SPIN-LIQUIDS ON THE KAGOME LATTICE: CHIRAL TOPOLOGICAL, AND GAPLESS NON-FERMI-LIQUID PHASE

ANDREAS W.W. LUDWIG (UC-Santa Barbara)

work done in collaboration with:

- Bela **Bauer** (Microsoft Station-Q, Santa Barbara)
- Simon **Trebst** (Univ. of Cologne)
- Brendan **Keller** (UC-Santa Barbara)
- Michele **Dolfi** (ETH Zuerich)

- arXiv-1303.6963 -

and, [for (gapped) "Chiral Spin Liquid (Kalmeyer Laughlin)" phase], also with:

- Guifre **Vidal** (Perimeter Inst.)
- Lukasz **Cincio** (Perimeter Inst.)

- arXiv-1401.3017 , Nature Communications (to appear).

INTRODUCTION

WHAT ARE "SPIN LIQUIDS" ?

Phases of quantum spin systems which don't order (at zero temperature) but instead exhibit unusual, often exotic properties.
[Loosely speaking: typically happens due to "frustration".]

In general, two cases:

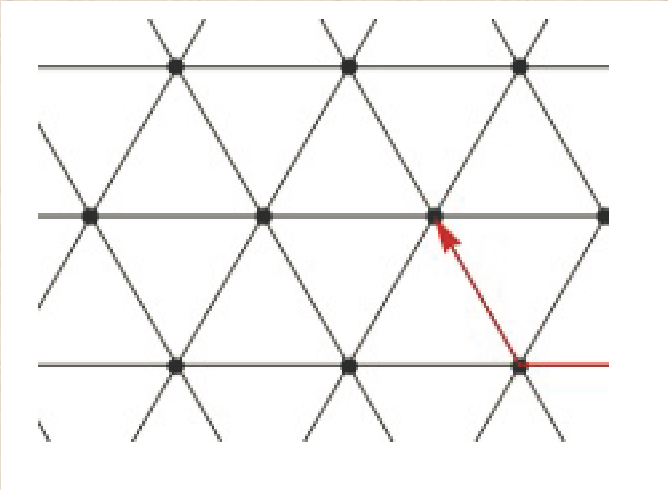
-(A): Gapped spin liquids (typically have some kind of topological order)

-(B): Gapless spin liquids (but gapless degrees of freedom are *not* the Goldstone modes of some spontaneous symmetry breaking)

SOME HISTORY:

- **Kalmeyer and Laughin (1987),** suggestion (not correct):

Ground state of s=1/2 Heisenberg quantum antiferromagnet on triangular lattice (which is frustrated) might break time reversal symmetry spontaneously, producing the Bosonic $\nu = 1/2$ Laughlin (fractional) quantum Hall state, a 'Chiral Spin Liquid'.



$$H = \sum_{<i,j>} \vec{S}_i \cdot \vec{S}_j$$

- Wen, Zee, Wilczek 1989; Baskaran 1989:

use the "**spin chirality operator**" $\chi_{ijk} := \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k)$

as an order parameter for chiral spin states.

Bosonic Laughlin quantum Hall state at filling $\nu = 1/2$

Wavefunction :

$$\psi(z_1, z_2, ..., z_n) = \prod_{i<j} (z_i - z_j)^2$$

Fact :

The edge of the $\nu = 1/2$ Bosonic Laughlin state is described by $SU(2)_1$ Conformal Field Theory having central charge $c = 1$

- a ‘Chiral Spin Liquid’ has appeared in the past

(i): in models with somewhat artificial Hamiltonians:
- Ch. Mudry (1989) , Schroeter, Thomale, Kapit, Greiter (2007); long-range interactions
- Yao+Kivelson (2007 – 2012); certain decorations of Kitaev’s honeycomb model

(ii): particles with topological bandstructure plus interactions:
- Tang et al. (2011), Sun et al. (2011), Neupert et al. (2011); “flat bands”
- Nielsen, Sierra, Cirac (2013)

(iii): SU(N) cases, cold atom systems: Hermele, Gurarie, Rey (2009).

Here I will describe the appearance of

- **(A):** the **Kalmeyer-Laughlin (gapped) ‘Chiral Spin Liquid’** (the Bosonic Laughlin quantum Hall state at filling $\nu = 1/2$),

as well as

- **(B):** a ***gapless spin liquid*** which is a ***non-Fermi Liquid*** with ***lines in momentum space*** supporting ***gapless SU(2) spin excitations*** replacing the Fermi-surface of a Fermi-liquid (sometimes called a “**Bose-surface**”),

in an ***extremely simple*** **model** of s=1/2 quantum spins with SU(2) symmetry and ***local short- range interactions.***

Notion of "Bose Surface" originates in work by: Matthew Fisher and collaborators

e.g.:
- Paramekanti, Balents, M. P. A. Fisher, Phys. Rev. B (2002);
- Motrunich and M. P. A. Fisher, Phys. Rev. B (2007);
- H.-C. Jiang et al. Nature (2013).

MODELS

“BARE-BONES” MODELS:

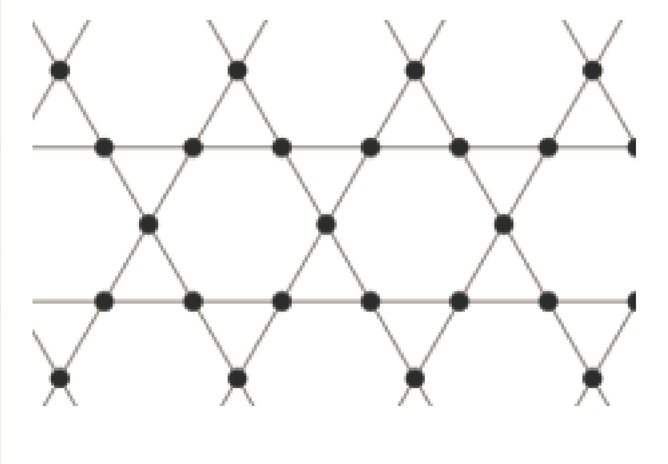
- Spin chirality operator $\chi_{ijk} := \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k)$

(breaks time-reversal and parity)

serves as an interaction term in the Hamiltonian on a lattice made of triangles:

- We consider: Kagome lattice (a lattice of corner-sharing triangles)

$$H = \sum_{ijk}^{triangles} J_{ijk} \; \chi_{ijk}$$

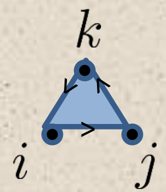


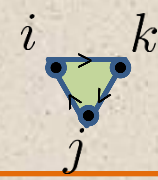
- QUESTION: What are the phases of this system?

TWO CASES:

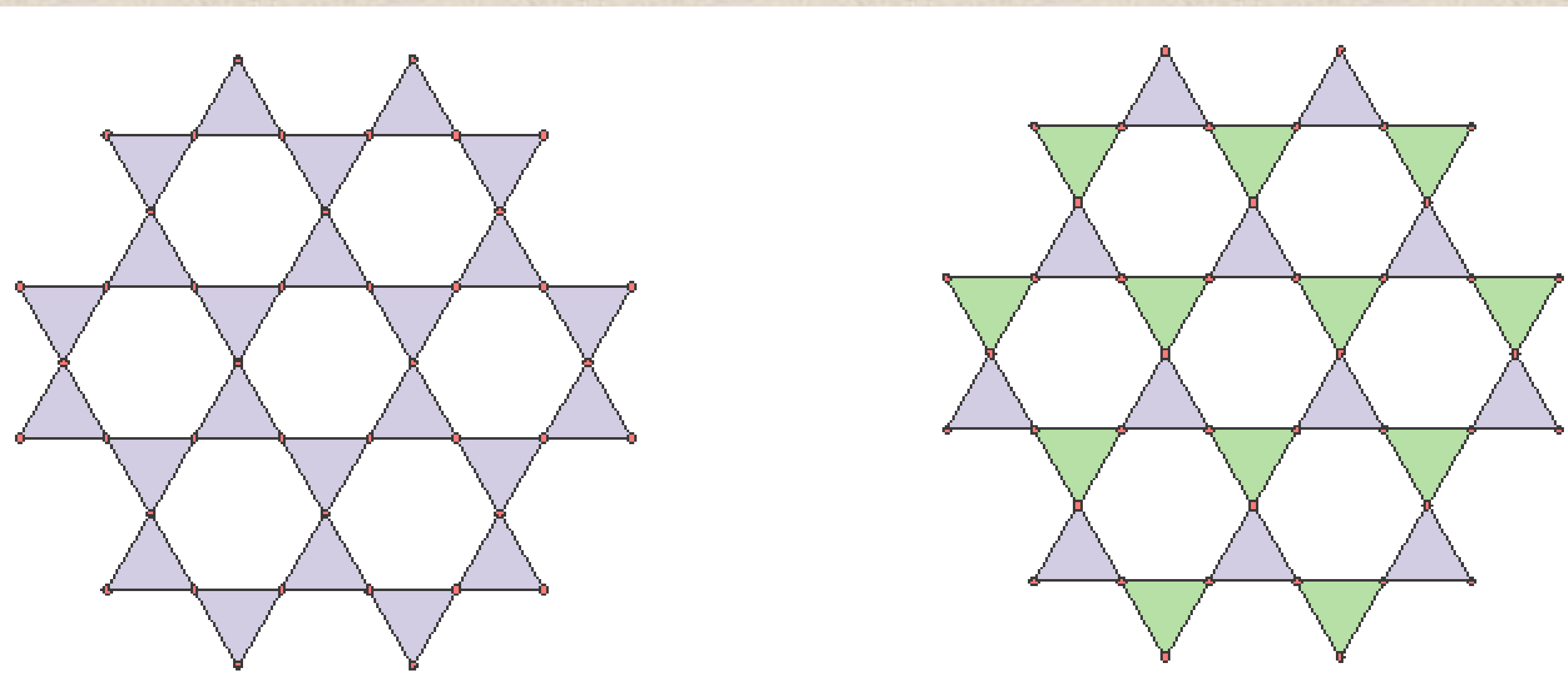
The lattice of the centers of the plaquettes of the Kagome lattice is a bipartite lattice
-> there are two natural models:

$$H = K \sum_{\triangle} \chi_{ijk} \pm K \sum_{\bigtriangledown} \chi_{ijk}$$

 = 

$= +\chi_{ijk}$ and  = 

$= -\chi_{ijk}$



Uniform ("Homogeneous") Staggered

HUBBARD-MODEL (MOTT INSULATOR) REALIZATION FOR THE UNIFORM PHASE :

One of the main messages of our work:

The uniform phase is the ground state of the simple (half-filled) Hubbard model on the Kagome lattice when a magnetic field is applied.

Since the **Hubbard model** is the minimal model describing typical Mott-insulating materials, this is a step towards finding the chiral spin liquid in standard electronic materials:

$$H = - \sum_{\langle i,j\rangle,\sigma=\pm 1} \left(t_{ij} c^{\dagger}_{i\sigma} c_{j\sigma} + t^{*}_{ij} c^{\dagger}_{j\sigma} c_{i\sigma} \right) + U \sum_{i} n_{i,+1} n_{i,-1} + \frac{h_z}{2} \sum_{i} (n_{i,+1} - n_{i,-1})$$

where $n_{i,\sigma} \equiv c^{\dagger}_{i,\sigma} c_{i,\sigma}$

$$\begin{aligned} t_{ij} &= \text{complex} \\ \text{and} \quad t_{ij} t_{jk} t_{ki} &= t^3 e^{i\Phi} \\ \text{where} \quad \Phi &= \text{magnetic flux through triangle} \end{aligned}$$

At half filling (one electron per site, of either spin), standard perturbation theory in (t/U) turns out to yield the following **spin-1/2 Hamiltonian**:

$$H = J_{HB} \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j + J_\chi \sum_{i,j,k \in \triangle} \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k) + $$
$$+ \quad h_z \sum_i \vec{S}_i^z$$

$$\text{where}: \quad J_{HB} = \frac{4t^2}{U}\left(1 - \frac{32t^2}{U^2} + ...\right), \quad J_\chi = \Phi \frac{24t^3}{U^2} + ...$$

First set Zeeman field to zero : $h_z = 0$ [put back later (will not change conclusions)]

Parametrize : $J_{HB} \equiv J\cos\theta, \quad J_\chi \equiv J\sin\theta;$ (from now on set $J = 1$)

Then :

$\underline{\theta = 0:}$ Heisenberg antiferromagnet on Kagome lattice

$$H = J_{HB} \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j$$

$\underline{\theta = \pi/2:}$ "Bare Bones" 3 – spin model

$$H = J_\chi \sum_{i,j,k \in \triangle} \chi_{ijk}$$

OUR RESULTS (from numerics): PHASE DIAGRAM

Heisenberg on Kagome

$$H = J_{HB} \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j$$

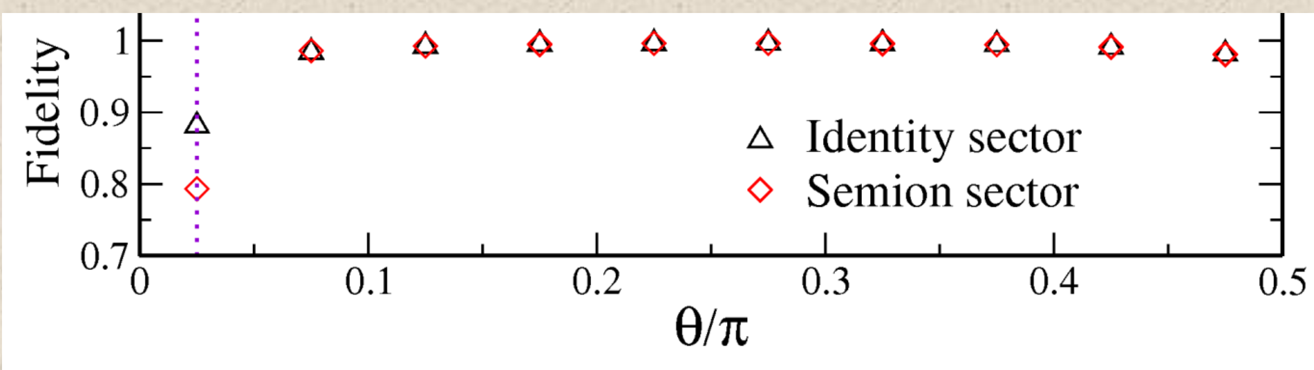
Bare Bones Model

$$H = J_\chi \sum_{i,j,k \in \triangle} \chi_{ijk}$$

Fidelity:

$$F(\theta) \equiv$$

$$\equiv \langle \Psi(\theta - \epsilon) | \Psi(\theta + \epsilon) \rangle$$



PREDICTION OF THE PHASES OF THE BARE-BONES MODEL (HEURISTIC):

TWO CASES:

