Hundred Years of Gauge Theories

hysikzentrum Bad Honnef

31 July 2018

Thomas Ryckman

Stanford University

Hermann Weyl:

symbolic construction
from the "purely infinitesimal"
&
gauge invariance

Are Weyl's intricate philosophical views of any possible relevance to current concerns, e.g.,

"The gauge principle is generally regarded as the most fundamental cornerstone of modern theoretical physics. In my view its elucidation is the most pressing problem in current philosophy of physics."

M. Redhead, "The Interpretation of Gauge Symmetry", in M. Kuhlmann, H. Lyre and A. Wayne (eds.), *Ontological Aspects of Quantum Field Theory*. London-Singapore-Hong Kong: World Scientific, 2002, 281-301 , 299.

?

Weyl's philosophy of symbolic construction
& role of invariance principles within it

1. *the infinitesimal agenda*: Leibniz, Riemann, Lie, Weyl
2. *subjectivity/objectivity*: the *problem of relativity*
3. symbolic construction: *arbitrariness of starting from "ego's immediate life of intuition" countered with principles of invariance*
4. goal: *a priori* mathematical framework for all possible types of covariant linear physical quantities; project *actual* upon *a priori* possible *background*
5. 1918 gauge principle: *from purely infinitesimal world-geometry*
6. 1929 gauge principle: *revise a priori framework for Dirac eq.*
7. "*Physics shouldn't depend on the physicist*" ?

"As far as I see, all *a priori* statements in physics have their origin in symmetry."* ‡

How "far" did Weyl see ?

* "Symmetry", *J. Washington Acad. Sciences* **28**, 1938.
‡ *Symmetry*. Princeton University Press, 1952.

Weyl meant something quite precise:

a priori statement in physics is an *a priori* specification

-- *of the possible linear covariant quantities,*

and

-- *of the range of their possible values* *a priori* statements seek to specify the *possible kinds* of covariant quantities that appear in physical theories -- since only these are *objective* quantities.

Identifying them is the solution to the "relativity problem":

> *"the relativity problem: to fix objectively a class of equivalent coordinatizations and to ascertain the group of transformations* ***S*** *mediating between them."*

The Classical Groups (1939), 16.

I. *the infinitesimal agenda*

G.W. Leibniz 1646-1716

B. Riemann 1826-1866

S. Lie 1842-1899

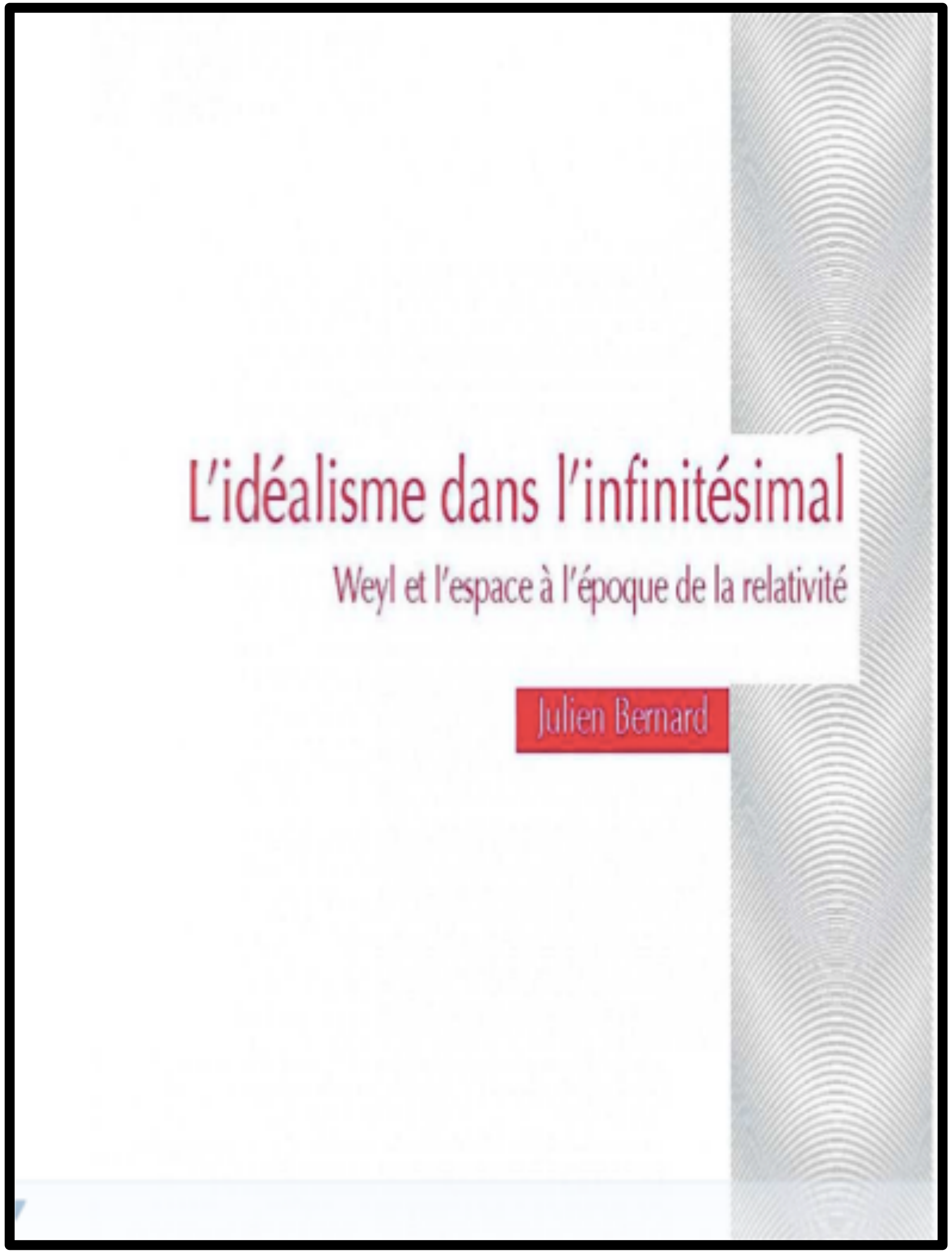
"the labyrinth of the continuum" 1672-86

