Proximity-induced magnetization dynamics, interaction effects, and phase transitions on a topological surface

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Work done in collaboration with:

- F. Nogueira @ *Theoretische Physik III, Ruhr-Uni Bochum*

Flavio S. Nogueira, Ilya Eremin, Phys. Rev. Lett. 109, 237203 (2012); Phys. Rev. B 88, 055126 (2013); Phys. Rev. B 90, 014431 (2014)

TPQM 2014, Vienna, 11.09.2014

Electrodynamics of the 3 dimensional insulator

Inside the usual insulator the action is

$$S_{EM} = \frac{1}{8\pi}\int d^3x dt \left(\varepsilon \mathbf{E}^2 - \frac{1}{\mu}\mathbf{B}^2 \right)$$

The integrand depends on geometry (easy to see if written in terms of electromagnetic tensor $F_{\mu\nu}$)

$$S_{EM} = \frac{1}{16\pi}\int d^3x dt \, F_{\mu\nu}F^{\mu\nu}$$

Summation over the repeated indices depends on the metric tensor (geometry)

What about topological insulators?

Electrodynamics of the topological insulator

In 2d+1 topological insulator (class A) there is another term

$$j_i = \sigma_H \epsilon^{ij} E_j \Rightarrow \frac{\partial \rho}{\partial t} = -\nabla \cdot \mathbf{j} = -\sigma_H \nabla \times \mathbf{E} = \sigma_H \frac{\partial B}{\partial t}$$

In a covariant form

$$j^\mu = \frac{\sigma_H}{2\pi} \epsilon^{\mu\nu\tau} \partial_\nu A_\tau$$

$\mu, \nu, \tau = 0, 1, 2$ are temporal and spatial indices

$$S_{\rm eff} = \frac{\sigma_H}{4\pi} \int d^2x \int dt A_\mu \epsilon^{\mu\nu\tau} \partial_\nu A_\tau$$

Description in terms of Chern-Simons topological FT

Electrodynamics of the topological insulator

In a 3d+1 Z_2 topological insulator (class AII) there is another term (θ-term)

$$S_\theta = \frac{e^2}{4\pi\hbar c^2}\int d^3 r dt \theta \varepsilon^{\mu\nu\sigma\tau}\partial_\mu A_\nu \partial_\sigma A_\tau = \frac{e^2}{2\pi\hbar c^2}\int d^3 r dt \theta \mathbf{E}\cdot\mathbf{B}$$

- does not depend on the metric but only on the topology of the underlying space
- serves as an alternative definition of the non-trivial topological insulator

X.-L. Qi, T. L. Hughes, and S.-C. Zhang, PRB 78, 195424 (2008)
A.M. Essin, J. E. Moore, and D. Vanderbilt, PRL 102, 146805 (2009)

Electrodynamics of the topological insulator

$$S_\theta = \frac{e^2}{4\pi\hbar c^2}\int d^3 r dt \theta \varepsilon^{\mu\nu\sigma\tau}\partial_\mu A_\nu \partial_\sigma A_\tau = \frac{e^2}{2\pi\hbar c^2}\int d^3 r dt \theta \mathbf{E}\cdot\mathbf{B}$$

- the value of θ is defined modulo 2π
- S_θ is an integral over a total derivative (no effect for θ = const.)
- matters at interfaces and surfaces, where θ changes
- for strong topological insulator $\theta=\pi$ (possibility to classify TI even in the presence of interactions)

Application of the Gauss-Theorem gives the CS term on the surface

$$S_\theta = \frac{e^2\theta}{2\pi\hbar c^2}\int d^2 r dt \varepsilon^{\nu\sigma\tau} A_\nu \partial_\sigma A_\tau$$