The lattice of the centers of the plaquettes of the Kagome lattice is a bipartite lattice
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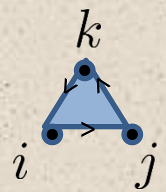
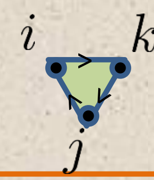
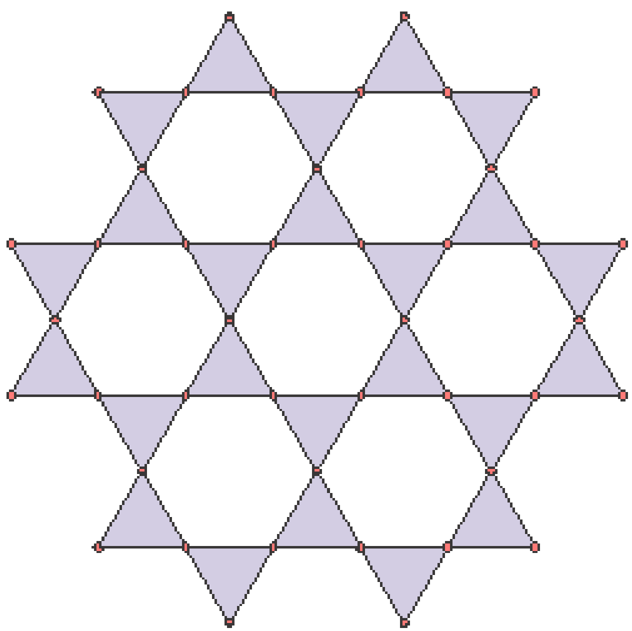
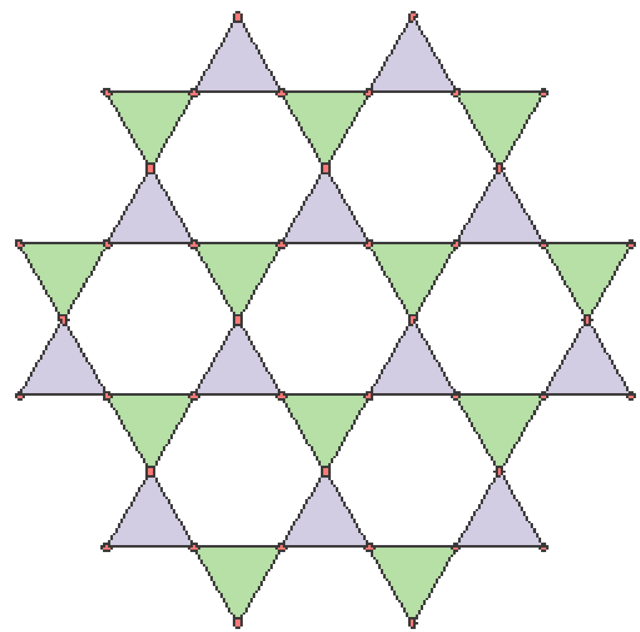
 = 

$= +\chi_{ijk}$ and  = 

$= -\chi_{ijk}$



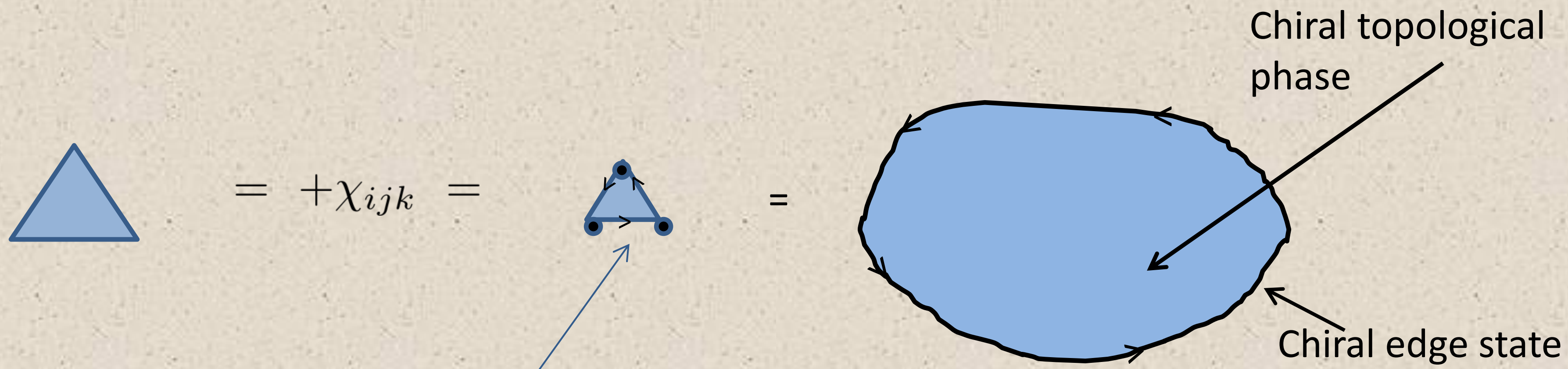
Uniform (“Homogeneous”)



Staggered

“NETWORK MODEL”

Think in terms of a “network model” to try to gain intuition about the behavior of the system:

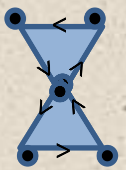


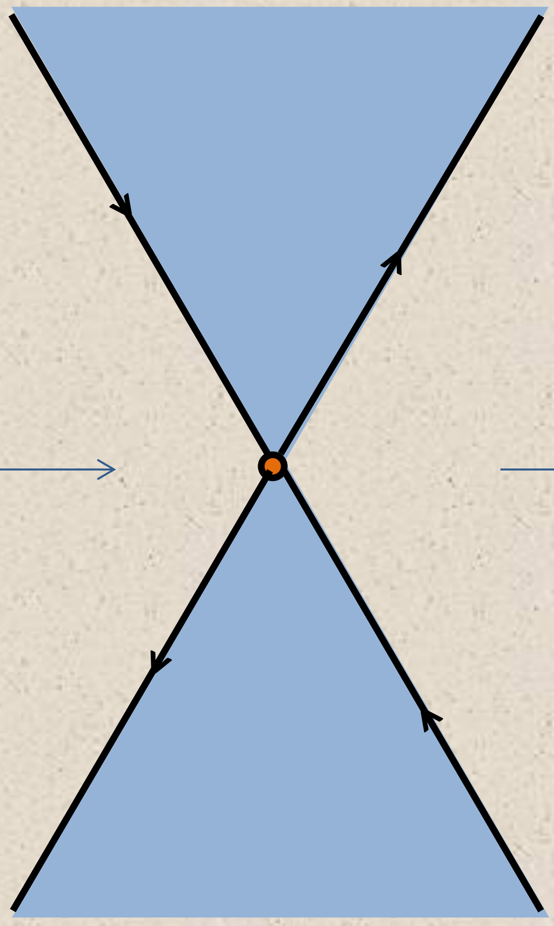
The 3-spin interaction on a triangle breaks time-reversal symmetry (and parity), but preserves SU(2) symmetry:

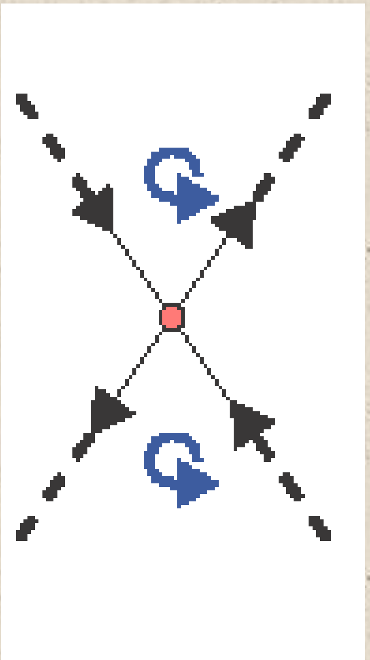
-> natural to view **<u>each triangle</u>** with 3-spin interaction as the **<u>seed</u>** <u>of a puddle of a</u> **<u>chiral topological phase</u>**
[which is expected to be the $\nu = 1/2$ Bosonic Laughlin state
– simplest state with broken Time-reversal and SU(2) symmetry].

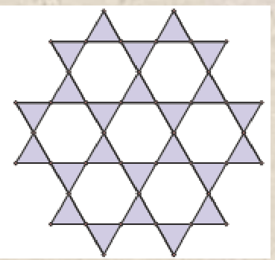
Joining two triangles (puddles) with a corner-sharing spin: a 2-channel Kondo effect

two triangles of equal chirality:

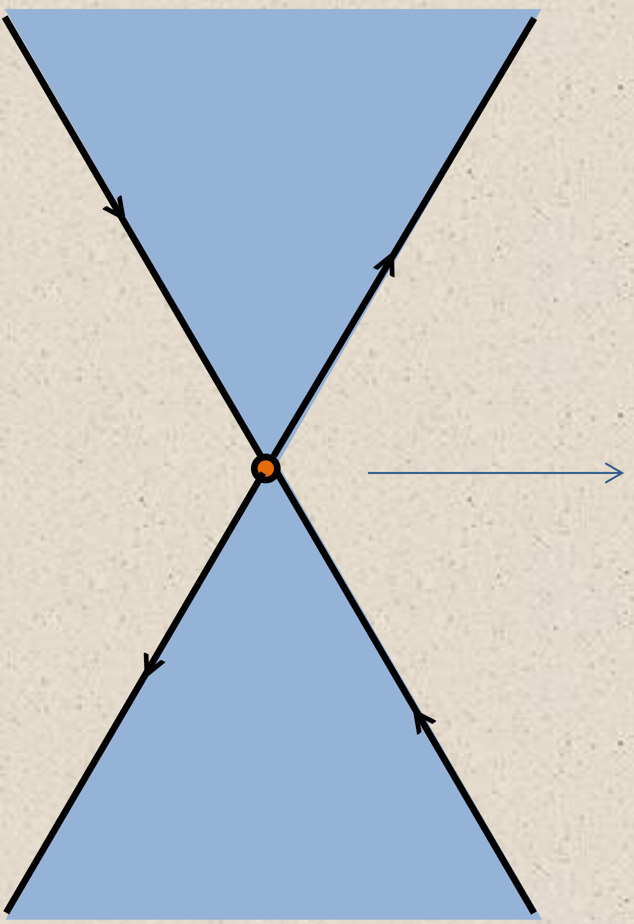




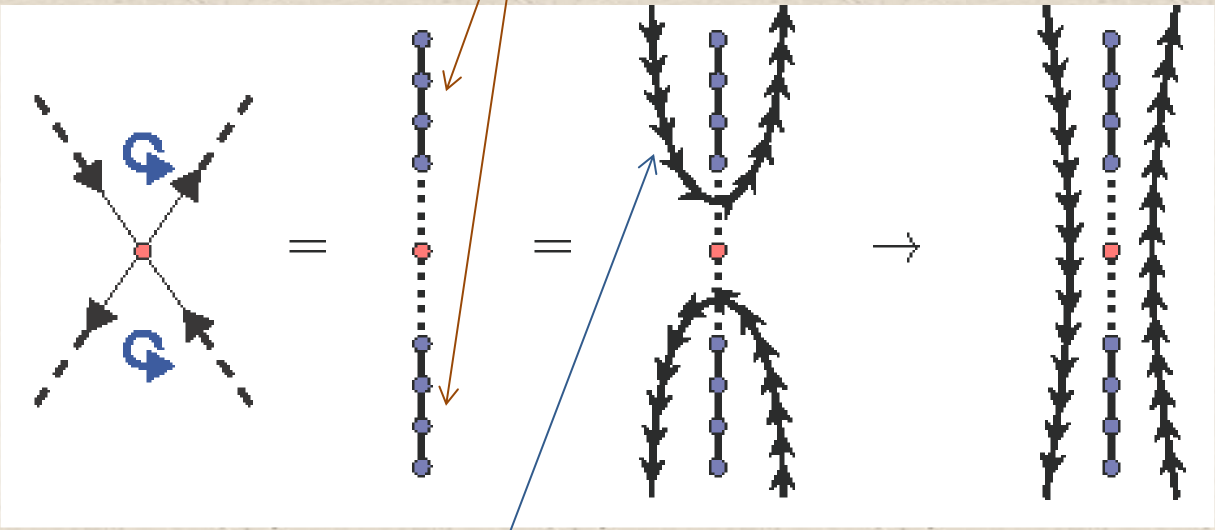




Joining two triangles (puddles) with a corner-sharing spin: a 2-channel Kondo effect



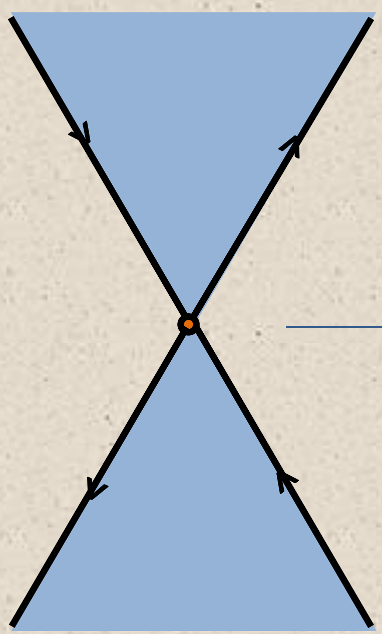
Two semi-infinite s=1/2 Heisenberg spin chains