-- continua merely *ideal*, not *real*; not resolvable into, nor composed from, determinate elements

-- infinitesimals are *fictions*; analysis is *ideal*

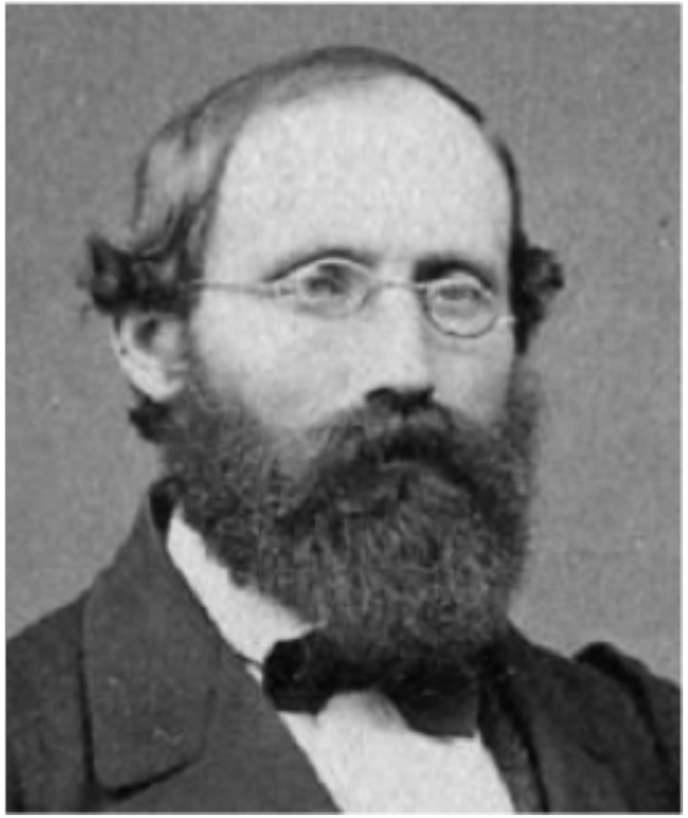
Specimen Dynamicum 1695

-- *heuristic:* force as infinitesimal element of action responsible for continuous changes in a body's state of motion *like Leibniz, Weyl's is also an "idealism in the infinitesimal"*



2015 book of Julien Bernard

the infinitely small as epistemologically primitive



"Questions concerning the immeasurably large, are, for the explanation of Nature, useless questions. It is quite otherwise however with questions concerning the immeasurably small. Knowledge of the causal connection of phenomena is based essentially upon the precision with which we follow them down into the infinitely small."

"*Über die Hypothesen, welche der Geometrie zu Grunde liegen*"
Probe-Vorlesung, 10 June 1854

distinct laws in the infinitely small

"One sees that in the passage from finitely separated points to ones infinitely close there is a complete leap (*Sprung*)and that to infinitely close points belong entirely other laws than those belonging to points at finite separation."

Lie-Engel, Theorie der Transformationsgruppen, Bd.III, 1893, 460

Lie groups can be linearized in passing to an infinitesimal group acting in the tangent space of the group identity.

“Leibniz- Riemann-Lie principle” (Weyl)

“The productivity shown by the differential calculus, by contiguous action [field] physics (*Nahewirkungsphysik*), and by Riemannian geometry certainly rests upon the principle: To understand the world, according to its form and content, from its behavior in the infinitely small, clearly because *all problems can be linearized in passing to the infinitely small.*”

Mathematische Analyse des Raumproblems. Vorlesungen gehalten in Barcelona und Madrid. 1923, 45

“Leibniz- Riemann-Lie principle” (Weyl)

“Die Ersetzung der endlichen Gruppe durch die infinitesimal – das ist wieder der ‘Rückgang aufs Unendlichkeleine’! – ist einer der Hauptgedanken der Lieschen Theorie.”

Mathematische Analyse des Raumproblems. 1923, 34

"Leibniz- Riemann-Lie principle" (Weyl)

"As the true lawfulness of nature, according to Leibniz's continuity principle, finds its expression in *Nahewirkungsgesetzen*, so the basic relations of geometry should concern only infinitely closely adjacent points *("Nahgeometrie" im Gegensatz zur "Ferngeometrie")*. *Only in the infinitely small may we expect to encounter elementary and uniform laws;* hence the world must be understood from its behavior in the infinitely small."

Philosophie der Mathematik und Naturwissenschaften, 1927/1949, 86.

- *“all problems can be linearized in passing to the infinitely small.”*

  “this fundamental fact of infinitesimal geometry, *viz.*, that with every point $P \in M$ there is associated a vector space V_P (one is tempted to call it the tangent space at P)….”

  “Similarity and Congruence: a chapter in the epistemology of science” Lecture, 1948-9, 162

- *to understand* ≈ *to construct* field structure starting from the infinitely small “infinitely small” : *immediate locus of transcendental subjectivity*

T_P : “horizon” within which construction *with Evidenz*

“Only the spatio-temporally coinciding and the immediate spatial-temporal neighborhood has a directly clear meaning exhibited in intuition. ... The philosophers may have been correct that our space of intuition bears a Euclidean structure, regardless of what physical experience says.

I only insist ... that to this space of intuition belongs the ego-center [*Ich Zentrum*] and that ... the relations of the space of intuition to that of physics, becomes vaguer the further the distance from the ego-center.” *”Geometrie und Physik”, Die Naturwissenschaften*, 1931

Weyl's group-theoretic solution to the new *Raumproblem* 1921-23

GR spacetimes permit group-theoretic characterization!

the old Helmholtz-Lie solution retains validity in the infinitely small if posed in terms of a group of rotations defined only in the homogeneous tangent space centered on each point $P \in M$.

For $n \geq 2$, vector rotations at P form a continuous group of infinitesimal linear transformations, the Lie algebra of $\mathbb{SO}(n, \mathbb{C})$

--> ***the "nature (Wesen) of space" at each point P is the same, and homogeneous.***

Weyl's group-theoretic solution to the new *Raumproblem* 1921-23

Metrical relations in neighborhood U of P defined on the assumption rotations at any point $P' \in$ U are obtained from rotations at P by a single linear congruence transformation (*length connection*) C taking P to P' by composition with rotations at P.