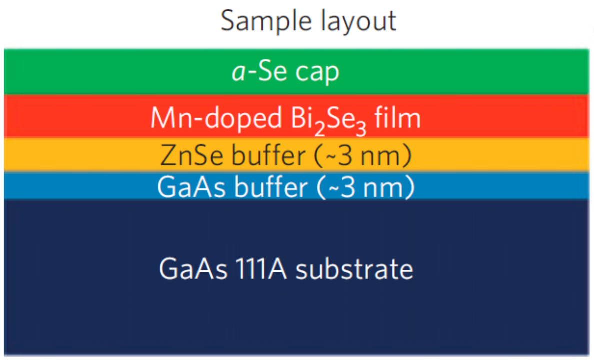
Outline

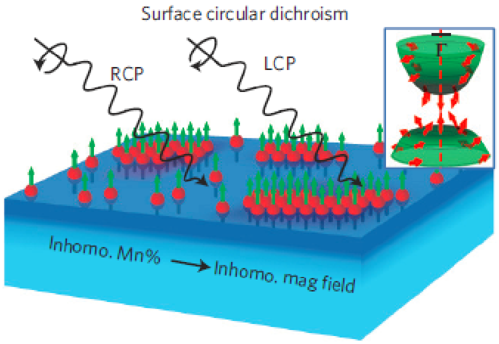
- **FM insulator/TI heterostructures**

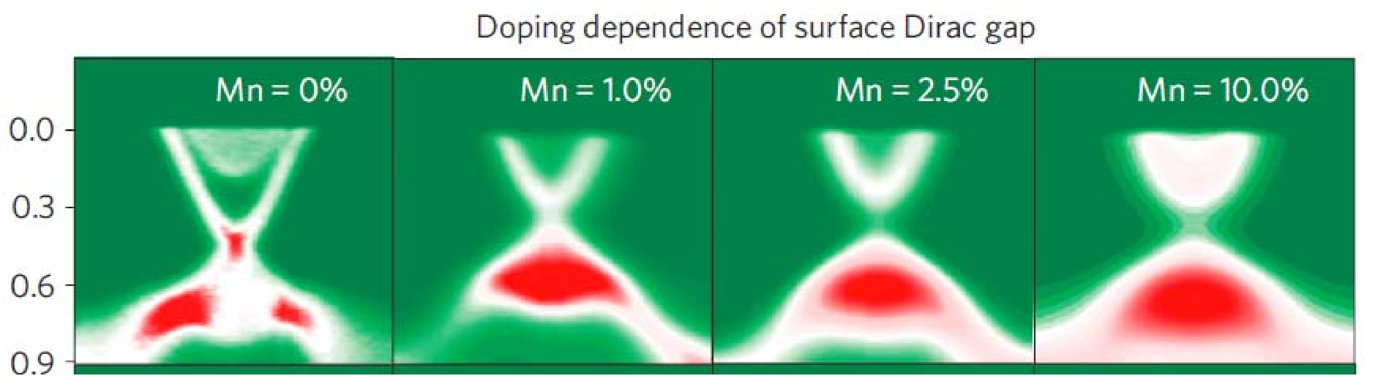
- **Interaction effects at the interface: dynamic generation of the Chern-Simons term**

- **Finite temperature and chemical potential effects**

ferromagnetic order in TI by doping with specific Elements (Mn,Fe,…)



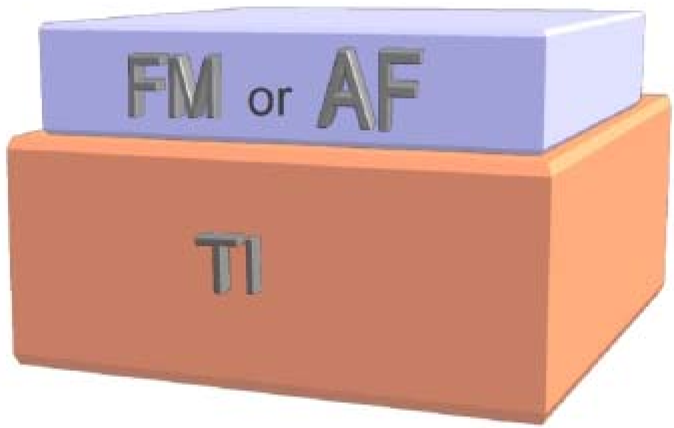




Exp.:Y. L. Chen et al., Science 329, 659 (2010); L. A. Wray et al., Nat. Phys. 7, 32 (2010); J. G. Checkelsky et al., Nat. Phys. 8, 729 (2012); S.-Y. Xu et al., Nat. Phys. 8, 616 (2012).

- hard to separate the surface and the bulk phases
- transport of a TI can be influenced by metallic overlayer or atoms
- crystal defects, magnetic scattering centers, as well as impurity states in the insulating gap

Proximity induced symmetry breaking



TI = Bi_2Se_3 or Sb_2Te_3

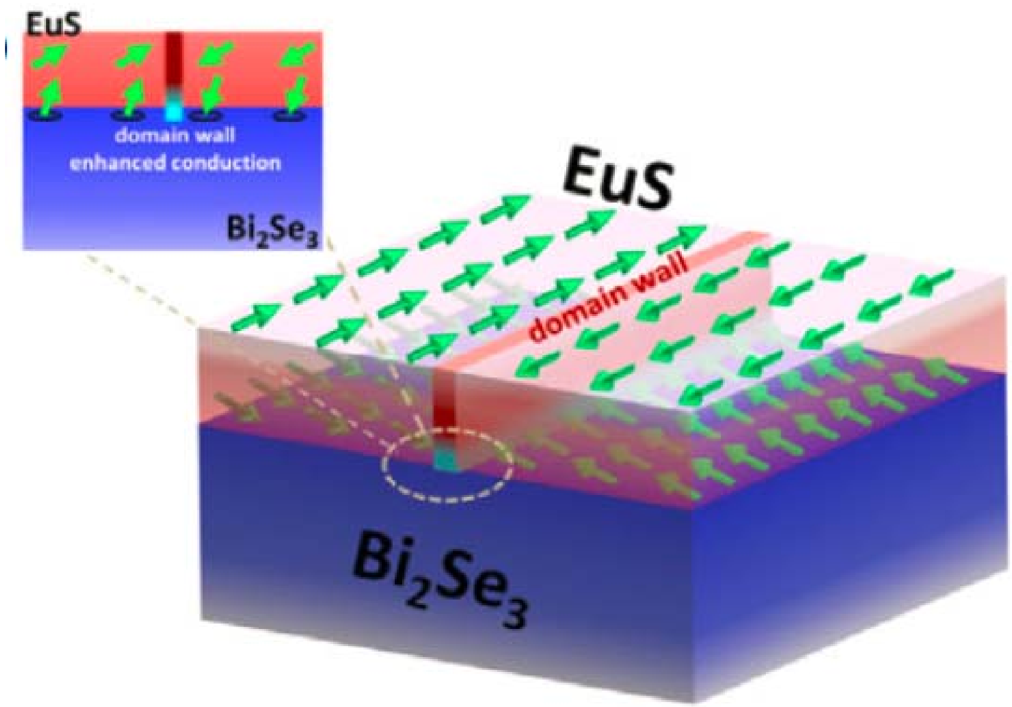
Material	Mag. order	$T_{c,N}$ (K)
EuO	FM	69.3
EuS	FM	16.6
EuSe	FM	pressure
MnSe	AF	247
MnTe	AF	307
$RbMnCl_3$	AF	99