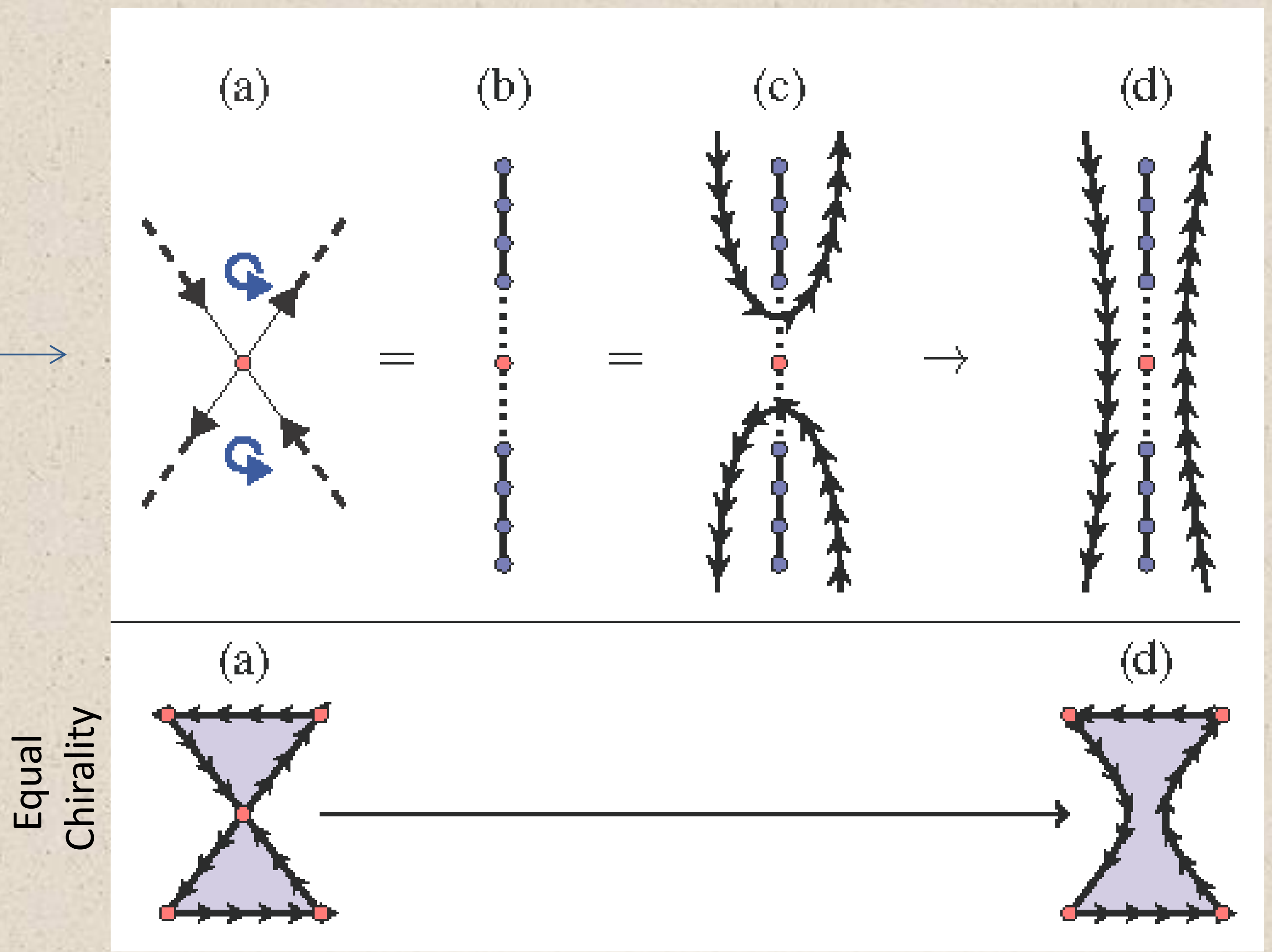


This is the same edge state $SU(2)_1$
as in the Bosonic Laughlin state

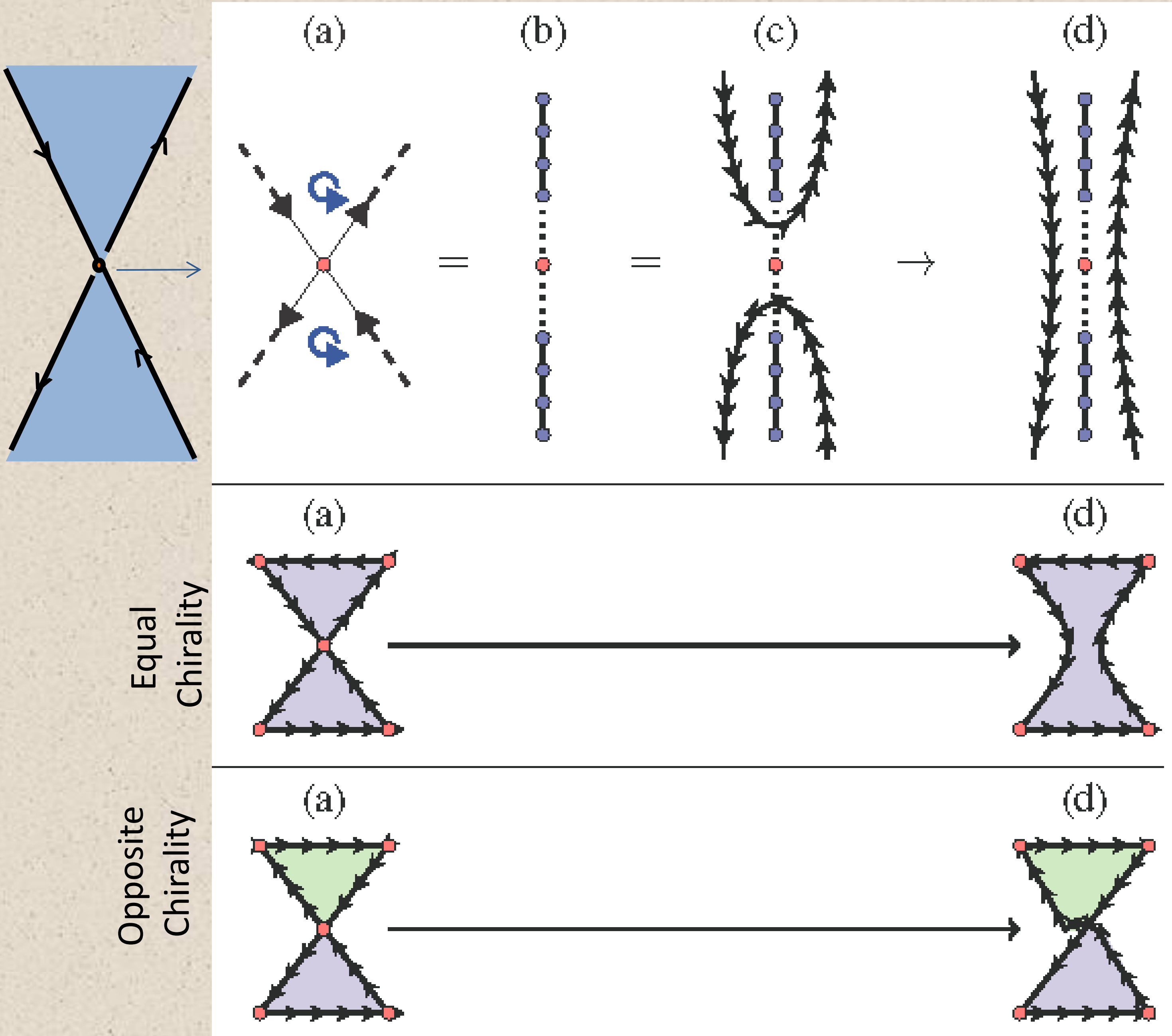
[I.Affleck+AWWL PRL(1992); S.Eggert+I.Affleck PRB (1992)]

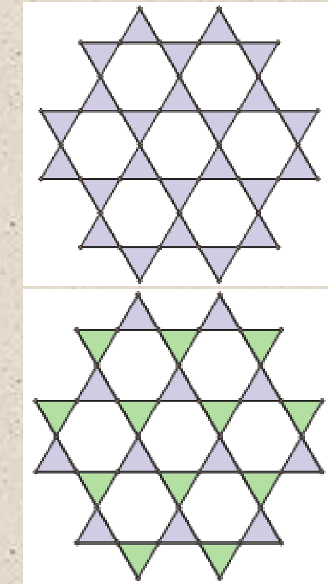
Joining two triangles (puddles) with a corner-sharing spin: a 2-channel Kondo effect





Joining two triangles (puddles) with a corner-sharing spin: a 2-channel Kondo effect

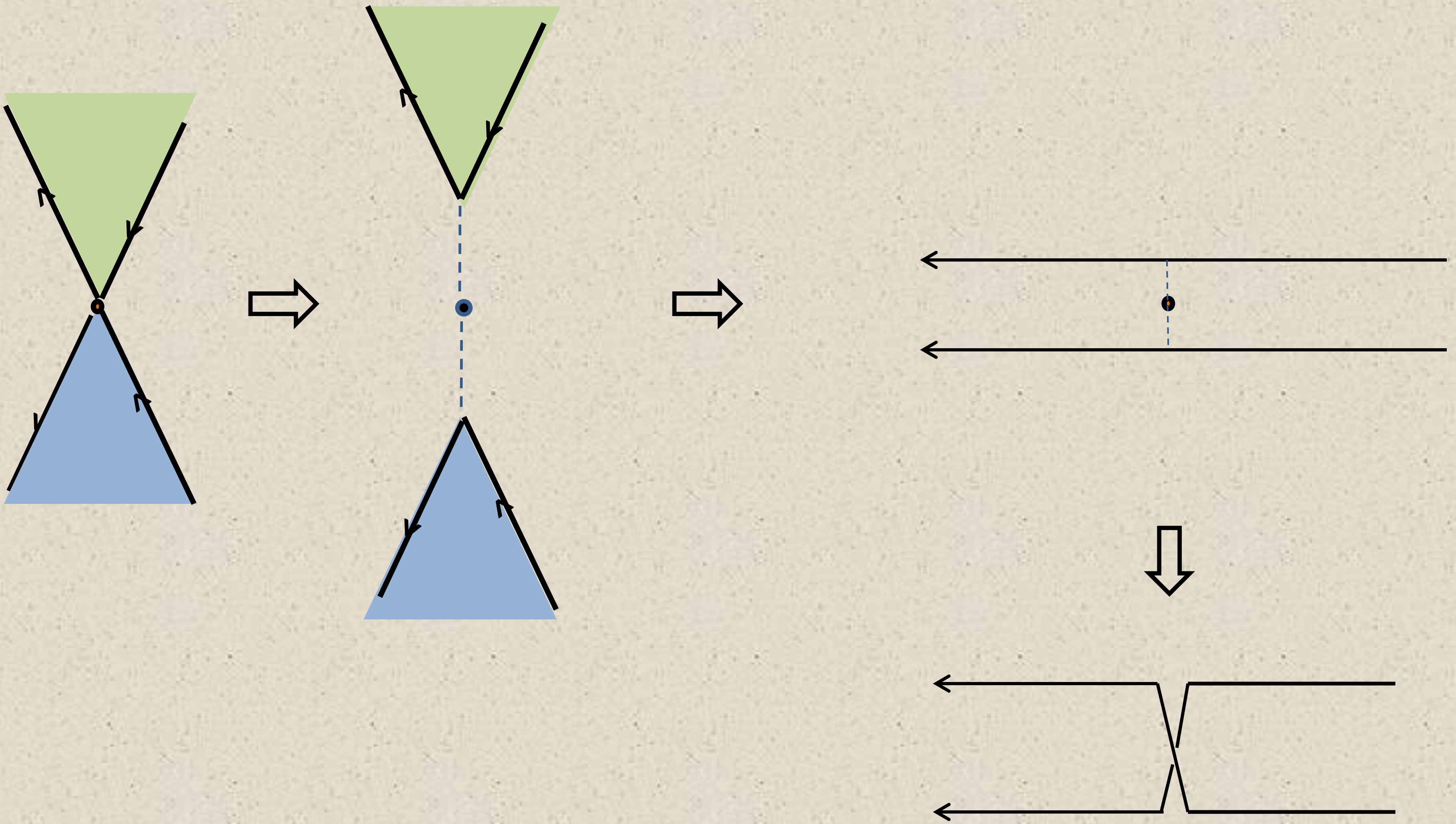




In both cases:

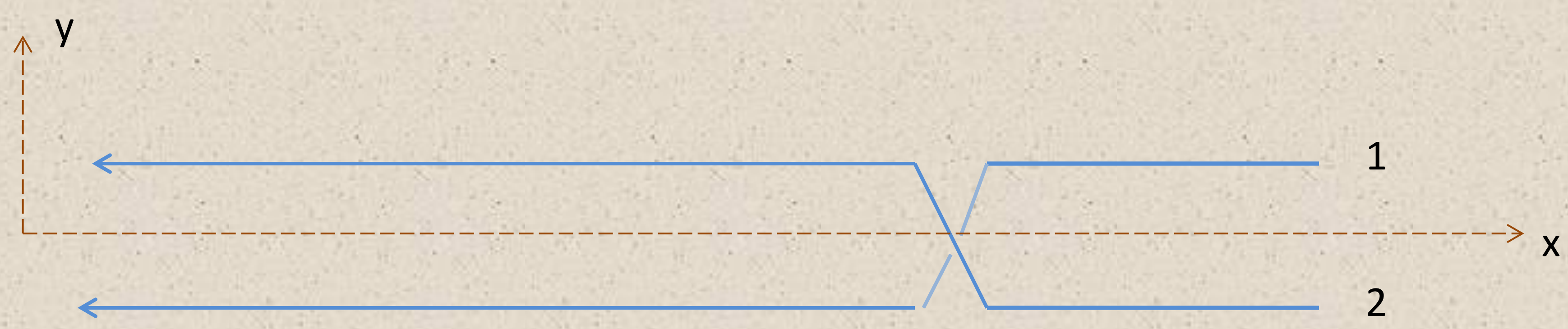
The two puddles join to form a larger puddle [surrounded by a single edge state]

Direct Analysis of the Case of Opposite Chirality Triangles:



[I.Affleck+AWWL PRL(1992); I. Affleck (Taniguchi Symposium, Japan,1993), J.Maldacena+AWWL, Nucl. Phys.B (1997)]

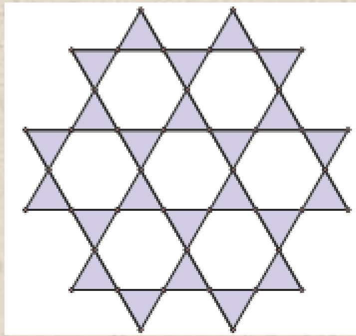
Protected by permutation symmetry 1 <-> 2:



forbids (RG-) relevant tunneling term:

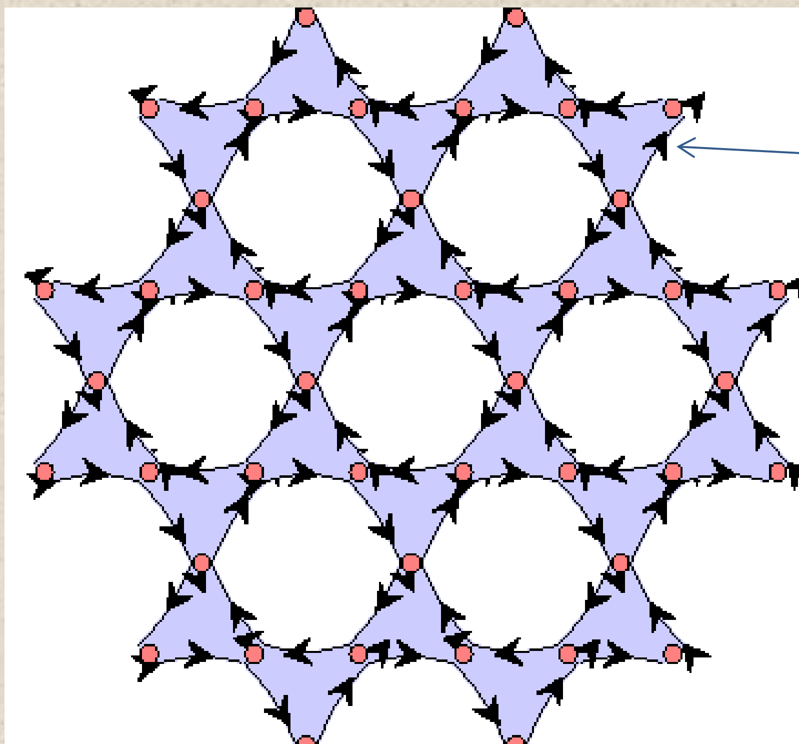
$$\epsilon^{\alpha\beta}\; g_{L1\alpha}(0)\; g_{L2\beta}(0) \;\to\; \epsilon^{\alpha\beta}\; g_{L2\alpha}(0)\; g_{L1\beta}(0) \;=\; (-1)\; \epsilon^{\alpha\beta}\; g_{L1\alpha}(0)\; g_{L2\beta}(0)$$

(A): Prediction for the nature of the Uniform (Homogeneous) Phase



using

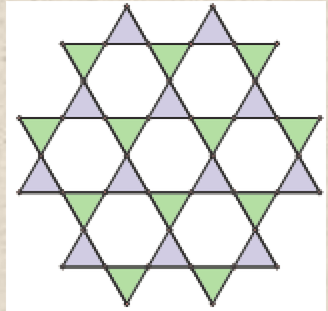
for each pair of corner-sharing triangles



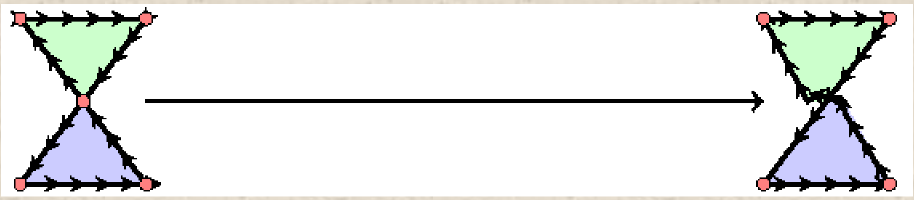
a single edge state described by $SU(2)_1$ conformal field theory surrounds the the system which is thus in the (gapped) Bosonic Laughlin quantum Hall state at filling $\nu = 1/2$ [described by SU(2)-level-one Chern Simons theory].

(See below: we have checked numerically the presence of edge state, torus ground state degeneracy, entanglement spectrum, S- and T-matrices, etc.).