C enables passing continuously from P to any other point $Q \in M$ so that all subgroups at each point have the same metric, i.e., are congruent to the special linear group $\mathbb{SL}(n)$.

--> ***"the orientation" of rotations at different points can vary, according to matter-energy sources***

Weyl's group-theoretic solution to the new *Raumproblem* 1921-23

Solution rests upon concept of infinitesimal group, *recast in language of linear vector spaces*. … compelling evidence that "mathematical simplicity and metaphysical originality (*Ursprünglichkeit*) are narrowly bound together".*

The purely infinitesimal solution to the new "Space Problem" & desire to find the "group theoretic foundation of the tensor calculus" led to purely mathematical research on representations of semisimple Lie groups and Lie algebras (1925-6)

* *"Das Raumproblem", Jahresbericht d. Deut.Math. Vereinigung* 31 (1922), "*auf diesem Felde mathmatische Einfachheit und metaphysische Ursprünglichkeit in enger Verbindung miteinander stehen.*"

MARP, 1923, p.82

first statement of Lie algebra structure of infinitesimal group

$$\Delta s_\gamma = \left(\frac{\partial \lambda^\gamma_\beta(s)}{\partial s_\alpha} - \frac{\partial \lambda^\gamma_\alpha(s)}{\partial s_\beta}\right) d s_\alpha d' s_\beta$$

nicht, wenn man s durch $\bar{s}$ ersetzt. Das ist aber zufolge der Relationen (15) in der Tat richtig; denn sie liefern den einfachen Ausdruck

$$\Delta s_\gamma = \sum_{\alpha\beta} c^\gamma_{\alpha\beta}\, \delta s_\alpha\, \delta' s_\beta\,.$$

Die Δs_γ sind übrigens nichts anderes als die Parameter der aus den beiden infinitesimalen Operationen $d x_i$, $d' x_i$ mit den Parametern δs und $\delta' s$ — vgl. Formel (18) — zusammengesetzten inf. Operation

$$\Delta x_i = (d' d - d d') x_i .$$

Damit ist nun auch rechnerisch der Zusammenhang zwischen der endlichen und der infinitesimalen Gruppe vollständig hergestellt. Unsere Normierung der Parameter hat zur Folge, wie die Integration der Gleichungen (16), (23) nach der Methode der sukzessiven Approximationen lehrt, daß die Funktionen $\lambda^\gamma_\alpha(s)$, $\psi_\alpha(s^0; s)$ analytisch von den Gruppenparametern s, bzw. von s^0 und s abhängen. Bei den λ handelt es sich sogar um Funktionen von ganz elementarem (Exponential)-Charakter, da die bestimmenden Differentialgleichungen konstante Koeffizienten besitzen.

Der Vollständigkeit halber seien noch einige Bemerkungen hinzugefügt über die *Konstitution* einer infinitesimalen Gruppe, ihre Zusammensetzung, welche durch das System der Zahlen $c^\gamma_{\alpha\beta}$ bestimmt ist. Diese sind schiefsymmetrisch in den Indizes α und β:

$$(24) \qquad c^\gamma_{\beta\alpha} = - c^\gamma_{\alpha\beta},$$

genügen aber außerdem noch gewissen einschneidenden Relationen, welche aus der Identität

$$[v_\alpha[v_\beta v_\gamma]] + [v_\beta[v_\gamma v_\alpha]] + [v_\gamma[v_\alpha v_\beta]] = 0$$

entspringen; sie lauten

$$(25) \qquad \sum_\varrho (c^\delta_{\alpha\varrho} c^\varrho_{\beta\gamma} + c^\delta_{\beta\varrho} c^\varrho_{\gamma\alpha} + c^\delta_{\gamma\varrho} c^\varrho_{\alpha\beta}) = 0 .$$

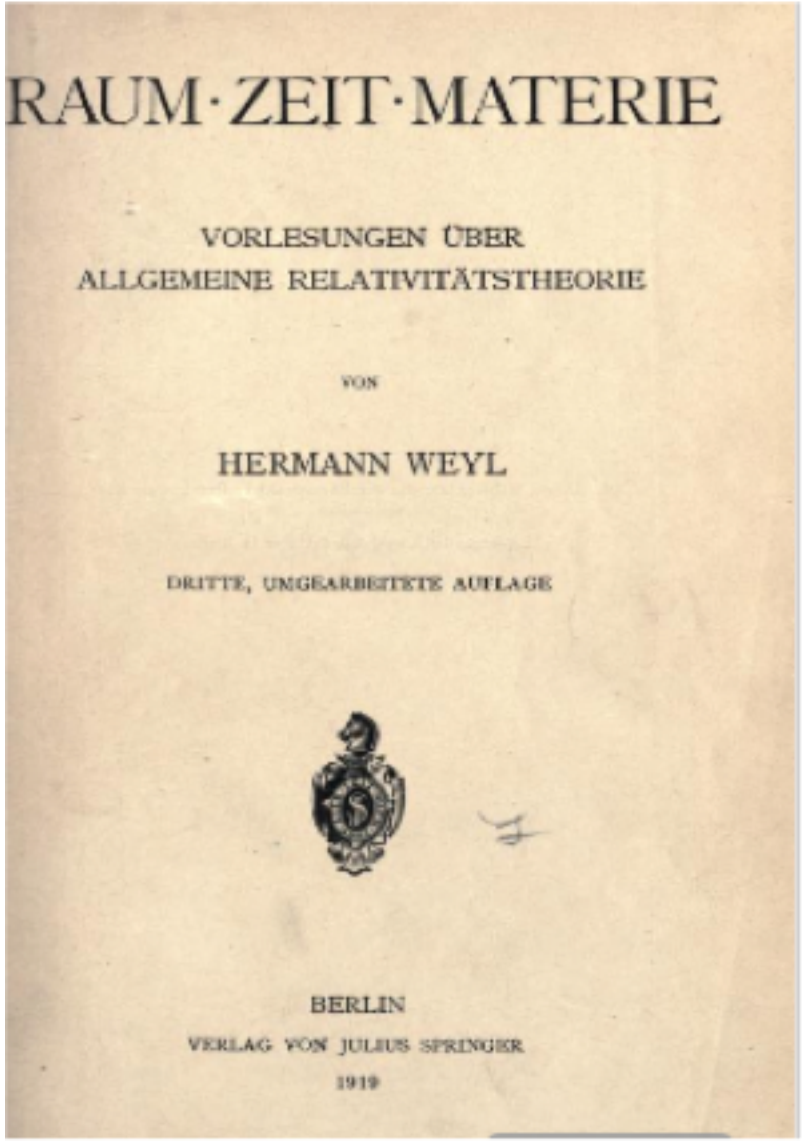
group-theoretic foundation of tensor calculus