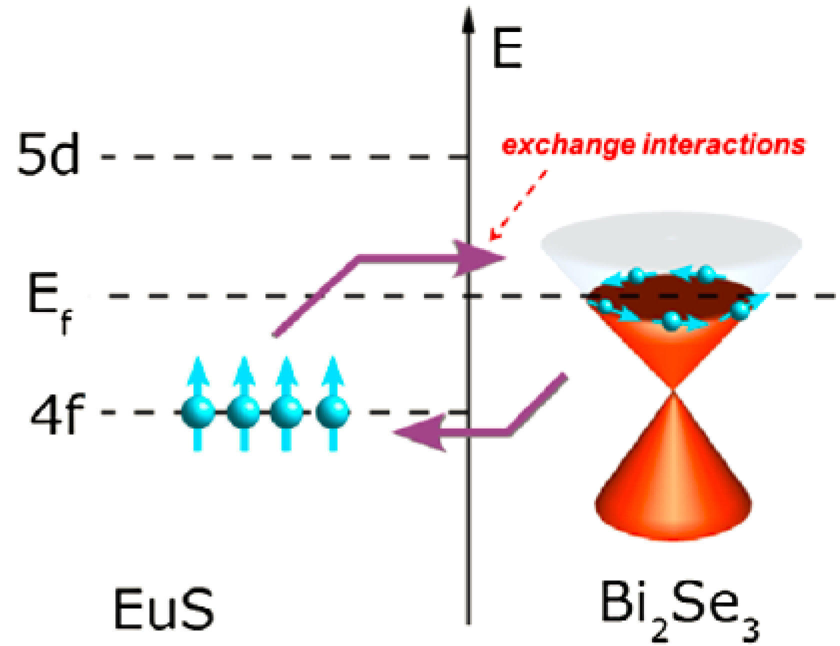
Candidate materials

- Ab initio calculations indicate that MnSe has good matching properties
  [W. Luo and X.-L. Qi, PRB **87**, 085431 (2013)]

S.V. Eremeev et al., PRB 88, 144430 (2014)

Proximity induced symmetry breaking





- EuS well behaved Heisenberg-like ferromagnetic insulator
-
- Local time-reversal symmetry breaking at the interface

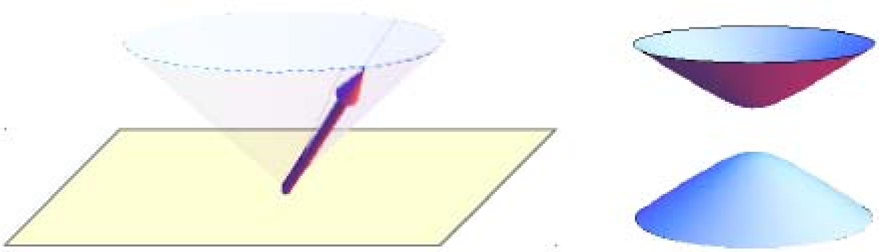
P. Wei et al. PRL 110, 186807 (2013);
Qi I. Yang et al., PRB 88, 081407(R) (2014)
L.D. Alegria et al., Appl. Phys. Lett. 105, 053512 (2014)
FMI($Y_3Fe_5O_{12}$)/TI: Lang et al., NanoLett. 14, 3459 (2014)

FI/TI Interface

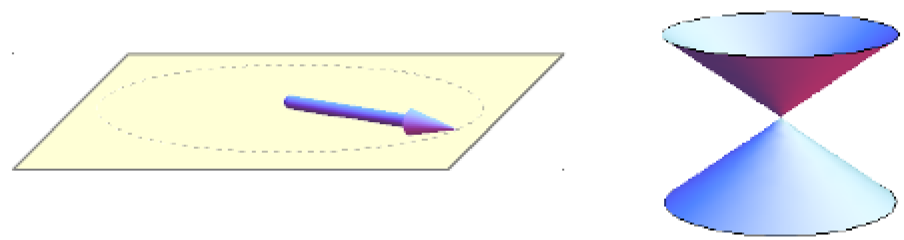
Mean-field type Hamiltonian at the interface

$$H = v_F(-i\hbar\nabla \times \hat{\mathbf{z}}) \cdot \boldsymbol{\sigma} - J(n_x\sigma_x + n_y\sigma_y) - J_\perp n_z \sigma_z$$

$$E_\pm = \pm\sqrt{(p_x - Jn_y)^2 + (p_y + Jn_x)^2 + J_\perp^2 n_z^2}, \qquad \mathbf{p} = \hbar v_F \mathbf{k}.$$



Out of plane magnetization: gapped Dirac spectrum



In-plane magnetization: gapless Dirac spectrum

Dirac point at $(Jn_y, -Jn_x)$

FI/TI Interface: vanishing out-of-plane magnetization

Electronic Lagrangian at the interface:

QED-like form in $d = 2 + 1$

$$\mathcal{L}_0 = \bar{\psi}[i\gamma_0\hbar\partial_t - i\vec{\gamma}\cdot(v_F\hbar\mathbf{\nabla} + iJ\mathbf{a})]\psi$$

vector potential $\mathbf{a} = (n_y, -n_x)$ $\gamma^0 = \sigma_z$, $\gamma^1 = -i\sigma_x$, and $\gamma^2 = i\sigma_y$

Add screened Coulomb interaction

$$\mathcal{H}_{\rm int} = \frac{g}{2}(\psi^\dagger\psi)^2 = \frac{g}{2}(\bar{\psi}\gamma^0\psi)^2$$

The full Lagrangian in terms of auxilary field a_0

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - J a\!\!\!/)\psi - \frac{J^2}{2g}a_0^2$$

FI/TI Interface: Effective action

(a) recall the situation $J_\perp \neq 0$ $\quad m = J_\perp \langle n_z \rangle$

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - J a\!\!\!/ - m)\psi - \frac{J^2}{2g} a_0^2$$

- Integrating out N fermionic degrees of freedom and expanding the action in terms of the components of the vector field

$$S_{\rm eff} = -N \,\mathrm{Tr} \ln(\partial\!\!\!/ - i J a\!\!\!/ + m) + \frac{J^2}{2g} \int d^3x a_0^2$$