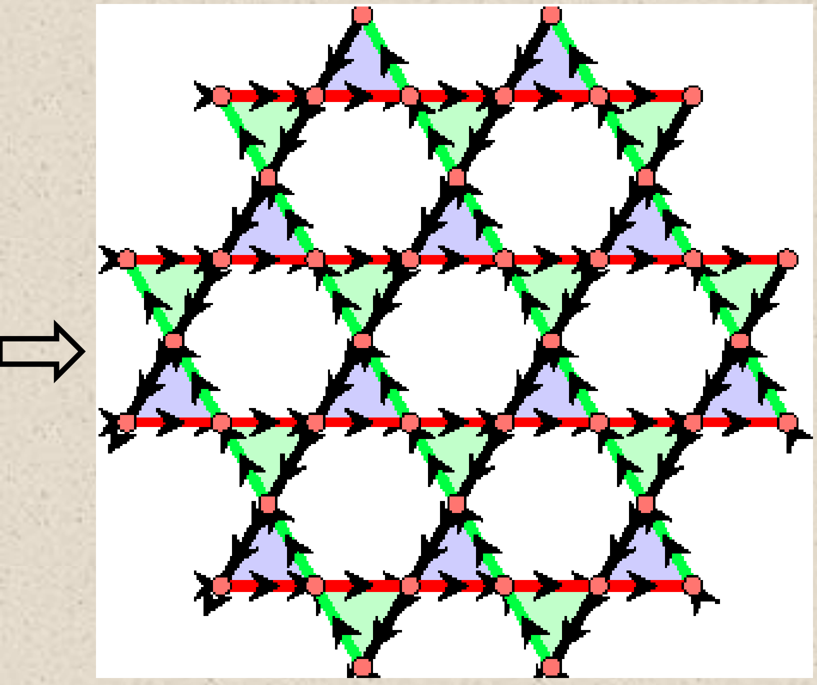
(B): Prediction for the nature of the Staggered Phase

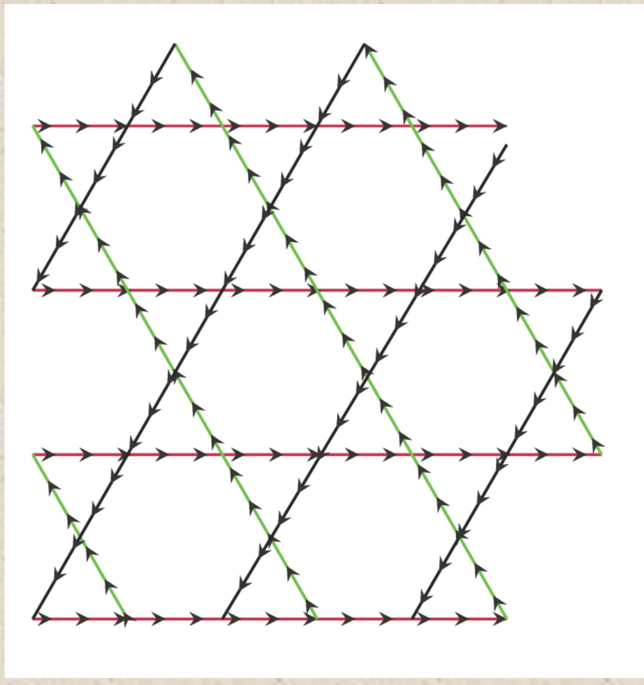


using



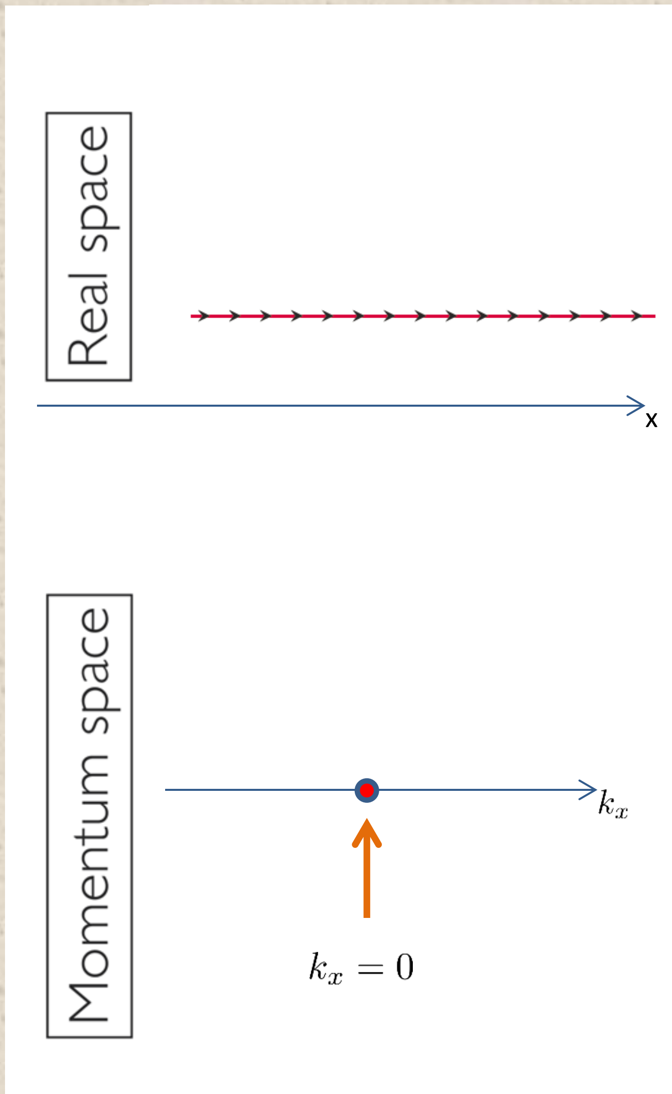
for each pair of *corner-sharing* triangles



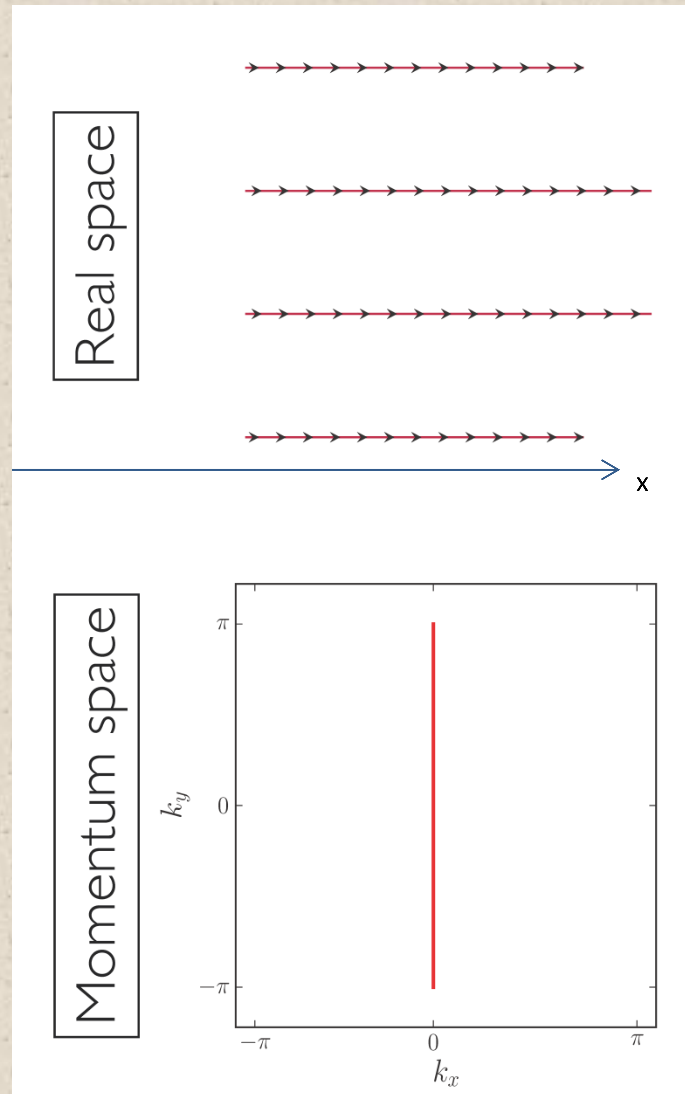


Three stacks of parallel lines of edge states, rotated with respect to each other by 120 degrees

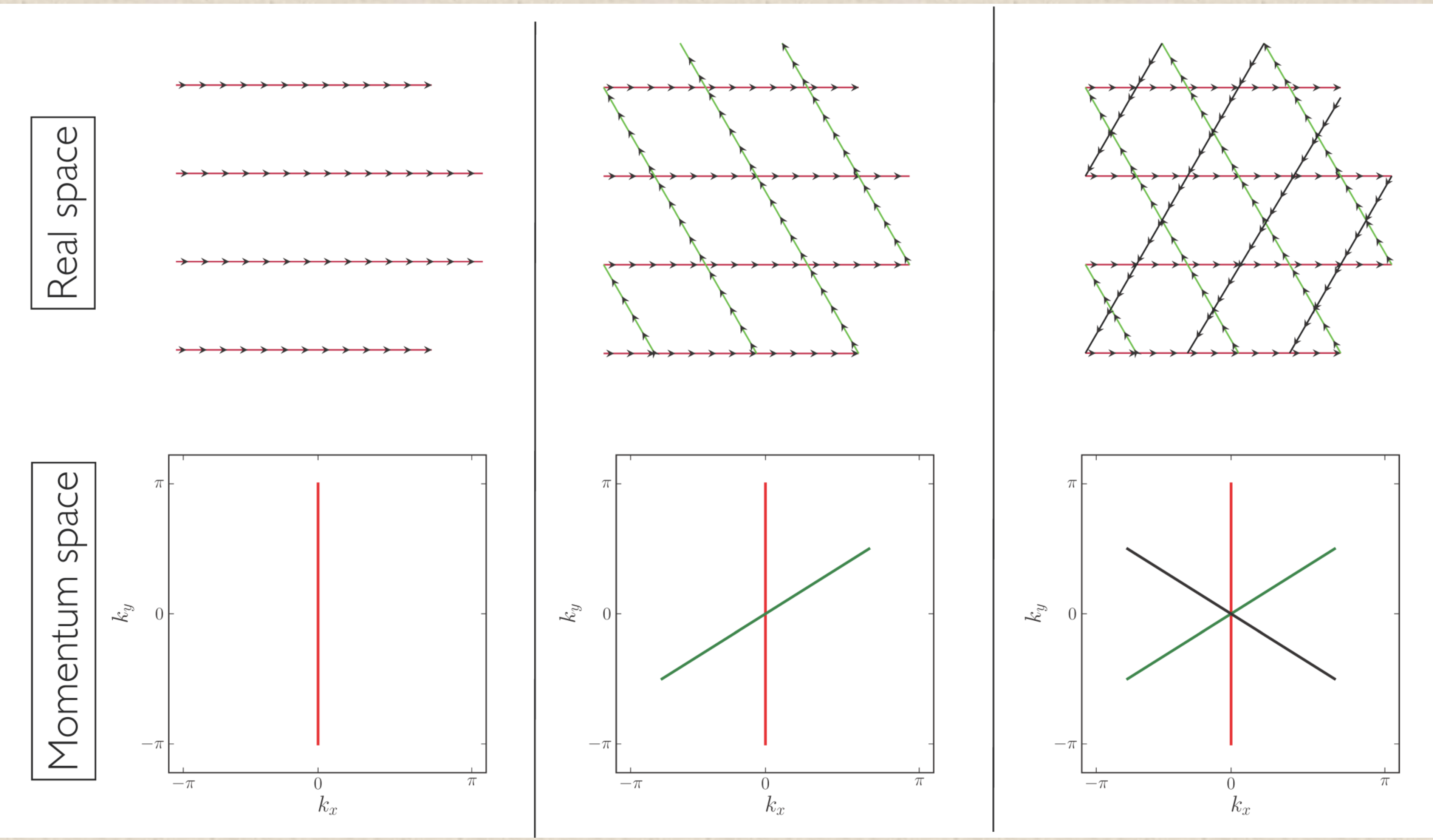
A single edge state
(in the “x-direction”):

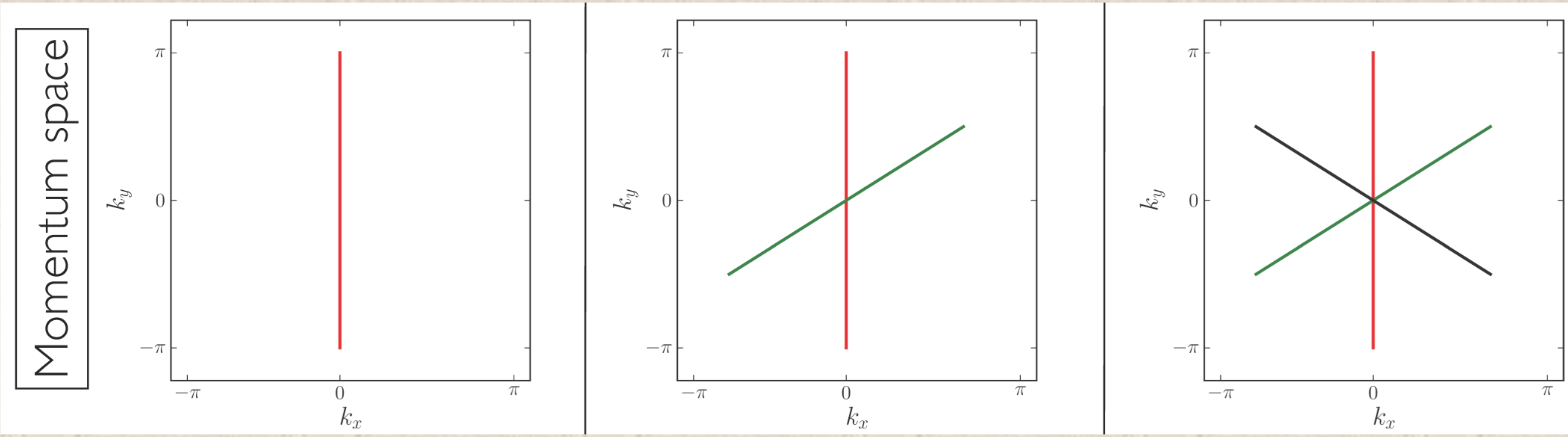


One stack of parallel edge states
(in the “x-direction”):

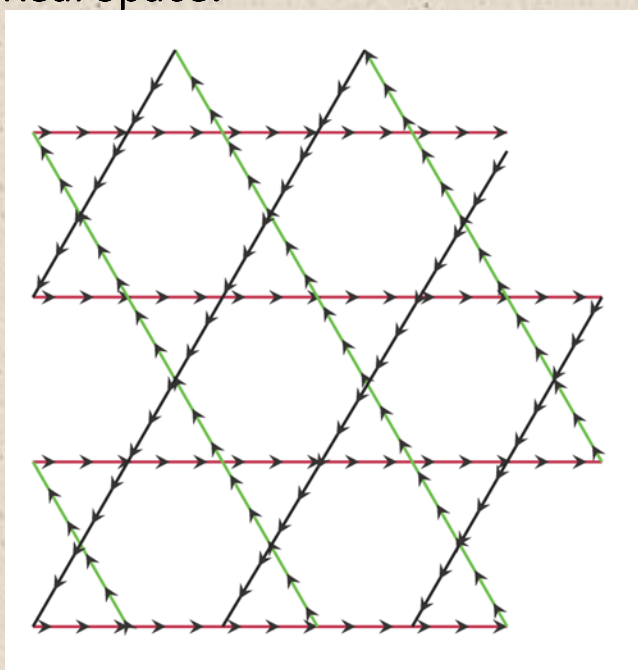


Three stacks of parallel edge states (rotated with respect to each other by 120 degrees):

