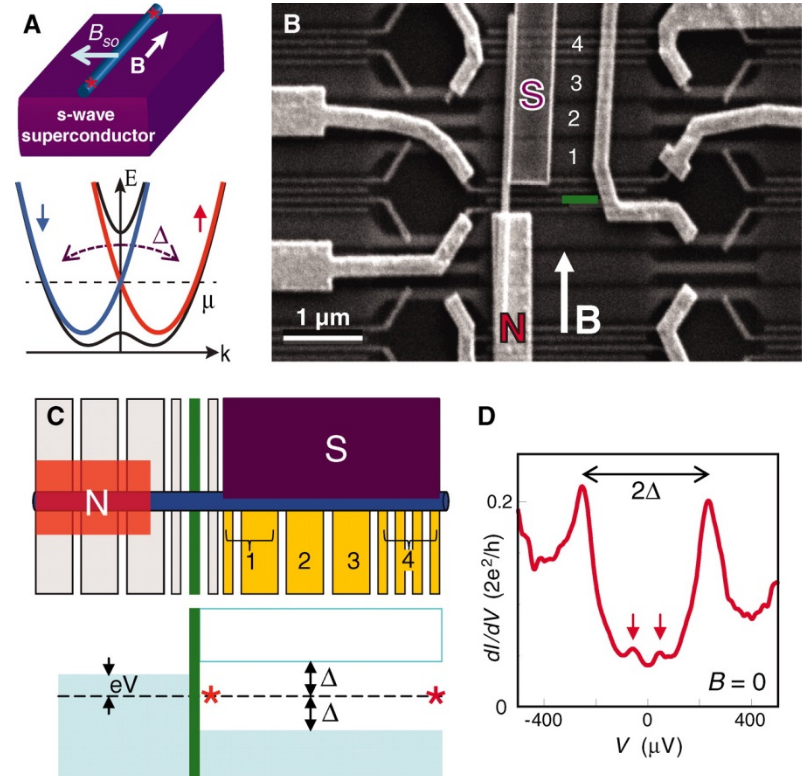
InSb nanowires expected to host Majoranas due to interplay of
- strong Rashba spin orbit field
- magnetic Zeeman field
- proximity-induced pairing

Oreg, Refael & von Oppen, PRL 2010
Lutchyn, Sau & Das Sarma, PRL 2010

Transport signature of Majoranas: Zero-bias conductance peak due to resonant Andreev reflection