- expanding the action in terms of the components of the vector field a_μ

$$S_{eff} \approx \frac{J^2}{8\pi} \int d^3x \sum_{i=1}^{N} \left[-\frac{1}{6|m_i|} f_{\mu\nu} f^{\mu\nu} + \frac{m_i}{|m_i|} \varepsilon_{\mu\nu\lambda} a^\mu \partial^\nu a^\lambda \right]$$

$$f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$$

FI/TI Interface: Effective action

(a) recall the situation $J_\perp \neq 0$

$$S_{eff} \approx \frac{J^2}{8\pi} \int d^3x \sum_{i=1}^{N} \left[-\frac{1}{6|m_i|} f_{\mu\nu} f^{\mu\nu} + \frac{m_i}{|m_i|} \varepsilon_{\mu\nu\lambda} a^{\mu} \partial^{\nu} a^{\lambda} \right]$$

- The first (Maxwell) term contains a dimensional coefficient
- the CS term is universal (depends on the sign of m), independent of the scale transformations

$$S_{CS} = \frac{J^2}{8\pi} \left(\sum_{i=1}^{N} \frac{m_i}{|m_i|} \right) \int d^3x \epsilon_{\mu\nu\lambda} a^{\mu} \partial^{\nu} a^{\lambda}$$

FI/TI Interface: Effective action

(a) recall the situation $J_\perp \neq 0$

$$S_{CS} = \frac{J^2}{8\pi}\left(\sum_{i=1}^{N} \frac{m_i}{|m_i|}\right)\int d^3x\, \epsilon_{\mu\nu\lambda} a^\mu \partial^\nu a^\lambda$$

- Suppose that N is even then one re-writes the Dirac Lagrangian in terms of N/2 four-component Dirac fermions using 4x4 γ matrices

$$\gamma^0 = \begin{pmatrix} \sigma_z & 0 \\ 0 & -\sigma_z \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} i\sigma_x & 0 \\ 0 & -i\sigma_x \end{pmatrix} \qquad \gamma^2 = \begin{pmatrix} i\sigma_y & 0 \\ 0 & -i\sigma_y \end{pmatrix}$$

- the chiral symmetry:

$$\gamma^3 = i\begin{pmatrix} 0 & I_2 \\ I_2 & 0 \end{pmatrix} \qquad \gamma^5 = i\begin{pmatrix} 0 & I_2 \\ -I_2 & 0 \end{pmatrix}$$

FI/TI Interface: Effective action

(a) the situation $J_\perp \neq 0$, N is even

$$S_{CS} = \frac{J^2}{8\pi}\left(\sum_{i=1}^{N}\frac{m_i}{|m_i|}\right)\int d^3x\epsilon_{\mu\nu\lambda}a^\mu\partial^\nu a^\lambda$$

- invariance under chiral transformations:

$$\psi \to e^{i\theta\gamma^3}\psi \qquad \psi \to e^{i\phi\gamma^5}\psi$$

$$\bar{\psi} \to \psi^\dagger e^{-i\theta\gamma^{3\dagger}}\gamma^0 = \bar{\psi}e^{i\theta\gamma^3} \qquad \bar{\psi} \to \psi^\dagger e^{-i\phi\gamma^{5\dagger}}\gamma^0 = \bar{\psi}e^{i\phi\gamma^5}$$

$$j^\mu = \bar{\psi}\gamma^\mu\psi$$ - current operator is invariant

$$m\bar{\psi}\psi$$ - Mass term is not invariant

- The mass breaks the chiral symmetry (not TRS and parity)
- The CS term is absent $\sum_i m_i/|m_i| = 0$

FI/TI Interface: Effective action

(a) the situation $J_\perp \neq 0$, N is odd $N = 2n + 1$

$$S_{CS} = \frac{J^2}{4\pi}\left(n + \frac{1}{2}\right)\frac{m}{|m|}\int d^3x \epsilon_{\mu\nu\lambda} a^\mu \partial^\nu a^\lambda$$

- Two-component Dirac fermions
- the broken symmetries are **TRS** and **mirror symmetry** N=2n+1

Mirror symmetry: $(x_0, x_1, x_2) \to (x_0, -x_1, x_2)$, $\psi \to \gamma^1\psi$, $\bar{\psi} \to -\bar{\psi}\gamma^1$
TR symmetry: $(x_0, x_1, x_2) \to (-x_0, x_1, x_2)$, $\psi \to \gamma^2\psi$, $\bar{\psi} \to -\bar{\psi}\gamma^2$

$$\theta = \pi, \quad \alpha = J^2$$

F.S. Nogueira and I. Eremin PRL109 (2012)

FI/TI Interface: Landau-Lifshitz equations

(a) $J_\perp \neq 0$

$$S_{CS} = \frac{NJ^2\theta}{8\pi^2} \int dt \int d^2r (n_y \partial_t n_x - n_x \partial_t n_y - 2\mathbf{n} \cdot \mathbf{E})$$

$\mathbf{E} = -\nabla a_0$ $\Rightarrow$ Electric field associated with screened Coulomb potential

$\mathbf{n} = (n_x, n_y, m/J_\perp)$

I. Garate and M. Franz, Phys. Rev. Lett. 104, 146802 (2010)
T. Yokoyama, J. Zang, and N. Nagaosa, PRB 81, 241410(R) (2010);
Ya. Tserkovnyak and D. Loss PRL 108, 187201 (2012)

$$\partial_t n_i = \epsilon_{ij} E_j$$ Spin-Hall response

To get the full magnetization dynamics

$$L_{FM} = \mathbf{b} \cdot \partial_t \mathbf{n} - \frac{\kappa}{2}\left[(\nabla \mathbf{n})^2 + (\partial_z \mathbf{n})^2\right] - \frac{r^2}{2}\mathbf{n}^2 - \frac{u}{4!}(\mathbf{n}^2)^2$$

the fluctuations in n_z around $\langle n_z \rangle$

FI/TI Interface: Landau-Lifshitz equations

(a) $J_\perp \neq 0$ the fluctuations in n_z around $\langle n_z \rangle$

$$S_{eff} \approx \frac{NJ^2}{8\pi}\int d^2rdt\left[-\frac{1}{6|m|}f_{\mu\nu}f^{\mu\nu}+\frac{m}{|m|}\varepsilon_{\mu\nu\lambda}a^\mu\partial^\nu a^\lambda + \frac{1}{|m|}\left[(\partial_t\tilde{n}_z)^2-(\nabla\tilde{n}_z)^2\right]\right]$$

$$\frac{\delta S_{eff}}{\delta n_i}=0$$

F.S. Nogueira and I. Eremin PRL109 (2012)

$$\partial_t\mathbf{n}=\gamma\left(\mathbf{n}\times\mathbf{H}_{eff}\right) + \frac{ZNJ^2}{2\pi\upsilon_F}\left[\mathbf{n}\times\mathbf{E}+\frac{1}{3|m|}\left(\mathbf{n}\cdot\mathbf{e}_z\right)\partial_t\mathbf{E}\right]$$