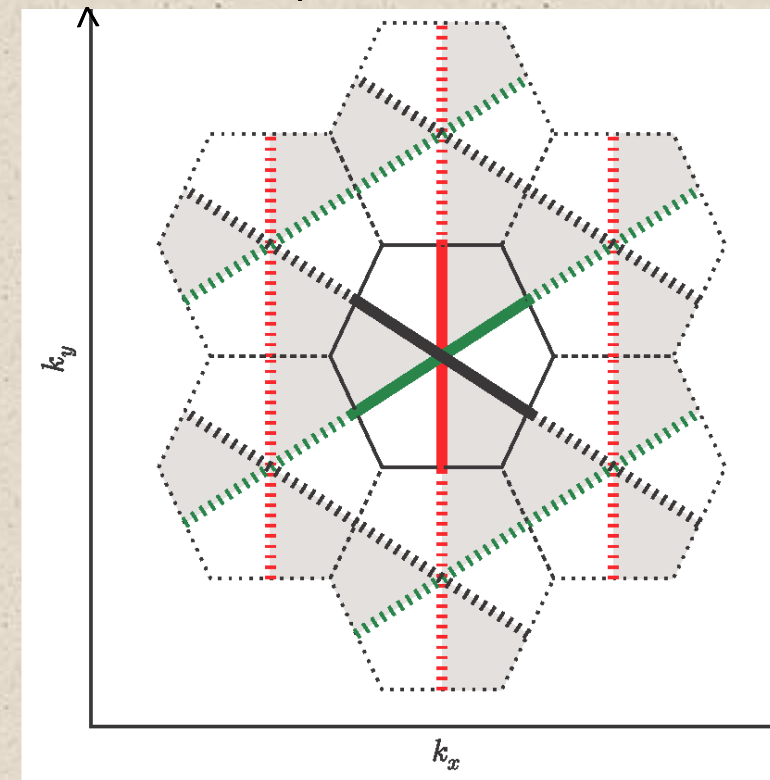




Real Space:

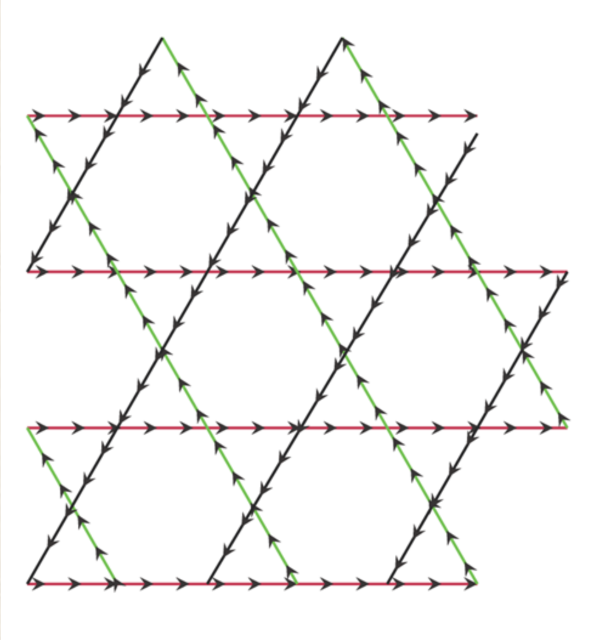


Momentum Space:

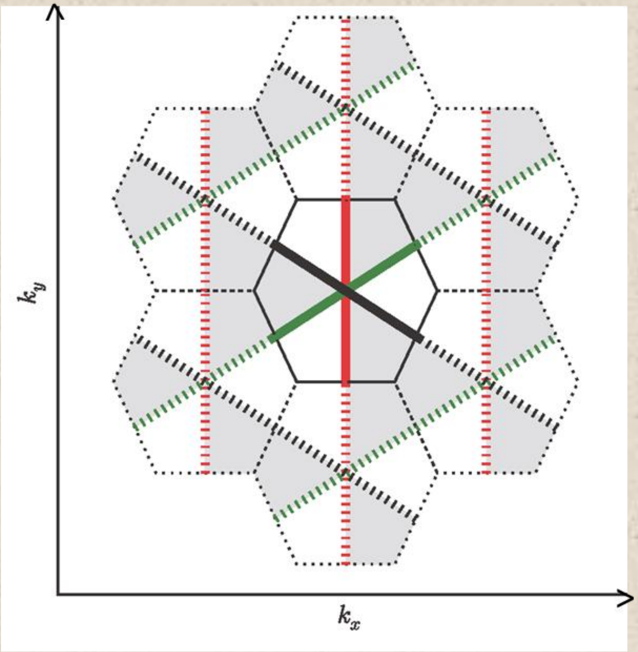


SYMMETRIES:

Real space:



Momentum space:



- Rotational symmetry (by 120 degrees)
- Reflection symmetry: y <-> -y
- (Reflection symmetry: x <-> -x) composed with (time-reversal symmetry)

CHECK NETWORK MODEL PICTURE IN THE CASE OF A TOY MODEL

Non-interacting Majorana Fermion Toy model:

CHECK NETWORK MODEL PICTURE IN THE CASE OF A TOY MODEL

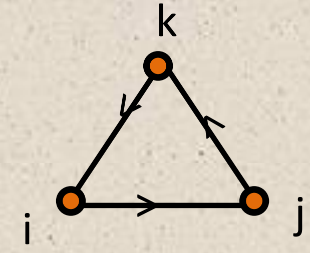
Non-interacting Majorana Fermion Toy model:

-> Replace { spin-1/2 operators $\vec{S}_i$ at the sites of the Kagome lattice } by { Majorana Fermion zero modes $\gamma_i (= \gamma_i^\dagger)$ }

-> On each triangle, replace:

Spin Chirality operator $\chi_{ijk} = \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k)$ by $\tilde{\chi}_{ijk} := i(\gamma_i \gamma_j + \gamma_j \gamma_k + \gamma_k \gamma_i)$

defining a notion of chirality for a triangle:



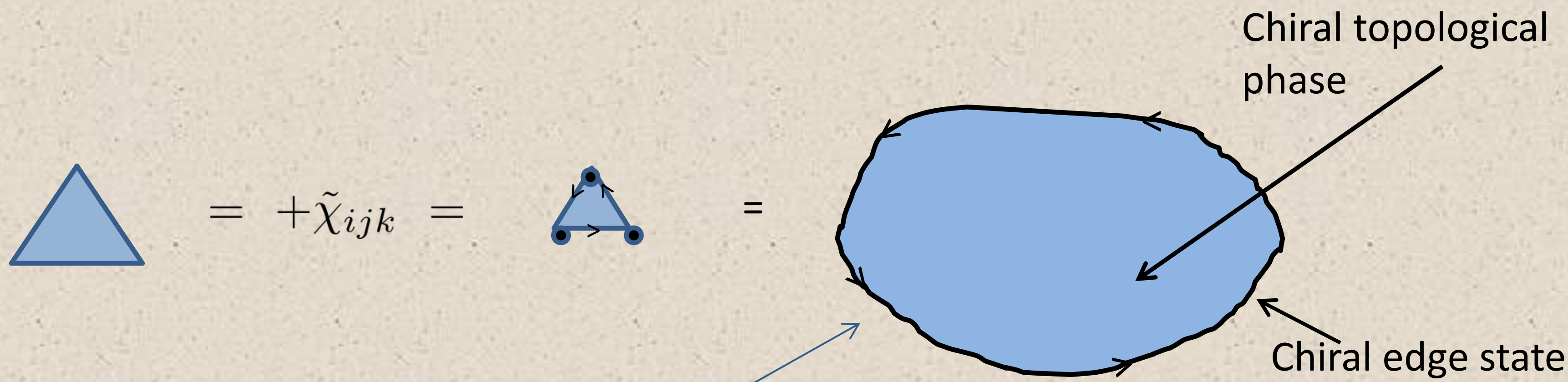
-> Hamiltonian as before (sum over triangles):

$$H = K \sum_{\triangle} \tilde{\chi}_{ijk} \pm K \sum_{\bigtriangledown} \tilde{\chi}_{ijk}, \qquad (K > 0)$$

"NETWORK MODEL"

(Non-interacting Majorana Fermion Toy model)

Think in terms of a "network model" to try to gain intuition about the behavior of the system:



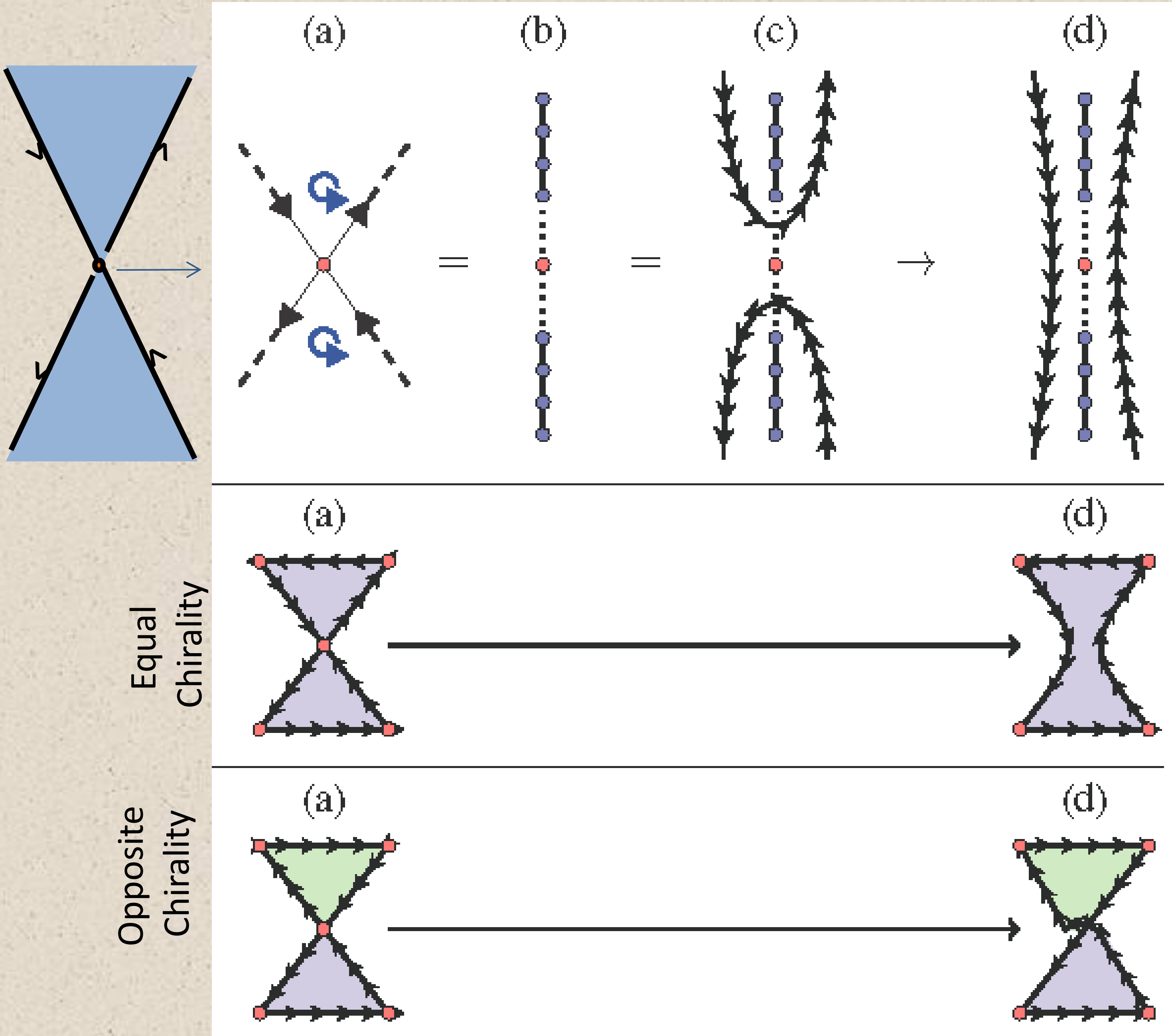
The 3-spin interaction on a triangle breaks time-reversal symmetry (and parity):

-> can view each triangle with 3-spin interaction as the seed (puddle) of a chiral topological phase [which is here the 2D $p_x + ip_y$ topological superconductor (symmetry class D), possessing a chiral Ising CFT edge theory (central charge c=1/2)]

[Grosfeld+Stern, PRB 2006; AWWL, Poilblanc, Trebst, Troyer, N.J.Phys. (2012)]

Joining two triangles (puddles) with a corner-sharing Majorana zero mode: resonant-level tunneling

[Kane+Fisher, 1992]

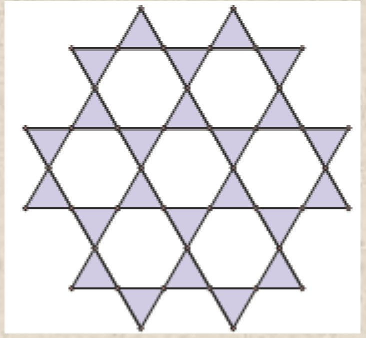


In both cases:

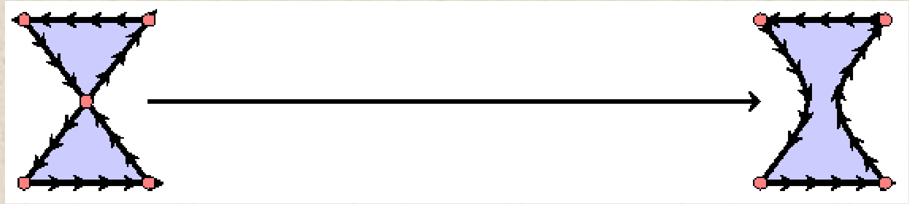
The two puddles join to form a larger puddle [surrounded by a single edge state]

(A): Prediction for the nature of the Uniform (Homogeneous) Phase

(Non-interacting Majorana Fermion Toy model)

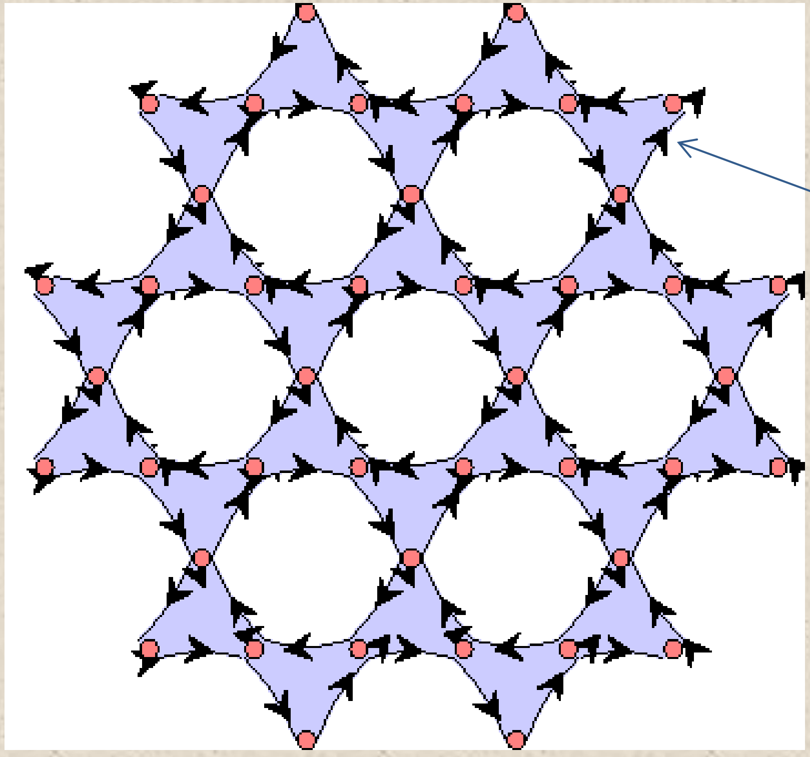


using



for each pair of corner-sharing triangles

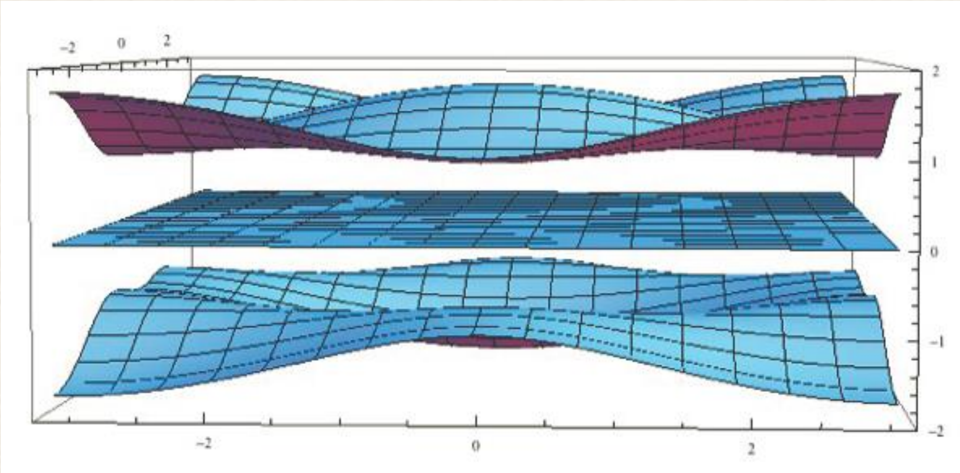
⇨



a single edge state described by Ising conformal field theory surrounds the the system which is thus is the 2D topological superconductor in symmetry class D (e.g. $p_x + ip_y$)