"The problem of obtaining an overview of all possible types of linear quantities in affine space is … nothing else but the *representation problem for the continuous group* G [= *GL(n,C)*] …
In geometrical and physical applications it always happens that a type of quantity is not characterized solely by tensor degree but in addition by symmetry conditions. The experiences of mathematicians and physicists suggests … that *there are no other linear quantities besides tensors* (whereby symmetry conditions are to be assumed in the tensor concept). This proposition, in which I perceive the proper group-theoretic justification of the tensor calculus, will … be proved in what follows."

"*Theorie der Darstellung der halbeinfacher Gruppen durch lineare Transformationen.* I" (1925) *problem to be addressed by symbolic construction*

"The immanent is absolute, i.e.,, exactly what it is as I have it and am able to bring its essence (*Wesen*) to givenness (*Gegebenheit*) before me in acts of reflection. … The given-to-consciousness (*Bewußtseins-Gegebene*) is the starting point at which we must place ourselves in order to comprehend the sense and the justification of the posit of reality (*Wirklichkeitsetzung*)." (3-4)

all 5 eds. (1918-1923)

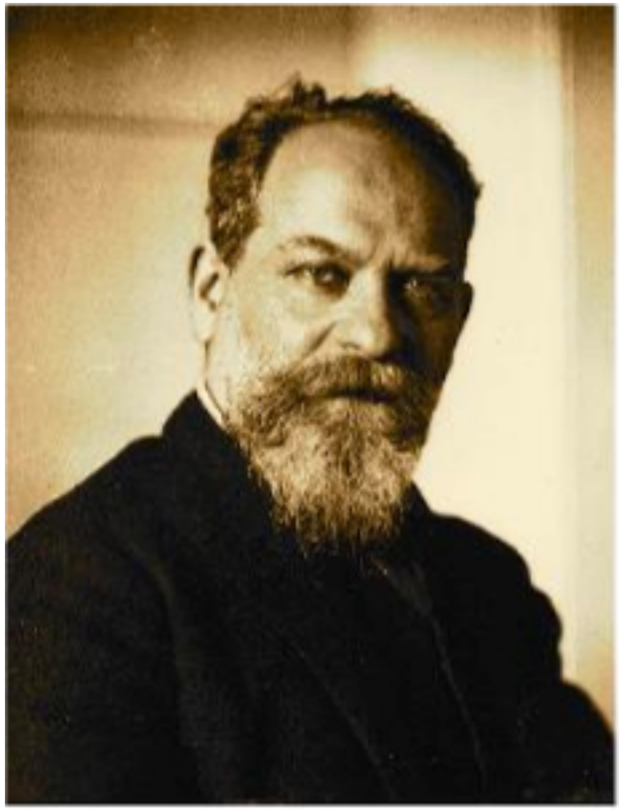


"idealism in the infinitesimal" (Weyl)

-- metaphysical/epistemological mandate: *comprehensibility of* physical world to be constructed starting from "the given-to-consciousness (*Bewußtseins-Gegebene*)".

-- realm of *Bewußtseins-Gegebene*" ("ego's immediate life of intuition") mathematically realized as tangent space in continua (Riemannian manifolds, Lie groups); locus of "ego-center" positing elementary linear relations in the "infinitely small" region surrounding $P \in M$

-- ***permits*** only local relations of comparison via linear connections *as Weyl knew in considerable detail, Husserlian phenomenology attempts to account for objective knowledge beginning with the "Bewußtseins-Gegebene",*



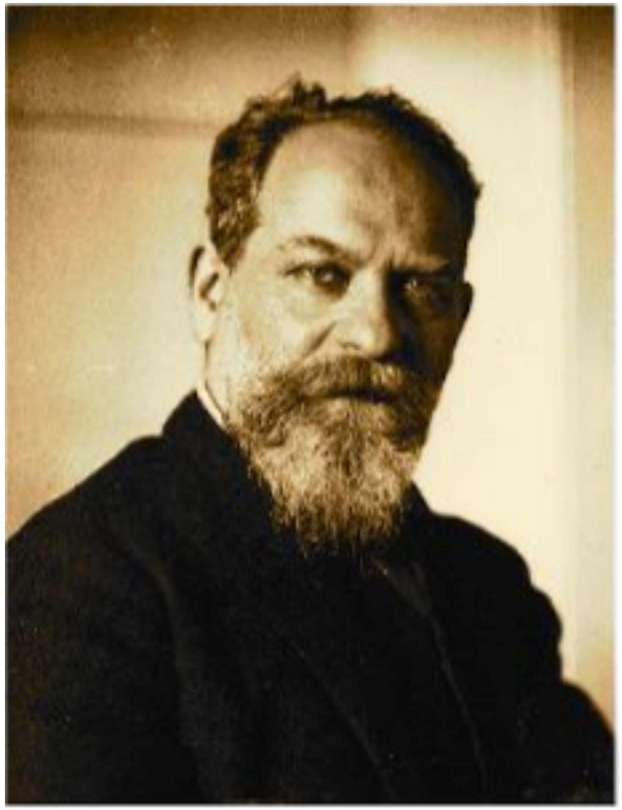
E. Husserl 1859-1938

"How can consciousness give or reach an object? ... How can natural science be made comprehensible insofar as, with each step, it supposes and posits knowledge of a Nature existing in itself?"