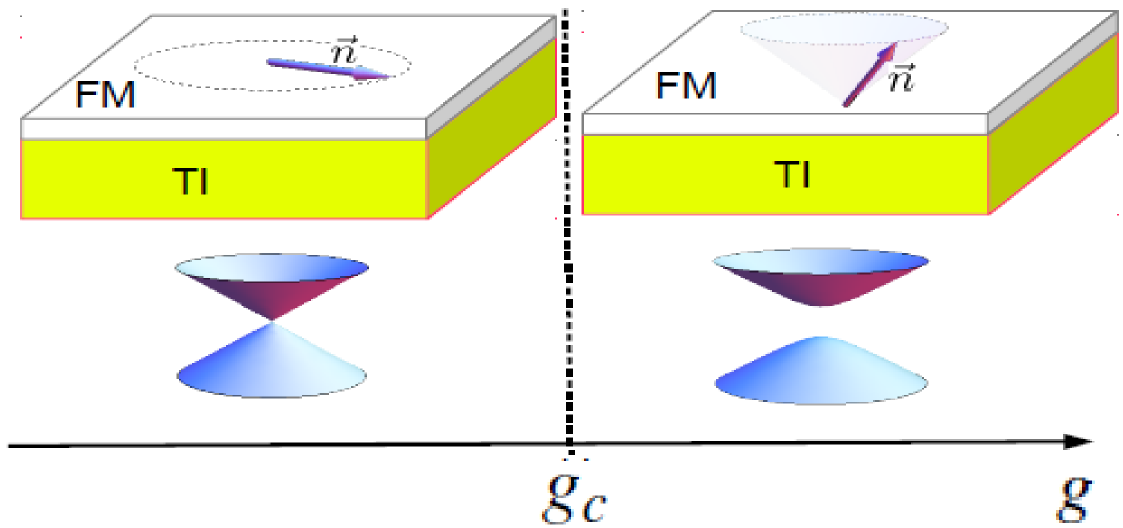
Landau-Lifshitz torque — Magnetoelectric torque

- Coupled to the equation determining the scalar potential $\frac{\delta S_{eff}}{\delta\varphi}=0$

Outline

- FM insulator/TI heterostructures
- **Interaction effects at the interface: dynamic generation of the Chern-Simons term**
- Finite temperature effects

FI/TI Interface: planar ferromagnet



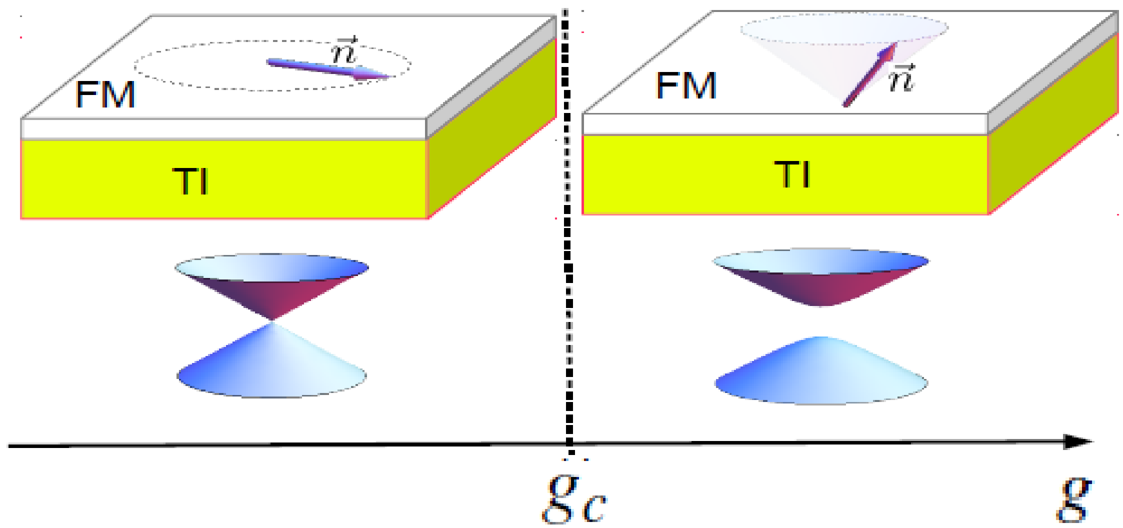
$$\mathcal{L}_e = \bar{\psi}(i\partial\!\!\!/ - J a\!\!\!/)\psi - \frac{J^2}{2g} a_0^2$$

⇒ Gap is dynamically generated due to spontaneous breaking of mirror and time-reversal symmetry

⇒ Competing exchange J and Coulomb interaction, g (or U), lead to a gap
$\Delta \sim \exp\left(-\frac{\text{const}(U-U_c)}{J^2}\right)$, rather than $\Delta \sim \exp\left(-\frac{\text{const}}{J^2}\right)$

$\text{const} > 0 \Longrightarrow$ gap vanishes discontinuously for $U < U_c$

FI/TI Interface: planar ferromagnet



$$\mathcal{L}_e = \bar{\psi}(i\partial\!\!\!/ - J a\!\!\!/)\psi - \frac{J^2}{2g} a_0^2$$

⇒ From effective action derive the propagator for the bosonic excitations (charge and spin fluctuations)