Bolech & Demler, PRL 2007
Law, Lee & Ng, PRL 2009
Flensberg, PRB 2010

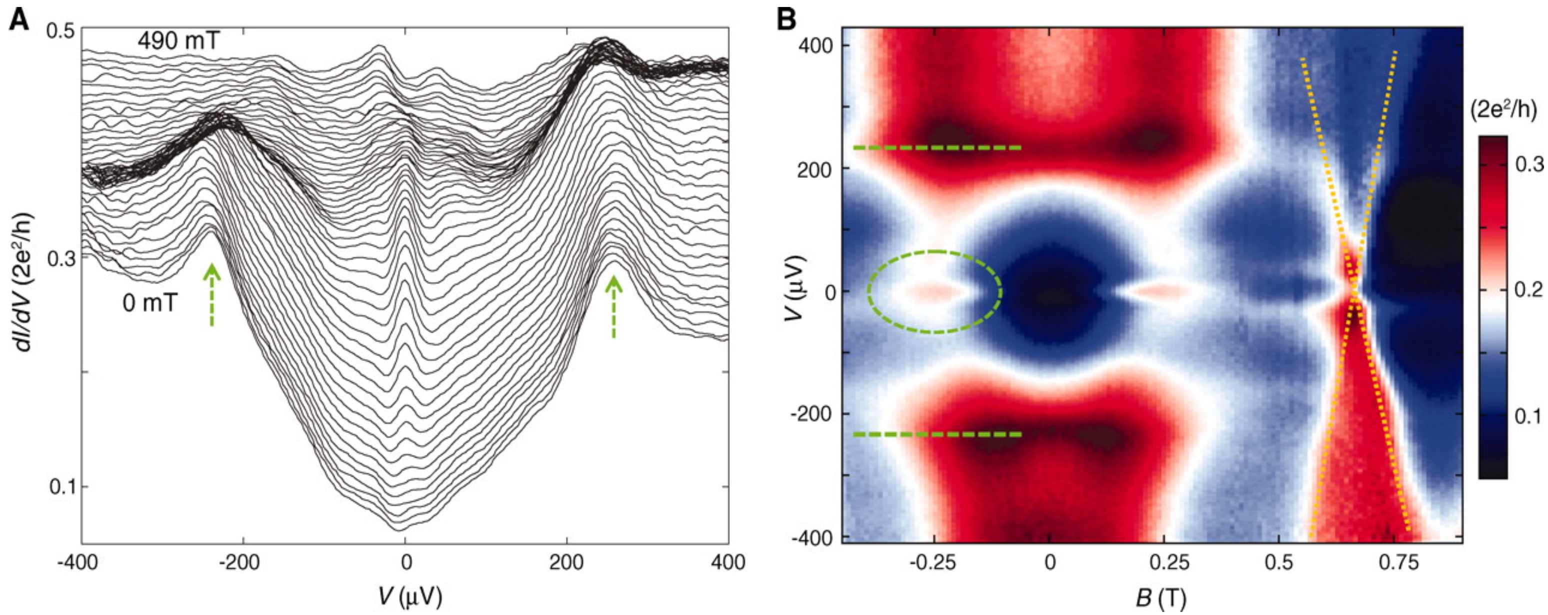
Mourik et al., Science 2012



See also: *Rokhinson et al., Nat. Phys. 2012; Deng et al., Nano Lett. 2012; Das et al., Nat. Phys. 2012; Churchill et al., PRB 2013*

Zero-bias conductance peak

Mourik et al., Science 2012

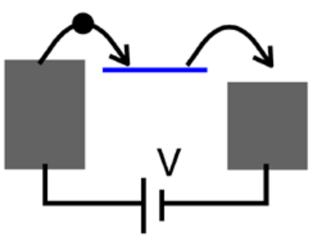


Possible explanations:

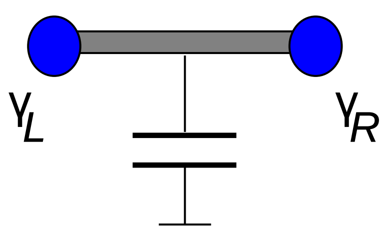
- Majorana state (most likely!)
- Disorder-induced peak — *Bagrets & Altland, PRL 2012*
- Smooth confinement — *Kells, Meidan & Brouwer, PRB 2012*
- Kondo effect — *Lee et al., PRL 2012*

Suppose that Majorana mode is realized…

- Quantum transport features beyond zero-bias anomaly peak? Coulomb interaction effects?
- Simplest case: Majorana single charge transistor
  - ‚Overhanging‘ helical wire parts serve as normal-conducting leads



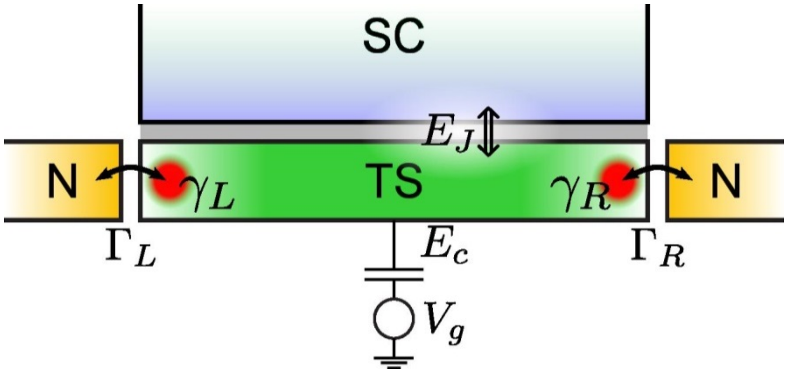
  - Nanowire part coupled to superconductor hosts pair of Majorana bound states
  - Include charging energy of this ‚dot‘



Majorana single charge transistor

Hützen et al., PRL 2012

- Floating superconducting ‚dot' contains two Majorana bound states tunnel-coupled to normal-conducting leads
- Charging energy finite



- Consider universal regime:
  - Long superconducting wire:
    Direct tunnel coupling between left and right Majorana modes is assumed negligible
  - No quasi-particle excitations:
    Proximity-induced gap is largest energy scale of interest