"Philosophie als strenge Wissenschaft", Logos I (1911), 299-300.

in Weyl this becomes *"the problem of relativity"*

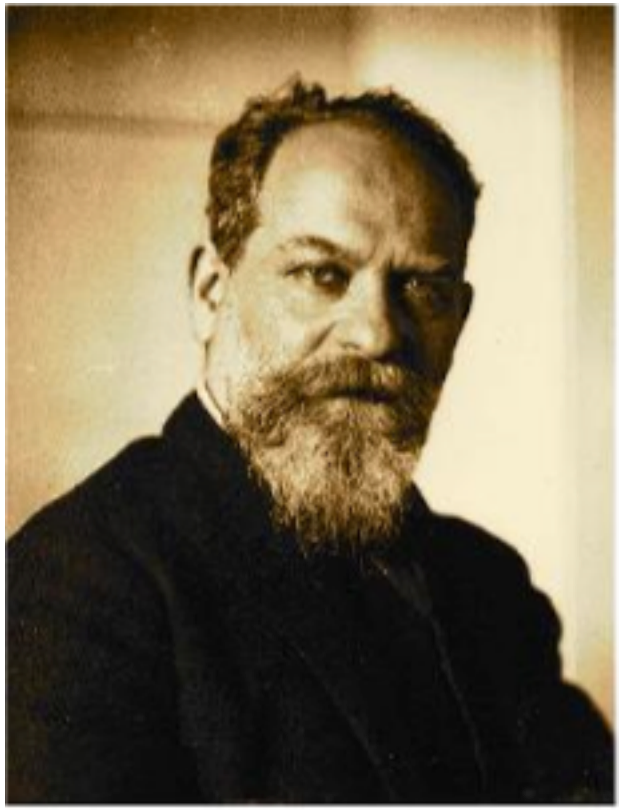
transcendental subjectivity/objectivity (Husserl)



E. Husserl 1859-1938

“*every existent* is *relative to transcendental subjectivity*. Transcendental subjectivity alone … exists ‘*in itself and for itself*’; and it exists, in itself and for itself, in a hierarchical order corresponding to the constitution that leads to the different levels of transcendental intersubjectivity.”

Formale und transzendentale Logik, 1929, § 103 *transcendental subjectivity/objectivity (Husserl)*



E. Husserl 1859-1938

"The existence of Nature *cannot* be the condition for the existence of consciousness, since Nature itself turns out to be a correlate of consciousness. Nature *is* only as constituted in regular concatenations of consciousness."

Husserl, *Ideen* I (1913): §51.

[marginal note in Husserl's copy 'A': "That will be misunderstood."]

transcendental subjectivity & problem of relativity (Weyl)

"Immediate experience is *subjective* and *absolute*. However hazy it may be, it is given in its very haziness and not otherwise. The objective world … that natural science attempts to crystallize by methods representing the consistent development of those criteria by which we experience reality in our everyday natural attitude – this *objective* world is of necessity *relative*; it can be represented by definite things (numbers or other symbols) only after a system of coordinates has been carried into the world. … Whoever desires the absolute must take subjectivity and egocentricity into the bargain; whoever feels drawn toward the objective faces the problem of relativity."

Philosophie der Mathematik und Naturwissenschaften, 1927/1949, 116.

symbolic construction:

"objective reality" not *given* (to consciousness) but *constructed* in symbols, on basis of arbitrary c.s.

"science concedes to idealism that its objective reality is not given but to be constructed *(nicht gegeben, sondern aufgegeben)*, and that it cannot be constructed absolutely but only in relation to an arbitrarily assumed coordinate system and in mere symbols."

Philosophie der Mathematik und Naturwissenschaften, 1927/1949, 117

“*symbolic construction*” (ca. 1925) from Weyl’s *Auseinandersetzung* with Hilbert

Hilbert: Questions of the truth or validity of individual mathematical statements replaced by metamathematical demand for consistency proof of the theory’s axioms, to be obtained in a “formal proof theory” in which proofs are rule-governed arrays of concrete and displayable formal signs.

D. Hilbert 1862-1943

Weyl dismissed Hilbert's metamathematical "game of formulae" as an adequate philosophical justification of the cognitive worth of mathematics. Instead, he *fused mathematics with physics*, locating the significance of mathematics in its application in theoretical physics.

"the significance of mathematics ultimately is that we can only design a theoretical picture of what *exists* (*des Seins*) against the background of the *possible.*"*

"In physics we ... apply an *a priori* construction of the possible, into which the actual is embedded on the basis of values of attributes indirectly determined by reactions"**

* *"Die heutige Erkenntnislage in der Mathematik"*, 1925.
** *Mind and Nature*, University of Pennsylvania Press, 1934.

Symbolic construction starts from “the general form of consciousness”

“The penetration of the This *(Hier-jetzt*) and the Thus *(So)* is the general form of consciousness; something *is* only in the indissoluble unity of intuition and sensation, in which continuous extension and continuous quality overlap. Phenomenologically one cannot get beyond this.”

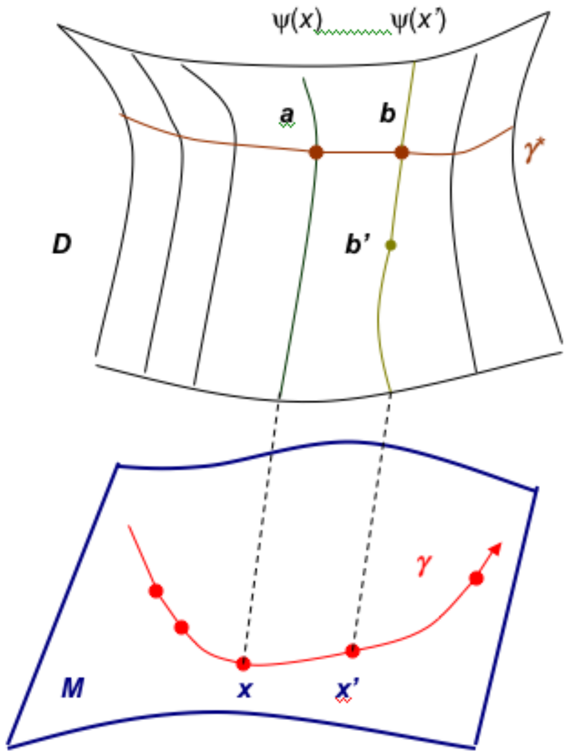
Philosophie der Mathematik und Naturwissenschaften, 1927, 93/1949, 130.