⇒ Compute the self-energy for the fermions and see what is the condition to have $\Sigma(0)\neq 0$

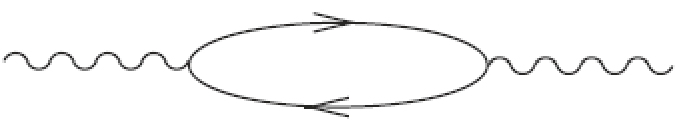
⇒ once it is non-zero it means the breaking of TRS and parity (generation of the Chern-Simons term)

FI/TI Interface: planar ferromagnet

Effective action from massles Dirac fermions

$$S_{\text{eff}} = \frac{1}{2}\int \frac{d^3p}{(2\pi)^3}\left[\Pi(p)\left(p^2\delta_{\mu\nu} - p_\mu p_\nu\right)a_\mu(p)a_\nu(-p) \; + \; \frac{J^2}{g}a_0(p)a_0(-p)\right]$$

$\Pi(p) = NJ^2/(16|p|)$ - vacuum polarization operator



integrate out a_0 fields

$$\chi(\omega,\mathbf{p}) = \frac{16}{NJ^2\sqrt{v_F^2\mathbf{p}^2-(\omega+i\delta)^2}}\left\{1 - \frac{Ngv_F^2\mathbf{p}^2}{(\omega+i\delta)^2}\left[1 + \frac{16}{Ng}\frac{\sqrt{v_F^2\mathbf{p}^2-(\omega+i\delta)^2}}{v_F^2\mathbf{p}^2}\right]\right\}$$

- ‘spin wave’ velocity is identical to the Fermi velocity
- no dynamics from the FI is included
- anomalous scaling dimension η=1 (different from 2+1 XY FM, η=0.04)

Planar FM: fermionic propagator

Schwinger-Dyson equation for the fermion propagator:

$$G^{-1}(p) = i\gamma_\mu p_\mu + J\int \frac{d^3k}{(2\pi)^3}\gamma_\mu G(p-k)D_{\mu\nu}(k)\Gamma_\nu(p,k)$$

$$D_{\mu\nu}(p) = \frac{1}{p^2\Pi(p)}\left\{\delta_{\mu\nu} + \left[\frac{g}{J^2}p^2\Pi(p)+1\right]\frac{p_\mu p_\nu}{\omega^2} - \frac{(p_\mu\delta_{\nu 0}+p_\nu\delta_{\mu 0})}{\omega}\right\}$$

- To determine *G(p)* approximately

$$G^{-1}(p) = Z(p)i\gamma_\mu p_\mu + \Sigma(p) \qquad \Gamma_\mu(p,k) = J\gamma_\mu$$

Look for the solution $\Sigma(p) = \Sigma(0) = m \ll |p| \ll \Lambda$. $\quad \Lambda \approx NJ^2/(\hbar v_F^2)$

Planar FM: self-consistent equation for the mass generation

The fermion mass modifies the vacuum polarization

⇓

Term in the photon propagator odd under parity and time-reversal may arise

$$D_{\mu\nu}^{\text{odd}}(p) = -32 \sum_i (m_i/N) \epsilon_{\mu\nu\lambda} p_\lambda / (N J^2 |p|^3)$$

For $m \ll |p| \ll \Lambda$ one gets the self-consistent equations for N masses

For N even ⇒ N/2 fermions have +m, and N/2 fermions have -m

For N odd ⇒ all N fermions acquire the mass +m

Planar FM: self-consistent equation for the mass generation

- N even (graphene-like): $N/2$ fermions have $m = +|m|$, while the remaining $N/2$ ones have $m = -|m|$ with

$$|m| = \left(\frac{\pi+1}{3\pi}\right)\frac{\hbar v_F}{a}\left(1 - \frac{U}{U_c}\right) \qquad U_c = \frac{4\pi^2}{\pi+1}\left(\frac{\hbar v_F}{a}\right)\left[1 - \frac{8}{\pi^2}\left(\frac{aJ}{\hbar v_F}\right)^2\right]$$

No CS term is generated because its coefficient is proportional to $\frac{1}{N}\sum_{a=1}^{N}\frac{m_a}{|m_a|} \Longrightarrow$ mirror and TR symmetries are overall conserved

Gap vanishes continuously at U_c

- N odd (TI): $g \sim Ua/t \sim Ua/(\hbar v_F)$, $a =$ lattice spacing

All $m > 0 \Longrightarrow$
$$m = \frac{hv_F}{a}\exp\left[-\frac{(\pi+1)\pi}{128a}\left(\frac{hv_F}{J}\right)^2(U - U_c)\right]$$

CS term is generated $\Longrightarrow$ mirror and TR symmetries are spontaneously broken [Nogueira and Eremin, PRB 88, 085126 (2013)]

Gap vanishes discontinuously at U_c

Outline

- Introduction: electrodynamics on the surface of a topological insulator

- FM insulator/TI heterostructures

- Interaction effects at the interface: dynamic generation of the Chern-Simons term

- **Finite temperature and chemical potential effects**

Finite temperature effects: shift of Curie temperature at the interface

⇒ FI/TI heterostructure

$$\langle n_z \rangle = 0 \text{ for } T = \boxed{\widetilde{T}_c = \frac{T_c}{1 + \frac{J^2 \ln 2}{\pi a_0 v_F^2}}} \Longrightarrow \widetilde{T}_c < T_c$$

[Nogueira and Eremin, PRL **109**, 237293 (2012)]

Estimate based on EuO $\Longrightarrow T_c \approx 70$ K and $\widetilde{T}_c \approx 54$ K

A downwards shift in the Curie temperature was recently observed in EuS thin films proximate to Bi_2Se_3

Exp.: P. Wei et al. PRL 110, 186807 (2013);

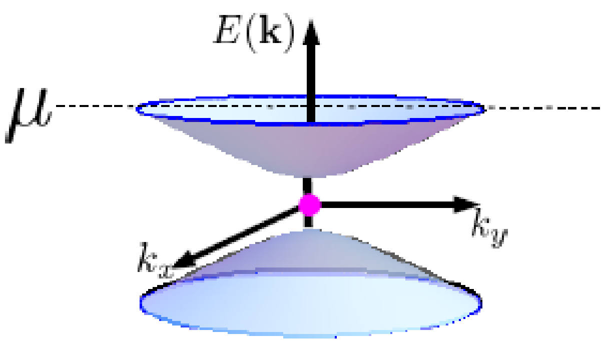
⇒ Temperature effects for the Chern-Simons term and Hall conductivity?