Hamiltonian: charging term

- Majorana pair: nonlocal fermion $d = \gamma_L + i\gamma_R$
- Condensate gives another zero mode
  - Cooper pair number N_c, conjugate phase ϕ
- Dot Hamiltonian (gate parameter n_g)

$$H_{island} = E_C\left(2N_c + d^+ d - n_g\right)^2$$

Majorana fermions couple to Cooper pairs through the charging energy

Tunneling

- Normal-conducting leads: effectively spinless helical wire
  - Applied bias voltage V = chemical potential difference
- Tunneling of electrons from lead to dot:
  - Project electron operator in superconducting wire part to Majorana sector
  - Spin structure of Majorana state encoded in tunneling matrix elements

Flensberg, PRB 2010

Tunneling Hamiltonian

Source (drain) couples to left (right) Majorana only:

$$H_t = \sum_{j=L,R} t_j c_j^+ \eta_j + h.c. \qquad \eta_j = \left(d \pm e^{-i\phi} d^+\right)/2$$

- respects current conservation
- Hybridizations: $\Gamma_j \sim \nu \left| t_j \right|^2$

Normal tunneling $\sim c^+ d, d^+ c$

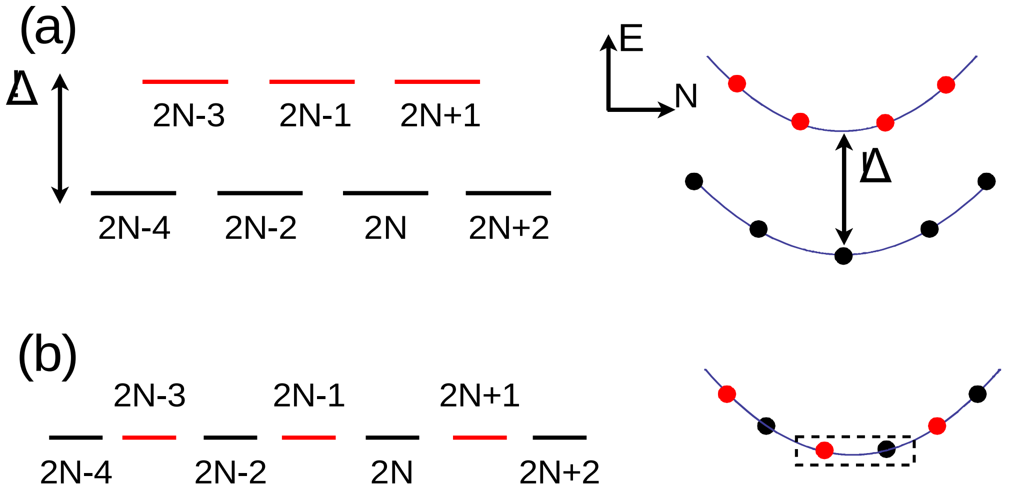
- Either destroy or create nonlocal d fermion
- Condensate not involved

Anomalous tunneling $\sim c^+ e^{-i\phi} d^+, d e^{i\phi} c$

- Create (destroy) both lead and d fermion
  & split (add) a Cooper pair

Absence of even-odd effect

- Without Majorana states: Even-odd effect
- With Majoranas: no even-odd effect!
  - Tuning wire parameters into the topological phase removes even-odd effect



picture from: Fu, PRL 2010

Noninteracting case: Resonant Andreev reflection

Bolech & Demler, PRL 2007
Law, Lee & Ng, PRL 2009

- $E_c=0$ Majorana spectral function

$$-\operatorname{Im} G^{ret}_{\gamma_j}(\varepsilon) = \frac{\Gamma_j}{\varepsilon^2 + \Gamma_j^2}$$

- T=0 differential conductance:

$$G(V) = \frac{2e^2}{h}\frac{1}{1+(eV/\Gamma)^2}$$

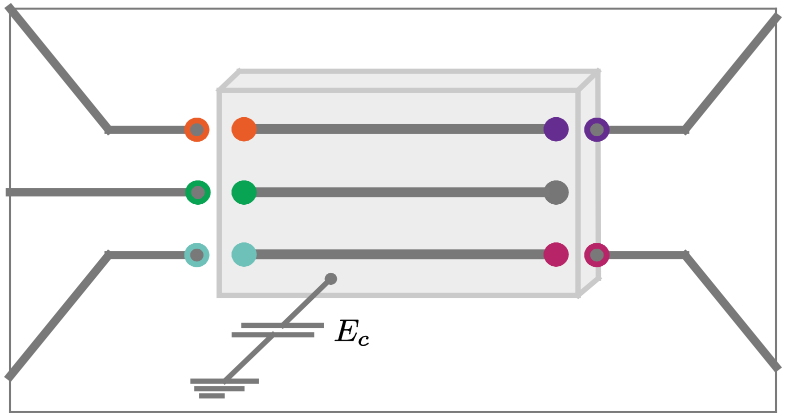
- Currents I_L and I_R fluctuate independently, superconductor is effectively grounded
- Perfect Andreev reflection via Majorana state
  - Zero-energy Majorana bound state leaks into lead