It requires a *coordinate system,* the "general form of consciousness"

"The necessity of the coordinate system goes back to the ultimate epistemological fact, the interpenetration of the *This* (here-now) and the *That.* This interpenetration is the general form of consciousness: only insofar as continuous extension and continuous quality coincide does something exist. This double nature of that which is real has the consequence that we can only draw up a theoretical picture of that which exists *against the background of the Possible.*"

"Time Relations in the Cosmos: Proper Time, Lived Time, and Metaphysical Time".
Lecture, Harvard, 1927.

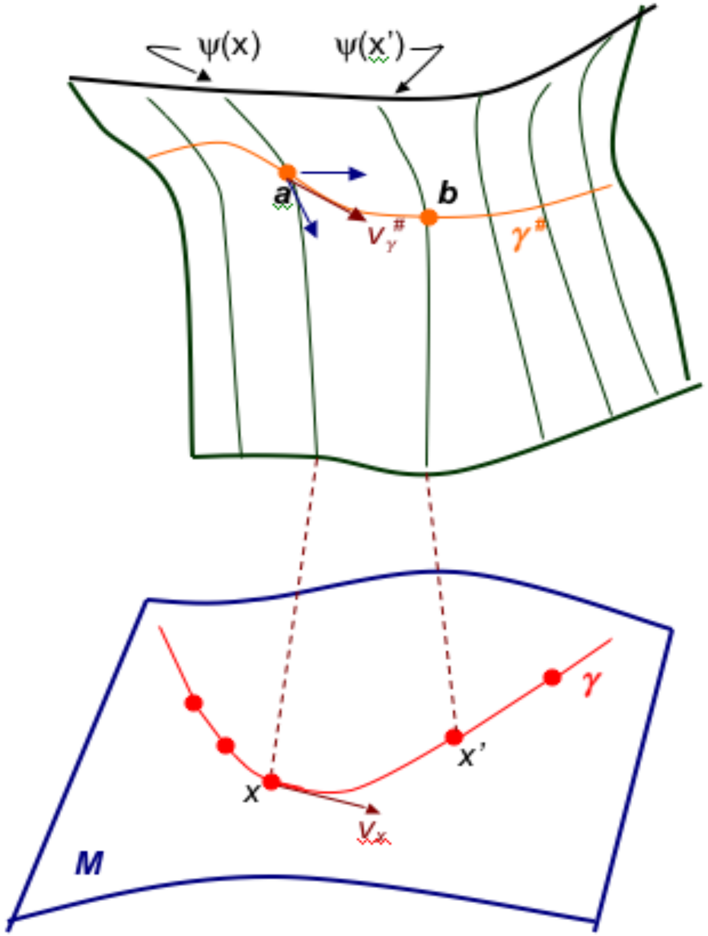
a linear connection tracks changes in continuous extension and continuous quality; e.g., parallel transport along horizontal lift γ'



Takes into account only change in *locus* ("*Hier-Jetzt*") (from x to x' in **M**); *quality* ("*So*") remains the same

- ***b*** in ψ(x′) is identified with ***a*** in ψ(x)

Connection is general rule analyzing total change into two components, change in This *(Hier-jetzt) and change in* Thus *(So)*



Change from ψ(x) to ψ(x′) along γ' analyzed into

- v_x (change in identity, "*Hier-jetzt*)
- $v_{\gamma'}$ (change in quality, "*So*")

problem of relativity (Weyl)

role of c.s. (as general form of consciousness) not completely eliminable

"… a coordinate system, or frame of reference has to be exhibited by an individual demonstrative act. The objectification, by elimination of the ego and its immediate life of intuition, does not fully succeed, and the coordinate system remains as the necessary residue of ego-extinction."

Philosophie der Mathematik und Naturwissenschaften, 1927/1949, 75.

problem of relativity (Weyl)

norm of objectivity

"Only if we are sure that the truth of the complete statement is no affected by free variation of the contingent factors and of those that are individually exhibited … have we a right to omit these factors from the statement and still to claim objective significance for it."

Philosophie der Mathematik und Naturwissenschaften, 1927/1949, 71.

symbolic construction: the answer to the *problem of relativity*

"To fulfill the demand of objectivity we construct an image of the world in symbols"

Philosophy of Mathematics and Natural Science, § 13: "The Problem of Relativity" p.77

objectivity ≡ invariance with respect to the group of automorphisms acting on the space

determining invariant relations requires construction via the arbitrary introduction of a coordinate system or set of labels, "self-created, distinctive, and always reproducible symbols"

"purely infinitesimal" world-geometry (Weltgeometrie)

"*Reine Infinitesimalgeometrie*", Mathematische Zeitschrift 2 (1918), 383-411.

"*Gravitation und Elektrizität*", Sitzungsberichte Akademie der Wissenschaften Berlin (1918), 465-80.

1918: *"purely infinitesimal" world-geometry*

an "inconsistency" of infinitesimal geometry of Einstein (Riemann)

Einstein (L-C) connection ∇

$$[\Gamma^a_{bc} = \langle dx^a, \nabla_{\partial_b}\partial_c\rangle, \quad \partial_a = \partial_{x^a} = \partial/\partial^{x^a}\,]$$

transports *direction* of a vector anholonomically, *length* holonomically.