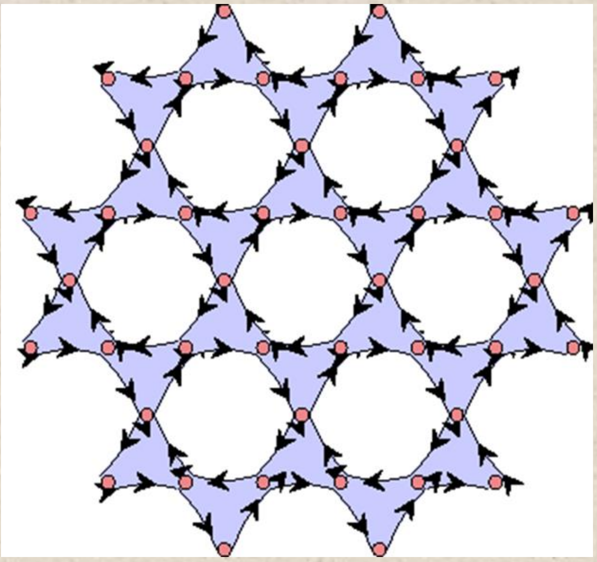
-- (Non-interacting) Fermion solution of the Uniform (Homogeneous) case:

[Ohgushi, Murakami, Nagaosa (2000)]



- Gapped spectrum
- Chern number of top and bottom bands is ± 1

-- **In agreement with prediction from Network Model:**



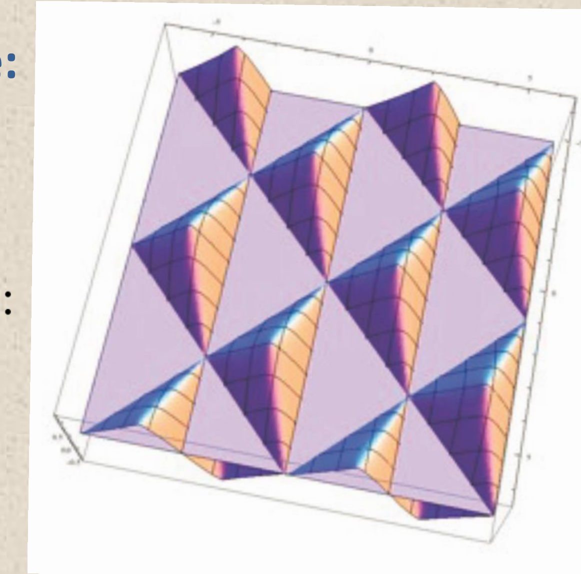
(B): Prediction (from Network) for the nature of the Staggered Phase

(Non-interacting Majorana Fermion Toy model)

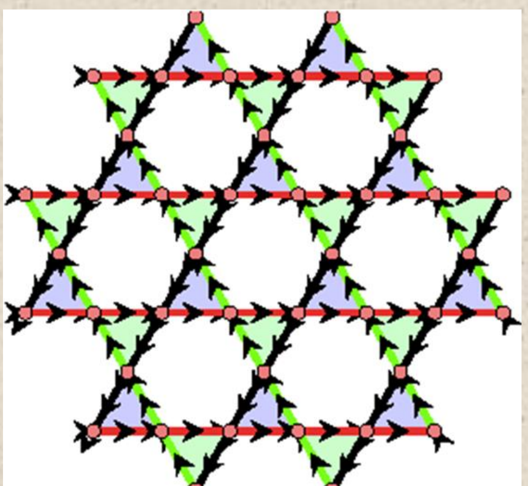
-- (Non-interacting) Fermion solution of the staggered case:

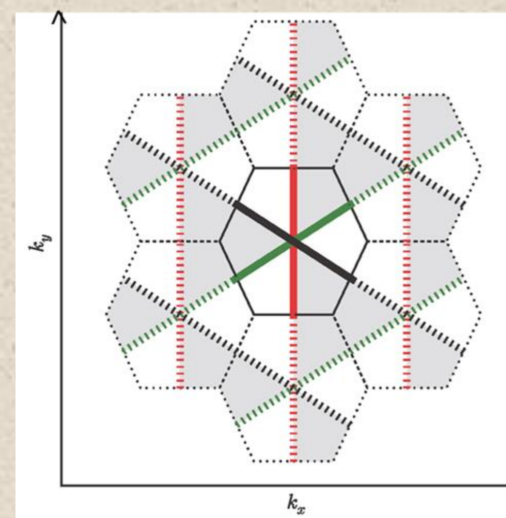
[Shankar, Burnell, Sondhi (2009)]

$$\text{Dispersion} \quad E(k_x, k_y) \text{ versus } (k_x, k_y):$$



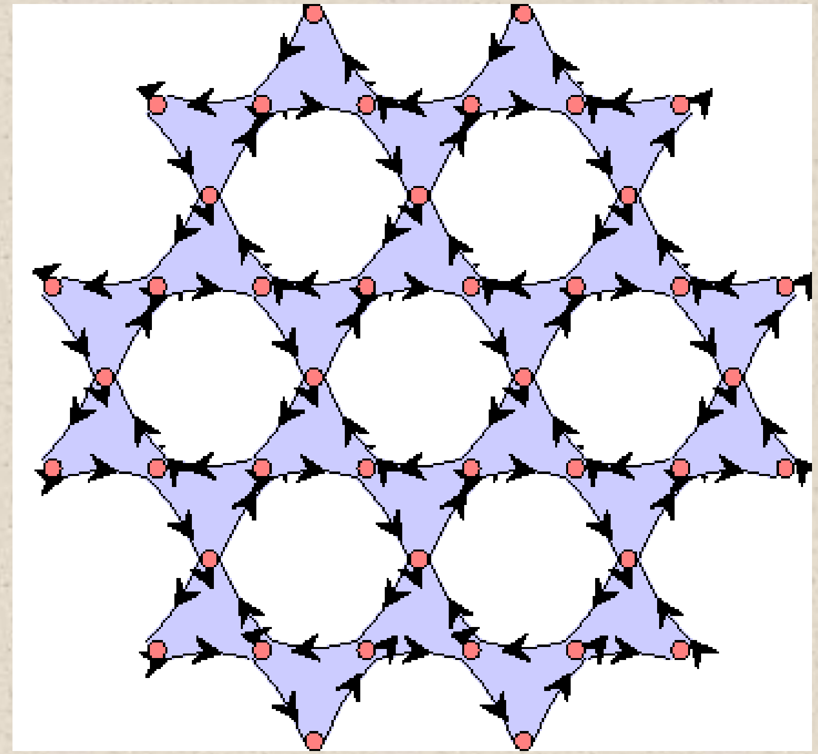
-- **In agreement with prediction from Network Model:**





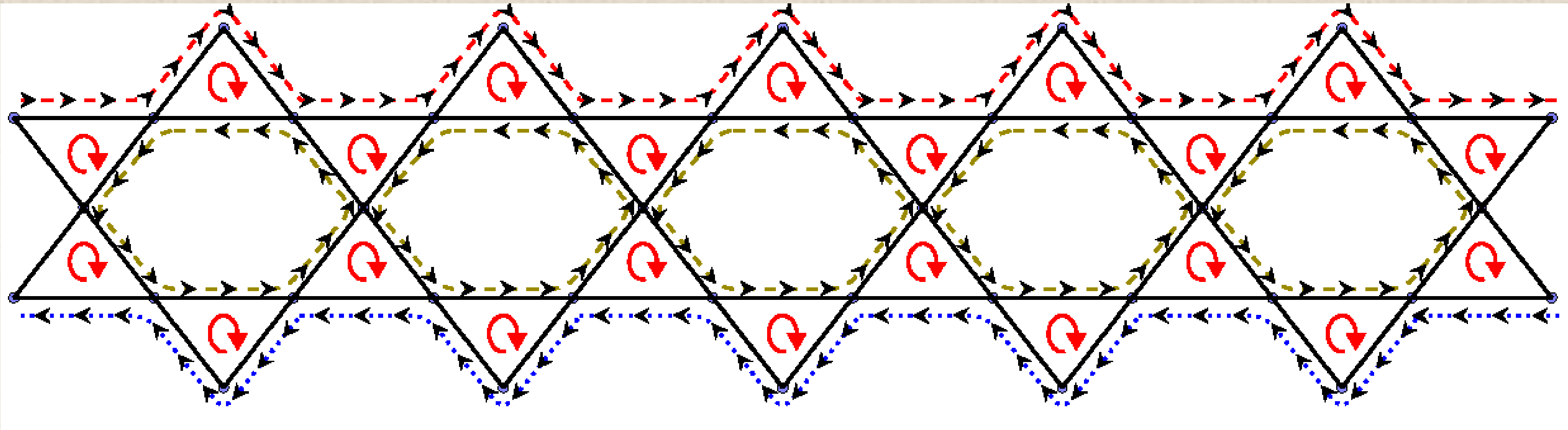
RETURN TO THE ORIGINAL MODEL OF S=1/2 SU(2) SPINS (NOT SOLVABLE):

(A): UNIFORM CASE - NUMERICAL RESULTS

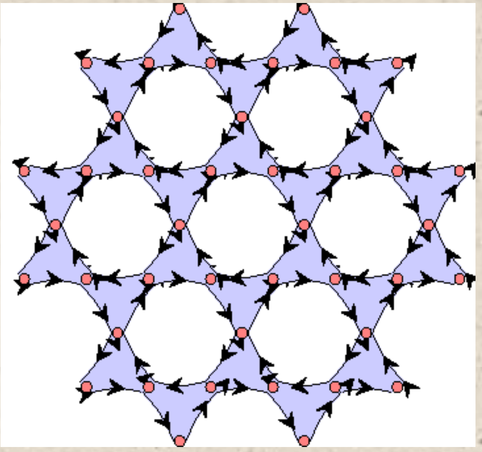


RETURN TO THE ORIGINAL MODEL OF S=1/2 SU(2) SPINS (NOT SOLVABLE):

(A): UNIFORM LADDER GEOMETRY ("thin torus", "thin strip" limits)



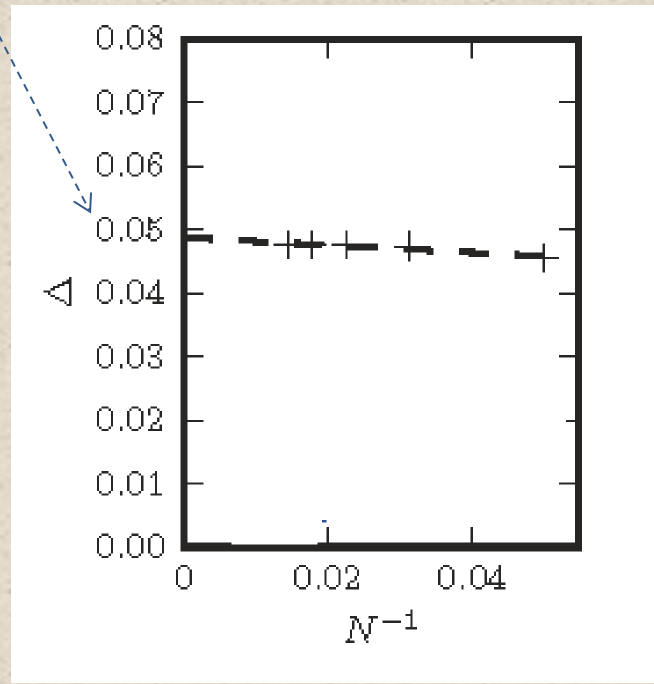
[recall 2D bulk model:



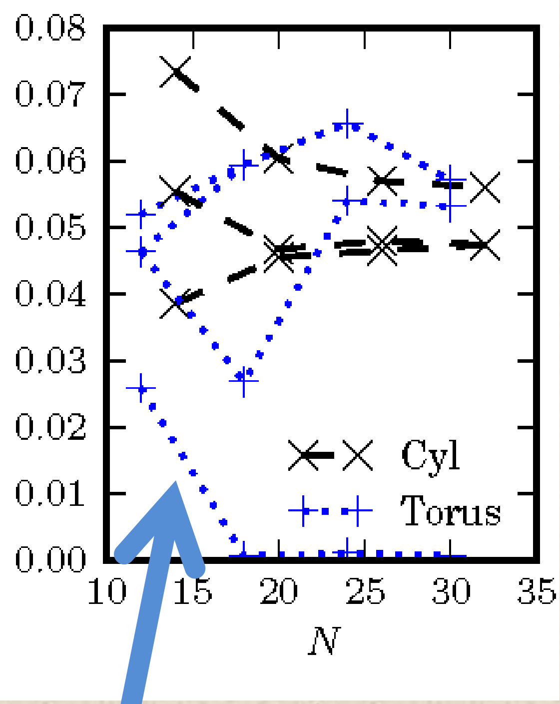
]

(A1): Numerical Results for Uniform phase: gap, and ground state degeneracy on torus

gap to 1st excited state: **cylinder** geometry



gaps to various low excited states:
torus **versus** **cylinder** geometry

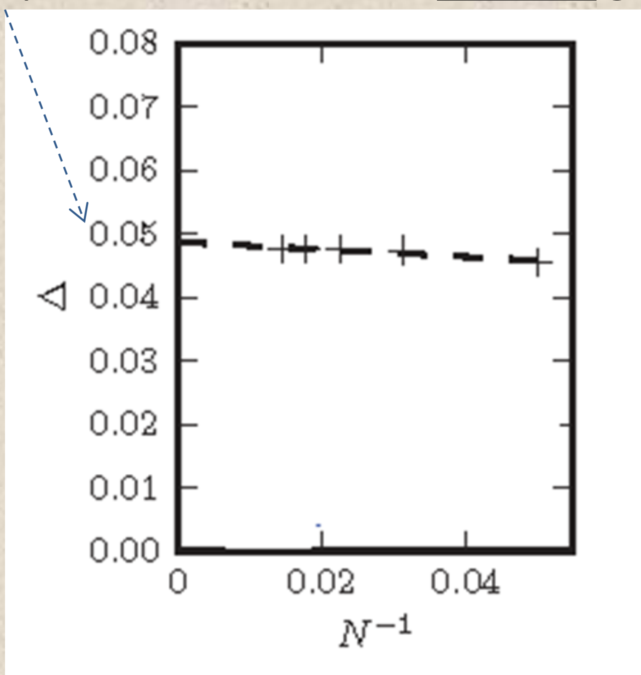


torus: 1 additional state becomes degenerate with the ground state !