Strong blockade: Electron teleportation

Fu, PRL 2010

- Peak conductance for half-integer n_g
- Strong charging energy then allows only two degenerate charge configurations
- Model maps to spinless resonant tunneling model
- Linear conductance (T=0): $G = e^2 / h$
- Interpretation: Electron teleportation due to nonlocality of d fermion

Topological Kondo effect



Beri & Cooper, PRL 2012
Altland & Egger, PRL 2013
Beri, PRL 2013
Altland, Beri, Egger & Tsvelik, PRL 2014
Zazunov, Altland & Egger, NJP 2014

- Now N>1 helical wires: M Majorana states tunnel-coupled to helical Luttinger liquid wires with g≤1
- Strong charging energy, with nearly integer n_g: unique equilibrium charge state on the island
- 2^{N-1}-fold ground state degeneracy due to Majorana states (taking into account parity constraint)
  - Need N>1 for interesting effect!

„Klein-Majorana fusion“

- Abelian bosonization of lead fermions
  - Klein factors are needed to ensure anticommutation relations between different leads
  - Klein factors can be represented by additional Majorana fermion for each lead
- Combine Klein-Majorana and ‚true‘ Majorana fermion at each contact to build auxiliary fermions, f_j
- All occupation numbers $f_j^+ f_j$ are conserved and can be gauged away
- purely bosonic problem remains…

Charging effects: dipole confinement

- High energy scales $> E_C$: charging effects irrelevant
  - Electron tunneling amplitudes from lead j to dot renormalize independently upwards $$t_j(E) \sim E^{-1+1/2g}$$
  - RG flow towards resonant Andreev reflection fixed point
- For $E < E_C$: charging induces ‚confinement'
  - In- and out-tunneling events are bound to ‚dipoles' with coupling $\lambda_{j \neq k}$: entanglement of different leads
  - Dipole coupling describes amplitude for ‚cotunneling' from lead j to lead k
  - ‚Bare' value $$\lambda_{jk}^{(1)} = \frac{t_j(E_C)\, t_k(E_C)}{E_C} \sim E_C^{-3+1/g}$$ large for small E_C

RG equations in dipole phase

- Energy scales below E_C: effective phase action

$$S=\frac{g}{2\pi}\sum_j\int\frac{d\omega}{2\pi}|\omega||\Phi_j(\omega)|^2-\sum_{j\neq k}\lambda_{jk}\int d\tau\cos(\Phi_j-\Phi_k)$$