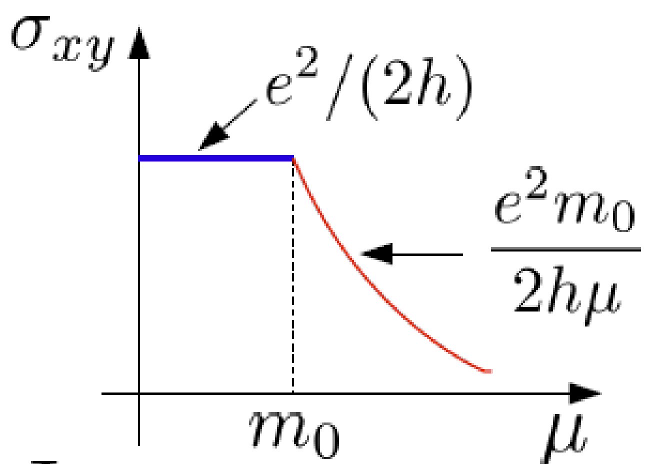
Effect of the temperatures on Chern Simons term

$$\mathcal{S}_{\rm CS} \approx \frac{\sigma(T,m)}{2} \int dt \int d^2 r \epsilon_{\mu\nu\lambda} a^\mu \partial^\nu a^\lambda$$

Set $\sigma(T,m) = NJ^2 \tilde{\sigma}(T,m)/(v_F^2 e^2)$

- $T = \mu = 0$: $\tilde{\sigma} = \sigma_{xy} = \frac{e^2}{2h}$
- $T = 0$ and $\mu \neq 0$:
  $\sigma_{xy}(0, m_0) = \frac{e^2}{2h}\left[\left(\text{sgn}(m_0) - \frac{m_0}{\mu}\right)\theta(|m_0| - \mu) + \frac{m_0}{\mu}\right]$





$$m_0 = J_\perp n_z$$

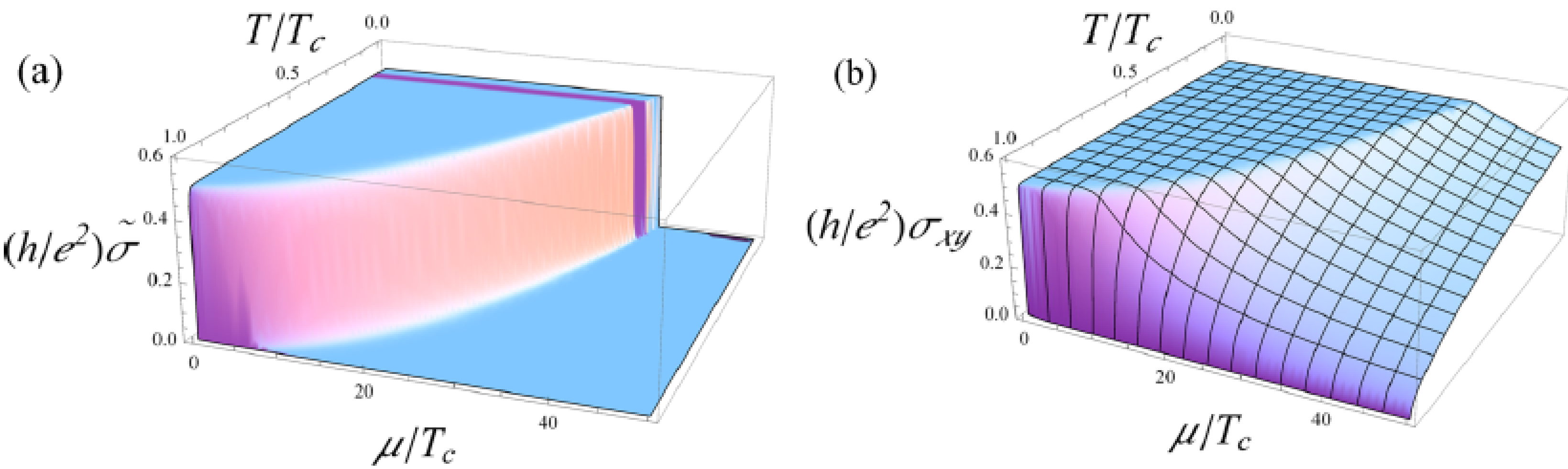
- Hall conductivity non-quantized and non-universal for $|m_0| < \mu$

Effect of the temperatures on Chern Simons term

$$\mathcal{S}_{\rm CS} \approx \frac{\sigma(T,m)}{2} \int dt \int d^2 r \epsilon_{\mu\nu\lambda} a^\mu \partial^\nu a^\lambda$$

- $T \neq 0$ and $\mu < |m|$:

$$\tilde{\sigma}(T,m) = \frac{e^2 \mathrm{sgn}(m) \sinh(|m|/T)}{2h[\cosh(|m|/T) + \cosh(\mu/T)]}$$



Conclusions:

TI/FI heterostructure:

for in-plane magnetization:

- For interacting Dirac fermions coupled to an in-plane exchange field there is a spontaneous breaking of parity and TRS due to a dynamical gap generation

$$m = \frac{hv_F}{a} \exp\left[-\frac{(\pi+1)\pi}{128a}\left(\frac{hv_F}{J}\right)^2 (U - U_c)\right]$$

- σ_{xy} is T and μ dependent and in the metallic phase ($\mu > m$), the Hall conductivity is not quantized and non-universal