Remedy: *length connection* $A = A_\mu dx^\mu$ [real-valued 1-form]

-- applied to tangent vector at P $(\dot{\gamma} = \dot{\gamma}^\mu \, \partial_\mu \in T_P M)$ & multiplied by initial length l_P at P yields increment at nearby point P',

$$\delta l = l_{P'} - l_P = l_P A_\mu \dot{\gamma}^\mu$$
$$l_{P'} = l_P(1 + l_P A_\mu \dot{\gamma}^\mu)$$

or at "finite distance" P_Q , $l_Q = l_P exp \int_\gamma A$.

Length curvature (*Streckenkrümmung*)

$$F = dA = \frac{1}{2}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right) dx^\mu \wedge dx^\nu$$

in general does not vanish

"In this theory all physical quantities have a world geometrical meaning."

Serendipitous unification of gravitation and E&M with identifications:

$A =$ 4-potential of electromagnetism
Faraday 2-form $F = dA$,
$dF = 0$, the two homogeneous Maxwell equations
up to Hodge duality

Demand invariance of generally covariant combined grav-electro. field eqs under simultaneous gauge transformations

$$A \mapsto A' = A + d\lambda$$
$$g \mapsto g' = e^{\lambda} g$$

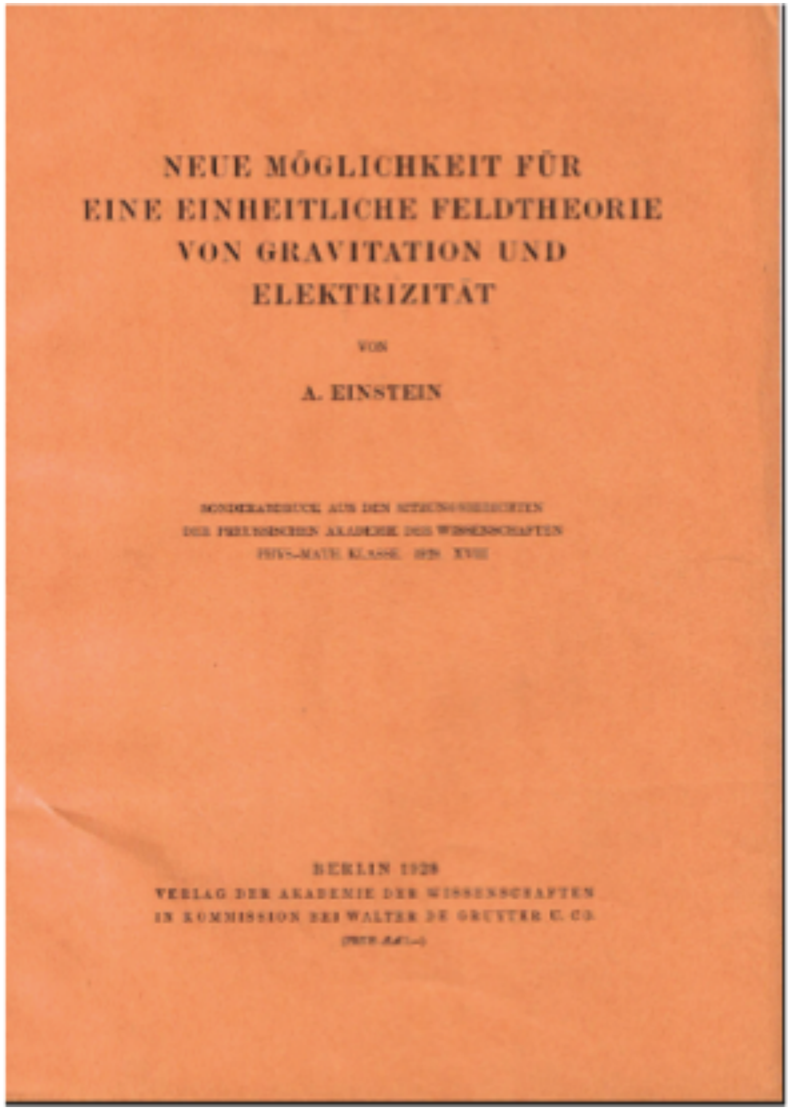
Weyl 1929: *the ”new gauge principle”*

1. “Gravitation and the electron.” *PNAS (USA)* **15:** 323-34;
   communicated March 7, 1929 (from Princeton)

2. “*Elektron und Gravitation* I.” *ZfP* **56:** 330-52;
   submitted 8 May 1929 (from Princeton)

3. “Gravitation and the electron.” *The Rice Institute Pamphlet* **16**: 280-95;
   lecture at Rice Institute 23 May 1929

Motivation: Einstein's *fernparallelismus* ?



"[Einstein] assumes *distant parallelism*, i.e., the axes in different points shall be so bound to one another that when one rotates, the axes in all other points automatically undergo the same rotation. I do not believe in this distant parallelism at all; there is no indication that Nature has availed herself of such an artificial geometry."

"Gravitation and the Electron", Rice Institute 1929, 286.

gauge principle "only to be understood" in the context of GR

"Da die Eichinvarianz eine willkürliche Funktion λ einschließt, hat sie den Charakter "allgemeiner" Relativität und kann natürlich nur in ihrem Rahmen verstanden werden."

"Elektron und Gravitation I." *ZfP* **56:** 331.

“only to be understood” in the context of GR

“This new principle of gauge invariance, which may go by the same name, has the character of general relativity since it contains an arbitrary function λ, and can certainly only be understood with reference to it.”

“Gravitation and the electron.” *PNAS (USA)* **15**: 324;

"only to be understood" in the context of GR

"The principle of gauge-invariance has the character of general relativity since it contains an arbitrary function λ, and can certainly only be understood it terms of it."

"Gravitation and the electron." *The Rice Institute Pamphlet* **16:** 285.

Why ?

-- appearance (in Dirac theory) of new type of linear physical quantity (two-component spinors) that is *not* a tensor

-- *requires new conceptual framework for symbolic construction:* construction of new linear quantity, retaining covariant linear quantities of gravitation, E&M.

-- new possibility space reveals new arbitrariness (egocentricity) that, as in GR, can be removed by a principle of invariance.