- One-loop RG equations

Lead DoS

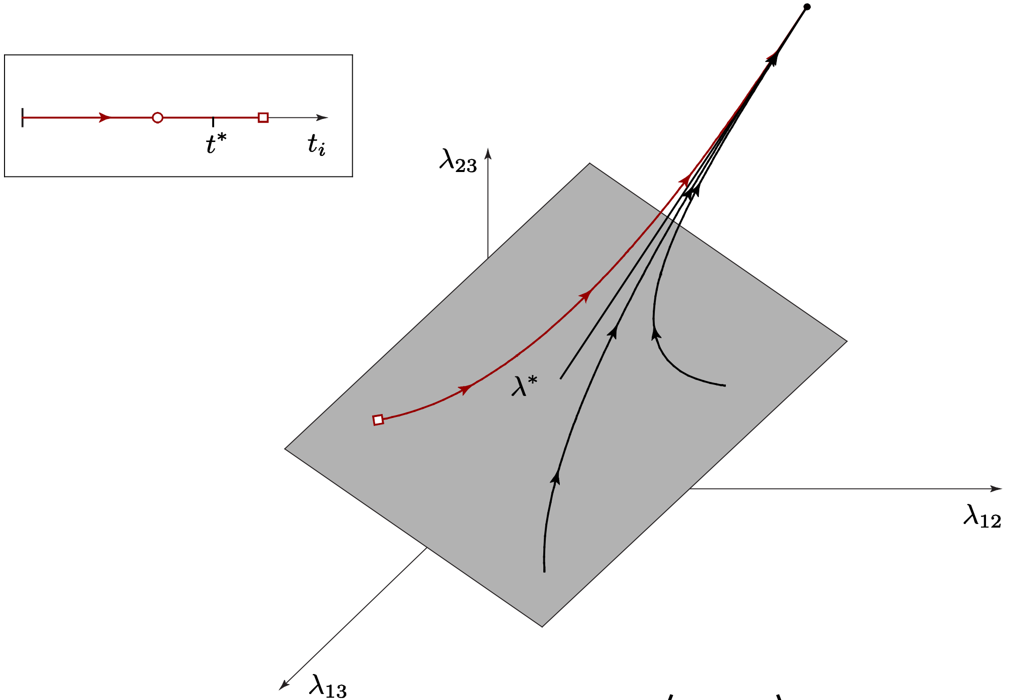
$$\frac{d\lambda_{jk}}{dl}=-(g^{-1}-1)\lambda_{jk}+\nu\sum_{m\neq(j,k)}^{M}\lambda_{jm}\lambda_{mk}$$

  - suppression by Luttinger liquid tunneling DoS
  - enhancement by dipole fusion processes

- RG-unstable intermediate fixed point with isotropic couplings (for M>2 leads)

$$\lambda_{j\neq k}=\lambda^*=\frac{g^{-1}-1}{M-2}\nu$$

RG flow



- RG flow towards strong coupling for $\left\langle \lambda^{(1)} \right\rangle > \lambda^*$
  Always happens for moderate charging energy
- Flow towards isotropic couplings: anisotropies are RG irrelevant
- Perturbative RG fails below Kondo temperature

$$T_K \approx E_C e^{-\lambda^* / \left\langle \lambda^{(1)} \right\rangle}$$

Topological Kondo effect

Beri & Cooper, PRL 2012

- Refermionize for g=1:

$$H = -i\int_{-\infty}^{\infty} dx \sum_{j=1}^{M} \psi_j^{+} \partial_x \psi_j + i\lambda \sum_{j\neq k} \psi_j^{+}(0) S_{jk} \psi_k(0)$$

- Majorana bilinears $S_{jk} = i\gamma_j \gamma_k$
  - ‚Reality' condition: SO(M) symmetry [instead of SU(2)]
  - nonlocal realization of ‚quantum impurity spin'
  - Nonlocality ensures stability of Kondo fixed point

Majorana basis $\psi(x) = \mu(x) + i\xi(x)$ for leads:

$SO_2(M)$ Kondo model

$$H = -i\int dx \mu^T \partial_x \mu + i\lambda \mu^T(0) \hat{S} \mu(0) + [\mu \leftrightarrow \xi]$$

Minimal case: M=3 Majorana states

- SU(2) representation of „quantum impurity spin“

$$S_j = \frac{i}{4}\varepsilon_{jkl}\gamma_k\gamma_l \qquad [S_1, S_2] = iS_3$$

- Spin S=1/2 operator, nonlocally realized in terms of Majorana states
  - can be represented by Pauli matrices
- Exchange coupling of this spin-1/2 to two SO(3) lead currents $\rightarrow$

  multichannel Kondo effect

Transport properties near unitary limit

- Temperature & voltages < T_K:
  - Dual instanton version of action applies near strong coupling limit
  - Nonequilibrium Keldysh formulation
- Linear conductance tensor

$$G_{jk} = e\frac{\partial I_j}{\partial \mu_k} = \frac{2e^2}{h}\left(1-\left(T/T_K\right)^{2y-2}\right)\left[\delta_{jk}-\frac{1}{M}\right]$$

  - Non-integer scaling dimension $y = 2g\left(1-\frac{1}{M}\right) > 1$ implies non-Fermi liquid behavior even for g=1
  - completely isotropic multi-terminal junction

Correlated Andreev reflection

- Diagonal conductance at T=0 exceeds resonant tunneling („teleportation“) value but stays below resonant Andreev reflection limit

$$G_{jj} = \frac{2e^2}{h}\left(1 - \frac{1}{M}\right) \Rightarrow \frac{e^2}{h} < G_{jj} < \frac{2e^2}{h}$$