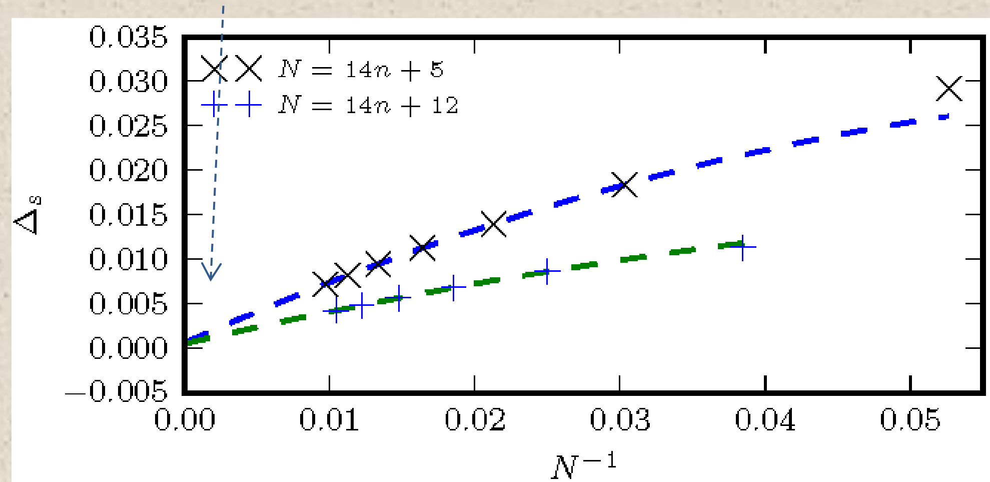
cylinder: non-degenerate ground state

(A2): Numerical Results for the Uniform phase: gapped on the cylinder, versus gapless on strip

gap to 1[st] excited state: **cylinder** geometry



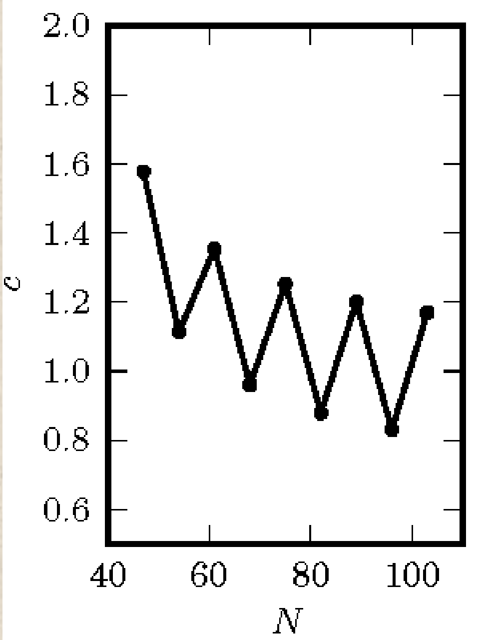
gapless: **strip** geometry (due to edge state)

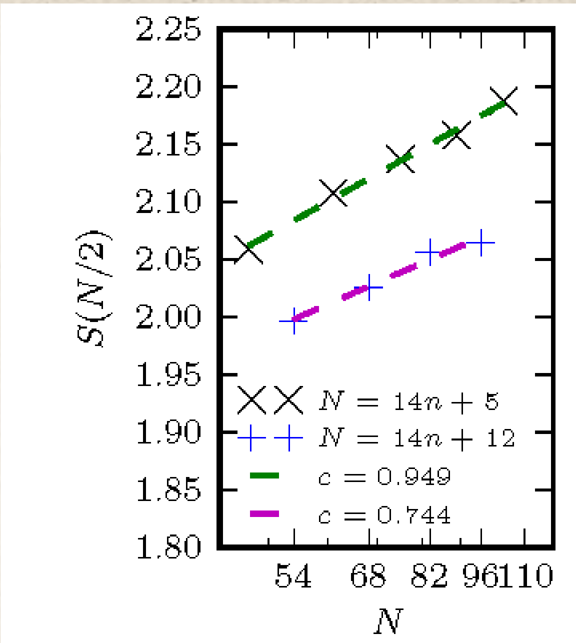


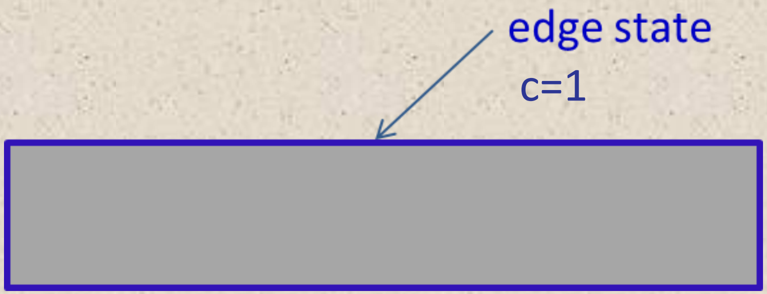
edge state

(A3): Numerical Results for the Uniform phase: entanglement entropy on a strip (c=1)

$$S(l) = S_0 + \frac{c}{6}\ln\left(\frac{2L}{\pi}\sin\frac{\pi l}{L}\right)$$ -> at center of the system: $l = \frac{L}{2}$ -> $$S(l) = S_0 + \frac{c}{6}\ln L$$

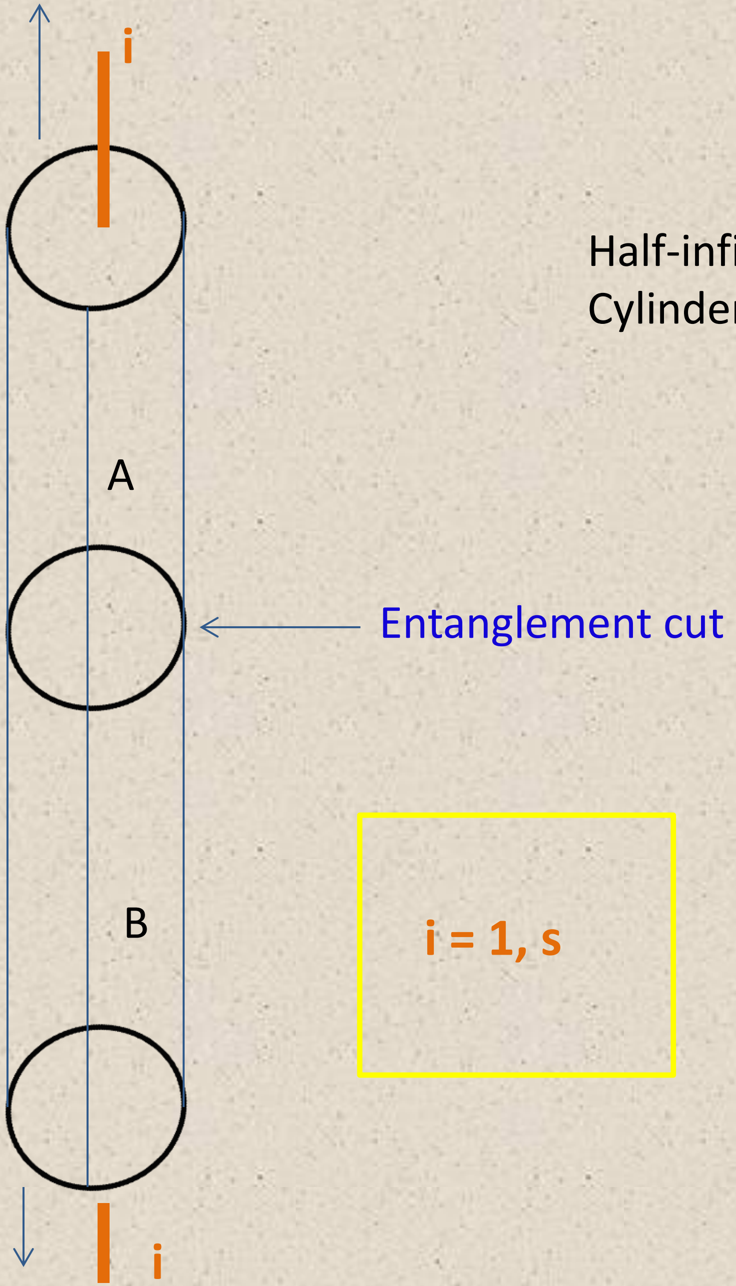




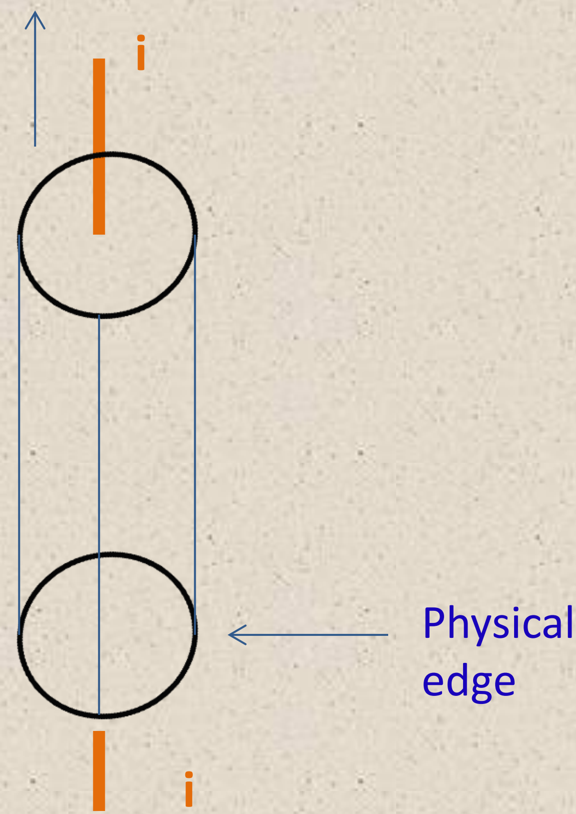


(A4): Numerical Results for the Uniform phase: Entanglement spectrum

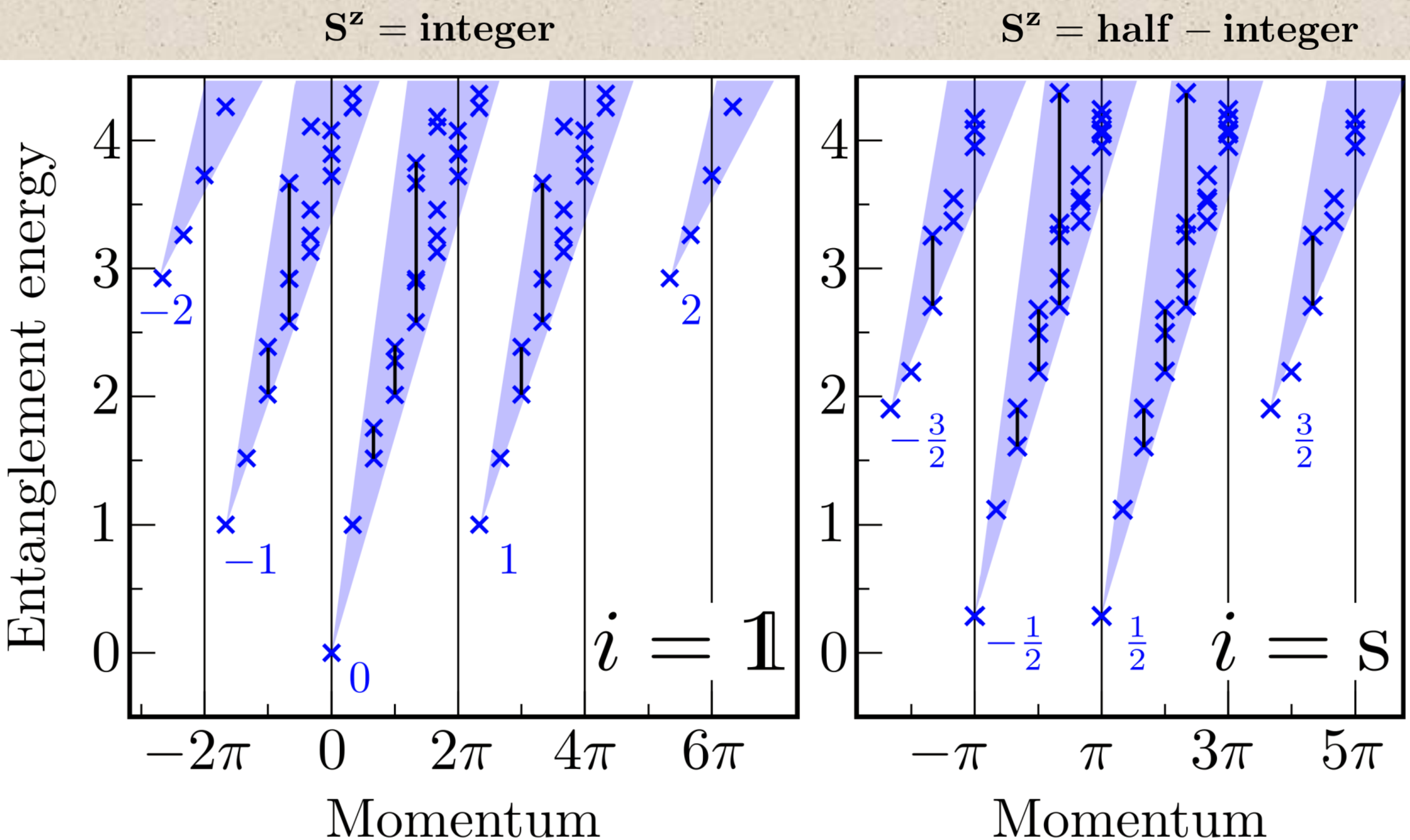
Infinite Cylinder:



Half-infinite Cylinder:



Entanglement spectrum:



Degeneracies (at fixed Sz): 1-1-2-3-5-… (=number of partitions of non-neg. integers)

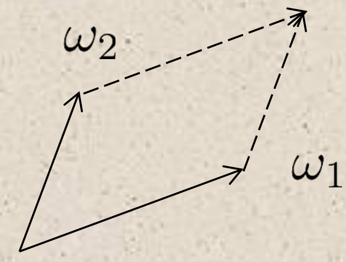
Aside:

$$
\begin{array}{rcl}
\text{number of partitions} & = & \underline{\textbf{U(1) Kac-Moody Algebra Degeneracies:}} \\
\underline{0} & 1 & |\mathbf{i}\rangle \\
\underline{1} & 1 & J^z_{-1}|\mathbf{i}\rangle \\
\underline{2} = \underline{1+1} & 2 & J^z_{-2}|\mathbf{i}\rangle, (J^z_{-1})^2|\mathbf{i}\rangle \\
\underline{3} = \underline{2+1} = \underline{1+1+1} & 3 & J^z_{-3}|\mathbf{i}\rangle, J^z_{-2}J^z_{-1}|\mathbf{i}\rangle, (J^z_{-1})^3|\mathbf{i}\rangle \\
\underline{4} = \underline{3+1} = \underline{2+2} = \underline{2+1+1} = \underline{1+1+1+1} & 5 & \ldots
\end{array}
$$

etc.