Weyl 1929 -- a tale of *three* connections*

1. *metric:* replace $\mathbb{GL}(4,\mathbb{R})$ by subgroup $\mathbb{SO}^+(1,3)$, locally isomorphic to $\mathbb{SL}(2,\mathbb{C})$, governing point-dependent rotations of local tetrads (*lokalen Achsenkreuzen*)

   infinitesimal parallel propagation of tetrads governed by connection

$$\mathcal{A} = \mathcal{A}^a_\mu dx^\mu \otimes T_a$$

   with values in the Lie algebra $\mathfrak{o}(1,3) = Lie\ \mathbb{SO}^+(1,3)$

* cf. Afriat, 2013

Weyl 1929

2. *spinors* $\psi \in \mathbb{C}^2$ transform under a group $\mathbb{W}(2,\mathbb{C})$
$= \{g \in \mathbb{GL}(2,\mathbb{C}) \colon |\det g| = 1\}$ slightly *larger* than $\mathbb{SL}(2,\mathbb{C})$

Parallel propagate by connection $\mathfrak{A}$ with values in
$\mathfrak{w}\,(2,\mathbb{C}) = Lie\ \mathbb{W}(2,\mathbb{C})$. Group homomorphism

$h \colon \mathbb{W}(2,\mathbb{C}) \to \mathbb{SO}^+(1,3)$ is key to Weyl's construction:

h leaves underdetermined the angular freedom $e^{i\lambda} \in \mathbb{U}(1) \cong (\mathbb{R},\cdot)$

Weyl 1929

3. “postulate of freedom” (*Math. Analyse des Raumprob.*1923)

in curved spacetime, local tetrads can rotate independently,
and if tetrads vary, so also should gauge factor λ:

“...in the general theory of relativity when we remove the restriction binding the local axis-systems to each other; we cannot avoid allowing the gauge factor to depend arbitrarily on position.”

p. 291 Rice lecture

Weyl 1929

3. the group homomorphism $h\colon \mathbb{W}(2,\mathbb{C}) \to \mathbb{SO}^+(1,3)$ determines the Lie algebra homomorphism

$$\mathfrak{h}\colon \mathfrak{w}(2,\mathbb{C}) \to \mathfrak{o}(1,3) \approx \mathfrak{sl}(2,\mathbb{C})$$

A *third* connection $A = A_\mu dx^\mu$ for residual $\mathbb{U}(1)$ freedom lying "in between" tetrads and spinors; in direction V, A yields infinitesimal generator $\langle A,V\rangle \in \mathbb{R}$ that multiplies λ to produce increment $\delta\lambda = \lambda\,\langle A,V\rangle$.

As in 1918, the same identifications can made:

$$F = dA = \frac{1}{2}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right) dx^\mu \wedge dx^\nu; \quad dF = d^2 A = 0,$$

i.e., the E&M field, its potential, and Maxwell's two homogenous equations *are a consequence of gauge invariance*

with general relativity,
“the principle of gauge invariance becomes self-evident”

“The local axis-system does not determine the components of ψ uniquely, but only with a [gauge factor] of absolute magnitude 1. … in the general theory of relativity when we remove the restriction binding the local axis-systems to each other, we cannot avoid allowing the gauge factor to depend arbitrarily on position. … The principle of gauge invariance becomes self-evident.”

“Gravitation and the Electron”, Rice Institute, 1929, 291.

problem of relativity (Weyl) revisited

the new norm of objectivity

"The quantitative description of nature requires two preliminary steps.
1) one has to assign coordinates to the points of space, and
2) one has to pick at every point *P* one of the local Cartesian frames [tetrads] *f*.

The laws of nature are independent of the arbitrariness involved in these two acts. …This analytic representation is different from the one adopted by Riemann and Einstein. The modification [(2)] is necessary if one wishes to include the Schrödinger-Dirac ψ of the electronic wave function into the scheme of general relativity."

"Similarity and Congruence: a chapter in the epistemology of science" Lecture, 1948-9, 163.

Weyl's action integral (*grav.; E&M; Dirac*) invariant under infinitesimal symmetries and their corresponding conserved quantities

1) infinitesimal rotations of frames: symmetry of $T_{\mu\nu}$
2) infinitesimal coord. transformation: 'quasi'-conservation of energy and and momentum
3) infinitesimal $\mathbb{U}(1)$ gauge transformations: conservation of charge

not a theory but a framework for constructing a still-to-be quantized theory to resolve problems with Dirac theory ("twice two-many energy levels, etc.").

"*Physics shouldn't depend on the physicist*" ?

-- general covariance and gauge invariance both introduce *arbitrary* mathematical degrees of freedom at each point *P* of space-time, *either* as functions of four independent variables (space-time coordinates) determined by the field laws *or* as arbitrary function of coordinates signifying an internal gauge symmetry.

-- the arbitrariness is understood *phenomenologically*, as each point indifferently can be considered the locus of subjectivity, an experiencing, constructing subject

“*Physics shouldn’t depend on the physicist*” ?

-- both coordinate and gauge transformations connect states that cannot be physically distinguished, both symmetries are not symmetries of nature but of the description of nature

-- invariance principles remove arbitrariness introduced by local starting point

-- both are demands of objectivity, that the constructed physical theory be independent from any particular starting point from which it is constructed

References

A. Afriat, 2013: “Weyl’s gauge argument.”
________, 2017: “Logic of gauge.”

J. Bernard, 2015. *L’idéalisme dans l’infinitésimal: Weyl et l’espace à l’époque de la relativité.*

C. Eckes, 2013: *Les groupes de Lie dans l’oeuvre de Hermann Weyl.*

E. Scholz, : 2004: “Hermann Weyl’s analysis of the ‘problem of space’ and the origin of gauge structures”.
________, 2005: “Local spinor structures in V. Fock’s and H. Weyl’s work on the Dirac equation.
________, 2011: “H. Weyl’s & E. Cartain’s proposals for infinitesimal geometry in the early 1920s.”
________, 2016: “The changing faces of the *Problem of Space* in the work of Hermann Weyl.”
________, 2017: “*Kommentar zum vorangehenden Text* Similarity and Congruence”
________, 2018: “Weyl’s search for a difference between ‘physical’ and ‘mathematical’ automorphisms.”