- Interpretation: Correlated Andreev reflection
- Remove one lead: change of scaling dimensions and conductance
- Non-Fermi liquid power-law corrections at finite T

Fano factor

Zazunov et al., NJP 2014

- Backscattering correction to current near unitary limit for $\sum_j \mu_j = 0$

$$\delta I_j = -\frac{e}{\hbar}\sum_k \left|\frac{\mu_k}{T_K}\right|^{2y-2}\left(\delta_{jk} - \frac{1}{M}\right)\mu_k$$

- Shot noise: $\tilde{S}_{jk}(\omega \to 0) = \int dt\, e^{i\omega t}\left(\langle I_j(t) I_k(0)\rangle - \langle I_j\rangle\langle I_k\rangle\right)$

$$\tilde{S}_{jk} = -\frac{2ge^2}{\hbar}\sum_l \left(\delta_{jl} - \frac{1}{M}\right)\left(\delta_{kl} - \frac{1}{M}\right)\left|\frac{\mu_l}{T_K}\right|^{2y-2}|\mu_l|$$

- universal Fano factor, but different value than for SU(N) Kondo effect

Sela et al. PRL 2006; Mora et al., PRB 2009

Majorana spin dynamics

Altland, Beri, Egger & Tsvelik, PRL 2014

- Overscreened multi-channel Kondo fixed point: massively entangled effective impurity degree remains at strong coupling: „Majorana spin“
- Probe and manipulate by coupling of Majoranas

$$H_Z = \sum_{jk} h_{jk} S_{jk}$$

  - ‚Zeeman fields‘ $h_{jk} = -h_{kj}$: overlap of Majorana wavefunctions within same nanowire
  - Couple to $S_{jk} = i\gamma_j \gamma_k$

Majorana spin near strong coupling

Bosonized form of Majorana spin at Kondo fixed point:

$$S_{jk} = i\gamma_j\gamma_k \cos[\Theta_j(0) - \Theta_k(0)]$$

- Dual boson fields $\Theta_j(x)$ describe ‚charge' (not ‚phase') in respective lead
- Scaling dimension $y_Z = 1 - \frac{2}{M}$ $\rightarrow$ RG relevant
- Zeeman field ultimately destroys Kondo fixed point & breaks emergent time reversal symmetry
- Perturbative treatment possible for $T_h < T < T_K$

dominant 1-2 Zeeman coupling: $T_h = \left(\frac{h_{12}}{T_K}\right)^{M/2} T_K$

Crossover SO(M)→SO(M-2)

- Lowering T below T_h → crossover to another Kondo model with SO(M-2) (Fermi liquid for M<5)
  - Zeeman coupling h_{12} flows to strong coupling → γ_1, γ_2 disappear from low-energy sector
  - Same scenario follows from Bethe ansatz solution