Flavio S. Nogueira, Ilya Eremin, Phys. Rev. Lett. 109, 237203 (2012); Phys. Rev. B 88, 055126 (2013); Phys. Rev. B 90, 014431 (2014)

Proximity effect between insulating ferromagnet and TI

- the axion term with uniform θ does not modify the Maxwell equations in the bulk

- but does modify the magnetization dynamics at the surface (magnetoelectric effect)

Ferromagnet Insulator/TI insulator heterostructure

- form of the Landau-Lifshitz equations

- Interaction effects

Quantum criticality on an AF topological surface

TPQM 2014, Vienna, 11.09.2014

Antiferromagnet on the surface of a topological insulator

Landau-Ginzburg Lagrangian for the AF in imaginary time:

$$\mathcal{L}_{\rm AF} = \frac{1}{2}[(\partial_\tau \mathbf{n})^2 + (\boldsymbol{\nabla}\mathbf{n})^2] + \frac{M^2}{2}\mathbf{n}^2 + \frac{\lambda}{4!}(\mathbf{n}^2)^2$$

$M^2 \propto g - g_c$, g is proportional to the AF exchange coupling
Electronic Lagrangian in imaginary time:

$$\mathcal{L}_e = \bar{\psi}(\not{\partial} - ig_1 \not{a} + g_2\sigma)\psi$$

Here $a^\mu = (\varphi, \mathbf{a})$, where $\mathbf{a} = (n_y, -n_x)$, and $\mathbf{n} = (n_x, n_y, \sigma)$. Thus,

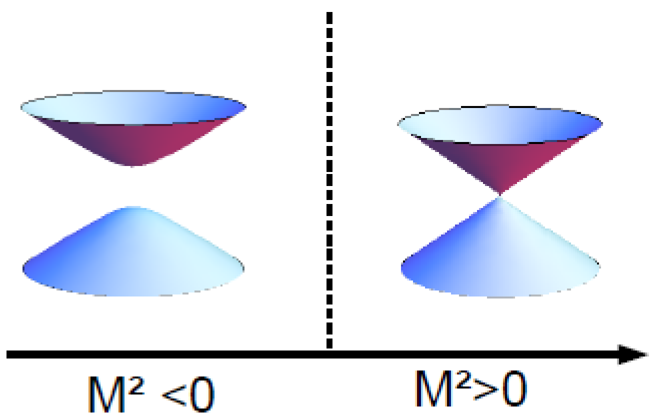
$$\mathcal{L}_{\rm AF} = \frac{1}{2}[(\partial_\mu \sigma)^2 + (\partial_\mu \mathbf{a})^2] + \frac{M^2}{2}(\sigma^2 + \mathbf{a}^2) + \frac{\lambda}{4!}(\sigma^2 + \mathbf{a}^2)^2$$

$\Longrightarrow$ In the absence of fermions, AF order occurs for $M^2 < 0$. The critical behavior belongs to the $O(3)$ universality class.

Antiferromagnet on the surface of a topological insulator

Lowest order calculation of the excitation spectra:

- Fluctuation-corrected mean-field theory.
- AF ordering occurs when $M^2 < 0 \Longrightarrow \sigma_0 = \langle\sigma\rangle = \sqrt{-6M^2/\lambda}$
- $\Longrightarrow$ fermions get gapped: $E(\mathbf{k}) = \sqrt{\mathbf{k}^2 + m_\psi^2}$, where $m_\psi^2 = -6g_2^2 M^2/\lambda$. Surface of the TI is insulating.



- AF excitations:

Longitudinal gapped mode $\Longrightarrow \omega_L(\mathbf{k}) = \sqrt{\mathbf{k}^2 + M^2 + \frac{\lambda m_\psi^2}{2g_2^2}}$

Magnon ($\Phi = n_x + i n_y$) transverse mode $\Longrightarrow \omega_T(\mathbf{k}) = |\mathbf{k}|$

Antiferromagnet on the surface of a topological insulator

Critical exponents from the renormalization group in $D = 3 - \epsilon$ dimensions

[F. S. Nogueira and I. Eremin, PRL **109**, 237293 (2012)]

- Longitudinal and transversal correlation lengths: $\xi \sim (g - g_c)^{-\nu}$ and
  $\xi_\perp \sim (g - g_c)^{-\nu_\perp} \sim \xi^{\nu_\perp/\nu}$
  $\nu \approx 1/2 + \epsilon[4(N+3)]^{-1}[(5/66)(3 - N + \sqrt{N^2 + 258N + 9}) + N]$
  $\nu_\perp \approx \nu + 3\epsilon/[4(N+3)]$
  For $D = 2$ and $N = 1$, we obtain $\boxed{\nu \approx 0.649}$ and $\boxed{\nu_\perp \approx 0.83}$
- Anomalous dimensions: $\langle \mathbf{n}_\parallel(x) \cdot \mathbf{n}_\parallel(0) \rangle \sim |x|^{-1-\eta_N}$,
  $\langle \sigma(x)\sigma(0) \rangle \sim |x|^{-1-\eta_N^\perp}$, $\quad \langle \psi(x)\bar{\psi}(0) \rangle \sim \frac{\gamma^\mu x_\mu}{|x|^{3+\eta_\psi}}$
  $\eta_N = N\epsilon/(N+3)$, $\eta_N^\perp = \epsilon$, and $\eta_\psi = \epsilon/[2(N+3)]$
  For $D = 2$ and $N = 1$: $\boxed{\eta_N = 1/4}$, $\boxed{\eta_N^\perp = 1}$, and $\boxed{\eta_\psi = 1/8}$.
  $\Longrightarrow$ *The Dirac fermions lead to an unconventional critical behavior featuring large anomalous dimensions.*

This is in stark contrast with the Landau-Ginzburg result, which yields in absence of fermions the exact value approaching

$\eta_N = \eta_N^\perp \approx 0.03$

corresponding to the $O(3)$ universality class