(A5): Numerical Results for the Uniform phase: Anyonic Bulk Excitations; Modular S- and T-Matrices (=Generators of the Modular Group)

Torus:



$$\{z \in \mathbf{C}| \quad z = z + \omega_1 = z + \omega_2\}$$

Ground State Wavefunctions on the Torus

anyons : a = 1, 2, ..., M (= number of anyons); a = 1(identity anyon)

Two Bases:

$$\textbf{Basis 1}: \quad |\Psi_a^{\omega_1}\rangle, \quad a = 1, 2, ..., M$$

$$\textbf{Basis 2}: \quad |\Psi_a^{\omega_2}\rangle, \quad a = 1, 2, ...M$$

$$\mathbf{S}-\textbf{matrix}: \quad |\Psi_a^{\omega_2}\rangle = \sum_b S_{ba}\ |\Psi_b^{\omega_1}\rangle,$$

$$\mathbf{T}-\textbf{matrix}: \quad |\Psi_a^{\omega_j}\rangle = \exp\{-i2\pi c/24\}\ \exp\{i2\pi h_a\}\ |\Psi_a^{\omega_j}\rangle, \quad \text{Dehn Twist}$$

[c central charge, h_a = "topological spin"]

Recall:

QuantumDimensions : $d_a = S^a_1 / S^1_1$

Total Quantum Dimension : $\mathcal{D} = 1/S^1_1 = \sqrt{\sum_a (d_a)^2}$

For the Bosonic Laughlin State:

$a = \mathbf{1}, \mathbf{s}; \quad d_1 = d_s = 1; \text{ hence} : \mathcal{D} = 2$

$c = 1(\text{central charge}); \ \mathrm{h}_1 = 1, \mathrm{h}_\mathrm{s} = 1/4$

$$T = e^{-i2\pi/24} \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}, \qquad S = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Numerical Results (system of 48 sites):

$$T_{num} = e^{-i(2\pi/24)\cdot 0.988} \begin{bmatrix} 1 & 0 \\ 0 & i \cdot e^{-i0.0021\pi} \end{bmatrix}, \ S_{num} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0.996 & 0.995 \\ 0.996 & -0.994 \cdot e^{-i0.0019\pi} \end{bmatrix}$$

$$c_{num} = 0.988$$

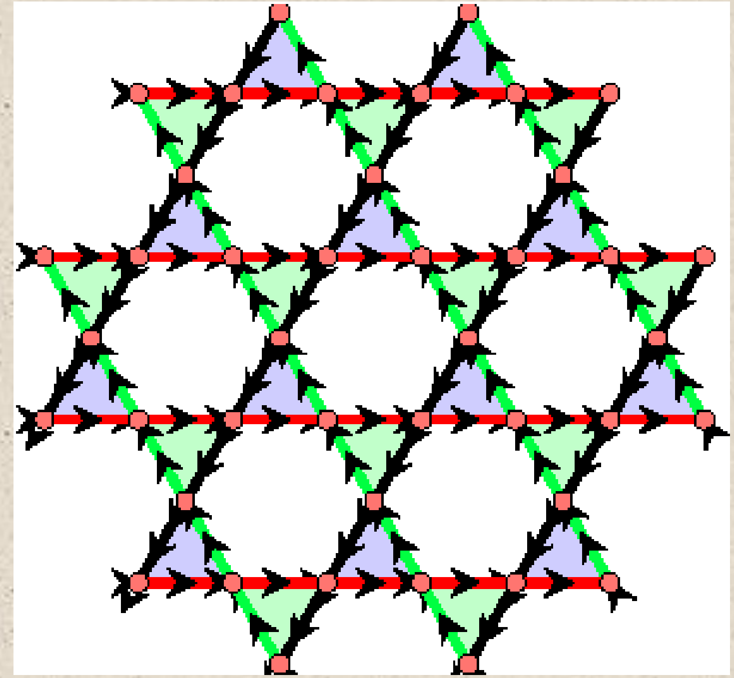
$$\mathcal{D}_{num} = 1/S_{11} = \sqrt{2}/0.996$$

(determines also topological entanglement entropy : $S = const.L - \gamma; \quad \gamma = \ln \mathcal{D}$)

Note: S_{num} is unitary -> the full set of anyon particles has been retained

RETURN TO THE ORIGINAL MODEL OF S=1/2 SU(2) SPINS (NOT SOLVABLE):

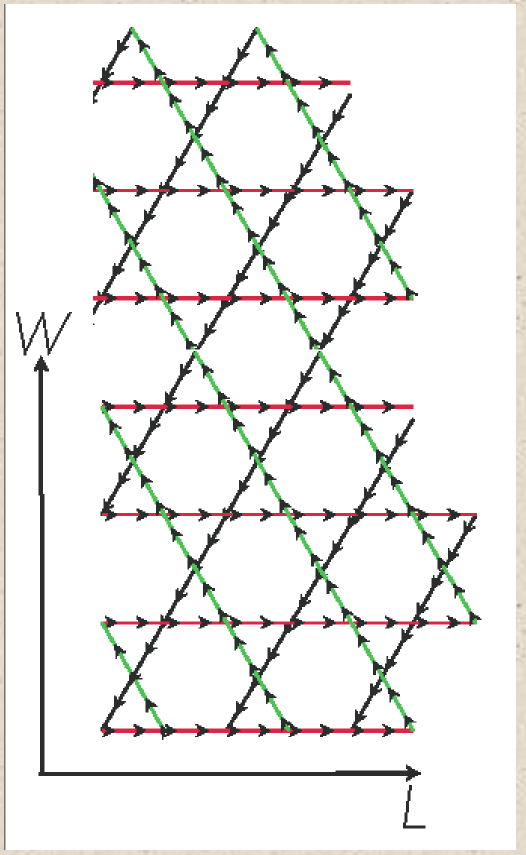
(B): STAGGERED CASE - NUMERICAL RESULTS



RETURN TO THE ORIGINAL MODEL OF S=1/2 SU(2) SPINS (NOT SOLVABLE):

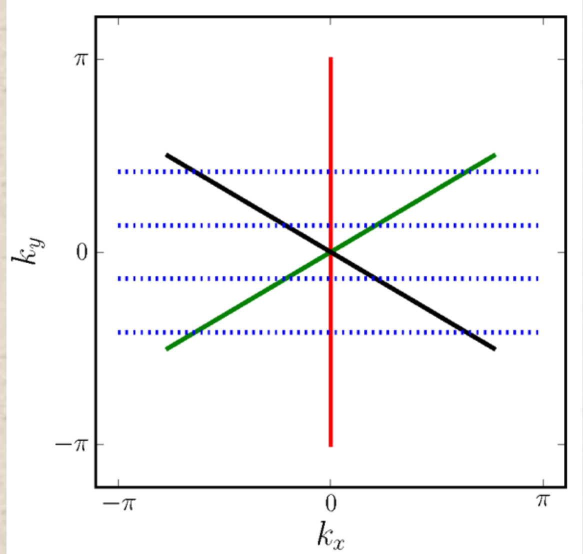
(B): STAGGERED CASE

Real (position) space:



L >> W

Momentum space:



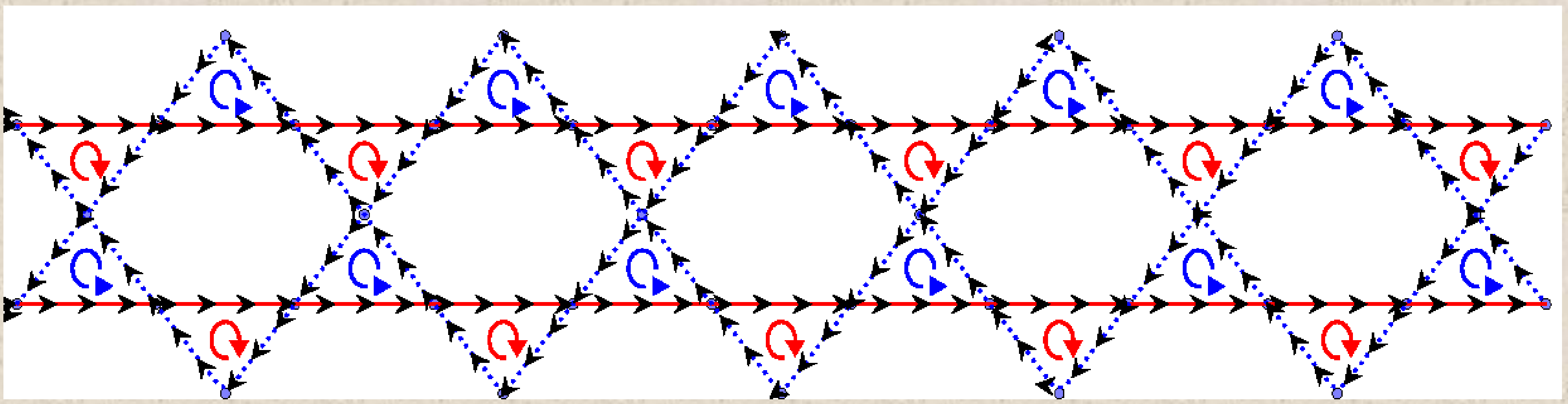
$$W = \#\text{ of } k_y \text{ points}$$

$$S = S_0 + \frac{W \cdot c}{6} \ln L$$

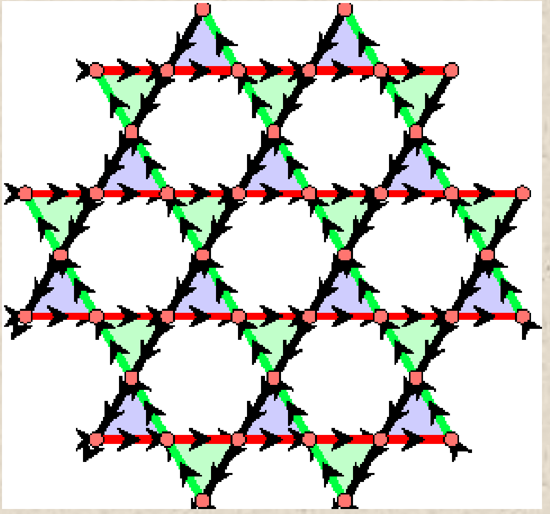
(bipartite entanglement cut: center of system)

(B): STAGGERED CASE -- W=2 LADDER GEOMETRY

Network model prediction: c = 2

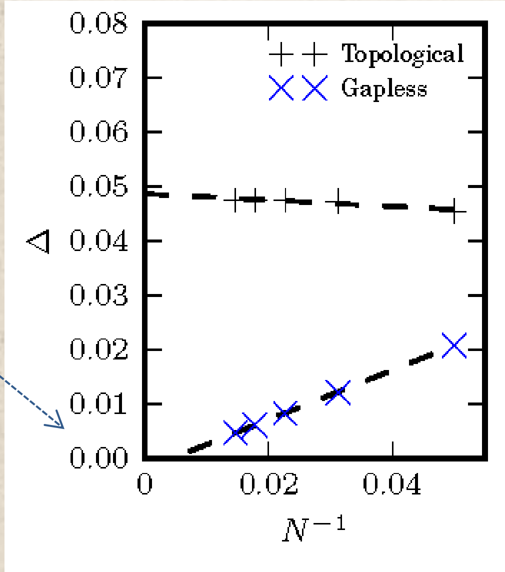


recall 2D bulk model:



(B): NUMERICAL RESULTS FOR THE STAGGERED CASE
-- W=2 LADDER GEOMETRY

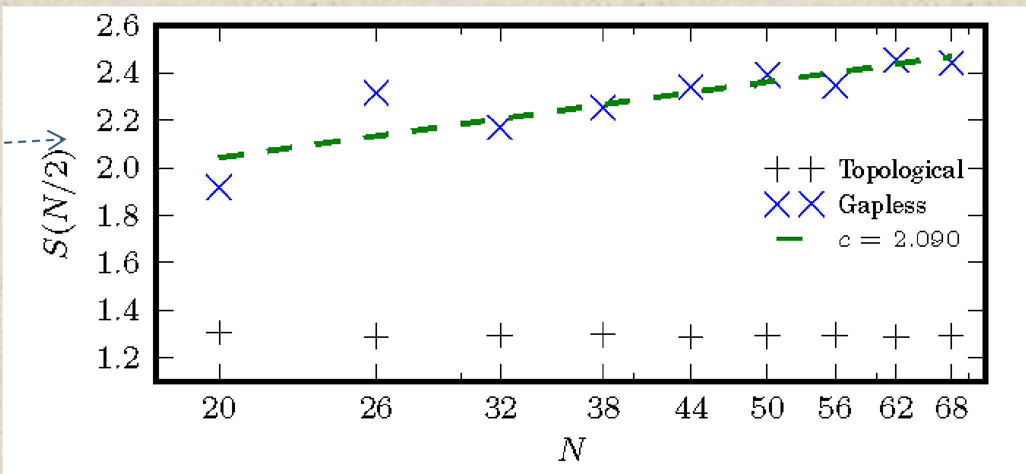
- **Vanishing gap** to 1st excited state: **cylinder** geometry



- Scaling of **entanglement entropy** with system size (center): **cylinder** geometry (**confirms c=2**)

(c=2)

$$S(l = N/2) = S_0 + \frac{c}{6} \ln N$$

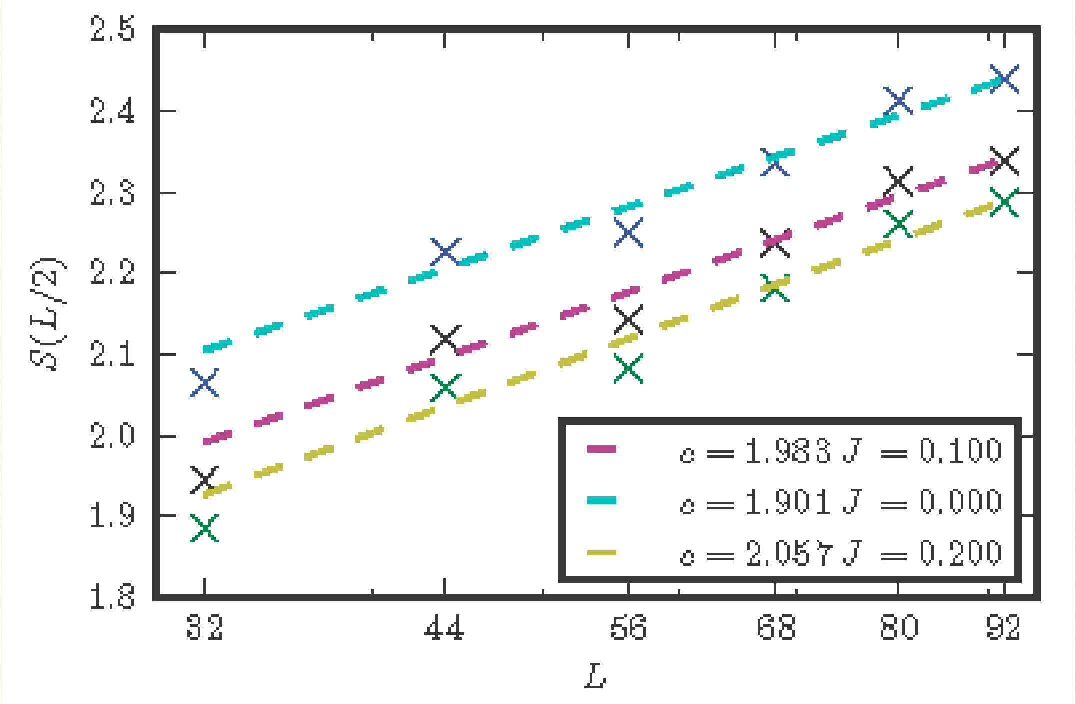


(B): STAGGERED CASE - W=2 LADDER GEOMETRY:

Stability to Heisenberg interactions (cylinder geometry)

Heisenberg interaction

$$H = K\sum_{\triangle}\chi_{ijk} \pm K\sum_{\nabla}\chi_{ijk} + J\sum_{i,j}\vec{S}_i\cdot\vec{S}_j$$



END