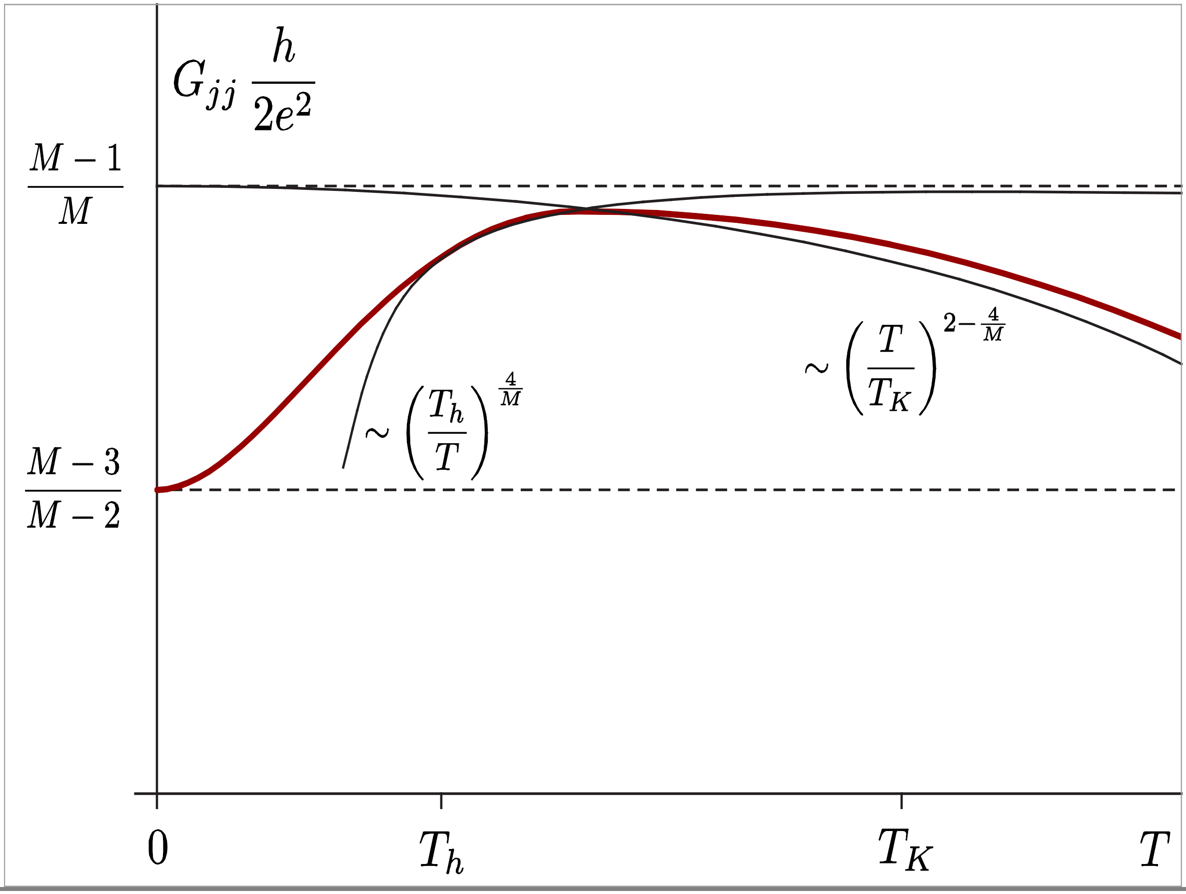
Altland, Beri, Egger & Tsvelik, JPA 2014

- Observable in conductance & in thermodynamic properties

SO(M)→SO(M-2): conductance scaling

for single Zeeman component $h_{12} \neq 0$ consider G_{jj} $(j \neq 1,2)$

(diagonal element of conductance tensor)



Multi-point correlations

- Majorana spin has nontrivial multi-point correlations at Kondo fixed point, e.g. for M=3 (absent for SU(N) case!)

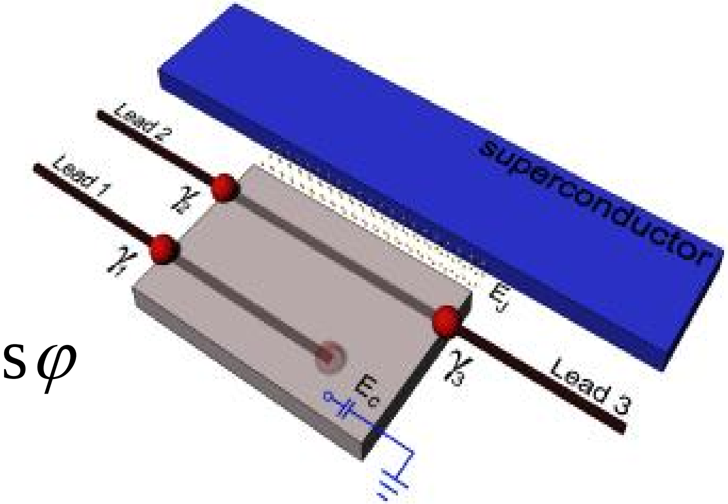
$$\left\langle T_\tau S_j(\tau_1) S_k(\tau_2) S_l(\tau_3) \right\rangle \sim \frac{\varepsilon_{jkl}}{T_K \left(\tau_{12}\tau_{13}\tau_{23}\right)^{1/3}}$$

- Observable consequences for time-dependent ‚Zeeman' field $B_j = \varepsilon_{jkl} h_{kl}$ with $\vec{B}(t) = \left(B_1 \cos(\omega_1 t), B_2 \cos(\omega_2 t), 0\right)$
  - Time-dependent gate voltage modulation of tunnel couplings
  - Measurement of ‚magnetization' by known read-out methods
  - Nonlinear frequency mixing $\left\langle S_3(t) \right\rangle \sim B_1 B_2 \cos[(\omega_1 \pm \omega_2)t]$
  - Oscillatory transverse spin correlations (for B_2=0)

$$\left\langle S_2(t) S_3(0) \right\rangle \sim B_1 \frac{\cos(\omega_1 t)}{(\omega_1 t)^{2/3}}$$

Adding Josephson coupling: Non Fermi liquid manifold

Eriksson, Mora, Zazunov & Egger, PRL 2014



$$H_{island} = E_C\left(2N_c + \hat{n} - n_g\right)^2 - E_J \cos\varphi$$

Yet another bulk superconductor: Topological Cooper pair box

Effectively harmonic oscillator for $E_J >> E_C$

with Josephson plasma oscillation frequency: $\Omega = \sqrt{8E_J E_C}$

Low energy theory

- Tracing over phase fluctuations gives two coupling mechanisms:
  - Resonant Andreev reflection processes

$$H_A = \sum_j t_j \gamma_j \left(\psi_j^+(0) - \psi_j(0) \right)$$

  - Kondo exchange coupling, but of $SO_1(M)$ type

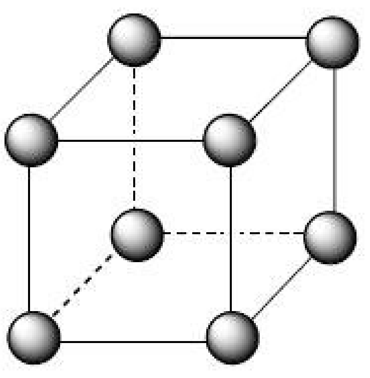
$$H_K = \sum_{j \neq k} \lambda_{jk} \left(\psi_j^+(0) + \psi_j(0) \right) \left(\psi_k^+(0) + \psi_k(0) \right) \gamma_j \gamma_k$$

- Interplay of resonant Andreev reflection and Kondo screening for $\Gamma < T_K$

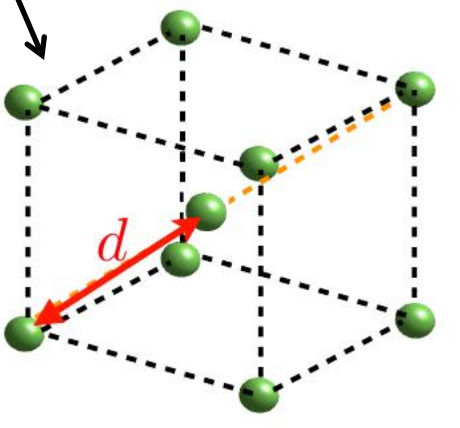
Quantum Brownian Motion picture

Abelian bosonization now yields (M=3)

$$H_A + H_K \propto -\sum_j \sqrt{\Gamma_j}\sin\Phi_j - \sqrt{T_K}\sum_{j\neq k}\cos\Phi_j\cos\Phi_k$$



Simple cubic lattice



bcc lattice

Quantum Brownian motion

- Leading irrelevant operator (LIO): tunneling transitions connecting nearest neighbors
- Scaling dimension of LIO from n.n. distance d

$$y_{LIO} = \frac{d^2}{2\pi^2}$$

Yi & Kane, PRB 1998

  - Pinned phase field configurations correspond to Kondo fixed point, but unitarily rotated by resonant Andreev reflection corrections
- Stable non-Fermi liquid manifold as long as LIO stays irrelevant, i.e. for $y_{LIO} > 1$

Scaling dimension of LIO

- M-dimensional manifold of non-Fermi liquid states spanned by parameters $\delta_j = \sqrt{\Gamma_j / T_K}$
- Scaling dimension of LIO

$$y = \min\left\{2, \frac{1}{2}\sum_{j=1}^{M}\left[1 - \frac{2}{\pi}\arcsin\left(\frac{\delta_j}{2(M-1))}\right)\right]\right\}$$

  - Stable manifold corresponds to y>1
  - For y<1: standard resonant Andreev reflection scenario applies
  - For y>1: non-Fermi liquid power laws appear in temperature dependence of conductance tensor

Conclusions

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- ‚Majorana quantum impurity spin‘ dynamics near strong coupling *Altland, Beri, Egger & Tsvelik, PRL 2014*
- Non-Fermi liquid manifold: coupling to bulk superconductor *Eriksson, Mora, Zazunov & Egger, PRL 2014*

THANK YOU FOR YOUR ATTENTION